CGP Education

California Mathematics

Course Two

Teacher's Edition

California Standards-Driven Program

California
Mathematics
Course Two

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Contents

This Textbook provides comprehensive coverage of all the California Grade 7 Standards. The Textbook is divided into eight Chapters. Each of the Chapters is broken down into small, manageable Lessons and each Lesson covers a specific Standard or part of a Standard.

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California Standard

Chapter 1 — The Basics of Algebra

California Standard

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California Grade Seven Mathematics Standards

The following table lists all the California Mathematics Content Standards for Grade 7 with cross references to where each Standard is covered in this Textbook. Each Lesson begins by quoting the relevant Standard in full, together with a clear and understandable objective. This will enable you to measure your progression against the California Grade 7 Standards as you work your way through the Program.

California Standard		Number Sense	Chapter
1.0		Students know the properties of, and compute with, rational numbers expressed in a variety of forms:	**2, 5, 8**
1.1		Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), compare rational numbers in general.	**2, 5**
1.2	★	Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.	**2, 5**
1.3		Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.	**2, 8**
1.4	★	Differentiate between rational and irrational numbers.	**2**
1.5	★	Know that every rational number is either a terminating or a repeating decimal and be able to convert terminating decimals into reduced fractions.	**2**
1.6		Calculate the percentage of increases and decreases of a quantity.	**8**
1.7	★	Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.	**8**
2.0		Students use exponents, powers, and roots and use exponents in working with fractions:	**2, 5**
2.1		Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.	**5**
2.2	★	Add and subtract fractions by using factoring to find common denominators.	**2**
2.3	★	Multiply, divide, and simplify rational numbers by using exponent rules.	**5**
2.4		Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.	**2**
2.5	★	Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.	**2**

California Standard		Algebra and Functions	
1.0		Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:	**1, 2, 3, 4, 5**
1.1		Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).	**1, 2, 3, 4**
1.2		Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.	**1**
1.3	★	Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.	**1**
1.4		Use algebraic terminology (e.g., variable, equation, term, coefficient, inequality, expression, constant) correctly.	**1, 5**
1.5		Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.	**1, 4**
2.0		Students interpret and evaluate expressions involving integer powers and simple roots:	**2, 5**
2.1		Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.	**2, 5**
2.2		Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.	**5**

California Grade Seven Mathematics Standards

3.0		Students graph and interpret linear and some nonlinear functions:	**4, 5, 7**
3.1		Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.	**5**
3.2		Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).	**7**
3.3	★	Graph linear functions, noting that the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.	**4**
3.4	★	Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the ratio of the quantities.	**4**
4.0	★	Students solve simple linear equations and inequalities over the rational numbers:	**1, 4**
4.1	★	Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.	**1, 4**
4.2	★	Solve multistep problems involving rate, average speed, distance, and time or a direct variation.	**4**

Measurement and Geometry

California Standard			
1.0		Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:	**3, 4, 7**
1.1		Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).	**4**
1.2		Construct and read drawings and models made to scale.	**3, 7**
1.3	★	Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.	**4**
2.0		Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:	**3, 7**
2.1		Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.	**3, 7**
2.2		Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.	**3, 7**
2.3		Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.	**7**
2.4		Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or [1 ft.^{2}] = [144 in.^{2}]; 1 cubic inch is approximately 16.38 cubic centimeters or [1 in.^{3}] = [16.38 cm^3]).	**7**
3.0		Students know the Pythagorean theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:	**3, 7**
3.1		Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) by using a compass and straightedge.	**3**
3.2		Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.	**3**

California Grade Seven Mathematics Standards

3.3	★	Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.	**3**
3.4	★	Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.	**3**
3.5		Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.	**7**
3.6	★	Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).	**7**

California Standard		**Statistics, Data Analysis, and Probability**	
1.0		Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:	**6**
1.1		Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.	**6**
1.2		Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).	**6**
1.3	★	Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.	**6**

California Standard	**Mathematical Reasoning**	
1.0	Students make decisions about how to approach problems:	**Illustrated throughout Program**
1.1	Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.	
1.2	Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.	
1.3	Determine when and how to break a problem into simpler parts.	
2.0	Students use strategies, skills, and concepts in finding solutions:	
2.1	Use estimation to verify the reasonableness of calculated results.	
2.2	Apply strategies and results from simpler problems to more complex problems.	
2.3	Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.	
2.4	Make and test conjectures by using both inductive and deductive reasoning.	
2.5	Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.	
2.6	Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.	
2.7	Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.	
2.8	Make precise calculations and check the validity of the results from the context of the problem.	
3.0	Students determine a solution is complete and move beyond a particular problem by generalizing to other situations:	
3.1	Evaluate the reasonableness of the solution in the context of the original situation.	
3.2	Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.	
3.3	Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.	

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Introduction

Helping You Teach

California Standards-Driven Grade 7 Program

Program Overview

This CGP Grade 7 Mathematics Program has been written specifically to provide comprehensive coverage of all the California State Standards and it provides teachers with all the necessary tools to help all students achieve success. It is designed to make it as **straightforward** as possible to deliver a complete and focused grade 7 Math course. It is **simply** and **clearly** organized while providing plenty of teaching support, including assistance for both new and second-subject teachers. It also has a flexible approach that will help you adapt the Program to suit different teaching methods, allowing you to provide Universal Access to meet the needs of a diverse student body. The result is a Program that:

- Is a Standards-Driven Program – focused purely on the California Standards for grade 7 with no extraneous content.
- Supplies you with an ideal pedagogical approach for grade 7 Math.
- Has a clear and simple structure.
- Provides useful and integral Program components that follow the same clear structure.
- Provides Universal Access – catering for a diverse student body.

A Research-Based Program

The Mathematics Framework for California Public Schools (2005) sets high expectations of students. The positive effects of high expectations have been well documented in education research, but high expectations without the teaching tools to back them up can only lead to disappointment (Horn, 2004). These two pages explain how, in the light of current educational theory, this grade 7 Program provides you with the tools you need to help your students succeed.

A <u>Pure</u> California Standards-Driven Program

The rich and rigorous California Standards are designed to equip students with a high level of mathematical comprehension (Framework 2005, page xiii), which poses a challenge to teachers when using a math Program. This Program:

- Has been **explicitly written** to match the California Grade 7 Mathematics Content Standards, using the Framework as a guide.
- Clearly states and explains the relevant Standards at the start of each Lesson in the **Textbook** so that students, teachers, and parents can check progress against the full list of grade 7 Standards at the front of the book.
- Contains notes in each Lesson in the **Teacher's Edition** on what students have previously studied and how a Lesson's content will be developed and expanded upon in later Chapters or Programs. This enables teachers to effectively prepare students for future math study.

<u>Balanced</u> Program providing concepts, procedures, and problem-solving

The Framework (2005, page 4) explains that students need a balanced instructional Program that creates an understanding of math concepts in addition to teaching procedural skills. It is also important that programs cultivate problem-solving abilities. This Program provides balanced, thorough coverage of all of these areas of math:

- The **Textbook** clearly introduces each concept before going on to teach the relevant algorithmic procedures. Problems, from simple and routine to more abstract, are then presented and modeled.
- The wraparound margins in the **Teacher's Edition** offer additional concept questions and additional examples that can be used to ensure students have understood the basic principles.
- **Section Explorations** allow students to analyze problems in an informal manner. This provides good preparation for the formal math techniques in the main teaching Lessons.
- **Chapter Investigations** pull together several concepts from the Chapters, and give students the opportunity to apply and enhance their problem-solving skills.

Building on <u>existing knowledge</u>

Research shows that building new understanding from what students already know is the best way to increase their knowledge (Burns 2002, page xii). This Program structures the explanations, worked examples, and questions so that they build on students' prior learning:

- Chapter 1 presents and expands upon content introduced in earlier grade levels on variables, expressions, and equations.
- Concepts are carefully explained in each Lesson so that students can relate them to existing ideas.
- This Program provides access for all students. Where appropriate, relevant background and concept prerequisites are provided to bridge gaps in prior learning. The **Preprogram Benchmark Test** and **Section Assessment Tests** will help you to decide whether additional support is necessary.

A Research-Based Program

The Program structure provides scaffolding

As students learn new skills, they need support. For students to reach a level where they can perform a task independently, the level of support should be steadily reduced — this model of teaching and learning is known as **scaffolding**. Research shows that when teachers provide instructional scaffolding, students can move from what they know to what they need to know (Ladson-Billings, 1994, page 124). This Program has several features that provide suitable scaffolding:

- The Lessons start by carefully explaining the concepts and modeling solutions to example problems, step by step. The examples have been carefully selected to increase in difficulty as each Lesson progresses.
- Warm-up sheets are provided for every Lesson in the Program on the **Teacher Resources CD-ROM**. These provide reinforcement of previously taught material, and include problems to help prepare students for the material to be taught in each new Lesson.
- Common student errors are outlined in the **Teacher's Edition** to enhance the scaffolding process.
- The Independent Practice questions in the **Textbook** (designed to be completed in class) give students responsibility for applying their skills, and allow for support and immediate feedback from the teacher.
- The questions in the **Homework Book** are designed for students to complete outside the classroom, and act as further reinforcement of newly acquired skills.

Universal Access notes provide additional scaffolding

- The suggested Universal Access activities frequently use manipulatives, such as algebra tiles, to demonstrate concepts in a less abstract manner. These manipulatives can be printed from the **Teacher Resources CD-ROM**. This provides another form of scaffolding to support students as they transfer their understanding from the concrete to the abstract.
- These approaches also provide you with opportunities to reach students with different learning modalities, such as visual or body-kinesthetic. Hutinger (2005) believes that it is important for teachers to address as many learning modalities as possible.

The Homework Book fosters independent learning

Homework is believed to help students improve their thinking skills, develop positive study habits, and take responsibility for their work (US Department of Education, 2002).

- The **Homework Book** provides homework for every Lesson, and allows students to continue their learning beyond the classroom.
- The selected answers at the back of the **Homework Book** allow student self-assessment, which helps to develop independent study skills.

The comprehensive Assessment System enhances teaching & learning

The Framework (2005, page 221) states that "regular and accurate assessment of student progress in mastering grade-level standards will be essential to the success of any instructional Program based on the mathematics content standards and this framework." The importance of this has been recognized in this Program, which contains a comprehensive suite of assessment tools:

- For each Lesson of the **Textbook**, the **Teacher's Edition** indicates which Lessons of the **Skills Review CD-ROM** are relevant if a student needs extra support.
- The **Preprogram Benchmark Test** and **End of Course Assessment Tests** measure students' skills at the start and end of the Program and assist you in allocating a level for each student.
- The **Section Assessment Tests** provide information on the skills students have mastered at different stages of the Program and allow you to modify their teaching appropriately.
- The **Assessment Test Generator CD-ROM** allows you to create personalized assessments for each student throughout the Program.

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Burns, M. 2002. *Lessons for Algebraic Thinking.* Math Solutions Publications.

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Horn, I. 2004. "Why do students drop advanced mathematics?" in *Educational Leadership, Volume 62 No. 3, November 2004*, ASCD.

Hutinger, P. 2005. *The Issue: Learning Modalities.* PBS Teacher Source.

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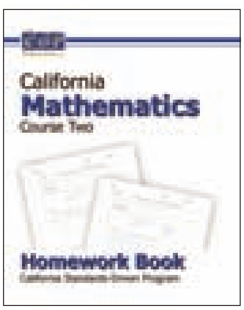
US Department of Education 2002. *Helping Your Child with Homework.* Educations Publications Center.

Program Components

Student Textbook

- Written in clear, straightforward language, with concepts explained clearly
- The Textbook is broken up into eight Chapters, each of which is divided into smaller Sections
- The Sections are then broken down into smaller, manageable Lessons, which are designed to be worked through in a typical 50-minute math class
- Information at the start of each Lesson shows students exactly which Standard is being covered, with clear objectives in everyday language
- Each Lesson contains core teaching, Guided Practice, and Independent Practice exercises
- Introductions and Round Ups in every Lesson summarize previous and future math learning
- Extra hints and examples are included in the margin to help students understand the material
- Section Explorations at the beginning of selected Sections give students a hands-on introduction to the material
- Chapter Investigations allow students to investigate real-life applications of the math material, and involve independent research

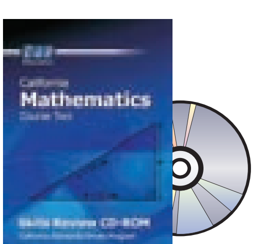


Homework Book

- Homework sheets relate directly to each Lesson of the Textbook
- The **Teacher's Edition** details which ability levels exercises are suitable for
- Includes worked examples and helpful hints, including guidance on how to tackle different types of problem
- Homework sheets are perforated so students can hand in homework for marking
- Contains notes for parents/guardians
- Contains an overview of each Chapter for students and parents/guardians
- Selected answers at the back of the book allow student self-assessment

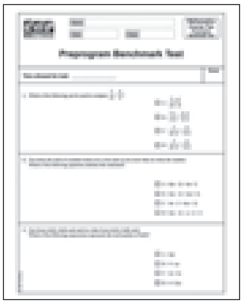
Teacher's Edition Textbook

- Teacher's Edition contains the main Student Textbook pages, with wraparound teacher margins
- Wraparound margins contain step-by-step teaching guidance with suggested projects, activities, and real-life applications — ideal for second-subject teachers or new teachers
- Each Lesson contains all the material you need in order to fill 50-minute math lessons
- Margins include guidance for teaching English Language Learners, Strategic Learners, and Advanced Learners
- Each Lesson is cross-referenced to the **Skills Review CD-ROM** and **Homework Book**, including level information for each homework sheet
- Includes answers to Guided Practice and Independent Practice exercises for easy reference
- Includes **Teacher Resources CD-ROM** that contains printable warm-up sheets for each Lesson, manipulatives, recording and reporting forms, and an assessment diagnostic guide



Skills Review CD-ROM

- Contains 48 printable handouts working up to grade 7 from the relevant grade 4, 5, and 6 material
- Suitable for all students, especially those who struggle with grade 7 concepts during Lessons
- Each handout includes teaching, examples, and exercises
- Contains teacher information on grades and Standards covered in each exercise
- Reliable — runs direct from the CD-ROM

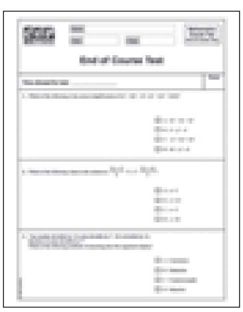


Preprogram Benchmark Test

- Start of Program assessment specifically designed to test whether students are ready for grade 7 Math
- Shows any areas students need to revisit before starting the course
- Helps you decide whether to use the Basic, Regular, or Advanced schedule for each student

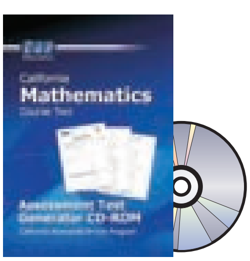
Section Assessment Test

- One test for every Section so you always know how well students have understood the material
- Includes space on each test to write the answers
- Tests cover all levels and types of question



End of Course Assessment Test

- End of Program assessment designed to test whether students have mastered grade 7 Math
- Assesses how well students have absorbed the whole course
- Helps you decide how well prepared students are for later math courses



Assessment Test Generator CD-ROM

- Create personalized tests for groups of students or for individuals
- Choose the questions you want, by Textbook Chapter and Section or by Standard
- Choose the length of test you require
- Allows you to customize difficulty of tests for different students
- Use in addition to the preprinted **Section Assessment Tests**

Solution Guide

- Contains fully worked solutions to all Guided Practice and Independent Practice Exercises in the **Textbook**, and all **Homework Book** pages
- Also contains fully worked solutions to **Section Assessment Tests** — shown on a copy of each Test for easy marking
- Gives detail on difficulty level and textbook references for all Section Assessment problems

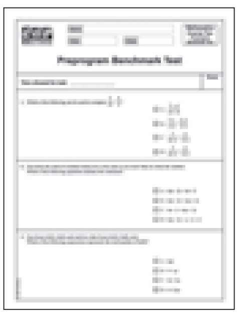
Using the Components

Start of course

Preprogram testing

Set your students the **Preprogram Benchmark Test**.

Refer to the diagnostic guide on the **Teacher Resources CD-ROM** to allocate a level to each student, which will help you decide which Exercises to set throughout the Program.

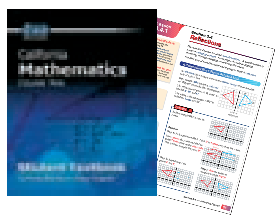


Delivering the course

Teach

Teach the Lesson using the clear explanations in the **Textbook**.

Use the suggestions in the **Teacher's Edition** wraparound margin to support students' learning.



Class exercises

Set a Warm-up worksheet at the start of the lesson to reinforce previous learning.

Work through Guided Practice exercises from the **Textbook** in class.

Towards the end of each Lesson, set Independent Practice exercises in class to check students' knowledge and understanding.

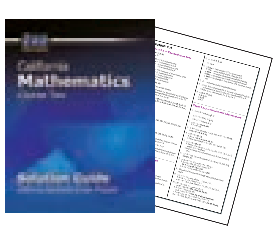
Homework

At the end of the Lesson, use the cross-references in the **Teacher's Edition** wraparound margin to set appropriate homework for each student from the **Homework Book**.

Teacher assistance

Solutions

The **Solution Guide** contains fully worked solutions to all of the exercises in the **Textbook**, the **Homework Book**, and all of the **Assessment Tests**.



Manipulatives and record-keeping

The **Teacher Resources CD-ROM** contains Warm-up worksheets for use at the start of every Lesson, and manipulatives for printing and distributing to students.

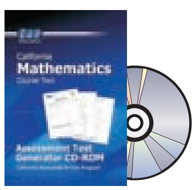
It also includes templates for Class Records and Student Progress Reports as well as diagnostic guidance for **Preprogram Benchmark**, **Section Assessment**, and **End of Course** tests.

Assessment

Section Assessment

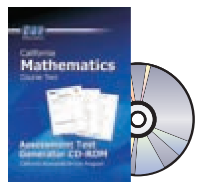
Set a **Section Assessment Test** at the end of the Section.

Alternatively, create tests tailored to a particular student's needs using the **Assessment Test Generator CD-ROM**.



Chapter Assessment

Create a Chapter Assessment Test at the end of the Chapter, using the **Assessment Test Generator CD-ROM**.



End of course

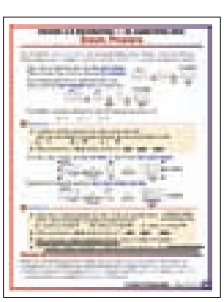
End of Course testing

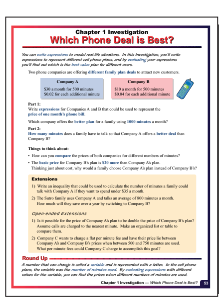
Set your students the **End of Course Assessment Test** to check they've all reached the necessary level.

Student support

Enrichment

Allocate time to the Section Explorations at the beginning of selected Sections and the Chapter Investigations at the end of each Chapter. These activities provide students with hands-on experience of math and allow them to investigate real-life applications of math.
The Investigations include open-ended activities involving independent research.

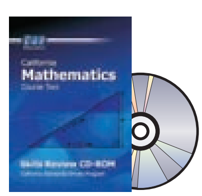


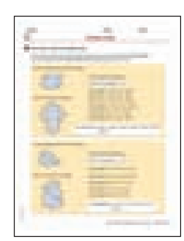


Skills Review

Use the **Skills Review CD-ROM** to supplement instruction.

Use the Skills Review worksheets to provide extra teaching and exercises for students who need further support or additional practice on a particular math topic.





For details on program support, visit **www.cgpeducation.com/training.asp**

Pacing the Program

Pacing the Program over one year

The Pacing Plan below shows the number of days allocated for each Chapter of the course. The traditional Pacing Schedule is designed to be taught over 160 days using 50-minute classes and is divided into Basic, Regular, and Advanced Schedules to allow you to adapt the course for students with different skills. The Block Schedule is intended for schools that have double lessons and is therefore designed to be taught over 80 days.

Full details of how you might pace the teaching of individual Chapters are given in the Pacing Guides located in the Teacher Notes at the start of each Chapter of the **Teacher's Edition**. Each Chapter Pacing Guide recommends which pages of the **Textbook** you should cover each day, and provides you with corresponding exercises to set, both in class and for homework.

Chapter	1	2	3	4	5	6	7	8	
Basic Schedule (Level 1) Regular Schedule (Level 2) Advanced Schedule (Level 3)	19	29	30	19	19	12	15	17	Total = 160 days
Block Schedule	9.5	14.5	15	9.5	9.5	6	7.5	8.5	Total = 80 days

Traditional Pacing Schedule (160-day course)

The three schedules below cover exactly the same topics from the **Textbook**, but assign different levels of exercises and activities — more straightforward problems in the Basic schedule and more probing ones in the Advanced. Therefore you can combine the three schedules in one class, and even switch between them if some students are strong in some areas, but need more help in others.

Basic Schedule (Level 1):
This schedule is designed for **Strategic Learners**. Students following the Basic Schedule are assigned the **Level 1** exercises identified in the **Teacher's Edition**. These exercises ask students to apply the concepts covered in class to straightforward problems.

In addition, one day per Chapter is assigned to an Investigation. Level 1 students should attempt the main part of each Investigation, which allows them to extend their learning, and to apply mathematical concepts to a real-world problem.

Regular Schedule (Level 2):
This schedule is designed for students who possess **typical mathematical skills and knowledge**.
Students following the Regular Schedule are assigned the **Level 2** exercises identified throughout the **Teacher's Edition**.
These questions ask students to apply the concepts covered in class to more complex problems.

In addition, one day per Chapter is assigned to an Investigation. This provides students with the opportunity to extend their learning, and to apply mathematical concepts to a real-world problem.

Advanced Schedule (Level 3):
This schedule is designed for students who possess **above-average mathematical skills and knowledge**. Students following the Advanced Schedule are assigned the **Level 3** exercises identified throughout the **Teacher's Edition**. These questions ask students to apply the concepts covered in class to more complex problems, and contain more instances where students are required to apply basic concepts to new contexts.

In addition, one day per Chapter is assigned to an Investigation plus extensions. Again students are given the opportunity to extend their learning, and to apply mathematical concepts to a real-world problem.
The Investigation extensions allow students to take the ideas behind the original problem further.

Block Pacing Schedule (80-day course)

The Block Schedule is designed for schools that have double-length lessons. Students following a Block Schedule will cover exactly the same material, at the same pace, but in half the number of lessons.

Use the Preprogram Benchmark Test to assess readiness for grade 7 Math

The **Preprogram Benchmark Test** covers material from previous grades relevant to grade 7. Each student's performance on the test will indicate whether they are ready to start grade 7 immediately or if they require some additional teaching on particular concepts. The diagnostic material is included on the **Teacher Resources CD-ROM**.

The results of this test will also be useful for grouping students within the class and can help determine whether students should follow the Basic, Regular, or Advanced Pacing Schedules. It may be that students are more proficient in some areas of math than others, so a combination of more than one schedule may be the best approach.

Accelerating the Program

If the students find some of the Sections very straightforward, you may want to accelerate the pace at which the material is delivered. This will mean that you will have more time to work on sections that the students find challenging. Alternatively, you could use the time to work on the open-ended extensions of the Chapter Investigations.

Acceleration can be achieved by the following:

- Spending less time working through examples with the class.
- Reducing the number of exercises you set students for the sections. It is important, however, to choose a good range of exercises, rather than simply setting the first few questions. For example, in **Lesson 3.2.2 (Drawing Shapes on the Coordinate Plane)**, it is important to make sure the selection of exercises includes problems on finding missing pairs of coordinates, and finding both the perimeter and area of shapes plotted on coordinate planes. You may wish to set students additional exercises as homework to ensure that they have adequate practice on a particular topic.

Decelerating the Program

If students are struggling with the material, you will need to decelerate the course, possibly by scheduling a study group or compressing non-Key Standards. This is particularly essential for the earlier Chapters, as these contain basic material on which later Lessons are based.

Deceleration can be achieved by the following:

- Use an alternative approach – Universal Access approaches are suggested in the margins throughout the **Teacher's Edition**.
- Provide and work through a greater number of examples in the classroom. Additional examples and their solutions are given in the wraparound margins in the **Teacher's Edition**.
- Use the **Skills Review CD-ROM**. The handouts contained on the CD-ROM work up to grade 7 from the relevant grade 4, 5, and 6 material, and may help students who struggle with grade 7 concepts during Lessons.

Pacing the Program over two years

The grade 7 course is usually completed in one year, and the pacing guidance given throughout the **Teacher's Edition** is based on this approach. The table on the right gives a schedule to pace the course over two years.

Year	Chapter				
1	1	2	3	4	
2	Year 1 Review	5	6	7	8

The Year 1 Review is intended to provide a bridge between the two years of study. It will give you an opportunity to remind students of the key concepts from Chapters 1–4, and a means of assessing their understanding. This should help you to identify areas of the remaining Chapters that students might need extra help with. We recommend that you devote approximately two weeks of lessons to the review, before starting work on Chapter 5. To conduct the Review we suggest that you:

- Use the **Skills Review CD-ROM** to go over key concepts from Chapters 1–4. These might be concepts that students found particularly difficult in Year 1, or ideas that will be extended in the remaining Chapters.
- Use the **Assessment Test Generator CD-ROM** to produce your own short tests. This will enable you to test students' knowledge of specific Standards or Sections of the **Textbook**.

Using the Teacher's Edition

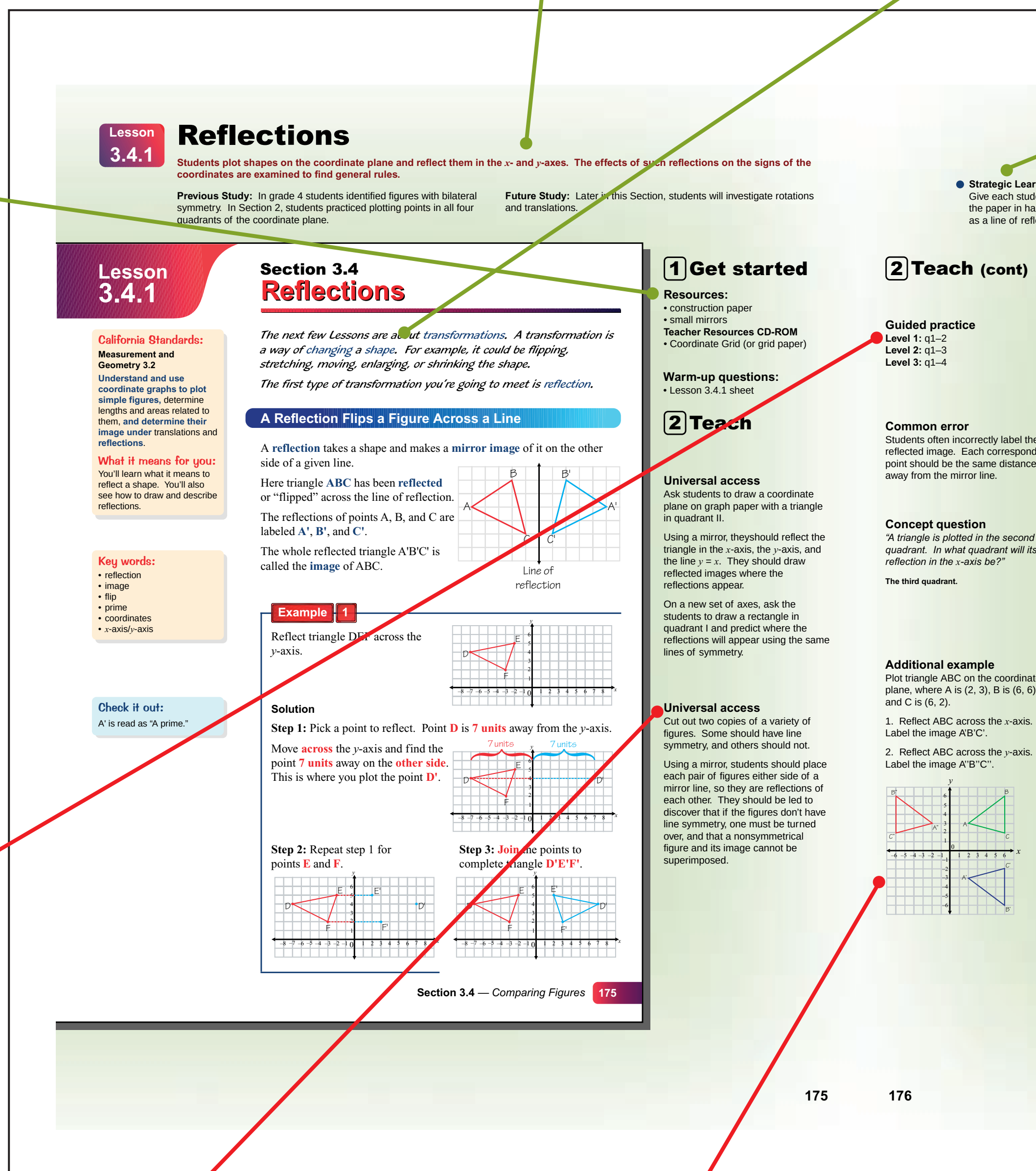
Get started

The Pacing Schedules on the previous two pages and the detailed Pacing Plan at the start of every Chapter show you where each Lesson fits within the overall grade 7 Program. Each Lesson is designed to be taught over a 50-minute lesson. On the first page of every Chapter there is a table that shows you how each Lesson fits into the K-12 Curriculum. This will give you a broad overview of the aims of the Lesson before you plan the day's teaching in more detail.

The introductory note gives an overview of the Lesson. Also included are notes on previous learning from earlier grades that this Lesson draws upon, and information about later math content that relies on the concepts taught in this Lesson.

Gather the resources identified, for use in the classroom and for the Universal Access approaches. Some resources are everyday items and some are manipulatives provided on the **Teacher Resources CD-ROM**, ready for printing and distributing to students.

Print out copies of the appropriate Warm-up worksheet from the **Teacher Resources CD-ROM** for each student to start the Lesson.



Teach

Work through the main student pages with the class. You may find it helpful to reproduce worked examples on the blackboard and ask students to come to the blackboard to complete the math work.

Set Guided Practice problems for the class, and work through the problems together.

The teacher margin contains information on level, to help you set problems appropriate for each student.

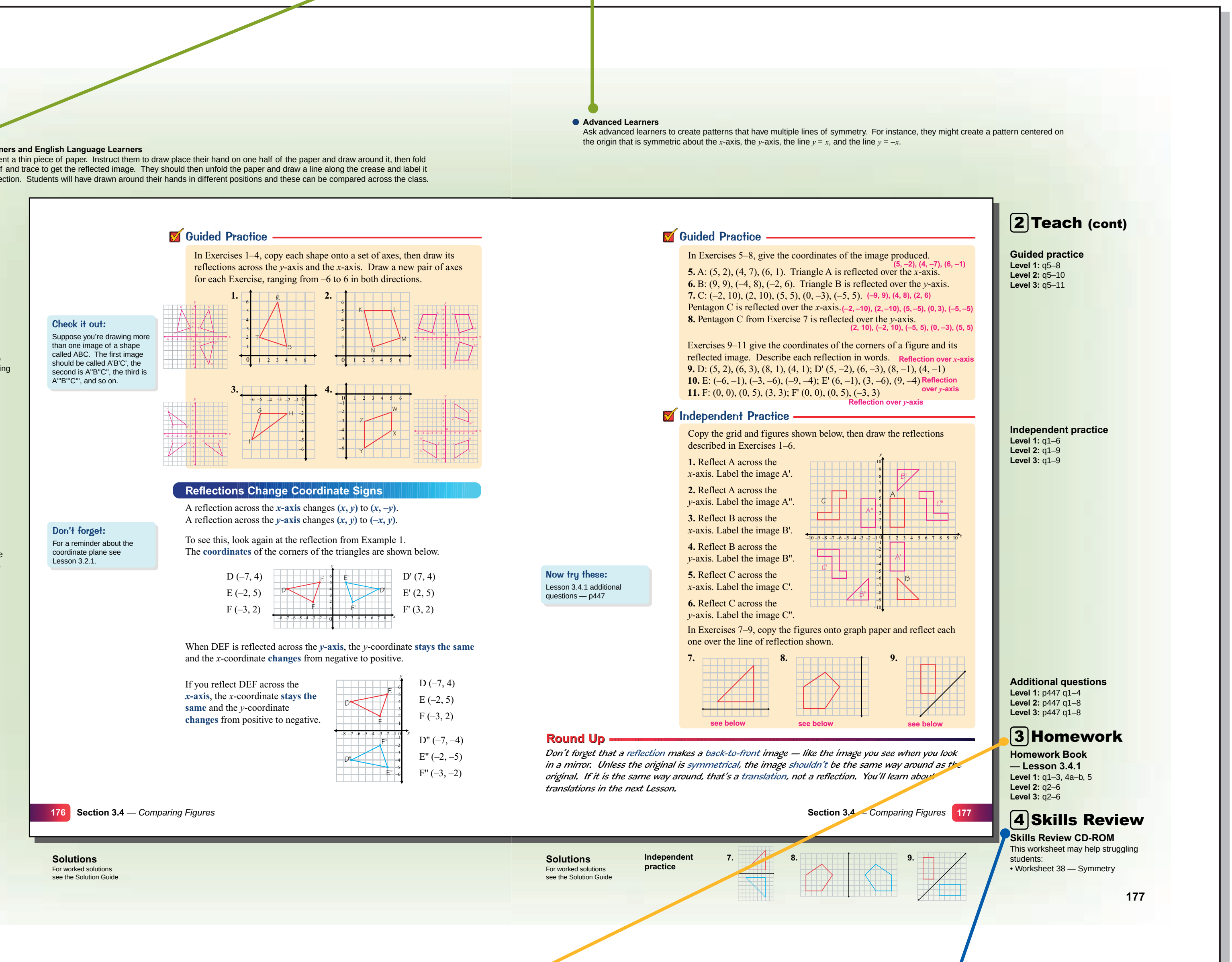
Use Universal Access approaches with students who require a hands-on approach to math concepts.

Universal Access approaches are indicated clearly throughout the Program.

Information and additional examples appear in the teacher margins next to the part of the student **Textbook** that they relate to.

At the beginning of each Lesson the relevant California Standard (or Standards) is clearly stated and explained in full so that you can easily measure the progression of your class through the K-12 Curriculum. Each Lesson begins with a clear learning objective so that the student is always aware of the aim of each Lesson and how it is linked to each Standard.

Refer to this information for an overview of issues that can affect Strategic Learners, English Language Learners, and Advanced Learners. This includes things like suggestions of particular approaches you may wish to adopt, or warnings of difficulties that students may encounter.



...ners and English Language Learners
...ent a thin piece of paper. Instruct them to draw place their hand on one half of the paper and draw around it, then fold ...f and trace to get the reflected image. They should then unfold the paper and draw a line along the crease and label it ...ection. Students will have drawn around their hands in different positions and these can be compared across the class.

Advanced Learners
Ask advanced learners to create patterns that have multiple lines of symmetry. For instance, they might create a pattern centered on the origin that is symmetric about the x-axis, the y-axis, the line $y = x$, and the line $y = -x$.

Guided Practice

In Exercises 1–4, copy each shape onto a set of axes, then draw its reflections across the y-axis and the x-axis. Draw a new pair of axes for each Exercise, ranging from –6 to 6 in both directions.

1. 2. 3. 4.

Check it out:
Suppose you're drawing more than one image of a shape called ABC. The first image should be called A'B'C', the second is A"B"C", the third is A'"B'"C'", and so on.

Reflections Change Coordinate Signs

A reflection across the **x-axis** changes (x, y) to $(x, -y)$.
A reflection across the **y-axis** changes (x, y) to $(-x, y)$.

To see this, look again at the reflection from Example 1.
The **coordinates** of the corners of the triangles are shown below.

D (–7, 4) D' (7, 4)
E (–2, 5) E' (2, 5)
F (–3, 2) F' (3, 2)

When DEF is reflected across the **y-axis**, the y-coordinate **stays the same** and the x-coordinate **changes** from negative to positive.

If you reflect DEF across the **x-axis**, the x-coordinate **stays the same** and the y-coordinate **changes** from positive to negative.

D (–7, 4)
E (–2, 5)
F (–3, 2)
D" (–7, –4)
E" (–2, –5)
F" (–3, –2)

Don't forget:
For a reminder about the coordinate plane see Lesson 3.2.1.

176 **Section 3.4** — *Comparing Figures*

Guided Practice

In Exercises 5–8, give the coordinates of the image produced.

5. A: (5, 2), (4, 7), (6, 1). Triangle A is reflected over the x-axis. (5, –2), (4, –7), (6, –1)
6. B: (9, 9), (–4, 8), (–2, 6). Triangle B is reflected over the y-axis. (–9, 9), (4, 8), (2, 6)
7. C: (–2, 10), (2, 10), (5, 5), (0, –3), (–5, 5). Pentagon C is reflected over the x-axis. (–2, –10), (2, –10), (5, –5), (0, 3), (–5, –5)
8. Pentagon C from Exercise 7 is reflected over the y-axis. (2, 10), (–2, 10), (–5, 5), (0, –3), (5, 5)

Exercises 9–11 give the coordinates of the corners of a figure and its reflected image. Describe each reflection in words.
9. D: (5, 2), (6, 3), (8, 1), (4, 1); D' (5, –2), (6, –3), (8, –1), (4, –1) Reflection over x-axis
10. E: (–6, –1), (–3, –6), (–9, –4); E' (6, –1), (3, –6), (9, –4) Reflection over y-axis
11. F: (0, 0), (0, 5), (3, 3); F' (0, 0), (0, 5), (–3, 3) Reflection over y-axis

Independent Practice

Copy the grid and figures shown below, then draw the reflections described in Exercises 1–6.

1. Reflect A across the x-axis. Label the image A'.
2. Reflect A across the y-axis. Label the image A".
3. Reflect B across the x-axis. Label the image B'.
4. Reflect B across the y-axis. Label the image B".
5. Reflect C across the x-axis. Label the image C'.
6. Reflect C across the y-axis. Label the image C".

In Exercises 7–9, copy the figures onto graph paper and reflect each one over the line of reflection shown.

7. see below **8.** see below **9.** see below

Now try these:
Lesson 3.4.1 additional questions — p447

Round Up
Don't forget that a reflection makes a back-to-front image — like the image you see when you look in a mirror. Unless the original is symmetrical, the image shouldn't be the same way around as the original. If it is the same way around, that's a translation, not a reflection. You'll learn about translations in the next Lesson.

Section 3.4 — *Comparing Figures* 177

2 Teach (cont)

Guided practice
Level 1: q5–8
Level 2: q5–10
Level 3: q5–11

Independent practice
Level 1: q1–6
Level 2: q1–9
Level 3: q1–9

Additional questions
Level 1: p447 q1–4
Level 2: p447 q1–8
Level 3: p447 q1–8

3 Homework
Homework Book — Lesson 3.4.1
Level 1: q1–3, 4a–b, 5
Level 2: q2–6
Level 3: q2–6

4 Skills Review
Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 38 — Symmetry

Solutions
For worked solutions see the Solution Guide

Solutions
For worked solutions see the Solution Guide

Independent practice 7. 8. 9.

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Homework

After each Lesson, set students appropriate work from the **Homework Book**. Problems suitable for each level of student are indicated here.

Skills Review

If students have struggled with any of the concepts in the Lesson, consider allocating an extra lesson to review fundamental skills.

The **Skills Review CD-ROM** contains handouts covering grade 7 material in a fresh, less formal way.

General Teaching Strategies

Many teaching skills, such as class management and an ability to communicate well with young people, are vital for teachers of all subjects. However, as a math teacher, you need to be aware of some important points that set this subject apart from the others.

Math is different from other subjects

Math teaching should contain a balance of skills

There are several different aspects to students' understanding of math material. This grade 7 Program fully covers three main aspects:

Concepts
Students with a **conceptual understanding** of a topic are more likely to remember the procedures in the long term. This understanding also forms a foundation upon which to build future concepts. If a student is competent at solving a basic type of problem, but is unable to apply the concept to a new situation, it is possible that their correct procedural knowledge is masking a lack of conceptual understanding.

Procedures
For many types of calculation, there is a **set of procedures** that students must work through to reach the correct solution. For example, students could learn to convert percents to decimals or fractions simply by memorizing a sequence of steps.

Problem-solving
Conceptual understanding and procedural knowledge prepare students to **solve problems independently**. Be sure to allocate plenty of problems that allow students to extend their knowledge into new areas and combine different skills to solve real-life application problems.

With this in mind, you should:

- **Make sure you are confident in your own conceptual understanding** — you will find helpful information in the Math Background sections of the teacher margins in the **Teacher's Edition**.
- **Always teach the concept before teaching any procedures**. Ask students questions that test their conceptual understanding — concept questions are included in the teacher margins.
- **Use the math in a wide range of contexts and problem types to check students really understand**. Each Chapter of this Program includes a variety of examples and exercises, and real-life applications are given in the wraparound margins. Second-subject teachers may be able to enrich their teaching by using examples from their first subject.

Math is a hierarchical subject — it builds on earlier concepts

In math, whether or not students understand a new concept depends very much on whether they have successfully mastered earlier concepts. Even a simple calculation, such as "8 + 3," requires a firm grasp of addition and an understanding of the base 10 number system. These ideas, in turn, rely on an understanding of what "3" and "8" actually represent.

The concepts integral to grade 7 are based on a complex web of underlying ideas. If students have these ideas in place, they are much more likely to understand the new concepts that you are trying to teach them.

With this in mind, you should:

- **Familiarize yourself with how the concepts you are teaching build on what students should already know**, and how they lay the foundations for what students will study in the future. This information is clearly set out on page B of the introduction to each Chapter (titled "How the Chapter fits into the K-12 curriculum"). More detailed information about previous and future study is given in the wraparound margin in the **Teacher's Edition**, at the start of each Lesson.
- **Consider which prerequisite concepts students could be missing** if they are struggling with a concept.
- Consider setting worksheets from the **Skills Review CD-ROM** if students lack confidence in fundamental techniques. References to appropriate handouts from the CD-ROM are given at the end of each Lesson.

Teaching approaches

Explaining and modeling

- Refer back to previous study where possible to help students make connections between Lessons. It's also important to make sure that all students understand the relevant mathematical terms and symbols.
- When modeling solutions to problems, talk through your reasoning at each step. This helps students to organize their own thinking in a logical manner.
- Where possible, use **manipulatives** to create a concrete approach to abstract concepts — this is particularly useful for strategic learners. Manipulatives should remain available throughout the lesson to support students in independent work.

Questioning and discussing

Skillful questioning keeps all students involved and is a crucial difference between a lively, interactive lesson and a lecture.

- Ask a mixture of open and closed questions. These should be carefully targeted at specific pupils so that students across the entire ability range can achieve success, while still being stretched.
- Ask questions that encourage students to think about the processes they have worked through to reach the answer. Concept questions are provided in the wraparound margins of the **Teacher's Edition**.
- In addition to being a means of teaching, questioning is also an assessment tool. The information it provides is particularly useful as it can be acted on immediately. By listening carefully to students' answers, you can pick up on misconceptions at an early stage and respond in a way that will push students' learning forward.

Effective grouping strategies

Sometimes, students will be able to understand a concept more easily if it is explained by a peer, rather than by a teacher. They may also feel more comfortable asking questions in a small group than in a general class discussion. Students will increase their own understanding of a concept by verbalizing it, and explaining their thought processes helps students to clarify them, as does listening to the thought processes of others.

- Set up seating arrangements so flexible groupings can be made (for example, rows that can be quickly pulled together for partner work, groups of four, and returned to rows for individual work).
- Grouping should be flexible and based on students' current needs. For example, students who are struggling with a concept may be temporarily grouped together. Grouping can also be cross-grade. For example, an advanced student may be grouped with students in a higher grade to work on an enrichment activity.
- Assign distinct roles and responsibilities for grouping activities such as:

  Facilitators who organize the team,

  Equipment managers who ensure that the group has everything they need to carry out the task,

  Reporters who record findings and report back to the rest of the class,

  Checkers who ensure that the math is correct.
- Be succinct but specific about responsibilities for each role. Rotate roles so each student learns each role. Give points or praise for cooperation, responsibilities performed, tasks accomplished. Have students self assess their group and themselves.
- Consider measures to check accountability during partner and group work (for example, grade both partners or entire group on the work of one student randomly chosen; or perform a notebook check or pop quiz).

The Chapter Investigations are particularly appropriate for group work. These Investigations require students to extend and apply what they have learned to new situations.

Student practice

Students should be given plenty of practice at applying newly learned concepts to a range of exercises. This should be done within the lesson with the rest of the class (using Guided Practice exercises from the **Textbook**), alone (Independent Practice exercises from the **Textbook**), and at home (**Homework Book**). Use the time when the class is working independently for focused teaching with individual students or groups, as necessary. Errors from classwork and homework should be used to identify misconceptions, which can be discussed with the students.

Providing Universal Access

Most classes are made up of students with a range of achievement levels. These pages describe the basic groups of learners that you will encounter and suggest how you can vary your teaching approach to address their diverse needs.

Strategic learners are achieving at below grade level

These students do not necessarily have special educational needs — they are mainly students who are achieving at below grade level for because of insufficient prior schooling, perhaps caused by poor attendance or relocation. Such students can be expected to achieve at grade level by following a Program that is carefully targeted to their needs.

Some students have special educational needs

These students find it even more of a challenge to meet the Standards, and need more **intensive help**. These students may be eligible for **special education services** and so may have an **individual education plan** (IEP). You should check if any of your students are in this situation. They may also be provided with **other assistance**, such as the services of a classroom assistant, or specialized equipment. This group also includes English Language Learners — see page xxviii for information about providing for these students.

Advanced learners need acceleration and enrichment

These students perform **significantly above grade level** and need acceleration and enrichment. The pace of instruction may be increased for them, and you should give opportunities to study material in greater depth. The suggested Pacing Plan for this Program can be accelerated to provide for these students (see page xix) to allow them more time for additional study.

Strategies for supporting students

Get to <u>know your students</u>

- Distribute a questionnaire to get to know each student as soon as they arrive in your class. Include a section with personal questions (for example, languages spoken at home, hobbies, foods, sports, music, after-school responsibilities) and a section on math attitudes (for example, "Do you like math?" "What's hard for you in math?")
- Introduce yourself to students and their parents in a letter including information about the languages you speak, your family heritage, education, and interests. Bring in pictures, if appropriate, and set up a small bulletin board for personal pictures of your family and the students' families if they choose to bring in photos.

Set the classroom stage for <u>comfort, predictability, support, and participation</u>

- **Establish predictable routines**. Give clear, but limited instructions on what students will see and do each day. Establish a consistent format for the class blackboard — for example, with "Today's Agenda" always on the upper left side and "Homework Assignment" always under the agenda. Keep daily routines — for example, students always hand in homework sheets as they walk in.
- **Establish, practice, and reward expected classroom norms**. For example, they should bring a notebook and two pencils every day, sit in an assigned seat, put everything under their desk except their notebook, pencil, and textbook, talk only during discussion times, and raise their hand to speak unless they are working with a partner or in groups. In particular, students who have attended school in other cultures may have different classroom norms and need explicit instructions on your expectations for appropriate behavior.
- **Establish notebook format and expectations**. Make a "model notebook" showing examples of what you expect to see. Check notebooks weekly at the beginning of the course; at least once per marking period later. Post notebook rubric and dates for notebook checks. Have students do a "self-check" before you check their notebook.

Students may need <u>more learning time</u>

Some students will need the pace of the Program reducing so that they have more time to learn fundamental concepts and skills. Ways of doing this, while still covering the whole Program, include:

- Scheduling study groups before or after school, or on weekends,
- Providing personalized assignments for students to complete at home, possibly with help from family members,
- Tutoring, ideally by a qualified teacher, but peer or cross-age tutoring are possibilities,
- Arranging additional math classes during the school day.

Check students' understanding frequently and analyze errors

Students who are struggling should be assessed regularly and the results of the assessments used to devise intervention strategies. These assessments may be formal tests, but could also take the form of verbal questioning or extra exercises. Examining a student's work will give you an insight into their thought processes and may help you see which points are confusing them. Assessment results will also help you to temporarily group students according to their current needs.

This Program features Assessment Tests aligned with each Section that are designed for students performing at grade level. However, if a student has difficulty with these tests, consider creating a personalized test using the **Assessment Test Generator CD-ROM**. For example, a shorter test may be better for students who find concentrating difficult, while tests consisting entirely of multiple-choice questions may be more suitable for other students.

Present material in a variety of ways

Many difficulties can be resolved, or even avoided, if material is presented in a different way. You should try to explain concepts both verbally and visually, and should relate them to concrete examples where possible. The Universal Access approaches in the **Teacher's Edition** margins are useful here and often use resources such as algebra tiles (provided on the **Teacher Resources CD-ROM**), to make abstract concepts more real. The **Skills Review CD-ROM** also provides a fresh teaching approach to topics, as each worksheet contains teaching, examples, and exercises.

Focus on key concepts

For students struggling with math, you should focus on helping them meet the **Key Standards**, which are marked in the "Mathematical Content Standards" on page vii and are pointed out in the **Teacher's Edition** margins. This will ensure they have adequate understanding on which further mathematical concepts can be built. The traditional Basic Schedule (see page xx) is designed to allow plenty of time for explaining concepts, and the suggested questions focus on developing basic skills.

Activities for Strategic Learners

Hands-on and conceptual activities

Ways of ensuring students have the opportunity to learn through participation without disruption in the classroom include:

- Use graphic organizers to illustrate key concepts and vocabulary relationships. Pictures of relationships may solidify memory links. You will find suggestions for relationships that can be shown graphically in the **Teacher's Edition** margin.
- Store manipulatives in plastic bags that can be quickly and easily distributed and collected. Use pictures and item names to reinforce names of manipulatives. When in groups, have the "equipment manager" retrieve and return manipulatives; the "checker" assure items are complete and in good order, and so on.
- Check frequently for understanding. For a "change-up", pass out individual boards with markers and eraser cloths to each student. Put a problem on the overhead, have each student copy the problem, then show their solution by holding up their board. Then show the correct solution on the blackboard in front of the whole class. Students should signal thumbs up if they are correct and thumbs down if they are incorrect. Let students help each other as needed, but confiscate boards that are improperly used.
- When you lecture, actively involve students in taking notes and making notes. Show students how to divide a regular notebook piece of paper in two columns lengthwise. Then label the columns "take notes" (where students copy what you write on the overhead or blackboard) on the left, and "make notes" on the right. The right side is for them to make notes in their own language or with their own examples to help clarify and remember what you said. Provide time for making notes and sharing notes with a partner and the class.
- Use "error correction" activities — for example, asking "What's wrong with this solution?" Use common errors, but not a specific student's work to avoid embarrassment. Common errors and Universal Access approaches are given in the teacher margins of the **Teacher's Edition**.

Vocabulary activities

- Consider using the last five minutes of a lesson to reflect on today's math, enter new vocabulary words into review file, and clarify the day's homework.
- Integrate a vocabulary review into weekly lessons. Also, create "preteaching" activities for upcoming vocabulary terms, using the vocabulary lists at the start of each Lesson. Try to expose students to new terms before they have to use them in a lesson.

Give students responsibility over their learning

- Provide a "Suggestion or Question" box and blank paper for students to write you a note asking for extra help, noting problems, asking you to speak slower, and so on. Make lists of classroom jobs and invite students to sign up (for example, decorating bulletin boards, handing out assessment tests, preparing or organizing manipulatives, and so on).

Providing for English Language Learners

The mathematical abilities of English Language Learners cover the **entire spectrum** — they may have special educational needs or be exceptionally gifted. Any learning barriers that their language skills present must be minimized.

It's important to avoid simply teaching the whole course in the students' first language — in order to be best prepared for the future they must become accustomed to learning in English. These two pages detail some techniques to help students understand the math content, and also highlight how this Program helps English Language Learners achieve their full potential.

Find out what students know and build on it

Many English Language Learners have **significant gaps** in their math background knowledge that hamper their understanding of new concepts. As with all students, it's important to find out what they already know and to use it as a **foundation** upon which to build new ideas — see pages xxvi and xxvii for more on providing Universal Access for all students.

The **Preprogram Benchmark Test** is designed to show the extent of students' math knowledge in preparation for the grade 7 course. To increase the validity of such tests for English Language Learners, it may be necessary to:

- simplify or paraphrase the directions,
- allow English Language Learners extra time,
- allow students to use dictionaries or word lists.

Information from alternative assessments, such as teacher observations, writing samples, and interviews may also be useful, as well as the questionnaires mentioned on page xxvi.

Give English Language Learners the best chance of following lessons

Communicate <u>clearly</u> with the class

When you're teaching, ensure that English Language Learners are included in lessons by thinking about how you communicate with the class:

- Use **simple vocabulary** and **short sentences**.
- Avoid using idioms and slang.
- Speak **slowly** and articulate clearly.
- Using **gestures** may also aid understanding.
- Wait at least three seconds after you ask a question of an English Language Learner and before you comment on their answer. This time allows them to translate and process language.
- Use a variety of ways of presenting material — diagrams, for example. The **Skills Review CD-ROM** and the Universal Access approaches in the **Teacher's Edition** will be useful for this.

<u>Prepare the classroom</u> for English Language Learners

- Determine all the languages spoken by the students in your classroom. Have them make posters with various translations of simple greetings in each language, then learn and use the greetings. Emphasize that speaking more than one language is an asset that will open doors later in life and they should continue to develop all the languages they speak.
- Assign each lower level English Language Learner a "language buddy" — someone who knows their primary language and has more advanced English language skills. Use language buddies for partner and group activities. When possible, rearrange groups each marking period to increase students' comfort level with all members of the class.
- Connect with school personnel who speak other languages. Ask for their help with communication, translation, insight, etc. Invite English Language Learners' parents to volunteer in the class.

Encourage English Language Learners to <u>communicate</u> in the classroom

- Teach students a simple set of hand gestures they can use to indicate four basic things:
  i) they agree with what was said,
  ii) they disagree,
  iii) they partially agree,
  iv) they don't understand what was said.

- Practice using these signals a lot during the first weeks. Ask for hand signals in response to not only what you say, but also what the students say. Hand signals need to be subtle so students won't hesitate to indicate when they disagree or don't understand.
- Maximize the opportunities for dialogue during each lesson. For many English Language Learners, class time will be the main time during the day that they speak English. Provide structures that build in student-to-student, student-to-teacher, and student-to-class dialogues. For example:
  i) Pair share: checking homework with a partner, sharing results with another pair of partners.
  ii) Review activities:
    - have Group 1 present a game for this week's vocabulary words,
    - Group 2 present the first two Guided Practice problems in today's Lesson,
    - Group 3 present solutions to yesterday's homework.

  Each member of each group must actively participate in the presentation in some way. Model what you expect the group to do before you assign an activity.
- Distinguish between talking time and silent working time. Allow students to speak in any language during "talking time" but always report findings in English. For students who are just beginning to speak English, expect and respect a "silent period" — that is, a period of some weeks or months before they are comfortable enough to talk aloud in English.

Concentrate on vocabulary and math terms

- Introduce new vocabulary before starting each lesson. Refer back to the meanings throughout the lesson to reinforce understanding. At the start of every Lesson in the Student and Teacher's **Textbooks** you will find a Key Words box highlighting the key math terms that students will need to know.
- Develop methods for reviewing vocabulary (for example, index card vocabulary files, notebook sections, a special folder for vocabulary, word walls). Many suggestions are given in the teacher margins throughout the **Teacher's Edition**.
- Have bilingual dictionaries available for each language spoken.
- English Language Learners may have particular difficulty with word problems. Teach them to identify information that they need to solve the problem (by underlining or circling), choose a strategy, work it out, check their work, and finally go back to the question and check the solution for reasonableness. Have students make up their own word problems, solve them, share with a partner, and check each other's work as part of the process of becoming comfortable with word problems.
- Expanding English Language Learners' vocabulary will be a constant task requiring energy, creativity, and consistency. Use the suggestions given in the teacher margins to model and use a variety of methods to develop vocabulary.

Create mixed groups of students

- See page xxv for more information on effective grouping strategies, including mixed groups to allow English Language Learners to participate in cooperative work.
- In particular, the Chapter Investigations provide an ideal opportunity for cooperative learning in small groups. This is useful for encouraging interaction between English Language Learners and first-language English speakers. English learners will benefit from hearing English spoken by their peers, and, with thoughtful grouping, will be in a less threatening situation for using new vocabulary.

Relate math to the real world

It's important that learning is relevant to students' **real-life experiences** — they need to be shown that what they are learning will be **valuable to them in the outside world**. Examples used should be meaningful to all students in the class. Many of the **examples, exercises, and investigations** in this Program apply the math concepts and skills to real-life situations.

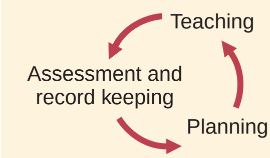
- English Language Learners should be given the opportunity to tackle real-life problems to ensure they develop an ability to reason effectively.
- The Chapter Investigations provide an ideal opportunity for **cooperative learning in small groups**. This is useful for encouraging interaction between English Language Learners and first-language English speakers.

Record-Keeping and Communication

Keeping student records

Why is it important?

Accurate records of student progress are vital. Well-organized assessment data can be used to **fine-tune future planning**. For example, your records may indicate that particular students have not met a learning objective and need **additional help**.



These elements should form a continuous cycle.

This could be achieved by:

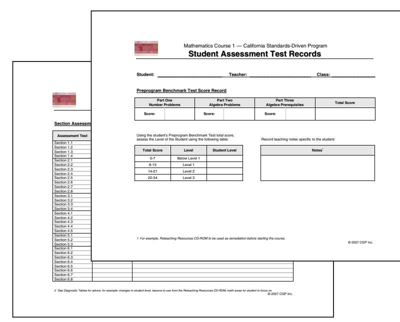
- Referring back to the **Textbook** to clarify any misunderstandings that students may have,
- Using a Universal Access approach from the **Teacher's Edition** margin,
- Using the **Skills Review CD-ROM** to reteach topics starting from earlier grade-level material.

Your records are likely to be useful to other educational professionals, such as the students' next teacher. They will also provide valuable evidence for reporting to parents and guardians and are a useful source of information to have to hand at parent-teacher conferences.

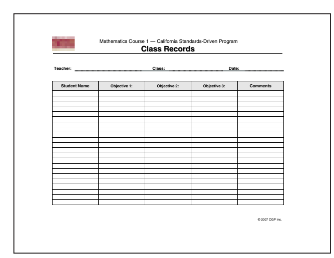
How to make it work for you

Whatever record-keeping system you use, it needs to be **manageable** and **straightforward**. Only information that will be genuinely useful for future planning or reporting should be included.

Two useful record-keeping sheets have been included on the **Teacher Resources CD-ROM**:



- **Student Assessment Test Records** — for recording students' achievements in the assessments completed throughout the course. Create one record sheet for each student to monitor their point scores on each test. This record will help you to use the scores from the **Preprogram Benchmark Test** to allocate a level to each student, and then to closely monitor any improvement or difficulties during each **Section Assessment Test**. Diagnostic Guides have been provided on the **Teacher Resources CD-ROM**, containing detailed advice on interpreting each Assessment Test. Keeping a record of test scores will help you decide if a student should be allocated a new level, or whether more practice is required.

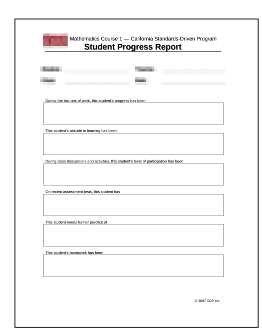


- **Class Records** — for recording students' progress towards **learning objectives**. The form is designed to be flexible — you can use the learning objectives from the start of each Section, or you can write your own, specifically to meet your students' needs.

Each of these sheets can be saved onto your own computer and completed on-screen for each class or individual student.

Communication between school and home

Good communication between students' homes and the school can improve your relationship with students' parents or guardians. This in turn can increase the effectiveness of your teaching to **raise achievement levels**. Ideally, there should be a **continuously open channel of communication**, rather than one that is limited to parent-teacher conferences.



You should provide regular information for parents and guardians on students' progress, achievements, classroom participation, and other relevant matters. You should also let parents and guardians know how they can contribute to their child's learning.

- The **Student Progress Report** form on the **Teacher Resources CD-ROM** makes it simple to keep parents and guardians up to date on their child's math progress. This reporting form should be used regularly, perhaps at the end of each Chapter of study. You should also save copies of previous Progress Reports in your files for each student.

It is very important that teachers are sensitive to variations in parents' literacy and numeracy skills.

Chapter 1

The Basics of Algebra

How Chapter 1 fits into the K-12 curriculum

Section 1.1 — Variables and Expressions		
Section 1.1 covers Algebra and Functions 1.1, 1.2, 1.3, 1.4 **Objective: To evaluate expressions using order of operations and various math properties**		
Previous Study In grade 6, students wrote and evaluated algebraic expressions using the order of operations. They also learned about the multiplicative inverse, and used the associative and commutative properties add and subtract whole numbers.	**This Section** Students are reintroduced to variables and expressions, and then use order of operations to evaluate numeric and variable expressions. Finally, students learn and use the identity, inverse, associative, and commutative properties.	**Future Study** In this Chapter, students will collect like terms and apply the distributive property when manipulating equations. In Section 2.4 they will use the order of operations to evaluate expressions involving exponents.

Section 1.2 — Equations		
Section 1.2 covers Algebra and Functions 1.1, 1.4, 4.1, Mathematical Reasoning 1.1, 2.1 **Objective: To write and solve two-step equations and to apply these techniques to real-life problems**		
Previous Study In grade 6, students wrote and solved one-step linear equations in one variable. Since grade 3, students have used appropriate units to quantify properties of objects, such as length, volume, and weight.	**This Section** Students are reintroduced to equations, and then solve one- and two-step equations. They then go on to write and solve equations related to real-life situations.	**Future Study** Later in this course students will use a similar method to write and solve two-step inequalities. In Algebra I, students solve systems of linear equations involving two variables.

Section 1.3 — Inequalities		
Section 1.3 covers Algebra and Functions 1.1, 1.4, 1.5 **Objective: To write two-step inequalities**		
Previous Study As early as grade 1, students used the symbols $<$, $=$, and $>$ to compare numbers. In grade 4, students used these symbols to compare fractions and decimals.	**This Section** Students review the symbols $<$ and $>$ and are introduced to the symbols $\leq$ and $\geq$. They then convert word problems to one- and two-step inequalities.	**Future Study** In Chapter 4 students will solve two-step inequalities in one variable. They will also write simple systems of inequalities. In Algebra I, they will solve inequalities involving absolute values.

Pacing Guide – Chapter 1

40- to 50-Minute Class Periods

If your class periods are 40-50 minutes, we recommend allowing **19 days** for teaching Chapter 1.
As well as the **15 days of basic teaching**, you have **4 days** remaining to allocate 4 of the 6 optional activities (to be delivered at any appropriate point during the Chapter).

The table shows the 15 teaching days as well as all of the **optional days** you may choose for Chapter 1, in the order we recommend.

Day	Lesson	Description
Section 1.1 — Variables and Expressions		
Optional		*Exploration — Variable Tiles*
1	1.1.1	Variables and Expressions
2	1.1.2	Simplifying Expressions
3	1.1.3	The Order of Operations
4	1.1.4	The Identity and Inverse Properties
5	1.1.5	The Associative and Commutative Properties
Optional		*Assessment Test — Section 1.1*
Section 1.2 — Equations		
Optional		*Exploration — Solving Equations*
6	1.2.1	Writing Expressions
7	1.2.2	Equations
8	1.2.3	Solving One-Step Equations
9	1.2.4	Solving Two-Step Equations
10	1.2.5	More Two-Step Equations
11	1.2.6	Applications Of Equations
12	1.2.7	Understanding Problems
Optional		*Assessment Test — Section 1.2*
Section 1.3 — Inequalities		
13	1.3.1	Inequalities
14	1.3.2	Writing Inequalities
15	1.3.3	Two-Step Inequalities
Optional		*Assessment Test — Section 1.3*
Chapter Investigation		
Optional		*Investigation — Which Phone Deal is Best?*

Accelerating and Decelerating

- To **accelerate** Chapter 1, allocate fewer than 4 days to the optional material, or cover the core Lessons in less than 15 days, as students may be familiar with much of the content from earlier grades. This will give you extra days to allocate to other Chapters.
  Note that you may use the remaining optional days at the end of the 160-day course.
- To **decelerate** Chapter 1, consider allocating more than 4 days to the optional Assessment Tests, Section Explorations, or Chapter Investigation, or spend longer teaching some Lessons. Also consider preparing students for difficult Lessons by reviewing previous coverage of math topics on related Skills Review Worksheets.
  Note that decelerating Chapter 1 will result in fewer days being available for teaching other Chapters.

90-Minute Class Periods

If you are following a block schedule with 90-minute class periods, we recommend allowing **9.5 days** for teaching Chapter 1.
The basic teaching material will take up **7.5 days**, and you can allocate the remaining **2 days** to the **optional material**.
To accelerate or decelerate a block schedule, follow the same advice as given above.

Section 1.1

Exploration — Algebra Tiles

Purpose of the Exploration

Students often have difficulty visualizing the meaning behind an algebraic expression. The use of algebra tiles helps students to see what an expression physically looks like. The tiles also assist students in understanding how to simplify an expression by collecting like terms.

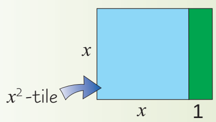
Resources
Teacher Resources CD-ROM
• Algebra Tiles

Strategic Learners

Using algebra tiles is very useful for strategic learners. They help students understand that the terms of an expression can be rearranged and collected together to form a simpler expression. Provide strategic learners with lots of additional practice at modeling expressions such as $x + 1$ and $4x + 7$ with the tiles.

Universal access

Advanced learners can also use x^2-tiles. This allows them to model expressions such as $x^2 + 3x + 1$, and calculations such as $x(x + 1)$.



Common error

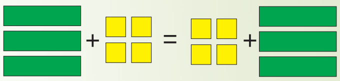
Students may have difficulty writing the term for each tile. They may know that there are three x tiles but not know what to write as the term. Remind students that the type of tile is the label that goes with the number.

Common error

Another problem is that students often leave out a set of parentheses when finding an area. An example would be when the length is $x + 1$ and the width is 2. Students might write $2 \times x + 1$.

Math background

Show students that the tiles can be grouped together in different ways. Addition is commutative so it doesn't actually matter if they write $3x + 4$ or $4 + 3x$.



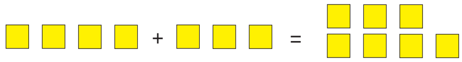
However, explain to students that it is best to write expressions with any constant terms at the end. This is the convention normally used.

Section 1.1 introduction — an exploration into: Algebra Tiles

You can write expressions using algebra tiles. This shows what an expression actually "looks like," and how you can simplify it. You'll be using two types of tile. Their areas represent their values —

1-tile: 1 [1] *x-tile:* 1 [x]

Here's the sum 4 + 3 shown using algebra tiles:

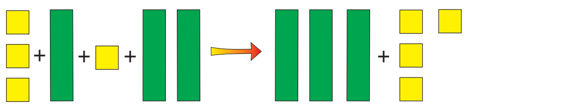


The example below shows how you can **simplify** algebra expressions by **rearranging the tiles.**

Example

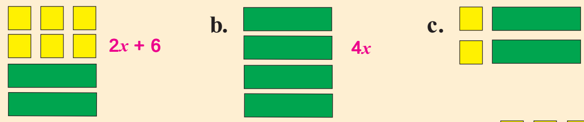
Show $3 + x + 1 + 2x$ using algebra tiles. Rearrange the tiles to simplify the expression.

Solution

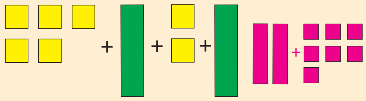


Exercises

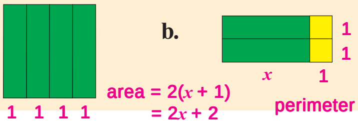
1. Use algebra tiles to model the following addition problems:
   a. $5 + 2$ b. $2 + 1$ c. $x + 2x$
2. Write the algebra expressions that are modeled by these tiles.



3. Rearrange these tiles to make a simplified expression:



4. Write an algebraic expression to represent the tiles in Exercise 3. Rewrite this as a simplified expression.
   $5 + x + 2 + x = 2x + 7$
5. Write expressions to represent the perimeter and area of these rectangles. a. b.



Round Up

When you're simplifying expressions, you have to organize them so the bits that are the same are all grouped together. You can then combine these — so 2x + 3x = 5x, and 1 + 2 = 3. What you can't do is combine different things together. For example, you could never add 4x and 7.

Lesson 1.1.1

Variables and Expressions

This Section reviews the concept of variables and their use in algebraic expressions. Students practice interpreting expressions containing variables and evaluating them when the values of the variables become known.

Previous Study: Students wrote and evaluated algebraic expressions in earlier grades.

Future Study: Students will write and solve two-step linear equations later in this Chapter.

Lesson 1.1.1

California Standards:

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., 3 less than a number, half as large as area A).

Algebra and Functions 1.4

Use algebraic terminology (e.g., variable, equation, **term, coefficient**, inequality, **expression, constant) correctly.**

What it means for you:

You'll learn how to use a letter or symbol to stand in for an unknown number, and how it can be exchanged for the real number when you know it.

Key words:

- variable
- expression
- evaluate

Section 1.1

Variables and Expressions

When you write out a problem in math you often need to include unknown numbers. You have to use a letter or symbol to stand in for an unknown number until you figure out what it is — you did this before in earlier grades. That's what a variable does. And that's what this Lesson is about.

A Variable Represents an Unknown Number

In algebra you'll often have to work with numbers whose values you don't know. When you write out math problems, you can use a **letter** or a **symbol** to stand in for the number. The letter or symbol is called a **variable**.

Coefficient → $2k + 4$ ← Constant

Variable ↗

The number that the variable is being multiplied by is called the **coefficient** — like the 2 above.

Any number not joined to a variable is called a **constant** — like the 4 above. It's called that because its value doesn't **change**, even if the value of the variable changes.

A **term** is a group of **numbers** and **variables**. One or more terms added together make an **expression**. For example, in the expression above, $2k$ is one term and 4 is another term. In the expression $3 + 4x - 5wyz$, the terms are 3, $4x$, and $-5wyz$.

An Expression is a Mathematical Phrase

Expressions are **mathematical phrases** that may contain **numbers**, **operations**, and **variables**. The operations act like a set of instructions that tell you what to do with the numbers and variables. For example, $2k + 4$ tells you to double k, then add four to it.

There are two types of expressions — **numeric** and **variable**.

- **Numeric expressions** have numbers in them, and often operations — but they don't include any variables:
  - → $5 + 13$
  - → $2 \cdot 5 - 6$
  - → $8 + 7 \div 6$
- **Variable expressions** have variables in them, and may also include numbers and operations:
  - → $5h$
  - → $7x - 2$
  - → $2k + 4$

Don't forget:

xy means "x multiplied by y." It's just the same as writing $x \times y$ or $x \cdot y$.

1 Get started

Resources:

- several oranges or other fruit
- some sheets of paper
- 10 pennies
- a cloth

Warm-up questions:

- Lesson 1.1.1 sheet

2 Teach

Concept question

"What is the role of a variable?"

A variable is used to represent a number. Variables are used when the value of the number is unknown.

(Variables are also used in representing general statements, for example $a + b = b + a$, and in expressing relationships, for example $A = lw$.)

Universal access

One way to introduce variables is to state the following:
"I have **some** sheets of paper."

"Some" in the English language is like x in algebra.

Universal access

The concept of a variable can be illustrated using concrete objects like fruit and vegetables. Students relate more to objects that they encounter every day, as opposed to letters or abstract symbols.

For example, place three oranges on a table and on the board write "3" next to a picture of an orange. The picture of an orange represents the object "orange," and the number 3 tells you how many there are. Take one orange away and change the number 3 to 2 on the board. After several examples, repeat using different letters in place of the picture.

- **Strategic Learners**
  The Universal Access activities on the previous page and next page are particularly suitable for helping strategic learners to develop their understanding of the concept of variables. Other symbols, such as stars and triangles, could also be used to represent unknown values.

- **English Language Learners**
  Review the terms that indicate operations (sum, difference, product, quotient, increased by, decreased by, added to, subtracted from, multiplied by, divided by). Students may find it useful to create a word bank of these phrases and add their own notes and examples. Warn students to take care with the phrase "subtracted from." 2 subtracted from x is "$x - 2$" — the numbers and symbols can't be translated in order.

2 Teach (cont)

Guided practice

Level 1: q1–8
Level 2: q1–8
Level 3: q1–8

Additional example

Identify whether the following are variable expressions. State how many variables each contains.

1) w

Variable expression. One variable.

2) 3

Not a variable expression. No variables.

3) $9fd$

Variable expression. Two variables.

4) +

Not a variable expression. No variables.

Common error

Students should avoid using letters that may be mistaken for other mathematical symbols. For example, variables such as I, o, and l could be mistaken for 0 and 1.

Guided Practice

Say whether the expressions in Exercises 1–8 are numeric or variable.

1. y	variable	**2.** $4(x - y)$	variable
3. $10 - 7$	numeric	**4.** 6^2	numeric
5. $8xy$	variable	**6.** $4(3 + 7)$	numeric
7. $9(5 - y)$	variable	**8.** x^2	variable

Expressions Can Be Described in Words

To show you understand an expression you need to be able to explain **what it means** in words. You can write a **word expression** to represent the numeric or variable expression.

Example 1

Write the variable expression $x + 5$ as a word expression.

Solution

In this question x is the **variable**. The rest of the expression tells you that five is being **added on** to the unknown number x.

So the expression becomes:

"**x is increased by five**" or

"**the sum of x and five**" or

"**five more than x**."

They all mean the same thing.

When you change a **variable expression** to a **word expression** you can say the **same thing** in several **different ways**.

+ Instead of "2 **added to** x" you could say "x **increased by** 2," "2 **more than** x," or "**the sum of** x and 2."

– "2 **subtracted from** x" means the same as "2 **less than** x" or "x **decreased by** 2."

× "x **multiplied by** 2" means the same as "**the product of** x and 2," "x **times** 2," or "**twice** x."

÷ And you could say either "x **divided by** 3" or "**one third** x."

As long as it **matches with the operation** that you're describing, you can use any of the phrases.

Check it out:
These four groups of phrases are describing the four operations you'll need: addition, subtraction, multiplication, and division.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Challenge students to find as many different ways as possible of writing a word expression to represent a variable expression. They should check that each other's phrases mean what they are meant to.

2 Teach (cont)

Don't forget:
You need to do the operations in the correct order.
The PEMDAS rule helps you remember this — it tells you that you need to deal with the parentheses first.
There's more on the PEMDAS rule in Lesson 1.1.3.

Example 2

Write the variable expression $4(w - 3)$ as a word expression.

Solution

In this expression w is the **variable**. The rest of the expression tells you that three is **subtracted** from w, and the result is **multiplied** by four.

So the expression becomes:

"**four times the the result of subtracting three from *w***" or

"**three subtracted from *w*, multiplied by four**."

They both mean the same thing.

Additional example

Translate these variable expressions into word expressions.

1) $-5c$

"Negative five times *c*" OR "The product of negative five and *c*."

2) $\frac{h}{9}$

"A number, *h*, divided by nine."

There are a variety of different ways to translate these expressions. These are just examples.

Guided Practice

Write a word expression for each of the variable expressions in Exercises 9–14.

9. $z - 10$ **"Ten less than *z*" or "*z* decreased by 10."**

10. $6b$ **"The product of six and *b*" or "*b* multiplied by six."**

11. $12h + 4$ **"12 times *h*, increased by four" or "four more than the product of 12 and *h*."**

12. $5(j + 6)$ **"Five times the sum of *j* and six."**

13. $6(4t)$ **"Six times the product of four and *t*."**

14. $2c + 4d$ **"The product of two and *c* added to the product of four and *d*."**

Check it out:
There is more than one possible word expression to describe each of these mathematical phrases.
As long as it means the same thing, any variation you use is fine.

Guided practice

Level 1: q9–11
Level 2: q9–13
Level 3: q9–14

When You Evaluate, You Swap Variables for Numbers

When you're given the actual numbers that the variables are standing in for you can **substitute** them into the expression. When you have substituted numbers for **all the variables** in the expression, you can work out its numerical value. This is called **evaluating the expression**.

Universal access

Cover seven pennies with a cloth so that the students don't know how many there are. Assign a variable to represent the number of pennies under the cloth. Place three more pennies next to the cloth and write an expression to represent the total number of pennies. Remove the cloth to find the value for the variable and evaluate the expression.

Example 3

Evaluate the expression $6f + 4$ when $f = 7$.

Solution

$6f + 4 = 6 \cdot 7 + 4$ **Substitute 7 in place of *f***

$= 42 + 4$ **Perform the multiplication first**

$= 46$ **Then do the addition**

Don't forget:
Multiplication and division come before addition and subtraction in the order of operations. That's why you do the multiplication first in this example.

Solutions

For worked solutions see the Solution Guide

2 Teach (cont)

Concept question

"The expression $2x + 3y$ is evaluated and its numerical value is found to be 12. What could the values of the variables be, assuming they are whole numbers?"

There are an infinite number of possible answers. For example: $x = 3$ and $y = 2$, $x = 6$ and $y = 0$, $x = 0$ and $y = 4$, $x = -3$ and $y = 6$.

Guided practice

Level 1: q15–20
Level 2: q15–21
Level 3: q15–22

Additional example

Evaluate the following expressions when $a = 3$ and $b = 6$.

1) ab

$3 \times 6 = 18$

2) $a + b$

$3 + 6 = 9$

3) $\frac{a}{b}$

$\frac{3}{6} = \frac{1}{2}$

4) $a - b$

$3 - 6 = -3$

Independent practice

Level 1: q1–7
Level 2: q1–9
Level 3: q1–10

Additional questions

Level 1: p430 q1–2, 9–10
Level 2: p430 q1–10
Level 3: p430 q1–12

3 Homework

Homework Book — Lesson 1.1.1

Level 1: q1–3, 5, 7
Level 2: q1–8
Level 3: q3–9

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 20 — Variables and Expressions

Example 4

Evaluate the expression $11(p - q)$ when $p = 7$ and $q = 3$.

Solution

$11(p - q) = 11(7 - 3)$ **Replace p and q with 7 and 3**
$= 11(4)$ **Do the subtraction**
$= 44$ **Do the multiplication to get the answer**

Guided Practice

Evaluate the expressions in Exercises 15–22, given that $j = 5$ and $t = 7$.

15. $6t - 4j$ **22**
16. $t + j$ **12**
17. $3(t - j)$ **6**
18. $3t + 7j$ **56**
19. jt **35**
20. $2(j - t)$ **–4**
21. $j + tj - 40$ **0**
22. $3jt - j + 20$ **120**

Independent Practice

Write word expressions for the variable expressions in Exercises 1–3.

1. $6y + 10$ **"10 more than the product of six and y."**
2. $3(p - 6)$ **"The product of three and six less than p."**
3. $18t$ **"The product of 18 and t."**

4. Joe and Bonita went fishing. Joe caught j fish and Bonita caught $j + 4$ fish. Write a sentence that describes the amount of fish Bonita caught compared with Joe. **Bonita caught four more fish than Joe.**

Evaluate the expressions in Exercises 5–7, given that $x = 3$.

5. $4x$ **12**
6. $1 - x$ **–2**
7. x^2 **9**

8. An orchard uses the expression $0.5w$ to work out how much money, in dollars, it will make, where w = the number of apples sold. How much money does the orchard make if 250 apples are sold? **$125**

9. Given that $a = -1$, $b = 3$, and $c = 2$, evaluate the expression $a^2 + ab + c^2$. **2**

10. A car rental company uses the expression $30 + 0.25m$ to calculate the daily rental price. The variable m represents the number of miles driven. What is the price of a day's rental if a car is driven 100 miles? **$55**

Now try these:
Lesson 1.1.1 additional questions — p430

Round Up

Variables are really useful — you can use them to stand in for any unknown numbers in an expression. When you know what the numbers are you can write them in — and then evaluate the expression. In Section 1.2 you'll see how expressions are the building blocks of equations.

Solutions
For worked solutions see the Solution Guide

Lesson 1.1.2

Simplifying Expressions

This Lesson covers two important skills for simplifying expressions — collecting like terms and using the distributive property.

Previous Study: In grade 6, students applied the distributive property to evaluate expressions.

Future Study: In the rest of this Chapter, students will collect like terms and apply the distributive property when solving equations.

Lesson 1.1.2

Simplifying Expressions

California Standards:

Algebra and Functions 1.3

Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, **distributive,** associative, commutative) and justify the process used.

Algebra and Functions 1.4

Use algebraic terminology (e.g., variable, equation, **term, coefficient,** inequality, **expression, constant) correctly.**

What it means for you:

You'll learn about some math techniques that will help you to simplify expressions.

Key words:

- like terms
- distributive property
- simplify

Check it out:

You can move the terms about because of the commutative and associative properties of addition. There's more about these in Lesson 1.1.5.

Check it out:

The minus signs in the expression stay with the 5 and the y they were originally connected to.

Remember — you can write subtractions as additions:

$x - 5 + 2y + 9 - y + 2x$

$= x + (-5) + 2y + 9 + (-y) + 2x$

This often makes it easier to collect like terms.

When you're evaluating an expression that's made up of many terms, it helps to simplify it as much as possible first. The fewer terms you have to deal with, the less likely you are to make a mistake. This Lesson is about two ways that you can simplify expressions.

Simplifying an Expression Makes It Easier to Solve

In math you'll come across some very **long expressions**. The first step toward solving them is to **simplify** them.

The first way to simplify an expression is to **collect like terms**. Like terms are terms that contain **exactly the same** variables.

These are like terms because they contain no variables — they're both constants.

$$h + 2 + 6h - 4$$

These are like terms because they both contain the same variable, h.

You need to bring **like terms** together to simplify the expression. When you do that, the plus or minus sign in front of a term stays with that term as it moves.

$$h + 2 + 6h - 4$$

First swap the positions of the "+ 2" term and the "+ 6h" term.

$$h + 6h + 2 - 4$$

Now all the terms containing the variable h are together and all the constants are together.

Then simplify the grouped terms.

$$7h - 2$$

You can't simplify this any more because there are no longer any like terms.

Example 1

Simplify the expression $x - 5 + 2y + 9 - y + 2x$.

Solution

$x - 5 + 2y + 9 - y + 2x$

$= (x + 2x) + (2y - y) + (-5 + 9)$ **Collect together the like terms**

$= 3x + y + 4$ **Simplify the parentheses**

Guided Practice

Simplify the expressions in Exercises 1–6 by collecting like terms.

1. $a + 4 + 2a$ $3a + 4$

2. $3r + 6 - 5r$ $6 - 2r$

3. $c - 2 + 4 \cdot c - 3$ $5c - 5$

4. $4x + 5 - x + 4 - 2x$ $x + 9$

5. $7 - k + 2k + 3 - k$ 10

6. $m + 4 - n + m - 2 - n$ $2m - 2n + 2$

1 Get started

Resources:

- Collection of books, papers, and other items

Teacher Resources CD-ROM

- Algebra Tiles

Warm-up questions:

- Lesson 1.1.2 sheet

2 Teach

Universal access

Gather up a disorganized bunch of papers, books, manipulatives, and weird items in your arms.

Tell students you need to "simplify this mess" by putting the books together in one pile, the papers together in a folder, the manipulatives in a bag, etc. Relate that to gathering up like terms and "simplifying expressions."

Concept question

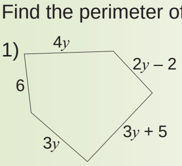
"Explain why the following expression can't be simplified any further by collecting like terms: $2y - x + y^2$"

There are no like terms in the expression (there are two terms containing y, but y isn't raised to the same power in both of them).

Additional example

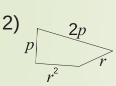
Find the perimeter of the following:

1)



$4y + (2y - 2) + (3y + 5) + 3y + 6 = 12y + 9$

2)



$2p + p + r + r^2 = 3p + r + r^2$

Guided practice

Level 1: q1–6

Level 2: q1–6

Level 3: q1–6

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  Use the Algebra Tiles from the **Teacher Resources CD-ROM** to model the distributive property. This is explained in more detail below.

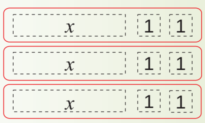
- **English Language Learners**
  Use whiteboards to review vocabulary. Do this by asking them to write a simple expression on their board and circle a coefficient, the variables, and each term in different colors. Remind students that there's an "invisible" coefficient of 1 next to every variable that doesn't otherwise have a coefficient ($x = 1x$).

2 Teach (cont)

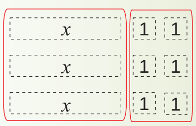
Universal access
Model the distributive property using the Algebra Tiles from the **Teacher Resources CD-ROM**.

For example, use tiles to model $3(x + 2)$ and $3x + 6$.

Model $(x + 2)$ three times.



Model $3x + 6$



The expressions are represented by exactly the same tiles — so the expressions are equivalent.

Additional example
Simplify the following expressions using the distributive property.

1) $10(9 + 3)$

$10 \bullet 9 + 10 \bullet 3 = 90 + 30 = 120$

2) $-8(4 + 6)$

$-8 \bullet 4 + (-8) \bullet 6 = -32 + -48 = -80$

3) $-6(11 - 8)$

$-6 \bullet 11 + (-6) \bullet (-8) = -66 + 48 = -18$

Common error
Students often forget that a negative sign outside the parentheses gets distributed to the numbers in the parentheses.

For example $-3(4 - y)$ means that -3 is multiplied by 4 and $-y$.

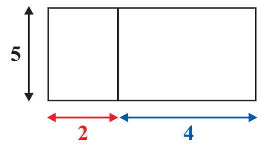
Don't forget:
The formula for the area of a rectangle is:
Area = Length × Width

Check it out:
No matter what numbers a, b, and c stand for, the rule will always be true.

In an expression like **$2(3m + 4) + m$** the **parentheses** stop you from **collecting like terms**. To **simplify** it any more you first need to **remove** the parentheses. You can use a property of math called the **distributive property** to do this.

Use the Distributive Property to Remove Parentheses

Look at this rectangle.



You can find its **total area** in two different ways.

- You could find the **areas** of both of the **smaller rectangles** and **add** them.
  Total area = **$(5 \bullet 2) + (5 \bullet 4)$** $= 10 + 20 =$ **30**
- Or you could find the total width of the whole rectangle by adding 2 and 4, and then multiply by the height. Total area = **$5(2 + 4)$** $= 5(6) =$ **30**

Whichever way you work it out you get the **same answer**, because both expressions represent the **same area**. This is an example of the **distributive property**.

So:

$$5 \bullet (4 + 2) = (5 \bullet 4) + (5 \bullet 2) = 20 + 10 = 30$$

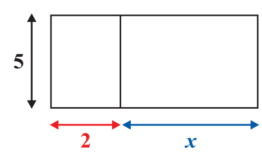
To use the distributive property you multiply the number outside the parentheses by every term inside the parentheses.

Algebraically the property is written as:

The Distributive Property
$$a(b + c) = ab + ac$$

The Distributive Property and Variable Expressions

Think about what would happen to the problem above if you didn't know one of the lengths.



Find its **total area** using the two different methods again.

- Adding the area of the two small rectangles:
  Total area = **$(5 \bullet 2) + (5 \bullet x)$** = **$10 + 5x$**
- Adding 2 and x to find the width and multiplying by the height:
  Total area = **$5(2 + x)$**

Again, you know that the two expressions have the **same value** because they represent the **same area**. So you can say that: **$5(2 + x) = 10 + 5x$**

You can use the **distributive property** to **multiply out** the parentheses in the first expression to get the second equivalent expression.

Advanced Learners
Ask students to create their own word problems that can be represented by the distributive property. Direct them to Independent Practice Exercise 8 for an example of the type of problem they could create.

Example 2

Write the expression $5(y - 3)$ without using parentheses.

Solution

$5(y - 3)$
$= (5 \bullet y) + (5 \bullet -3)$ **Multiply both y and -3 by 5**
$= 5y - 15$ **Simplify the multiplications**

You don't know what number y is standing in for, so this is the most that you can simplify the expression.

Guided Practice

Write the expressions in Exercises 7–14 in a new form using the distributive property.

7. $6(w + 5)$ $6w + 30$
8. $4(7 - h)$ $28 - 4h$
9. $-2(d + 2)$ $-2d - 4$
10. $-3(f - 3)$ $9 - 3f$
11. $-1(3 - x)$ $x - 3$
12. $-2(-3 - m)$ $6 + 2m$
13. $y(9 + t)$ $9y + ty$
14. $3(3 + k) + 2(k - 0)$ $9 + 5k$

Don't forget:
If you're multiplying the contents of parentheses by a negative number, remember to include the negative sign when you multiply each term.
For example:
$-3(2 - 1) = (-3 \bullet 2) + (-3 \bullet -1)$
$= -6 + 3 = -3$

Independent Practice

Simplify the expressions in Exercises 1–3 by collecting like terms.

1. $2p + 5 + 2p$ $4p + 5$ **2.** $3 + 4x - 5 + x$ $5x - 2$ **3.** $5h + 7 - h - 3 + 2h$ $6h + 4$

4. Keon and Amy are collecting leaves for a project. On a walk Keon finds k leaves and Amy finds 6. Then Keon finds another $4k$ leaves, and Amy loses 2. Write an expression to describe how many leaves they ended up with, then simplify it fully. $k + 6 + 4k - 2$, $5k + 4$

Using the distributive property, write the expressions in Exercises 5–7 in a new form.

5. $8(y + 1)$ $8y + 8$ **6.** $5(w - 9)$ $5w - 45$ **7.** $-h(t - 4)$ $4h - ht$

8. Damian makes \$7 an hour working at the mall. Last week Damian worked for 12 hours. This week he will work for x hours.
a) Write an expression using parentheses to describe how much money he will have made altogether? $\$7(12 + x)$
b) Given that $x = 8$, evaluate your expression using the distributive property. \$140

Simplify the expressions in Exercises 9–12 as far as possible.

9. $2(m + 1) + 1(3 - m)$ $m + 5$
10. $3(b + 2 - b)$ 6
11. $2(h + 4) - 4(h - 1)$ $12 - 2h$
12. $x(2 + y) + 3(x + y)$ $5x + xy + 3y$

Now try these:
Lesson 1.1.2 additional questions — p430

Round Up

Collecting like terms and using the distributive property are both really useful ways to simplify an expression. Simplifying it will make it easier to evaluate — and that will make solving equations easier later in this Chapter.

2 Teach (cont)

Common error
Students often only multiply the first term in the parentheses. Each term in the parentheses must be multiplied by the expression outside.

Guided practice
Level 1: q7–10
Level 2: q7–12
Level 3: q7–14

Concept question
"What numbers could be placed in the space in this expression so that the result of evaluating it is negative?"

2(3 + ___)

Any number less than –3 will work.

Independent practice
Level 1: q1–7
Level 2: q1–9
Level 3: q1–12

Additional questions
Level 1: p430 q1–4
Level 2: p430 q1–7
Level 3: p430 q1–10

3 Homework

Homework Book — Lesson 1.1.2
Level 1: q1, 4, 6, 7a–d
Level 2: q2, 3, 5–8
Level 3: q3–10

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
• Worksheet 20 — Variables and Expressions
• Worksheet 22 — Distributive Property

Solutions
For worked solutions see the Solution Guide

Lesson 1.1.3 The Order of Operations

This Lesson is about using the order of operations to evaluate numeric and variable expressions. The focus is on evaluating expressions with parentheses, and on performing multiplications and divisions, then additions and subtractions, from left to right.

Previous Study: In grade 6, students used the order of operations to evaluate expressions containing operations and positive and negative integers.

Future Study: Students will use the order of operations in Section 1.2 for solving equations. They will also revisit the order of operations in Section 2.5, where they will evaluate expressions involving exponents.

1 Get started

Resources:
- Equipment for cooking rice

Warm-up questions:
- Lesson 1.1.3 sheet

2 Teach

Concept question

"Half of 4 is subtracted from 6. Write this as an expression that can be evaluated by following the order of operations. Explain whether parentheses are necessary."

6 – 4 ÷ 2. This could also be written as 6 – (4 ÷ 2), but division is done before subtraction anyway, so the parentheses are not necessary.

Math background

Students should understand that mathematical rules, like the order of operations, were developed by people — and are just useful conventions that have been agreed upon.

The order of operations rules were set up so that each expression would have a unique value.

Universal access

Rather than using PEMDAS as an acronym, GEMA (Grouping; Exponents; Multiplication and division; Addition and subtraction) can be used.

This emphasizes the fact that there are many grouping symbols besides parentheses — for example, square root symbols, horizontal fraction bars, brackets, etc.

It also makes it more likely that students will remember that multiplication and division have equal priority, as do addition and subtraction.

The disadvantage of GEMA, however, is that it doesn't specifically indicate where division and subtraction fit in.

Lesson 1.1.3 The Order of Operations

California Standards:
Algebra and Functions 1.2
Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.

What it means for you:
You'll learn about the special order to follow when you're deciding which part of an expression to evaluate first.

Key words:
- parentheses
- exponents
- PEMDAS

Check it out:
Another way to remember the order of operations is using the GEMA rule:
Grouping — any symbol that groups things, like parentheses, fraction bars, or brackets.
Exponents.
Multiplication and Division – done from left to right.
Addition and Subtraction – done from left to right.
Use either PEMDAS or GEMA — whichever one you feel happier with.

Check it out:
You'll learn about exponents in Section 2.4.

When you have a calculation with more than one operation in it, you need to know what order to do the operations in.

For example, if you evaluate the expression 2 • 3 + 7 by doing "multiply 2 by 3 and add 7," you'll get a different answer from someone who does "add 7 to 3 and multiply the sum by 2." So the order you use really matters.

There's a set of rules to follow to make sure that everyone gets the same answer. It's called the order of operations — and you've seen it before in grade 6.

The Order of Operations is a Set of Rules

An expression can contain lots of **operations**. When you evaluate it you need a set of rules to tell you **what order** to deal with the different bits in.

Order of operations — the PEMDAS Rule

()[]{}	**P**arentheses	First do any operations inside parentheses.
x^2 y^7	**E**xponents	Then evaluate any exponents.
× ÷	**M**ultiplication or **D**ivision	Next follow any multiplication and division instructions from left to right.
+ –	**A**ddition or **S**ubtraction	Finally follow any addition and subtraction instructions from left to right.

When an expression contains multiplication and division, or addition and subtraction, do first whichever comes first as you read from left to right.

9 ÷ 4 • 3 Divide first, then multiply. 9 • 4 ÷ 3 Multiply first, then divide.

9 + 4 – 3 Add first, then subtract. 9 – 4 + 3 Subtract first, then add.

Following these rules means that there's only one correct answer. Use the rules each time you do a calculation to make sure you get the **right answer**.

Example 1

What is 8 ÷ 4 • 4 + 3?

Solution

Follow the order of operations to decide which operation to do first.

8 ÷ 4 • 4 + 3	**There are no parentheses or exponents**
= 2 • 4 + 3	**Do the division first**
= 8 + 3	**Then the multiplication**
= 11	**Finally do the addition to get the answer**

You do the division first as it comes before the multiplication, reading from left to right.

- **Strategic Learners**
  Illustrate "order" (as in doing things in the correct order) by preparing a cup of cooked rice using a pot, a jug of water, measuring cup, hotplate, and bag of rice. Ask a student to give you directions for preparing a cup of cooked rice. Follow the instructions exactly. For example, if they say, "put the rice in the pan," set the bag of rice in the pan.

- **English Language Learners**
  Use the think-pair-share approach to evaluate expressions following the order of operations. Students first attempt to evaluate an expression by themselves, then they discuss the order of operations they have used with a partner. This allows English language learners to prepare the language needed before presenting their solution to the class.

Guided Practice

Evaluate the expressions in Exercises 1–6.

1. $3 - 2 + 6 - 1$ **6** **2.** $6 \div 2 + 1$ **4** **3.** $4 + 3 - 2 + 7$ **12**
4. $2 + 5 \cdot 10$ **52** **5.** $40 - 10 \div 5 \cdot 6$ **28** **6.** $5 + 10 \div 10$ **6**

Always Deal with Parentheses First

When a calculation contains **parentheses**, you should deal with any operations inside them **first**. You still need to follow the **order of operations** when you're dealing with the parts inside the parentheses.

Don't forget:
Remember to show all your work step by step to make it clear what you're doing.

Example 2

What is $10 \div 2 \cdot (10 + 2)$?

Solution

The **order of operations** says that you should deal with the operations in the **parentheses** first — that's the **P** in **PEMDAS**.

$10 \div 2 \cdot (10 + 2)$
$= 10 \div 2 \cdot 12$ **Do the addition in parentheses**
$= 5 \cdot 12$ **Then do the division**
$= \mathbf{60}$ **Finally do the multiplication**

You do the division first here because it comes first reading from left to right.

Guided Practice

Evaluate the expressions in Exercises 7–14.

7. $10 - (4 + 3)$ **3** **8.** $(18 \div 3) + (2 + 3 \cdot 4)$ **20**
9. $10 \div (7 - 5)$ **5** **10.** $41 - (4 + 2 - 3)$ **38**
11. $10 \cdot (2 + 4) - 3$ **57** **12.** $(5 - 7) \cdot (55 \div 11)$ **–10**
13. $6 \cdot (8 \div 4) + 11$ **23** **14.** $32 + 2 \cdot (16 \div 2)$ **48**

PEMDAS Applies to Algebra Problems Too

The order of operations still applies when you have calculations in **algebra** that contain a mixture of **numbers** and **variables**.

Example 3

Simplify the calculation $k \cdot (5 + 4) + 16$ as far as possible.

Solution

$k \cdot (5 + 4) + 16$
$= k \cdot 9 + 16$ **Do the addition within parentheses**
$= \mathbf{9k + 16}$ **Then the multiplication**

2 Teach (cont)

Guided practice
Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Common error
Students often do multiplication before division, and addition before subtraction, rather than using the left-to-right rule. Emphasize the fact that these have equal priority, or introduce the GEMA acronym (see previous page).

Concept question
"Explain the two mistakes that have been made in this work:
$12 + 5 \cdot (10 - 4 \div 2)$
$= 12 + 5 \cdot (6 \div 2)$
$= 12 + 5 \cdot 3$
$= 17 \cdot 3$
$= 51$"

The order of operations within the parentheses hasn't been followed correctly (the subtraction has been done before the division). Then once the parentheses have been evaluated, the addition has been done before the multiplication.

Guided practice
Level 1: q7–10
Level 2: q7–12
Level 3: q7–14

Additional examples
1) $50 \div 5 - (6 \cdot 3)$

$50 \div 5 - (6 \cdot 3)$
$= 50 \div 5 - 18$
$= 10 - 18$
$= -8$

2) $22 - 4 \cdot (8 - 4)$

$22 - 4 \cdot (8 - 4)$
$= 22 - 4 \cdot 4$
$= 22 - 16$
$= 6$

Common error
Students might try to follow the order multiplication/division, then addition/subtraction rigidly, ignoring any parentheses.

The order of operations must be followed inside any parentheses before performing the remainder of the calculation. This may mean that some additions/subtractions need to be done before multiplications/divisions.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to make up a multiple-choice problem using at least three different operations. They should provide three wrong answers and one correct answer. The wrong answers should be ones you would get if you did not use the correct order of operations. They should then exchange problems with a partner and solve each other's. The students should identify the order of operations error that would result in each incorrect answer.

2 Teach (cont)

Guided practice

Level 1: q15–18
Level 2: q15–19
Level 3: q15–20

Additional examples

Let $n = -3$ and $k = 6$.
Evaluate the expressions below.

1. $12 + nk - 3$

$= 12 + (-3) \bullet 6 - 3 = 12 + (-18) - 3$
$= -6 - 3 = -9$

2. $n(k + 4) + 16$

$= -3 \bullet (6 + 4) + 16 = -3 \bullet 10 + 16$
$= -30 + 16 = -14$

3. $2 \bullet (n + k) + k \bullet 2$

$= 2 \bullet (-3 + 6) + 6 \bullet 2 = 2 \bullet 3 + 6 \bullet 2$
$= 6 + 6 \bullet 2 = 6 + 12 = 18$

Common error

Students are likely to make errors if they try to perform more than one step in a calculation at a time.

Encourage them to work in the "funnel method" when calculating. Each step is written out vertically until the expression is funneled down to one result. For example:

$-3 \bullet (6 + 4) + 16$
$= -3 \bullet 10 + 16$
$= -30 + 16$
$= -14$

Independent practice

Level 1: q1, 4–7, 10–11
Level 2: q1–7, 10–12
Level 3: q1–12

Additional questions

Level 1: p430 q1–4
Level 2: p430 q1–6
Level 3: p430 q1–9

3 Homework

Homework Book — Lesson 1.1.3

Level 1: q1, 2, 4a–c, 5, 10
Level 2: q2–8
Level 3: q3–9, 11

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
• Worksheet 20 — Variables and Expressions
• Worksheet 21 — Order of Operations

Guided Practice

Simplify the expressions in Exercises 15–20 as far as possible.

15. $5 + 7 \bullet x$ **$5 + 7x$**
16. $2 + a \bullet 4 - 1$ **$4a + 1$**
17. $3 \bullet (y - 2)$ **$3y - 6$**
18. $10 \div (3 + 2) - r$ **$2 - r$**
19. $20 + (4 \bullet 2) \bullet t$ **$8t + 20$**
20. $p + 5 \bullet (-2 + m)$ **$p - 10 + 5m$**

Independent Practice

1. Alice and Emilio are evaluating the expression $5 + 6 \bullet 4$.
Their work is shown below.

Alice	Emilio
$5 + 6 \bullet 4$	$5 + 6 \bullet 4$
$= 11 \bullet 4$	$= 5 + 24$
$= 44$	$= 29$

Emilio has the right answer because he has used the correct order of operations: he has done the multiplication before the addition.

Explain who has the right answer.

Now try these:
Lesson 1.1.3 additional questions — p430

The local muffler replacement shop charges \$75 for parts and \$25 per hour for labor.
2. Write an expression with parentheses to describe the cost, in dollars, of a replacement if the job takes 4 hours. **$75 + (4 \bullet 25)$**
3. Use your expression to calculate what the cost of the job would be if it did take 4 hours. **\$175**

Evaluate the expressions in Exercises 4–7.

4. $2 + 32 \div 8 - 2 \bullet 5$ **-4**
5. $4 + 7 \bullet 3$ **25**
6. $7 + 5 \bullet (10 - 6 \div 3)$ **47**
7. $3 \bullet (5 - 3) + (27 \div 3)$ **15**

8. Paul buys 5 books priced at \$10 and 3 priced at \$15. He also has a coupon for \$7 off his purchase. Write an expression with parentheses to show the total cost, after using the coupon, and then simplify it to show how much he spent. **$(5 \bullet 10) + (3 \bullet 15) - 7$. He spent \$88.**

9. Insert parentheses into the expression $15 + 3 - 6 \bullet 4$ to make it equal to 48. **$(15 + 3 - 6) \bullet 4$**

Simplify the expressions in Exercises 10–12 as far as possible.

10. $x - 7 \bullet 2$ **$x - 14$**
11. $y + x \bullet (4 + 3) - y$ **$7x$**
12. $6 + (60 - x \bullet 3)$ **$66 - 3x$**

Round Up

If you evaluate an expression in a different order from everyone else, you won't get the right answer. That's why it's so important to follow the order of operations. This will feature in almost all the math you do from now on, so you need to know it. Don't worry though — just use the word PEMDAS or GEMA to help you remember it.

Solutions

For worked solutions see the Solution Guide

Lesson 1.1.4

The Identity and Inverse Properties

This Lesson formally introduces students to the identity and inverse properties of addition and multiplication. Students are then shown how knowledge of these properties allows them to justify some steps in their work.

Previous Study: In grade 6, students were taught to cross-multiply by multiplying both sides of an equation by a multiplicative inverse.

Future Study: In Chapter 2, students will use the idea of multiplicative inverses to divide by fractions.

Lesson 1.1.4

The Identity and Inverse Properties

California Standards:
Algebra and Functions 1.3
Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) **and justify the process used.**

What it means for you:
You'll learn how to use math properties to show why the steps of your work are reasonable.

Key words:
- justify
- identity
- inverse
- reciprocal
- multiplicative
- additive

When you're simplifying and evaluating expressions you need to be able to justify your work. To justify it means to use known math properties to explain why each step of your calculation is valid.

The math properties describe the ways that numbers and variables in expressions behave — you need to know their names so that you can say which one you're using for each step.

The Identity Doesn't Change the Number

There are two identity properties — one property for **addition** and one property for **multiplication**:

> **The Additive Identity = 0**
> For any number, a, $a + 0 = a$.

Adding **0** to a number **doesn't change it**. For example:

$5 + 0 = 5$
$x + 0 = x$

This is called the **identity property of addition**, and **0** is called the **additive identity**.

> **The Multiplicative Identity = 1**
> For any number, a, $a \bullet 1 = a$.

Don't forget:
Writing x is exactly the same as writing $1x$ or $1 \bullet x$.

Multiplying a number by **1 doesn't change it**. For example:

$1 \bullet 7 = 7$
$1 \bullet x = x$

This is called the **identity property of multiplication**, and **1** is called the **multiplicative identity**.

Guided Practice

1. What do you get by multiplying 6 by the multiplicative identity? 6

2. What do you get by adding the additive identity to $3y$? $3y$

Complete the expressions in Exercises 3–6.

3. $x + \_\_ = x$ $x + 0 = x$

4. $\_\_ + 0 = h$ $h + 0 = h$

5. $k \bullet \_\_ = k$ $k \bullet 1 = k$

6. $1 \bullet \_\_ = t$ $1 \bullet t = t$

1 Get started

Resources:
Teacher Resources CD-ROM
- Number Line

Warm-up questions:
- Lesson 1.1.4 sheet

2 Teach

Universal access

Introduce the idea of identities by writing part of an equation on the board:

x ______ $= x$

Ask the students to suggest what could go in the space so that the equation is true.

Additional examples

1) What do you get by multiplying 0.3 by the multiplicative identity?
0.3

2) What do you get when adding the additive identity to 0.3?
0.3

Complete the expressions below:

3) $-x + \_\_\_ = -x$
$-x + 0 = -x$

4) $-h \bullet \_\_\_ = -h$
$-h \bullet 1 = -h$

Guided practice

Level 1: q1–6
Level 2: q1–6
Level 3: q1–6

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
The number line approach described in the Universal access section below could be used to introduce the concept of additive inverses. Circle diagrams could be used to introduce multiplicative inverses — for instance, what must one-quarter of a circle be multiplied by to get one full circle? Also, what must four full circles be multiplied by to get one circle?

- **English Language Learners**
Help students understand what is meant by the term "justifying" by asking them to divide a piece of paper in two lengthwise. Then they should list four activities they did after school yesterday on the left side, such as eating dinner. On the right side, next to each activity, they should write a reason/**justification** for doing the activity.

2 Teach (cont)

Universal access

Use a number line to model the inverse property of addition.

Find the original number on the number line. Then count how many steps you need to take to get back to zero. If you have to move right, you're adding a positive number, so the additive inverse is positive. Moving left means the additive inverse is negative.

Additional examples

What is the additive inverse of the following?

1) $-h$ **h**
2) h **$-h$**
3) 0.5 **–0.5**
4) –0.5 **0.5**

Guided practice

Level 1: q7–10
Level 2: q7–12
Level 3: q7–12

Concept question

"Say whether the following statements are true or false:
1) The multiplicative identity is the same for every number. **True**
2) The additive inverse is the same for every number. **False**
3) The additive identity is the same for every number. **True**
4) The multiplicative inverse is the same for every number." **False**

Concept question

"Does zero have both an additive inverse and a multiplicative inverse?"

Zero has an additive inverse (which is 0, because 0 + 0 = 0).
Zero does not a multiplicative inverse. (Whatever you multiply zero by, you get 0. Also, you can't divide by zero — it's undefined.)

The Inverse Changes the Number to the Identity

There are two inverse properties — one for **addition** and one for **multiplication**. Different numbers have different **additive inverses** and different **multiplicative inverses**.

The Additive Inverse Adds to Give 0

The **additive inverse** is what you add to a number to get **0**.

Don't forget:
Remember — adding a negative number can be rewritten as a subtraction:
2 + –2 = 2 – 2 = 0

The additive inverse of 2 is –2 ⇨ $2 + -2 = 0$

The additive inverse of –3 is 3 ⇨ $-3 + 3 = 0$

The additive inverse of $\frac{1}{4}$ is $-\frac{1}{4}$ ⇨ $\frac{1}{4} + -\frac{1}{4} = 0$

The Additive Inverse of *a* is –*a*.
For any number, a, $a + -a = 0$.

Guided Practice

Give the additive inverses of the numbers in Exercises 7–12.

7. 6 **–6**
8. 19 **–19**
9. –5 **5**
10. –165 **165**
11. $\frac{1}{7}$ **$-\frac{1}{7}$**
12. $-\frac{2}{3}$ **$\frac{2}{3}$**

The Multiplicative Inverse Multiplies to Give 1

The **multiplicative inverse** is what you multiply a number by to get **1**. So, a number's **multiplicative inverse** is **one divided by the number**.

Don't forget:
A multiplicative inverse is sometimes called a reciprocal.

The multiplicative inverse of 2 is $1 \div 2 = \frac{1}{2}$.

The multiplicative inverse of 7 is $1 \div 7 = \frac{1}{7}$.

To check, $2 \cdot \frac{1}{2} = \frac{2 \cdot 1}{2} = \frac{2}{2} = 1$ and $7 \cdot \frac{1}{7} = \frac{7 \cdot 1}{7} = \frac{7}{7} = 1$.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to create and evaluate their own expressions, using the inverse and identity properties of multiplication and addition during their evaluation. They could then exchange expressions with a partner, and identify where the inverse and identity properties have been used.

The Multiplicative Inverse of a is $\frac{1}{a}$.

For any nonzero number, a, $a \cdot \frac{1}{a} = 1$.

Guided Practice

Give the multiplicative inverses of the numbers in Exercises 13–16.

13. 2 $\frac{1}{2}$ **14.** 10 $\frac{1}{10}$

15. –4 $-\frac{1}{4}$ **16.** –5 $-\frac{1}{5}$

Fractions Have Multiplicative Inverses Too

When you multiply two **fractions** together, you multiply their **numerators** and their **denominators** separately.

For example : $\frac{1}{2} \cdot \frac{3}{4} = \frac{1 \cdot 3}{2 \cdot 4} = \frac{3}{8}$

If you **multiply** a **fraction** by its **multiplicative inverse**, the product will be **1** — because that's the definition of a multiplicative inverse.

For a fraction to equal 1, the numerator and denominator must be **the same**. So when a fraction is multiplied by its inverse, the product of the numerators must be the same as the product of the denominators.

For example: $\frac{3}{4} \cdot \frac{4}{3} = \frac{3 \cdot 4}{4 \cdot 3} = \frac{12}{12} = 1$

You can say that for any two non-zero numbers a and b, $\frac{a}{b} \cdot \frac{b}{a} = \frac{ab}{ab} = 1$.

So, the multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$.

The **multiplicative inverse**, or **reciprocal**, of a **fraction** is just the fraction **turned upside down**.

Example 1

Give the multiplicative inverse of $\frac{1}{4}$.

Solution

$\frac{4}{1}$, or 4, is the **multiplicative inverse** of $\frac{1}{4}$.

To check your answer **multiply** it by $\frac{1}{4}$.

$\frac{1}{4} \cdot \frac{4}{1} = \frac{1 \cdot 4}{4 \cdot 1} = \frac{4}{4} = 1$.

So 4 is the multiplicative inverse of $\frac{1}{4}$.

Don't forget:
Any fraction where the numerator is the same as the denominator is equal to 1.
So,
$\frac{12}{12} = \frac{9}{9} = \frac{4}{4} = \frac{1}{1} = 1$

Don't forget:
Any number divided by one is equal to itself. So any whole number can be written as a fraction by putting it over 1.

So $6 = \frac{6}{1}$ and $4 = \frac{4}{1}$.

2 Teach (cont)

Additional examples
Give the multiplicative inverse of:
1) 1
1

2) $-x$
$-\frac{1}{x}$

3) 0.25
4

4) –0.2
–5

Guided practice
Level 1: q13–16
Level 2: q13–16
Level 3: q13–16

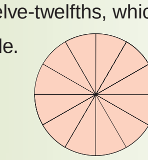
Math background
Multiplying a fraction by a number is the same as finding a fraction of that number.

So $\frac{3}{4} \cdot \frac{4}{3}$ is $\frac{3}{4}$ of $\frac{4}{3}$.

To get 1, it makes sense that one of the fractions being multiplied must be greater than 1 (an improper fraction) and the other must be less than 1 (a proper fraction).

Universal access
Show using diagrams that fractions with the same numerator and denominator are equal to 1.

$\frac{12}{12}$ is twelve-twelfths, which is one-whole.



Additional examples
Complete each expression to represent the property shown.

1) –12 + ___ = 0 Inverse
–12 + 12 = 0

2) –12 + ___ = –12 Identity
–12 + 0 = –12

3) –12 • ___ = 1 Inverse
$-12 \cdot -\frac{1}{12} = 1$

4) –12 • ___ = –12 Identity
–12 • 1 = –12

Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Guided practice
Level 1: q17–20
Level 2: q17–20
Level 3: q17–20

Additional example
Simplify the expression
$8(4 + (m \cdot 1)) + 0$. Justify your work.

$8(4 + m \cdot 1) + 0$
$= 8(4 + m) + 0$
The identity property of multiplication
$= 32 + 8m + 0$
The distributive property
$= 32 + 8m$
The identity property of addition

Guided practice
Level 1: q21–22
Level 2: q21–24
Level 3: q21–24

Independent practice
Level 1: q1–6, 9–10
Level 2: q1–10
Level 3: q1–12

Additional questions
Level 1: p431 q1–5
Level 2: p431 q1–8, 11–12
Level 3: p431 q1–14

3 Homework

Homework Book — Lesson 1.1.4
Level 1: q1–5,
Level 2: q1, 2, 5, 7, 9, 10
Level 3: q1, 2, 5, 7–10

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 21 — Order of Operations
- Worksheet 22 — Distributive Property

Guided Practice

Give the multiplicative inverses of the fractions in Exercises 17–20.

17. $\frac{2}{5}$ $\frac{5}{2}$ **18.** $\frac{1}{10}$ $\frac{10}{1}$ or 10.

19. $-\frac{3}{4}$ $-\frac{4}{3}$ **20.** $-\frac{1}{2}$ $-\frac{2}{1}$ or –2.

You Can Use Math Properties to Justify Your Work

To **justify** your work you need to use **known math properties** to **explain** why each step of your calculation is **valid**.

Check it out:
To justify a step of your work you can write beside it which of the known math properties you have used.

Example 2

Simplify the expression $4(2 - \frac{1}{4}x)$. Justify your work.

Solution

$4(2 - \frac{1}{4}x)$
$= 4 \cdot 2 - 4 \cdot \frac{1}{4}x$ — The distributive property
$= 8 - 1x$ — The inverse property of multiplication
$= \mathbf{8 - x}$ — The identity property of multiplication

Guided Practice

Simplify the expressions in Exercises 21–24. Justify your work.

21. $m \cdot 1 + 6$ $m + 6$ **22.** $d + 0 - d + 9$ 9

23. $\frac{1}{3}(9 + 3f)$ $3 + f$ **24.** $-5(2 - 4 \cdot \frac{1}{4}a) + 10$ $5a$

Independent Practice

Complete the expressions in Exercises 1–4.

1. $1 \cdot \_\_ = 5$ 5 **2.** $\_\_ + 0 = -2$ –2
3. $2.5 \cdot \_\_ = 2.5$ 1 **4.** $-0.5 + \_\_ = -0.5$ 0

Give the additive and multiplicative inverses of the numbers in Exercises 5–8.

5. 6 $-6, \frac{1}{6}$ **6.** –7 $7, -\frac{1}{7}$
7. $\frac{5}{7}$ $-\frac{5}{7}, \frac{7}{5}$ **8.** $-\frac{2}{3}$ $\frac{2}{3}, -\frac{3}{2}$

Simplify the expressions in Exercises 9–12. Justify your work.

9. $a + a - a$ a **10.** $\frac{1}{4} \cdot 4 \cdot d$ d
11. $5(\frac{1}{5} + 2 - n)$ $11 - 5n$ **12.** $2(\frac{1}{2}x + 4 + 0) - 1(5 - 5 + 7)$ $x + 1$

Now try these:
Lesson 1.1.4 additional questions — p431

Round Up

The identity property and the inverse property are two math properties you'll need to use in justifying your work. Justifying your work is explaining how you know that each step is right. In the next Lesson you'll cover two more properties that can be used in justifying your work.

Solutions
For worked solutions see the Solution Guide

Lesson 1.1.5

The Associative and Commutative Properties

This Lesson formally introduces the associative and commutative properties of multiplication and addition. Students are given the opportunity to use these properties, together with those from previous Lessons, to simplify expressions and justify their work.

Previous Study: In earlier grades, students have used the commutative and associative properties for the addition and subtraction of whole numbers and for justifying work.

Future Study: Students will continue to use the abstract way of thinking developed in this Lesson to help make sense of complex situations through later grades.

Lesson 1.1.5

The Associative and Commutative Properties

California Standards:
Algebra and Functions 1.3
Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

What it means for you:
You'll learn about some more math properties that will help you to justify your work.

Key words:
- associative property
- commutative property
- justify

There are two more properties you need to know about to help simplify and evaluate expressions. They're the associative and commutative properties. They allow you to be a little more flexible about the order you do calculations in.

You used these properties already in earlier grades. It's important to know their names and to practice using them to justify your work.

The Associative Properties

If you **change** the way that you group numbers and variables in a **multiplication or addition** expression, you **won't change the answer**.

$$7 + (4 + 2) = (7 + 4) + 2 = 13$$
$$4 \bullet (y \bullet 5) = (4 \bullet y) \bullet 5 = 20y$$

The numbers and variables don't move — only the parentheses do.

These are the **associative properties of addition and multiplication**. In math language they are:

The Associative Properties

Addition:	$(a + b) + c = a + (b + c)$
Multiplication:	$(ab)c = a(bc)$

Sometimes changing the grouping in an expression using the associative property allows you to simplify it.

Example 1

Simplify the expression $(h + 12) + 13$ using the associative property.

Solution

$(h + 12) + 13$
$= h + (12 + 13)$ — **The associative property of addition**
$= h + 25$ — **Do the addition**

Example 2

Simplify the expression $15(10y)$ using the associative property.

Solution

$15 \bullet (10 \bullet y)$
$= (15 \bullet 10) \bullet y$ — **The associative property of multiplication**
$= 150y$ — **Do the multiplication**

1 Get started

Resources:
- small objects such as buttons or cubes

Warm-up questions:
- Lesson 1.1.5 sheet

2 Teach

Universal access
Discuss the idea that when you have several numbers to add, there's often one particular order that's easiest to add them in.
For example, when doing 7 + 6 + 19, it's easier to do
7 + (6 + 19) = 7 + 25 = 32, than to do (7 + 6) + 19.
It's the associative rule than lets you group the numbers in the most convenient order.

Multiplication works the same.
For instance 8•6•5 might be more easily worked out as 8•(6•5) than (8•6)•5.

Common error
Students often move the numbers as well as the parentheses when illustrating the associative property. The numbers stay in position while the groups change.

- **Strategic Learners**
  Use counters to model the associative and commutative properties of addition, and use rectangular arrays to illustrate the associative and commutative properties of multiplication. Model **the format** you want them to use in their notebooks when they justify problems.

- **English Language Learners**
  As you increase the complexity of the problems, be sure English language learners have the vocabulary to write the justifications. Use language buddies or groups to help those who need support. Review and use **hand signals** for **agree**, **disagree**, **partially agree**, and **don't understand** while modeling the justification of work for the class.

2 Teach (cont)

Guided practice
Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Universal access
Use rectangular arrays of objects, such as buttons, to illustrate multiplication facts, for instance 3 × 5. Rotate the array through 90° and discuss whether this illustrates the same multiplication fact. This should lead to students understanding that numbers can be multiplied in either order.

Additional example
Simplify this expression, justifying your work.
$(3 \cdot k \cdot 2) + 3p + (2 \cdot k \cdot 4)$

$= (3 \cdot 2 \cdot k) + 3p + (2 \cdot 4 \cdot k)$ Commutative property of multiplication (twice)
$= 6k + 3p + 8k$ Simplification
$= 6k + 8k + 3p$ Commutative property of addition
$= (6k + 8k) + 3p$ Associative property of addition
$= 14k + 3p$ Simplification

Concept question
"Explain and give an example of why there isn't a commutative property of division."

You can't change the order in division. The following example shows why it doesn't work:
10 ÷ 2 = 5, and, 2 ÷ 10 = 0.2

Universal access
Students could remember the commutative property by thinking of two numbers "commuting," or changing places.

The associative property can be remembered by thinking of two numbers "associating," then one leaving to "associate" with another.

Guided practice
Level 1: q7–10
Level 2: q7–11
Level 3: q7–12

Guided Practice

Simplify the expressions in Exercises 1–6. Use the associative property.

1. $10 + (15 + k)$ **$25 + k$**   **2.** $5(7d)$ **$35d$**   **3.** $(x + 5) + 7$ **$x + 12$**
4. $-3(4f)$ **$-12f$**   **5.** $(y + 13) + (20 + m)$ **$y + 33 + m$**   **6.** $0.5(3p)$ **$1.5p$**

The Commutative Properties

When you're adding numbers together it doesn't matter what order you add them in — the answer is always the same.

For example: $\boxed{10} + \boxed{14} = \mathbf{24}$ and $\boxed{14} + \boxed{10} = \mathbf{24}$

Also, when you're multiplying numbers it doesn't matter what order you multiply them in — the answer is always the same.

For example: $\boxed{4} \cdot \boxed{x} \cdot \boxed{2} = \mathbf{8x}$ and $\boxed{2} \cdot \boxed{4} \cdot \boxed{x} = \mathbf{8x}$

The numbers and variables move around, but the answer doesn't change — these are the **commutative properties**. Algebraically they're written as:

The Commutative Properties

Addition:	$a + b = b + a$
Multiplication:	$ab = ba$

Check it out:
You can't use the associative or commutative properties on a subtraction or a division problem. But you could use the inverse property to change it into an addition or multiplication problem first. Subtraction is the same as adding a negative number, and division is the same as multiplying by a fraction.
So $x - y = x + -y$
and $x \div y = x \cdot \frac{1}{y}$.
When they're expressed in this way, you can use the associative and commutative properties.

Example 3

Simplify the expression $18v + 9 + 2v + 4$. Justify your work.

Solution

$18v + 9 + 2v + 4$
$= 18v + 2v + 9 + 4$ **The commutative property of addition**
$= (18v + 2v) + (9 + 4)$ **The associative property of addition**
$= \mathbf{20v + 13}$ **Do the additions**

Example 4

Simplify the expression $4 \cdot n \cdot 9$ using the commutative property.

Solution

$4 \cdot n \cdot 9$
$= 4 \cdot 9 \cdot n$ **The commutative property of multiplication**
$= (4 \cdot 9) \cdot n$ **The associative property of multiplication**
$= \mathbf{36n}$ **Do the multiplications**

Guided Practice

Simplify the expressions in Exercises 7–12.

7. $7 + j + 15$ **$22 + j$**   **8.** $11 \cdot x \cdot 20$ **$220x$**
9. $3 + 4t + 7 + t$ **$10 + 5t$**   **10.** $3 \cdot -y \cdot 4$ **$-12y$**
11. $5 + q + 3 + -q$ **8**   **12.** $-2 \cdot f \cdot -3$ **$6f$**

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Even advanced students may forget the names of these properties. To help the students remember them, ask them to make up a rhyme or mnemonic describing the properties that they have met over the last few Lessons. Their rhyme/mnemonic should describe the effect of each property. The rhymes/mnemonics can be shared with the class and used as memory aids.

You Can Use the Properties to Justify Your Work

Check it out:
To justify your work means to use a math property to explain why each step of your calculation is valid.

You can use **all the properties** together to **justify** the work you do when solving a math problem.

Example 5

Simplify the expression $3(x + 5 + 2x)$. Justify your work.

Solution

$3(x + 5 + 2x)$
$= 3x + 15 + 6x$ — **The distributive property**
$= 3x + 6x + 15$ — **The commutative property of addition**
$= \mathbf{9x + 15}$ — **Collect like terms**

Don't forget:
The math properties that you've seen in the last two Lessons are:
- the distributive property and the
- associative,
- commutative,
- identity, and
- inverse

properties of multiplication and addition.

And you've also seen how to collect like terms.

Example 6

Simplify the expression $(x \bullet y) \bullet \frac{1}{y}$. Justify your work.

Solution

$(x \bullet y) \bullet \frac{1}{y}$
$= x \bullet (y \bullet \frac{1}{y})$ — **The associative property of multiplication**
$= x \bullet 1$ — **The multiplicative inverse property**
$= \mathbf{x}$ — **The identity property of multiplication**

Guided Practice

Simplify the expressions in Exercises 13–16. Justify your work.

13. $2(\frac{1}{2}h)$ ***h***
14. $(c + -2) + 2$ ***c***
15. $5(1 + t + 3 + 2t)$ **20 + 15*t***
16. $p \bullet 5 \bullet \frac{1}{p} \bullet 2$ **10**

Independent Practice

Simplify the expressions below using the associative properties.

1. $55 + (7 + z)$ **62 + *z***
2. $6 \bullet (10 \bullet t)$ **60*t***
3. $-3(7k)$ **–21*k***
4. $(-22 + q) + (q + 30)$ **8 + 2*q***

Simplify the expressions below. Use the commutative properties.

5. $7 + f + 3 + f$ **10 + 2*f***
6. $9 \bullet y \bullet 4$ **36*y***
7. $2 + a + 18 + b$ **20 + *a* + *b***
8. $-3 \bullet c \bullet 4 \bullet -h$ **12*ch***

Now try these:
Lesson 1.1.5 additional questions — p431

Simplify the expressions in Exercises 9–12. Justify your work.

9. $-5 + m + 5$ ***m***
10. $(r \bullet \frac{1}{2}) \bullet 2$ ***r***
11. $2(x + 5 + -x)$ **10**
12. $(\frac{1}{p} \bullet 3) \bullet p \bullet 2(4 + p - 4)$ **6*p***

Round Up

The associative and commutative properties are two more math properties. They're all important tools to use when you're simplifying expressions. By saying which property you are using in each step, you can justify your work.

Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Additional example

Identify the properties used in simplifying this expression.

3(5 + 8) + 6
= 3 • 5 + 3 • 8 + 6
= 15 + 24 + 6
= 15 + (24 + 6)
= 15 + 30 = 45

Distributive property: Both numbers in the parentheses were multiplied by three.
Associative property of addition: The order remained the same and parentheses were placed around 24 and 6.

Guided practice

Level 1: q13–14
Level 2: q13–15
Level 3: q13–16

Independent practice

Level 1: q1–2, 5–7, 9–11
Level 2: q1–11
Level 3: q1–12

Additional questions

Level 1: p431 q1–5
Level 2: p431 q1–11
Level 3: p431 q1–14

3 Homework

Homework Book — Lesson 1.1.5

Level 1: q1–7
Level 2: q1–9
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 23 — Associative Property
- Worksheet 24 — Commutative Property

Section 1.2 Exploration — Solving Equations

Purpose of the Exploration

Students often have difficulty with the concept of equality in an equation. This Exploration emphasizes that when something is done to one side of an equation, it must also be done to the other. The exploration also helps students to understand that isolating a variable can allow you to determine its value.

Resources

Teacher Resources CD-ROM

• Algebra Tiles

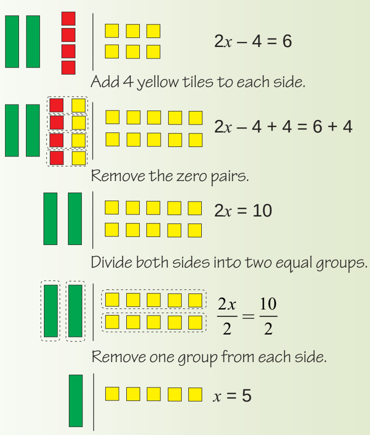
Strategic & EL Learners

Strategic learners may benefit from some simple addition and subtraction problems using the tiles and making zero pairs. They may also have difficulty modeling the problems. If this is the case, set up the equations for students.

The term "vertical" may be unfamiliar to English language learners. The vertical line is used to represent the equals sign.

Universal access

Students can extend this Exploration to model and solve two-step equations, such as $2x - 4 = 6$. Exactly the same methods are used:



Common errors

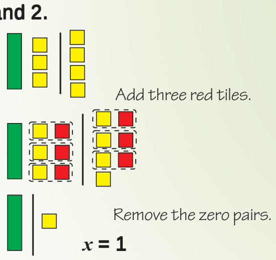
Students may encounter difficulty with the concept of making a zero pair. Remind students that opposites will cancel each other out to make zero.

Another potential problem is dividing into equal groups. Students may want to always divide the tiles into two equal groups. Remind them that the number of green tiles determines the number of groups.

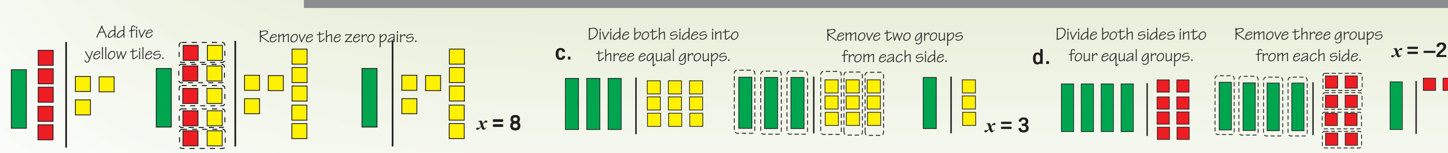
Solutions

1 and 2.

a.



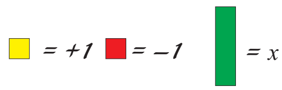
b.



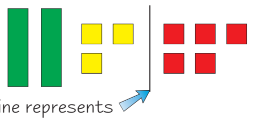
Section 1.2 introduction — an exploration into:

Solving Equations

You can use algebra tiles to model algebra equations. You can also use them to model how you solve equations to find the value of x. The tiles on the right will be used to represent algebra equations:



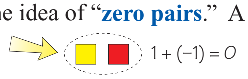
Here is the algebra equation **$2x + 3 = -5$** modeled using algebra tiles:



This vertical line represents the equals sign.

To solve an equation, you need to get **one green "x" tile** by itself on one side of the equation, and only **red and yellow tiles** on the other. This way you'll know the value of **one green "x" tile**.

You'll often need to use the idea of "**zero pairs**." A yellow and a red tile together make **zero**. This is called a **zero pair**.

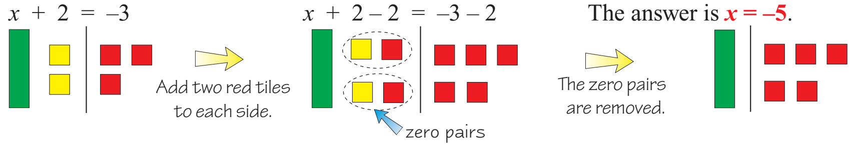


Example

Use algebra tiles to model and solve the equation $x + 2 = -3$.

Solution

$x + 2 = -3$ $\quad\quad$ $x + 2 - 2 = -3 - 2$ $\quad\quad$ The answer is $x = -5$.

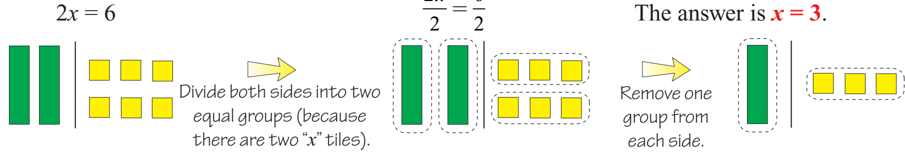


Example

Use algebra tiles to model and solve the equation $2x = 6$.

Solution

$2x = 6$ $\quad\quad$ $\frac{2x}{2} = \frac{6}{2}$ $\quad\quad$ The answer is $x = 3$.



Exercises

1. Use algebra tiles to model these equations.
   a. $x + 3 = 4$ **b.** $x - 5 = 3$ **c.** $3x = 9$ **d.** $4x = -8$ see below
2. Use algebra tiles to solve the equations above. see below

Round Up

When you solve an equation, the aim is always to get the variable on its own, on one side of the equation. In this Exploration, the variable was the green tile "x." To get it on its own, you have to do exactly the same to both sides of the equation — or else the equation won't stay balanced.

Lesson 1.2.1

Writing Expressions

In this Lesson, students write math expressions to represent verbal descriptions and word problems. The Lesson covers both one- and two-step expressions, and requires students to use their knowledge of the order of operations to write unambiguous expressions.

Previous Study: In earlier grades, and in Section 1.1, students developed an understanding of how variables are used to represent unknown values.

Future Study: Later in this Section, students will use these skills to write variable equations in order to solve real-life word problems.

Lesson 1.2.1

California Standards:

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities **that represents a verbal description** (e.g., three less than a number, half as large as area A).

What it means for you:

You'll learn how to change word descriptions into math expressions.

Key words:

- addition
- subtraction
- product
- quotient

Don't forget:

There are lots of different phrases that describe the four operations. You could come across any of them, so you need to remember which of the operations each phrase refers to. For a reminder, see Lesson 1.1.1.

Section 1.2
Writing Expressions

You can use both a word expression and a math expression to describe the same situation. In Section 1.1 you practiced changing numeric and variable expressions into word expressions. This Lesson is all about doing the reverse — changing word expressions into math expressions.

Variable Expressions Describe Word Expressions

To write an **expression** to represent a **word sentence** you need to figure out what the word sentence actually **means**.

Example 1

Write a variable expression to describe the sentence
"A number, x, is increased by five."

Solution

The phrase "a number, x" is telling you that x is the **variable** being used. The words "**increased by**" are telling you that something is being **added on** to the variable x. In this case it is the number **five**.

So the sentence "A number, x, is increased by five" translates as $x + 5$.

Example 2

Write a variable expression to describe the sentence
"Nine is multiplied by a number, k."

Solution

The operation phrase being used is "**multiplied by**." The rest of the sentence tells you that it is the number **nine** and the variable k that are being multiplied together.

So "Nine is multiplied by a number, k" translates as $9 \bullet k$ or $9k$.

Guided Practice

Write variable expressions to describe the phrases in Exercises 1–5.

1. Six more than a number, h. $h + 6$
2. Seven is decreased by a number, m. $7 - m$
3. A number, g, divided by 11. $\frac{g}{11}$
4. The product of a number, w, and 10. $w \bullet 10$
5. A number, k, divided into four equal parts. $\frac{k}{4}$

1 Get started

Resources:
- blank index cards

Teacher Resources CD-ROM
- Number and Operator Tiles

Warm-up questions:
- Lesson 1.2.1 sheet

2 Teach

Universal access

Provide students with a set of Number and Operator Tiles from the **Teacher Resources CD-ROM**. Students can match the cards to chunks of the word expression, then arrange the cards in the correct order.

Common error

Students may try to translate the word expression from left to right directly — this often won't work. Expressions with the words "sum," "difference," "product," "quotient," and "less" than cannot be directly translated left to right.
The numbers or variables need to be correctly positioned on either side of the operation sign.

Additional examples

Write variable expressions to describe the following phrases.
1) Four subtracted from a number, k.
$k - 4$
2) The sum of four and a number, k.
$4 + k$
3) The product of 4 and a number, k.
$4k$

Guided practice

Level 1: q1–5
Level 2: q1–5
Level 3: q1–5

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Have partners make a set of **math expression** index cards. One writes the **word expressions** (such as, "the difference between six and two"), and the other writes the **translations** ("6 – 2"). Supply a list of variable expressions to ensure they cover essential vocabulary. These can be used for the games described in the Universal access sections below and on the previous page.

- **English Language Learners**
  Provide a bank of operation words, such as "multiply" and "sum." Students should add their own examples, which can be used to check their understanding of the terms.

2 Teach (cont)

Universal access

Students can use cards they have created (see the Strategic Learners section above) to play games such as Pelmanism (matching pairs from memory).

Another game that can be played involves pairs of students dealing out six cards to each player, leaving the rest face down in a pile. Students then take turns choosing a card from their partner's hand and trying to make pairs of matching expressions and translations. If they can't make a pair, they have to take a card from the face-down pile. The winner is the student with the most pairs once all the cards have been played.

Additional example

There are two shelves. The top shelf contains three fewer mugs than the bottom shelf. Write an expression for the number of mugs contained on both shelves.

$t + (t - 3)$
$= 2t - 3$, where t is the number of mugs on the bottom shelf.

Guided practice

Level 1: q6–7
Level 2: q6–9
Level 3: q6–9

You Need to Sort Out the Important Information

You'll often need to write a **math expression** as the first step toward solving a **word problem**. That might include **choosing variables** as well as working out what **operations** the words are describing.

Example 3

Carla and Bob have been making buttons to sell at a fund-raiser. Carla made four more than Bob. Write an expression to describe how many buttons they made between them.

Solution

First you need to work out what the expression you have to write must describe. In this case the expression must describe the **total number of buttons made by Carla and Bob**.

See if there is an unknown number in the question: **you don't know how many buttons Bob made**. You don't know how many Carla made either, but you can say how many she made compared with Bob, so you only need one variable.

Assign a letter or symbol to the unknown number: **let b = the number of buttons Bob made**.

Then you need to identify any operation phrases: **"more than" is an addition phrase**.

Carla made four more buttons than Bob, so she made $b + 4$ buttons. Which means that together they made $b + 4 + b$ buttons.

This expression can be simplified to $2b + 4$.

Don't forget:
To simplify this expression you can collect like terms, as you did in Lesson 1.1.2.
If you collect the two b terms together you get $2b$.

Guided Practice

Write variable expressions to describe the sentences in Exercises 6–9. Use x as the variable in each case, and say what it represents.

6. A rectangle has a length of 2 inches. What is its area? **$2x$, where x is the width.**

7. Jenny has five fewer apples than Jamal. How many apples does Jenny have in total? **$x - 5$, where x is the number of apples Jamal has.**

8. The student council is selling fruit juice at the prom for \$0.75 a glass. How much money will they take? **\$0.75x, where x is the number of glasses sold.**

9. A gym charges \$10 per month membership plus \$3 per visit. What is the cost of using the gym for a month? **\$(10 + 3x), where x is the number of visits.**

Some Expressions Describe More than One Operation

You can also translate sentences with **multiple operations** in the same way. You just need to spot all the separate operations and work out **what order** to write them in.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to devise word problems where one amount can be given by writing an expression in terms of another. The word problem in Example 3 can be used as a model for this.

Example 4

Write a variable expression to describe the phrase "ten decreased by the product of a number, y, and two."

Solution

In this question the phrase contains **two different operation phrases**, so you need to work out which operation is carried out **first**.

The two operations here are "**decreased by**," which is a **subtraction** phrase, and "**product**," which is a **multiplication** phrase. You're told to subtract the product from 10. So you need to work out the **product** first — this is the product of y and **2**, which is $2y$.

Now you have to subtract this product, $2y$, from **10**.

So the phrase "ten decreased by the product of a number, y, and 2." translates as $10 - 2y$.

Guided Practice

Write variable expressions to describe the phrases in Exercises 10–14.

10. Five more than twice a number, q. $2q + 5$

11. Sixteen divided by the sum of a number, m, and 7. $\frac{16}{m+7}$

12. Twenty decreased by a quarter of a number, j. $20 - \frac{j}{4}$

13. The product of 7 and six less than a number, t. $7(t - 6)$

14. The product of a number, k, and the sum of 5 and a number, x. $k(5 + x)$

Don't forget:
You might need to include parentheses in your expression to show which operation needs to be done first.

Independent Practice

Write variable expressions to describe the phrases in Exercises 1–5.

1. The product of six and a number, h. $6h$

2. A number, y, decreased by eleven. $y - 11$

3. A fifteenth of a number, p. $\frac{p}{15}$

4. Nine more than twice a number, w. $2w + 9$

5. Sixteen increased by the product of a number, k, and three. $16 + 3k$

6. A pen costs half as much as a ruler. Write an expression to describe how much the pen costs, using r as the cost of the ruler. $0.5r$ or $\frac{r}{2}$

7. Peter has three fewer cards than Neva. Write an expression to describe how many cards they have together, using c as the number of cards Neva has. $2c - 3$

Now try these:
Lesson 1.2.1 additional questions — p431

Round Up

Changing word expressions into algebra expressions is all about spotting the operation phrases and working out what order the operations need to be written in. Writing expressions is the first step toward writing equations — a skill that you'll use when solving problems later in this Section.

2 Teach (cont)

Additional example

There are n tables in a restaurant. Each table has four chairs around it. There are also six chairs in a waiting area. Write an expression for the total number of chairs.

$4n + 6$

Guided practice

Level 1: q10–12
Level 2: q10–13
Level 3: q10–14

Common error

Students may forget to include parentheses as they write an expression from left to right. They should test that their variable expression matches the word expression by substituting simple numbers and following the order of operations.

Independent practice

Level 1: q1–5
Level 2: q1–6
Level 3: q1–7

Additional questions

Level 1: p431 q1–4, 7–8
Level 2: p431 q1–10
Level 3: p431 q5–13

3 Homework

Homework Book — Lesson 1.2.1

Level 1: q1, 2, 3, 5, 8
Level 2: q1–9
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 20 — Variables and Expressions

Solutions

For worked solutions see the Solution Guide

Lesson 1.2.2

Equations

In this Lesson students are taught that equations are made up of two expressions of equal value. They write two-step variable equations to represent statements and the information in word problems. The Lesson also covers formulas, and how they relate to equations.

Previous Study: In grade 6, students wrote and solved one-step linear equations.

Future Study: Students will solve two-step equations later in this Chapter. In Chapter 4 students will meet systems of equations.

1 Get started

Resources:

- Individual whiteboards
- Models of regular solids

Teacher Resources CD-ROM

- Balance
- Equation Mat

Warm-up questions:

- Lesson 1.2.2 sheet

2 Teach

Universal access

To help students understand that there is a balance between the left and right sides of an equation, use a two-sided balance — either a real one, or the one from the **Teacher Resources CD-ROM**.

Place ten pennies on one side of the scale and three on the other side. Ask: "Does this represent an equation?" No.
"What do we need to add to the three to get an equation?" Seven pennies. This idea can be used as a starting point for writing equations.

Additional examples

Classify these statements as expressions or equations.

1) $4x$
Expression

2) m
Expression

3) $3c - 1 = 19$
Equation

4) $12 = 4 - y$
Equation

5) $10k + 2x$
Expression

Concept question

"Explain how an expression differs from an equation."

An equation has an equals sign and two sides. An expression is a mathematical phrase with no equals sign.

Lesson 1.2.2

Equations

California Standards:

Algebra and Functions 1.4
Use algebraic terminology (e.g., variable, **equation**, term, coefficient, inequality, **expression**, constant) **correctly.**

What it means for you:
You'll learn what an equation is, and how it's different from an expression.

Key words:

- equation
- expression
- formula

In Lesson 1.1.2 and Lesson 1.2.1 you learned how to write expressions. Writing equations takes writing expressions one step further — equations are made up of two expressions joined by an equals sign.

An Equation Has an Equals Sign

An **equation** is made up of **two expressions** joined together by an **equals sign**. The equals sign is **really important** — it tells you that the expressions on each side of the equation have **exactly the same value**.

$$5x + 8 = 10x - 12$$

Expression 1... ...is equal to... ...Expression 2.

Numeric Equations Contain Only Numbers

Numeric equations contain **only numbers** and **operations**. For example, $3 + 2 = 5$ and $(6 \cdot 4) + 3 = 31 - 4$ are both numeric equations.

If both sides of the equation do have the **same value**, then the equation is said to be **true**, or **balanced**.

Example 1

Prove that $(10 \cdot 4) + 4 = (8 \cdot 11) \div 2$ is a true equation.

Solution

To show that this is a true equation, you need to **evaluate** both sides. Treat them as **two expressions**, and **evaluate** them both according to the **order of operations**.

$(10 \cdot 4) + 4 = (8 \cdot 11) \div 2$ **First simplify the parentheses**

$40 + 4 = 88 \div 2$ **Then complete the work**

$44 = 44$

Both sides equal 44, so the equation is true.

Don't forget:
The order of operations is Parentheses, Exponents, Multiplication and Division, then Addition and Subtraction — PEMDAS.

The **numbers** and **operations** are **different** on the **left-hand** and **right-hand sides** of the equation. But the **value** of both sides is the **same**.

- **Strategic Learners**
  Allow students time to make up their own numeric equations to ensure they understand the concept that both sides must have the same value. Use the Equation Mat from the **Teacher Resources CD-ROM** to emphasize the fact that equations have two halves. Make word problems and equations relevant to student's hobbies and interests in order to capture their attention.

- **English Language Learners**
  Put increasingly complex equations on the overhead, and using whiteboards ask students to list the operations on the left side of the equation, the variables on the right side, the like terms, etc.

Guided Practice

Prove that the equations in Exercises 1–6 are true by evaluating both sides.

1. $4 + 5 = 9 + 0$ **9 = 9**

2. $4 \cdot 5 = 60 \div 3$ **20 = 20**

3. $4 + 2 \cdot 3 = 3 \cdot 2 + 4$ **10 = 10**

4. $6 - 4 \div 2 = 12 \div 2 - 2$ **4 = 4**

5. $8 + 6 \cdot 3 = 2(10 + 3)$ **26 = 26**

6. $20 \div 4 - 5 = 14 \div 2 - 7$ **0 = 0**

Variable Equations Contain Numbers and Variables

Variable equations contain **variables** as well as numbers and operations. There may be variables on **either or both sides** of the equation.

For example, $2x + 2 = 6$ and $3x = 2y$ are both variable equations.

The **same rules** that apply to numeric equations also apply to variable equations. The expressions that make up the two sides of the equation still have to be **equal in value**.

The two equations above are both true if $x = 2$ and $y = 3$.

$$2x + 2 = 6 \qquad 3x = 2y$$
$$2 \cdot 2 + 2 = 6 \qquad 3 \cdot 2 = 2 \cdot 3$$
$$6 = 6 \qquad 6 = 6$$

Don't forget:
x is a variable — that means that it could stand for any number. But this equation is only true when it stands for 2. So $x = 2$ is called a solution of the equation.

Writing Equations Involves Writing Expressions

To **write an equation** you write **two expressions** that have the same value and join them with an **equals sign**. One of the expressions will often just be a number.

Example 2

Write an equation to describe the sentence "Eight increased by the product of a number, k, and two is equal to twenty-four."

Solution

The phrase "**is equal to**" represents the **equals sign**. It also **separates** the **two expressions** that make up the two sides of the **equation**.

One expression is "Eight increased by the product of a number, k, and two." This turns into the expression $8 + 2k$.

The other expression is just a number, **24**.

So the sentence "Eight increased by the product of a number, k, and two is equal to twenty-four" turns into the equation $8 + 2k = 24$.

2 Teach (cont)

Guided practice
Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Additional examples
Are these equations true?

1) $4 + 3 = 9 - 2$
7 = 7 — Yes

2) $12 - 5 + 4 = 3 \cdot 2 + 6$
11 = 12 — No

3) $9 + 5 \cdot 2 = 7(1 + 3)$
19 = 28 — No

4) $12 \div 4 - 3 = 16 \div 8 \cdot 0$
0 = 0 — Yes

Additional examples
Which number makes the equation true?

1) $3 + h = 18$ **h = 15**

2) $14 - k = 5$ **k = 9**

3) $6s = 42$ **s = 7**

4) $35 \div g = 7$ **g = 5**

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Provide students with regular solids, such as rectangular prisms, labeled with the surface area and total edge length (it doesn't matter if these aren't accurate). Then ask students to write equations for the surface area and total edge length using suitable variables.

2 Teach (cont)

Guided practice
Level 1: q7–10
Level 2: q7–11
Level 3: q7–11

Common error
Students often confuse the idea of an expression with an equation. It is critical to have students understand that the equals sign makes the difference. Expressions can be evaluated when we know the value of the variable. Equations can be used to find the value of the variable.

Universal access
Students could use local newspaper advertisements, menus, and price lists to provide subject material to base word problems on. They could then exchange their problems with a partner and solve each other's.

Common error
Students often have difficulty pulling out the important information from word problems. On a copy of the problem, they should practice underlining all the variables and operational keywords. Encourage students to cross out any information that is "filler." Have students explain what they are being asked to find.

Additional example
A rectangular riding ring requires 150 yards of fencing to completely surround it. It has a length of 40 yards and a width of w yards. Write an equation to represent this information.

$150 = 80 + 2w$ $(75 = 40 + w)$

Guided practice
Level 1: q12–14
Level 2: q12–15
Level 3: q12–15

Guided Practice

Write an equation to describe each of the sentences in Exercises 7–11.

7. Five less than a sixth of m is equal to 40. $\frac{m}{6} - 5 = 40$

8. Five more than the product of six and d is equal to ten. $6d + 5 = 10$

9. Four increased by the product of three and t is equal to 40. $3t + 4 = 40$

10. Nine less than the product of six and r is equal to 11. $6r - 9 = 11$

11. Two times y is equal to y divided by four. $2y = \frac{y}{4}$

To Write an Equation, Identify the Key Information

When math problems are described using words, you'll often be given lots of **extra information** as part of the question. You need to be able to extract the **important** information and use it to **set up an equation** — just like you set up an expression.

Check it out:
It doesn't matter exactly what Sarah is selling here, or what she buys with her profit. The important thing is how the numbers in the situation relate to each other — and that's what the equation is describing.

Example 3

Sarah has been selling lemonade. The lemonade cost her \$9 to make, and she sold each glass for \$0.75. She made \$20 profit, which she is going to use to buy a necklace. Write an equation to describe this information. Use x to represent the number of glasses she sold.

Solution

Sarah made \$20 profit. So the price of one glass (\$0.75) multiplied by the number of glasses she sold (x), minus the amount the lemonade cost her to make (\$9), is equal to 20.

So you can write $0.75x - 9 = 20$.

Guided Practice

Write an equation to describe each of the situations in Exercises 12–15.

12. Javier spent \$20 at the gas station. He bought a drink for \$3 and spent the rest, \$$d$, on gas. $d + 3 = 20$

13. Jane is wrapping a parcel. She needs 15 feet of string to tie it up. A roll of string is p feet long. She uses exactly three rolls. $3p = 15$

14. Sam takes 12 sheets of paper to write an essay on. The essay is $2h$ pages long. He has k spare sheets left to put back. $2h + k = 12$

15. A telephone company charges \$0.05 a minute for local calls and \$0.10 a minute for long-distance calls. Asuncion makes one local and one long-distance call. Each call is y minutes long. Her calls cost a total of \$4. $0.05y + 0.10y = 4$

Solutions
For worked solutions see the Solution Guide

A Formula is an Equation That States a Rule

A **formula** is a specific type of **equation** that sets out a **rule** for you. It explains how some **variables** are related to each other. For example:

Area of a rectangle. $A = l \bullet w$ Product of length and width.

Example 4

Write a formula for the perimeter of a square.

Solution

To calculate the **perimeter** of a square you multiply its **side length** by **4**.

Choose variables to use:

Let P = perimeter of the square, and let s = side length.

So the formula becomes $P = 4 \bullet s$.

The formula shows the **relationship** between a **square's side length and perimeter**. The formula works for **any square** at all — if you are given the value of **one** of the variables you can always **calculate the other**.

Check it out:

You can use any letters or symbols you like for the variables. But make sure you use the same ones all through your work, and remember to say what they stand for.

Guided Practice

Use the formula to calculate the missing values in Exercises 16–18.

16. Rectangle area = length • width — length = 4 cm, width = 0.5 cm
Area = 2 cm²

17. Speed = distance ÷ time — distance = 8 miles, time = 2 h
Speed = 4 mi/h

18. Length in cm = 2.54 • (length in in.) — length in in. = 10
Length in cm = 25.4

Independent Practice

1. Which of a) and b) is an expression? Which is an equation? How do you know? a) $2w - 6 = 21$ b) $5c + 3$
a) is an equation, as it has an equals sign, b) is an expression — it has no equals sign.

2. Given that Distance = Speed • Time, calculate the distance traveled when a car goes 55 mi/h for 8 hours. **440 miles**

Write an equation to describe each of the sentences in Exercises 3–5.

3. Six more than x is equal to four. $x + 6 = 4$

4. The product of h and two is equal to 40 decreased by h. $2h = 40 - h$

5. Ten increased by the result of dividing five by t is equal to nine. $10 + \frac{5}{t} = 9$

6. Mike earned $100 working for h hours in a restaurant. He earns $10 an hour, and received $30 in tips. Write an equation using this information. $10 \bullet h + 30 = 100$

Now try these:

Lesson 1.2.2 additional questions — p432

Round Up

The equals sign in an equation is very important — it tells you that both sides of the equation have exactly the same value. In the next Lesson you'll see how to solve an equation that you've written.

2 Teach (cont)

Concept question

"Explain how you know a formula is an equation rather than an expression."

Formulas have an equals sign and two sides — this makes them equations. Expressions are mathematical phrases with no equals sign.

Additional example

A formula is used to convert temperature from degrees Celsius to degrees Fahrenheit :

Temperature in Fahrenheit = $\frac{9}{5} \bullet$ (Temperature in Celsius) + 32

What is the temperature in degrees Fahrenheit when it is 20 °C?

Temperature in Fahrenheit
$= \frac{9}{5} \bullet (20) + 32$
$= 36 + 32$
$= 68$ °F

Guided practice

Level 1: q16–18
Level 2: q16–18
Level 3: q16–18

Independent practice

Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Additional questions

Level 1: p432 q1–7
Level 2: p432 q1–11
Level 3: p432 q1–14

3 Homework

Homework Book — Lesson 1.2.2

Level 1: q1–4, 7
Level 2: q1–5, 7–9
Level 3: q3–10

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 25 — Writing Linear Equations

Solutions

For worked solutions see the Solution Guide

Lesson 1.2.3

Solving One-Step Equations

This Lesson reviews the method for solving one-step linear equations to find the value of a variable. It emphasizes the use of inverse operations to isolate a variable.

Previous Study: In grade 6, students wrote and solved one-step linear equations in one variable.

Future Study: In the next Lesson, students use these skills to solve two-step equations in one variable. They will also use a similar method to solve two-step inequalities in Chapter 4.

1 Get started

Resources:
- Reference books/internet computers
- Counters (in two colors)

Teacher Resources CD-ROM
- Equation Mat

Warm-up questions:
- Lesson 1.2.3 sheet

2 Teach

Common error

Students often hold the belief from previous work in math that the equals sign means "do something."
Use the activity below to emphasize that equations have two sides of the same importance.

Universal access

Use a seesaw analogy to introduce the idea of doing the same to both sides of an equation to keep it balanced:

Equations are like two people sitting on a balanced seesaw. If one person gets off and the other stays on, the balance is gone and one person crashes to the ground.
Likewise, if weight is added to one side and not the other, the balance is lost.

When you solve equations, you have to do the same thing to both sides. If weight is added to one side then the same amount of weight has to be added to the other.

Concept question

"Why don't we add 4 to the expression $x - 4$ to find x?"

Because $x - 4$ is an expression, not an equation. You can't find the value of a variable from an expression.

Lesson 1.2.3

Solving One-Step Equations

California Standards:

Algebra and Functions 4.1

Solve two-step **linear equations** and inequalities **in one variable over the rational numbers,** interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

What it means for you:

You'll learn how to solve an equation to find out the value of an unknown variable.

Key words:
- solve
- isolate
- inverse

Solving an equation containing a variable means finding the value of the variable. It's all about changing the equation around to get the variable on its own.

Do the Same to Both Sides and Equations Stay True

The **equals sign** in an equation tells you that the two sides of the equation are of **exactly equal value**. So if you do the **same thing** to both sides of the equation, like add five or take away three, they will still have the same value as each other.

$$4 + 6 = 9 + 1$$

Add 5 to both sides.

$$4 + 6 + 5 = 9 + 1 + 5$$

Then simplify.

$$15 = 15$$

All three are **balanced equations**.

You Can Use This to Find the Value of a Variable

To get a variable in an equation on its own you need to do the **inverse operation** to the operation that has already been performed on it.

- If a variable has had a number **added** to it, **subtract** the same number from both sides. $+ \rightarrow -$
- If a variable has had a number **subtracted** from it, **add** the same number to both sides. $- \rightarrow +$
- If a variable has been **multiplied** by a number, **divide** both sides by the same number. $\times \rightarrow \div$
- If a variable has been **divided** by a number, **multiply** both sides by the same number. $\div \rightarrow \times$

For example:

$$y - 5 = 33$$

y has had 5 subtracted from it, so add 5 to both sides.

$$y - 5 + 5 = 33 + 5$$

$$y + 0 = 38$$

$$y = 38$$

You've got the variable alone on one side of the equation, so now you know its value.

Don't forget:

This is using the inverse properties of addition and multiplication.

$$a + -a = 0$$

$$a \cdot \frac{1}{a} = 1$$

You came across these in Lesson 1.1.4.

- **Strategic Learners**
  Review opposite/inverse operations. Start with pairs of simple calculations, such as $6 \times 2 = 12$, and $12 \div 2 = 6$, to illustrate how inverse operations "undo" each other. Use the Equation Mat activity in the Universal access section below.

- **English Language Learners**
  Prepare students for solving equations by reviewing words and phrases that mean "solve" — for example, "evaluate," "find the value of the variable," "find x."

Reverse Addition by Subtracting

When a variable has had something added to it, you can **undo the addition** using **subtraction**.

Example 1

Find the value of x when $x + 15 = 45$.

Solution

$x + 15 = 45$

$x + 15 - 15 = 45 - 15$ **Subtract 15 from both sides**

$x = 30$ **Simplify to find x**

Reverse Subtraction by Adding

When a variable has had something taken away from it, you can **undo the subtraction** using **addition**.

Example 2

Find the value of k when $k - 17 = 10$.

Solution

$k - 17 = 10$

$k - 17 + 17 = 10 + 17$ **Add 17 to both sides**

$k = 27$ **Simplify to find k**

Example 3

Find the value of g when $-10 = g - 9$.

Solution

$-10 = g - 9$

$-10 + 9 = g - 9 + 9$ **Add 9 to both sides**

$-1 = g$ **Simplify to find g**

Check it out:
Here the variable is on the right-hand side of the equals sign. But that doesn't matter — as long as it's on its own. You've still found its value by isolating it.

Guided Practice

Find the value of the variable in Exercises 1–8.

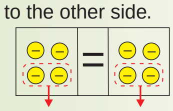
1. $x - 7 = 14$ $x = 21$
2. $70 = t + 41$ $t = 29$
3. $f + 13 = 9$ $f = -4$
4. $g - 3 = -54$ $g = -51$
5. $y - 14 = 30$ $y = 44$
6. $22 = 14 + d$ $d = 8$
7. $4.5 = 9 + v$ $v = -4.5$
8. $-6 = b - 4$ $b = -2$

2 Teach (cont)

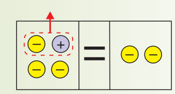
Universal access

Use the Equation Mat and counters of two colors to show how to solve one-step equations. Counters of one color are positive, and counters of the other color are negative.

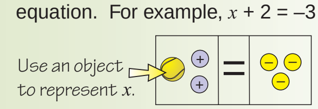
If you add or remove counters from one side, you have to do exactly the same to the other side.



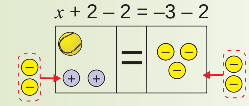
A positive counter pairs with a negative counter on the same side to make zero. You can add or remove these pairs.



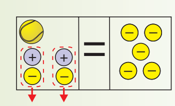
Then use the mat to model solving an equation. For example, $x + 2 = -3$



$x + 2 - 2 = -3 - 2$



$x = -5$



Common error

Students often make an error when the variable is on the right-hand side of the equals sign.

For example, to solve $19 = j + 65$ students will often subtract 19 from both sides. This arises from always seeing the variable on the left-hand side. Demonstrate solving equations with the variable on both sides.

Guided practice

Level 1: q1–4
Level 2: q1–6
Level 3: q1–8

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to use data from reference books, or from the internet, to write their own one-step equations. These can be solved by a partner. (Direct students to Independent Practice Exercises 1, 8, and 22 for inspiration.)

2 Teach (cont)

Additional examples

Solve the equations.

1) $3k = -12$ — $k = -4$

2) $-15v = 45$ — $v = -3$

3) $60 = 12d$ — $d = 5$

4) $-36 = -3w$ — $w = 12$

Concept question

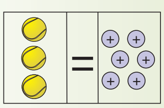
"How does dividing by the number in front of the variable isolate the variable?"

A number divided by itself is 1. 1 multiplied by the variable is equal to the variable itself (by the identity property of multiplication).

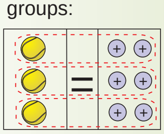
Universal access

Use the Equation Mat to model multiplication in one-step equations.

For example: $3y = 6$



Separate the counters into three equal groups:



$y = 2$

Additional examples

Solve the equations.

1) $\frac{1}{3}a = 30$ — $a = 90$

2) $80 = \frac{4}{5}k$ — $k = 100$

3) $\frac{f}{8} = 10$ — $f = 80$

4) $-\frac{3}{2}h = 15$ — $h = -10$

Reverse Multiplication by Dividing

When a variable in an equation has been multiplied by a number, you can **undo the multiplication** by **dividing** both sides of the equation by the same number.

$$2y = 18$$

y has been multiplied by 2, so start by dividing both sides by 2.

$$2y \div 2 = 18 \div 2$$

Then simplify.

$$y = 9$$

Example 4

Find the value of b when $20b = 100$.

Solution

$20b = 100$

$20b \div 20 = 100 \div 20$ — **Divide both sides by 20**

$b = 5$ — **Simplify to find b**

Reverse Division by Multiplying

When a variable in an equation has been divided by a number, you can **undo the division** by **multiplying** both sides of the equation by the same number.

$$\frac{d}{2} = 50$$

d has been divided by 2, so start by multiplying both sides by 2.

$$\frac{d}{2} \cdot 2 = 50 \cdot 2$$

Then simplify.

$$d = 100$$

Check it out:

Fractions represent divisions.

So $1 \div 2$ and $\frac{1}{2}$ mean exactly the same thing. You can write either.

Example 5

Find the value of t when $t \div 4 = 6$.

Solution

$t \div 4 = 6$

$t \div 4 \cdot 4 = 6 \cdot 4$ — **Multiply both sides by 4**

$t = 24$ — **Simplify to find t**

Guided Practice

Find the value of the variables in Exercises 9–16.

9. $3k = 18$ $k = 6$ **10.** $b \div 3 = 4$ $b = 12$

11. $h \div 5 = -3$ $h = -15$ **12.** $-9y = 99$ $y = -11$

13. $q \div 8 = \frac{1}{2}$ $q = 4$ **14.** $10t = -55$ $t = -5.5$

15. $d \div -2 = -4$ $d = 8$ **16.** $240 = 8m$ $m = 30$

Independent Practice

1. The Sears Tower in Chicago is 1451 feet tall, which is 405 feet taller than the Chrysler Building in New York. Use the equation $C + 405 = 1451$ to find the height of the Chrysler Building. 1046 feet

Find the value of the variable in Exercises 2–7.

2. $k + 7 = 10$ $k = 3$ **3.** $c + 10 = -27$ $c = -37$

4. $s + 4 = -7$ $s = -11$ **5.** $70 = 5 + b$ $b = 65$

6. $h + 0 = 14$ $h = 14$ **7.** $32 = 11 + a$ $a = 21$

8. The Holland Tunnel in New York is 342 feet longer than the 8216-foot-long Lincoln Tunnel. Use the equation $H - 342 = 8216$ to find the length of the Holland Tunnel. 8558 feet

Find the value of the variable in Exercises 9–14.

9. $x - 7 = 13$ $x = 20$ **10.** $41 = m - 35$ $m = 76$

11. $p - 13 = -82$ $p = -69$ **12.** $t - 27 = 37$ $t = 64$

13. $100 = g - 18$ $g = 118$ **14.** $-7 = y - 2$ $y = -5$

15. Marlon buys a sweater for \$28 that has \$17 off its usual price in a sale. Write an equation to describe the cost of the sweater in the sale compared with its usual price. Then solve the equation to find the usual price of the sweater. $x - 17 = 28$, $x = \$45$

Find the value of the variable in Exercises 16–21.

16. $5c = 80$ $c = 16$ **17.** $v \div 7 = 3$ $v = 21$

18. $22x = -374$ $x = -17$ **19.** $h \div -2 = 4$ $h = -8$

20. $-3k = -24$ $k = 8$ **21.** $-27 = f \div 3$ $f = -81$

22. The tallest geyser in Yellowstone Park is the Steamboat Geyser. Reaching a height of 380 feet, it is twice as high as the Old Faithful Geyser. Use the equation $2F = 380$ to find the height reached by the Old Faithful Geyser. 190 feet

Now try these:
Lesson 1.2.3 additional questions — p432

Round Up

Solving an equation tells you the value of the unknown number — the variable.
To solve an equation all you need to do is the reverse of what's already been done to the variable. That way you can isolate the variable. Just remember that you need to do the same thing to both sides. That's what keeps the equation balanced.

Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Guided practice
Level 1: q9–12
Level 2: q9–14
Level 3: q9–16

Concept question
"Does the equation 0x = 2 have a solution?"

No — 0 multiplied by a number is always 0. It can't be 2.

Independent practice
Level 1: q1–5, 8–11, 16–19
Level 2: q1–12, 16–21
Level 3: q1–22

Additional questions
Level 1: p432 q1–10
Level 2: p432 q1–12
Level 3: p432 q4–12

3 Homework

Homework Book — Lesson 1.2.3
Level 1: q1–5, 7–9
Level 2: q2, 4–10
Level 3: q2, 4a–b, 5a–b, 6–10

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
- Worksheet 26 — Solving Linear Equations

Lesson 1.2.4

Solving Two-Step Equations

In this Lesson, students learn to solve two-step equations by applying inverse operations. They apply the order of operations rules to identify the last operation that would be done to the variable, and undo that one first.

Previous Study: In grade 6, students wrote and solved one-step linear equations in one variable. This was reviewed in the previous Lesson.

Future Study: In future Lessons, students will write and solve two-step equations in response to real-life problems. They will also use a similar method to solve two-step inequalities in Chapter 4.

1 Get started

Resources:
- Counters, in two colors

Teacher Resources CD-ROM
- Equation Mat
- Calendar

Warm-up Questions:
- Lesson 1.2.4 sheet

2 Teach

Universal access

When presenting an equation, make the equals sign a different color from the rest of the problem.
This emphasizes that equations have two sides.

Math background

You can do the process in the other order, but it often leads to trickier math. For instance, $7x + 3 = 17$ could be solved by first dividing the equation by 7:

$$7x + 3 = 17$$
$$\frac{7x}{7} + \frac{3}{7} = \frac{17}{7} \quad \text{divide each term by 7}$$
$$x = \frac{17}{7} - \frac{3}{7} \quad \text{then subtract to isolate } x$$
$$= \frac{14}{7} = 2$$

Universal access

Use the Equation Mat from the **Teacher Resources CD-ROM** to model the solution of two-step equations.

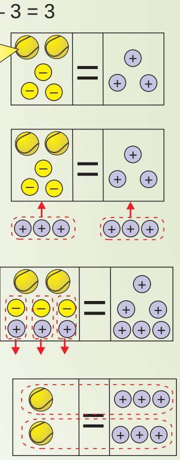
For example: $2x - 3 = 3$

use an object to represent x

$2x - 3 + 3 = 3 + 3$

$2x = 6$

$x = 3$



Lesson 1.2.4

Solving Two-Step Equations

California Standards:

Algebra and Functions 4.1

Solve two-step linear equations and inequalities **in one variable over the rational numbers**, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

What it means for you:

You'll learn how to solve an equation that involves more than one operation to find out the value of an unknown variable.

Key words:
- solve
- isolate
- inverse

Don't forget:

You need to remember to think of PEMDAS or GEMA. That way you'll know what order the operations have been done in — and what order to go in to reverse them.

When you have an equation with two operations in it, you need to do two inverse operations to isolate the variable. But other than that, the process is just the same as for solving a one-step equation.

Two-Step Equations Have Two Operations

A **two-step equation** is one that involves **two different operations**.

$$7 \bullet x + 3 = 17$$

first operation — second operation

You need to perform two inverse operations to isolate the variable.

It's easiest to **undo** the operations in the **opposite order** to the way that they were done. It's like taking off your shoes and socks. You normally put on your socks first and then your shoes. But when you're removing them you go in the **reverse order** — you take your shoes off first, and then your socks.

Here x is first multiplied by seven, and then the product has three added to it.

$$7x + 3 = 17$$

So to isolate the variable, first subtract three from both sides...

$$7x + 3 - 3 = 17 - 3$$
$$7x = 14$$

...and then divide both sides by seven.

$$7x \div 7 = 14 \div 7$$
$$x = 2$$

Example 1

Find the value of d when $4d + 6 = 38$.

Solution

In the equation $4d + 6 = 38$ the variable d has first been **multiplied** by 4, and 6 has then been **added** to the product. So to isolate the variable you must first **subtract** 6 from both sides, and then **divide** both sides by 4.

$$4d + 6 = 38$$
$$4d + 6 - 6 = 38 - 6 \quad \text{Subtract 6 from both sides}$$
$$4d = 32$$
$$4d \div 4 = 32 \div 4 \quad \text{Divide both sides by 4}$$
$$d = 8$$

- **Strategic Learners**
  Make an analogy between unlocking doors and solving equations. To solve a one-step equation, you need to unlock the front door to find what you need. To solve a two-step equation, you need to unlock the front door, and then unlock another door inside the house to find what you need.
- **English Language Learners**
  The Equation Mat approach described on the previous page is particularly suitable for English Language Learners.

Example 2

Find the value of h when $3h - 11 = 25$.

Solution

In the equation $3h - 11 = 25$ the variable h has first been **multiplied** by 3, and 11 has then been **subtracted** from the product.
So to isolate the variable you must first **add** 11 to both sides, and then **divide** them both by 3.

$$3h - 11 = 25$$
$$3h - 11 + 11 = 25 + 11 \quad \text{Add 11 to both sides}$$
$$3h = 36$$
$$3h \div 3 = 36 \div 3 \quad \text{Divide both sides by 3}$$
$$h = 12$$

Guided Practice

Find the value of the variables in Exercises 1–6.

1. $2x + 8 = 12$ $x = 2$
2. $10 + 5y = 25$ $y = 3$
3. $5t - 6 = 34$ $t = 8$
4. $7f - 19 = 30$ $f = 7$
5. $60 = 8b + 12$ $b = 6$
6. $34 = 4p - 10$ $p = 11$

Follow the Same Procedure with All the Operations

You can use this method for any **two-step equation**. Just perform the **inverse** of the two operations in the **opposite order** to the order in which they were done.

Sometimes the order in which the operations are performed is less obvious, and you'll need to think more carefully about it.

Check it out:

This equation could have been written as $(r \div 4) - 6 = 13$. But division takes priority over subtraction, so the parentheses aren't needed.

Example 3

Find the value of r when $r \div 4 - 6 = 13$.

Solution

In the equation $r \div 4 - 6 = 13$, the variable r has first been **divided** by 4, and 6 has then been **subtracted** from the quotient.
So to isolate the variable you must first **add** 6 to both sides, and then **multiply** both sides by 4.

$$r \div 4 - 6 = 13$$
$$r \div 4 - 6 + 6 = 13 + 6 \quad \text{Add 6 to both sides}$$
$$r \div 4 = 19$$
$$r \div 4 \cdot 4 = 19 \cdot 4 \quad \text{Multiply both sides by 4}$$
$$r = 76$$

2 Teach (cont)

Concept question

"If you double a number, and then subtract 5, you are left with 7. What number did you start with? Explain how you solved this."

**6. First, undo the "subtract 5," by adding 5 (7 + 5 = 12).
Then undo the doubling by dividing by 2 (12 ÷ 2 = 6).**

Additional examples

Find the value of the variables in the equations below:

1) $5t + 6 = 31$ **$t = 5$**

2) $7p - 3 = 18$ **$p = 3$**

3) $21 = 12c + 45$ **$c = -2$**

Guided practice

Level 1: q1–6
Level 2: q1–6
Level 3: q1–6

Universal access

Some students have difficulty identifying the correct operations to use when solving equations. Prepare a sheet full of tables that the student can use for each problem.

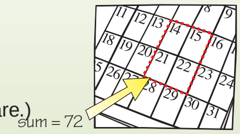
For example: $-4g - 14 = 16$

Operation in equation	Inverse operation
– 14	+ 14
× (–4)	÷ (–4)

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask advanced learners to investigate how a mathematical puzzle works. Using a page from a calendar with the dates arranged in a grid (one is provided in the **Teacher Resources CD-ROM**), one student chooses any four days that make a square, and finds their sum. Another student then deduces which four days were chosen by solving this equation: sum = $4n + 16$, where n is the top-left number. Ask students to figure out how this works. ($4n + 16$ is equal to $(n) + (n + 1) + (n + 7) + (n + 8)$ — this represents the four numbers in the square.)
  Finally, ask students to devise their own similar mathematical puzzles.



2 Teach (cont)

Concept question

"Do these equations have different solutions? Explain your answer."
$6x + 3 = 12$ and $6(x + 3) = 12$

Yes. The parentheses in the second equation change the order of operations. They mean that the 6 multiplies by both the x and the 3, not just the x. In the first equation, $x = 1.5$. In the second, $x = -1$.

Guided practice

Level 1: q7–10
Level 2: q7–11
Level 3: q7–12

Additional examples

Find the value of the variables in the equations below:

1) $t \div 6 + 5 = 13$ **$t = 48$**

2) $y \div (6 + 5) = 3$ **$y = 33$**

Independent practice

Level 1: q1–7, 11–13
Level 2: q1–14
Level 3: q1–16

Additional questions

Level 1: p432 q1–6
Level 2: p432 q1–10
Level 3: p432 q1–12

3 Homework

Homework Book — Lesson 1.2.4

Level 1: q1, 2, 4, 7, 8
Level 2: q2–10
Level 3: q3–11

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 26 — Solving Linear Equations

Check it out:

The parentheses are needed here because addition doesn't take priority over division. If the parentheses were left out, this equation would have a different solution — because of the order of operations rules.

Example 4

Find the value of v when $(v + 2) \div 7 = 3$.

Solution

In the equation $(v + 2) \div 7 = 3$, the variable v and 2 have first been **added** together, and then their sum has been **divided** by 7. So to isolate the variable you must first **multiply** both sides by 7, and then **subtract** 2 from both sides.

$$(v + 2) \div 7 = 3$$
$$(v + 2) \div 7 \cdot 7 = 3 \cdot 7 \quad \text{Multiply both sides by 7}$$
$$v + 2 = 21$$
$$v + 2 - 2 = 21 - 2 \quad \text{Subtract 2 from both sides}$$
$$v = 19$$

Guided Practice

Find the value of the variables in Exercises 7–12.

7. $x \div 2 + 8 = 9$ **$x = 2$**
8. $d \div 7 + 4 = 6$ **$d = 14$**
9. $k \div 3 - 15 = 30$ **$k = 135$**
10. $y \div 4 - 3 = 12$ **$y = 60$**
11. $9 = g \div 2 - 6$ **$g = 30$**
12. $(j + 20) \div 5 = 3$ **$j = -5$**

Now try these:

Lesson 1.2.4 additional questions — p432

Independent Practice

In Exercises 1–4, say which order you should undo the operations in.

1. $x \div 3 + 7 = 20$ **Undo the addition, then the division.**
2. $21x - 12 = 44$ **Undo the subtraction, then the multiplication.**
3. $11 = x \div 10 - 5$ **Undo the subtraction, then the division.**
4. $14 = 2 \cdot (2 + x)$ **Undo the multiplication, then the addition.**

Find the value of the variables in Exercises 5–16.

5. $4h + 2 = 22$ **$h = 5$**
6. $2r + 11 = -13$ **$r = -12$**
7. $10b - 5 = 55$ **$b = 6$**
8. $5w - 15 = 10$ **$w = 5$**
9. $14 = 2 + 3c$ **$c = 4$**
10. $-10 = 2n - 2$ **$n = -4$**
11. $m \div 4 + 6 = 11$ **$m = 20$**
12. $d \div 2 + 9 = -9$ **$d = -36$**
13. $p \div 7 - 4 = 2$ **$p = 42$**
14. $f \div 3 - 17 = -20$ **$f = -9$**
15. $10 = 5 + a \div 10$ **$a = 50$**
16. $-20 = q \div 2 - 12$ **$q = -16$**

Round Up

Solving a two-step equation uses the same techniques as solving a one-step equation. The important thing to remember with two-step equations is to do the inverse operations in the reverse of the original order. This same method applies to every equation, no matter how many steps it has. Later in this Section you'll use this technique to solve real-life problems.

Solutions

For worked solutions see the Solution Guide

Lesson 1.2.5

More Two-Step Equations

In this Lesson, students solve equations containing fractions by converting the fraction to a multiplication and a division and using the method for solving two-step equations. They are also taught to check solutions by substituting them into the original equation.

Previous Study: In the previous Lesson, students learned the basic method for solving two-step equations with one variable.

Future Study: In future Lessons, students will write and solve two-step equations in response to real-life problems. They will also use a similar method to solve two-step inequalities in Chapter 4.

Lesson 1.2.5

More Two-Step Equations

California Standards:

Algebra and Functions 4.1

Solve two-step linear equations and inequalities **in one variable over the rational numbers**, interpret the solution or solutions in the context from which they arose, and **verify the reasonableness of the results.**

What it means for you:

You'll learn how to deal with fractions in equations, and how to check that your answer is right.

Key words:

- fraction
- isolate
- check

When you have a fraction in an equation, you can think of it as being two different operations that have been merged together. That means it can be solved in the same way as any other two-step equation.

Fractions Can Be Rewritten as Two Separate Steps

Fractions can be thought of as a combination of **multiplication** and **division**. You might see what is essentially the same expression written in several different ways. For example:

$\frac{3}{4}x$ $\qquad$ $\frac{3x}{4}$

$(3 \cdot x) \div 4$

$\frac{1}{4} \cdot 3x$ $\qquad$ $3 \cdot \frac{1}{4} \cdot x$

All five expressions are the same.

Deal with a Fraction in an Equation as Two Steps

Because a fraction can be split into **two steps**, an equation with a fraction in it can be solved using the two-step method.

Using the example above:

$$\frac{3}{4}x = 6$$
$$3x \div 4 = 6$$

First split the expression into two separate operations: here x is first multiplied by 3, and then divided by 4.

$$3x = 24$$

Then solve as a two-step equation.

$$x = 8$$

Example 1

Find the value of a when $\frac{2}{3}a = 6$.

Solution

$$\frac{2}{3}a = 6$$

$2a \div 3 = 6$ — **Split the expression into two operations**

$2a = 18$ — **Solve as a two-step equation**

$$\mathbf{a = 9}$$

Check it out:

Another way to do this is to multiply both sides by the reciprocal of the fraction. Multiplying a fraction by its reciprocal gives a product of 1 — so it "gets rid of" the fraction.

To find the reciprocal of a fraction you invert it.

So the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$$\frac{2}{3}a = 6$$
$$\frac{3}{2} \cdot \frac{2}{3}a = 6 \cdot \frac{3}{2}$$
$$a = 9$$

For more on reciprocals see Lesson 1.1.4.

1 Get started

Resources:

- index cards with equivalent forms of fractional expressions on them

Warm-up questions:

- Lesson 1.2.5 sheet

2 Teach

Concept question

"If $x = 2$, will the value of $\frac{10}{4}x$ be bigger or smaller than 2? Explain your answer."

Bigger.
x is multiplied by 10 and divided by 4.
Or, $\frac{10}{4}x$ is the same as $2.5x$.

Universal access

Make index cards with equivalent forms of fractional expressions on them. Students can then play Pelmanism with the cards.

Concept question

"In the equation $\frac{6x}{5} = 30$ what operations are done to x? What operations would you need to do to each side of the equation to isolate x?"

x is multiplied by 6, and the product divided by 5 (or x is divided by 5 and the quotient is multiplied by 6, or x is multiplied by six-fifths). Discuss the alternatives and the fact that they are equivalent.

To isolate x, you need to perform the inverse operations: divide by 6 and multiply by 5 (in either order).
Or divide by six-fifths (which is the same as multiplying by five-sixths).

Math background

Remind students that the variable x actually has the coefficient 1.
So $1x = x$.

You're not actually getting rid of the coefficient of a variable when you're solving an equation — you're converting it to 1.

- **Strategic Learners**
  Develop the "unlocking doors" analogy introduced in the last Lesson. Checking a solution can be thought of as "relocking the doors." The doors are relocked in the reverse of the order in which they were unlocked.

- **English Language Learners**
  Have students identify the two operations that are in each two-step equation, and also the "opposite/inverse" operations that will be used to solve the equations.

2 Teach (cont)

Guided practice

Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Additional examples

Solve these examples and check your solutions:

1) $4x + 10 = 50$ $x = 10$

Check: 4 • 10 + 10 = 40 + 10 = 50

2) $-7m + 14 = 28$ $m = -2$

Check: –7 • –2 + 14 = 14 + 14 = 28

3) $36 = 8y + 20$ $y = 2$

Check: 8 • 2 + 20 = 16 + 20 = 36

Common error

The check can be seen as just a formality for students. A common error is shown in the check below.

$x = 2 \Rightarrow 8x + 3 = 20$
$8(2) + 3 = 20$
$16 + 3 = 20$
$20 = 20$

The check is wrong but the student writes 20 = 20 anyway. Reinforce the idea that the check is an important part of solving equations.

Don't forget:

The number on top of a fraction is called the numerator.

$\frac{2}{5}$

The number on the bottom of a fraction is called the denominator.

Here is another example — this one has a more complicated numerator.

Example 2

Find the value of h when $\frac{h+2}{4} = 3$

Solution

$$\frac{h+2}{4} = 3$$

The whole expression $h + 2$ is being divided by 4 — the fraction bar "groups" it. Put it in parentheses here to show that this operation originally took priority.

$$(h + 2) \div 4 = 3$$

Split the expression into two operations

$$h + 2 = 12$$

Solve as a two-step equation

$$h = 10$$

Guided Practice

Find the value of the variables in Exercises 1–6.

1. $\frac{1}{2}a = 2$ ***a* = 4**

2. $\frac{3}{4}q = 33$ ***q* = 44**

3. $\frac{2}{3}v = 4$ ***v* = 6**

4. $\frac{4}{1}r = -8$ ***r* = –2**

5. $6 = \frac{2}{5}s$ ***s* = 15**

6. $\frac{2c}{3} = 6$ ***c* = 9**

Check Your Answer by Substituting It Back In

When you've worked out the value of a variable you can **check your answer** is right by **substituting** it into the original equation.

Once you've substituted the value in, **evaluate** the equation — if the equation is still **true** then your calculated value is a **correct solution**.

$$3x + 2 = 14$$
$$3x + 2 - 2 = 14 - 2$$
$$3x = 12$$
$$3x \div 3 = 12 \div 3$$
$$x = 4$$

First solve the equation to find the value of x.

Now substitute the calculated value back into the equation.

$$3x + 2 = 14, \quad x = 4$$
$$3(4) + 2 = 14$$
$$12 + 2 = 14$$
$$14 = 14$$

Then evaluate the equation using your calculated value.

As both sides are the same, the value of x is correct.

Don't forget:

If the equation isn't true when you've substituted in your solution, look back through your work to find the error.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to use the ages of friends, family, or celebrities to make up age-related problems. For instance, "I'm 13 years old. I'm half the age my favorite movie star was four years ago. How old is my favorite movie star?" Then ask the students to represent the problems by equations, which they can then solve. For example, movie star age = m; $13 = \frac{1}{2}(m-4)$.

Check it out:
It might seem like needless extra work to check your solution, but it's always worth it just to make sure you've got the right answer.

Example 3

Check that $c = 8$ is a solution of the equation $10c + 15 = 95$.

Solution

$10c + 15 = 95$
$10(8) + 15 = 95$ — **Substitute 8 into the equation**
$80 + 15 = 95$
$95 = 95$

The equation is still true, so $c = 8$ is a solution of the equation $10c + 15 = 95$.

Guided Practice

Solve the equations below and check your answers are correct.

7. $12m + 8 = 56$ **$m = 4$**
8. $22 + 3h = 34$ **$h = 4$**
9. $56 = 18 + 19v$ **$v = 2$**
10. $16 - 4g = -28$ **$g = 11$**
11. $3 - 6x = 9$ **$x = -1$**
12. $5y - 12 = 28$ **$y = 8$**

Independent Practice

Find the value of the variables in Exercises 1–6.

1. $\frac{3}{4}d = 24$ **$d = 32$**
2. $\frac{4}{5}k = 8$ **$k = 10$**
3. $-\frac{2}{3}b = 14$ **$b = -21$**
4. $27 = \frac{3}{2}w$ **$w = 18$**
5. $22 = n \bullet \frac{2}{5}$ **$n = 55$**
6. $\frac{5t}{10} = 4$ **$t = 8$**

Solve the equations in Exercises 7–10 and check your solution.

7. $2x + 4 = 16$ **$x = 6$**
8. $3r - 6 = -12$ **$r = -2$**
9. $6 = v \div 4 + 2$ **$v = 16$**
10. $\frac{3}{4}c = 15$ **$c = 20$**

11. For each of the equations, say whether a) $y = 3$, or b) $y = -3$, is a correct solution.
Equation 1: $10 - 2y = 16$ **b) is a correct solution, a) is not.**
Equation 2: $-\frac{2}{3}y = -2$ **a) is a correct solution, b) is not.**

For each equation in Exercises 12–14, say whether the solution given is a correct one.

12. $x \div 2 + 4 = 9$, $x = 10$. **Yes.**
13. $3x - 9 = 12$, $x = 4$. **No ($x = 7$ would be correct).**
14. $8 = 5x - 7$, $x = 3$. **Yes.**

Now try these:
Lesson 1.2.5 additional questions — p433

Round Up

You can think of a fraction as a combination of two operations. So a fraction in an equation can be treated as two steps. And don't forget — when you've found a solution, you should always substitute it back into the equation to check that it's right.

2 Teach (cont)

Common error

Sometimes students check their solutions correctly, but when their check leads to an untrue equation, they don't know what to do next.

$x = 2 \Rightarrow 8x + 3 = 20$
$8(2) + 3 = 20$
$16 + 3 = 20$
$19 = 20$

Emphasize the importance of checking back through their work until the error is discovered. One way of giving students practice at doing this is by supplying some incorrectly solved equations and asking students to analyze them to find the errors. The additional example below is suitable for this.

Guided practice

Level 1: q7–10
Level 2: q7–11
Level 3: q7–12

Additional example

A student has attempted to solve the equation below. Their work is shown:
$3x - 6 = 12$
$3x - 6 + 6 = 12$
$3x = 12$
$\frac{3x}{3} = \frac{12}{3} \Rightarrow x = 4$
Check their solution. If it's incorrect, find the error and find the correct solution.

6 was added to only one side, not both. x is actually 6.

Independent practice

Level 1: q1–3, 7–11
Level 2: q1–4, 7–12
Level 3: q1–14

Additional questions

Level 1: p433 q1–3, 7–8
Level 2: p433 q4–12
Level 3: p433 q4–10, 13–14

3 Homework

Homework Book — Lesson 1.2.5

Level 1: q1, 2, 6–8
Level 2: q2–10
Level 3: q2–10

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
- Worksheet 26 — Solving Linear Equations

Solutions

For worked solutions see the Solution Guide

Lesson 1.2.6

Applications of Equations

In this Lesson, students apply their equation-solving skills to real-life problems. They are also encouraged to examine the solutions they get to real-life problems and consider whether they are of a reasonable size and if they make sense in the context of the question.

Previous Study: Earlier in this Chapter, students developed the skills necessary to solve two-step equations in one variable.

Future Study: In Algebra I, students will solve systems of linear equations involving two variables. Checking the reasonableness of solutions is a process that students should use across all topics.

1 Get started

Warm-up questions:
- Lesson 1.2.6 sheet

2 Teach

Common error
Some students are able to understand and solve the problems using non-algebraic methods — however, they may not see the connection between what they have done and the equations that can be used to represent the problem. The Universal access approach given below is useful for these students.

Universal access
Using copies of the problems and equations on this page, select one problem and have students cover up the equation given. Ask them to read through the problem, circling key words and numbers, then to answer the question non-algebraically.

Next, ask them to describe what they did to get the answer, and to model the same operations algebraically to form an equation. This shows that the equation contains the same operations as they used.

Additional example
Sean spends $10 on supplies for his lemonade stand. This is enough to make 90 cups of lemonade.
If he can sell them all, how much should he charge per cup to make $8 profit?

Let p = price per cup
$90p - \$10 = \$8$
$90p = \$18$
$p = \$0.20 = 20¢$

Lesson 1.2.6

Applications of Equations

California Standards:

Algebra and Functions 4.1
Solve two-step linear equations and inequalities **in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.**

Mathematical Reasoning 2.1
Use estimation to verify the reasonableness of calculated results.

What it means for you:
You'll see how to use equations to help solve real-life math problems, and how to check if your answer is sensible.

Key words:
- model
- check
- reasonable
- sensible

Equations can be really useful in helping you to understand real-life situations. Writing an equation can help you sort out the information contained in a word problem and turn it into a number problem.

Equations Can Describe Real-Life Situations

An equation can help you to **model** a real-life situation — to **describe** it in math terms. For example:

- You've just had your car repaired. The bill was $280.
- You know the parts cost $120.
- You know the mechanic charges labor at $40 per hour.
- You want to know how long the mechanic worked on your car.

Let h = number of hours worked by mechanic. ← *1. Choose a variable.*
$40h + 120 = 280$ ← *2. Write an equation.*
$40h = 160$ ← *3. Solve the equation.*
$h = 4$

So you know the mechanic must have worked on your car for **4 hours**.

You can use an equation to help you describe almost any situation that involves numbers and **unknown numbers**.

Example 1

At the school supply store, Mr. Ellis bought a notebook costing $3 and six pens. He spent $15 in total. Find the price of one pen, p.

Solution

First **write out** the **information** you have:
Total spent = $15
Cost of notebook = $3
Cost of six pens = $6p$

You know that **six pens** and the **notebook** cost a total of **$15**. So you can write an **equation** with the cost of each of the items bought on one side, and the total spent on the other.
$6p + 3 = 15$

Now you have a **two-step equation**. You can find the cost of one pen by **solving** it.

$6p + 3 = 15$
$6p = 12$
$p = 2$ **One pen costs $2.**

Check it out:
When you've solved your equation you'll need to decide if the solution needs units.
In this example you're figuring out the price of a pen in dollars, so your answer is $2.

You'll see more about how to find the right units for your answer in Lesson 1.2.7.

- **Strategic Learners**
  Encourage strategic learners to make a brief list after reading each problem. They should note each piece of information and what their variables stand for. They should also make a note of what the problem is asking them to find. A checklist, like the one described in the Universal access approach on the next page, may also be useful for strategic learners.

- **English Language Learners**
  Allow students to work with language buddies or in groups to tackle word problems. Suggest that they solve the problems in two stages. First, ask them to identify the question being asked, decide on a variable, and make an equation representing the information. Then stop them and check that all students agree on this. If so, ask them to continue partner work to choose a strategy, solve, and check for reasonableness.

Guided Practice

Write an equation to describe each of the situations in Exercises 1–3. Then solve it to find the value of the variable.

1. Emily is seven years older than Ariela. The sum of their ages is 45. How old is Ariela? $A + A + 7 = 45, A = 19$

2. A sale rack at a store has shirts for $9 each. Raul has $50 and a coupon for $4 off any purchase. How many shirts can he buy? $9S - 4 = 50, S = 6$

3. The price for renting bikes is $15 for half a day, then $3 for each additional hour. How many hours longer than half a day can you keep a bike if you have $24? $15 + 3h = 24, h = 3$

You Need to Check That Your Answer is Reasonable

When you've **solved** an equation that describes a real-life problem, you need to look at your answer carefully and see if it is **reasonable**. Here are two important things to think about:

1) Does Your Answer Make Sense?

You must always check that the answer **makes sense** in the context of the question. For example:

An orchard charges $1.10 for **a pound** of apples. You have $8.25. How many pounds of apples can you buy?

- Set up an equation to describe the problem:
  Number of pounds = 8.25 ÷ 1.10 = 7.5

→ **This is a reasonable answer as the orchard will happily sell you half a pound of apples.**

But if you change the problem slightly:
A store charges $1.10 for **a bag** of apples. You have $8.25.
How many bags of apples can you buy?

- **Number of bags = 8.25 ÷ 1.10 = 7.5.**

→ **This is no longer a reasonable answer — the store wouldn't sell you half a bag of apples. You could only buy 7 bags.**

Don't forget:

You might need to round your answer up or down, depending on the question.

In this example, you calculate that you can afford 7.5 bags — but you can only buy **whole bags**. You can't afford to buy 8 bags — so it's sensible to round your answer down to 7.

If instead you were working out how many bags you needed for a recipe and your answer came out as 7.5, then you would round it up. You would buy 8 bags, because you want **at least** 7.5.

There's a lot more about rounding, and how to round reasonably, in Section 8.3.

2) Is Your Answer About the Right Size?

The **size of your answer** has to make sense in relation to the question that is being asked. For example:

→ If you're finding the height of a mountain, and your answer is **5 feet**, it's **not reasonable**.

→ If you're finding the height of a person, and your answer is **5000 feet**, that's **not reasonable** either.

If the size of your answer doesn't seem **reasonable** then it's really important to go back and **check your work** to see if you've made an error somewhere.

2 Teach (cont)

Guided practice
Level 1: q1–2
Level 2: q1–3
Level 3: q1–3

Additional examples
Say which of the following are sensible solutions. For any that aren't, give a more sensible alternative.

1) 3.7 buses needed to be chartered for a school trip.

Not sensible. You can't charter part of a bus. 4 buses would have been needed.

2) I can just afford to buy 6.1 CDs with the money I've saved.

Not sensible. You can't buy 0.1 of a CD. 6 is more sensible (you couldn't afford 7).

3) I need to buy 4.8 feet of ribbon for a craft project.

This is sensible. You are likely to be able to buy a fraction of a foot of ribbon.

Concept question
"John expected the tax on his $70 purchase to be about $5. When he solved an equation he calculated the tax to be only $0.05. Since the two numbers are so different, what should John do?"

**John should substitute his answer back into the equation and check.
If this doesn't give him a true equation, he should rework his solution. He may also need to check that the equation he is using represents the situation correctly.**

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to work with a partner to list examples of situations in which the answer would need rounding up or rounding down to a whole number. Ask students to then construct problems that produce answers that need to be treated in each of these ways.

2 Teach (cont)

Universal access

Create a checklist for students to use when they think they have solved a problem. For example:

1. Check the solution by substituting it back into the equation.
2. Include units if needed.
3. See if your answer is about the size you would have expected.
4. Think about whether the answer makes sense in the context of the problem.

Guided practice

Level 1: q4–5
Level 2: q4–6
Level 3: q4–7

Independent practice

Level 1: q1–2, 4
Level 2: q1–5
Level 3: q1–5

Additional questions

Level 1: p433 q1–3
Level 2: p433 q1–5
Level 3: p433 q1–7

3 Homework

Homework Book — Lesson 1.2.6
Level 1: q1, 2, 5, 6
Level 2: q1–7
Level 3: q1–8

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
• Worksheet 26 — Solving Linear Equations
• Worksheet 29 — Rates

Example 2

Kea is going to walk 1.5 miles at a steady speed of 3 miles per hour. She works out how long it will take using the work shown.
Is her answer reasonable?

Distance = 1.5 miles
Speed = 3 mi/hour
Time = 1.5 × 3 = 4.5 hours

Solution

Given that Kea's walk is only 1.5 miles long and she walks at 3 mi/h, **4.5 hours is not a reasonable answer** — it is much too long.

(Kea multiplied the distance of the walk by her speed. She should **divide** the distance by the speed instead: **Time = 1.5 ÷ 3 = 0.5 hours**.)

Check it out:
If Kea walked at 3 miles per hour for 1 hour she'd go 3 miles. So to cover half that distance would take her half the time — 0.5 hours.

Guided Practice

4. Pete is buying trading cards. One card costs 20¢. He says 10 cards will cost $20. Is this a sensible answer? Explain why or why not. see left

5. Six friends earn $87 washing cars. How much will each one get if they split it evenly? Is your answer reasonable in the context of the question? see left

6. A yard has a 150-foot perimeter. Fencing is sold in 40-foot rolls. Write an equation to describe the number of rolls, n, you need to buy to fence the yard. Solve the equation. Is your answer reasonable in the context of the question? $40n = 150$, $n = 3.75$. This isn't reasonable: you couldn't buy part of a roll. You would have to buy 4.

7. Ana is $\frac{5}{6}$ as tall as T.J., who is 174 cm tall. Write an equation to describe Ana's height, A. Solve it. Is the size of your answer reasonable? $A = \frac{5}{6} \cdot 174 = 145$ cm. This is a reasonable height.

4. This is not reasonable — his answer is much too big. 10 cards will cost $2.

5. Each will get $14.50. This is reasonable, as they can get fractions of dollars each (and it seems about the right size).

Independent Practice

Write an equation to describe each situation in Exercises 1–2, and solve the equation to answer the question.

1. Don has spent $474 ordering sticks for his hockey team. A stick costs $50. Shipping costs $24. How many did he buy? $50x + 24 = 474$, 9 sticks.

2. Tiana is saving up to buy a fishing rod. The rod costs $99 with tax. She already has $27, and can afford to save another $12 each week. How long will it take her to save enough for the rod? $27 + 12x = 99$, 6 weeks.

3. Joy went to the fabric store to buy ribbon. She got f feet, and spent $5. The ribbon cost 80¢ a foot. Write an equation to describe how much she got. Solve it. Is your answer reasonable in the context of the question? see below

4. Mike is asked to multiply 5 by $\frac{1}{2}$. He says the answer is 10. Is this reasonable in the context of the question? Explain why or why not. see below

5. Two friends run a dog walking service, each walking the same number of dogs. Write and solve an equation to show how many dogs, d, each friend walks if they walk nine dogs between them. Is your answer reasonable? see below

Now try these:
Lesson 1.2.6 additional questions — p433

Round Up

Equations can help you to understand situations. They can also help you to describe a real-life math problem involving an unknown number and come up with a solution. But don't forget to always think carefully about whether the answer is a reasonable one in relation to the question.

Solutions

For worked solutions see the Solution Guide

Independent practice
3. $0.8f = \$5$, $f = 6.25$ feet. This is reasonable: the store is likely to be happy to sell her part of a foot of ribbon.
4. This isn't reasonable: when you multiply a number by a fraction less than 1, the answer should be smaller than the original number.
5. $2d = 9$, $d = 4.5$. This isn't reasonable. You couldn't walk part of a dog.

Lesson 1.2.7 Understanding Problems

In this Lesson, students are made aware that when solving real-life problems, they often won't have perfect information at hand — they might have more than they need, or be missing a vital piece. The Lesson also looks at analyzing units to find the correct ones for a solution.

Previous Study: Since grade 3, students have been developing their ability to use appropriate units to quantify the properties of objects, such as length, volume, and weight.

Future Study: The skills developed in this Lesson should continue to be used by students when solving word problems throughout the rest of grade 7, and through later grades.

Lesson 1.2.7 Understanding Problems

California Standards:

Algebra and Functions 4.1

Solve two-step linear equations and inequalities **in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.**

Mathematical Reasoning 1.1

Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

What it means for you:

You'll learn how to spot which pieces of information are important in answering a question, and how to check that your answer has the correct units.

Key words:

- relevant
- irrelevant
- unit

Math problems are full of all kinds of details. The challenge is to work out which bits of information you need and which bits you don't need. To be able to do this you need to understand exactly what the question is asking.

You Can't Solve a Problem with Information Missing

Sometimes a piece of **information** needed to solve a real-life problem will be **missing**. You need to be able to read the question through and **identify** exactly what vital piece of information is missing.

Example 1

Brian's mechanic charged \$320 to fix his car. The bill for labor was \$157.50. How many hours did the mechanic work on the car?

Solution

The question tells you that Brian's total bill for labor was \$157.50. But to use this piece of information to work out how many hours the mechanic worked on the car you would also need to know what the **mechanic's hourly rate** was, as **hours worked = bill for labor ÷ hourly rate**.

You can't solve the problem as **the mechanic's hourly rate is missing**.

Guided Practice

In Exercises 1–4 say what piece of information is missing that you need to solve the problem.

1. Samantha is 20 inches taller than half Adam's height. How tall is Samantha? **Adam's height**

2. A coffee bar charges \$2 for a smoothie. Sol buys a smoothie and a juice. How much is his check? **The cost of a juice**

3. Erin has \$36 and is going to save a further \$12 a week. How many weeks will it take her to save enough for a camera? **The camera's price**

4. A box contains 11 large tins and 17 small tins. A large tin weighs 22 ounces. What is the weight of the box? **The weight of a small tin**

Some Information in a Question May Not Be Relevant

You will often come across real-life problems that contain **more information** than you need to find a solution. Information that you **don't need** to solve a problem is called **irrelevant information**.

You need to be able to **sort out** the information you do need from the information you don't. A good example of this is a question where you have to **pick out** the information that you need from a **table**.

1 Get started

Warm-up questions:
- Lesson 1.2.7 sheet

2 Teach

Concept question

"You want to find out the total price for an orchestra to all go on a train journey. What is the minimum amount of information that you need to know?"

Price of each ticket (or where they are traveling from and to, and at what time, so you can find out the price), the numbers of adults and children, whether there are discounts for groups.

Universal access

Some students find it difficult to identify the key facts in a word problem. They should underline (or copy) names and numbers that are in the problem. From this they should list what they know and what they want to know. This helps them define a variable and write an equation.

Guided practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1, 3–4

Math background

You can only find the value of one variable from a single equation. If you have an equation containing two unknowns, you can't find their values. (You'd need an additional equation to do this — and this would be a system of equations. There's more on this in Section 4.1.)

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  Give strategic learners a question to which they might want to know the answer. For instance, "What is the total cost of buying some cartons of juice?" Then give a list of facts: the price per carton, the size of the cartons, the number of cartons bought, the rate of tax. Ask students to identify which facts would be important to know from the list.

- **English Language Learners**
  When reading problems, ask students to circle (or make a note of) words they don't understand. Arrange for these students to work with language buddies or in groups to provide support.

2 Teach (cont)

Universal access

Some students may have difficulty locating information in a table. Before solving problems, ask them to find specific pieces of information from a table. For instance, "What is the volume of a can of blue paint?"

Model finding the correct columns and rows and then reading down and across them. The following exercise gives students practice at reading tables.

Put this table on the board:

Type of cheese	Fat (per 100 g)	Carbohydrates (per 100 g)	Protein (per 100 g)
Feta	21 g	4 g	14 g
Cheddar	33 g	1 g	25 g
Brie	28 g	0 g	21 g
Swiss	28 g	5 g	27 g

Ask students the following questions about the table:

1) Which cheese contains no carbohydrates? **Brie**
2) How much protein is contained in 100 g of cheddar cheese? **25 g**
3) Which two cheeses contain equal amounts of fat? **Brie and Swiss**
4) How many grams of carbohydrates are contained in 50 g of feta and 100 g of Swiss cheese altogether? **7 g**

Guided practice

Level 1: q5
Level 2: q5–6
Level 3: q5–7

Example 2

At the hardware store Aura spent $140 on paint. She bought four cans of blue paint and spent the rest of the money on green paint. Use the table below to calculate how many liters of green paint she bought.

Aura only bought blue paint and green paint. So you only need the circled data in these two rows to answer the question.

Color of paint	Volume of can (l)	Price of can ($)
Blue	1	20
Yellow	2	35
Red	1	20
Green	1.5	30

Solution

To answer the question you need the **price of a can of blue paint**, and the **volume and price of a can of green paint**. The volume of cans of blue paint is **irrelevant**, as is the information about red and yellow paint.

- First work out how much Aura spent on **blue paint**. You know that she bought four cans of blue paint that cost $20 each. So she spent $80 on blue paint. That means she spent $140 – $80 = **$60 on green paint**.
- Each can of green paint is $30. So she bought $60 ÷ $30 = **2 cans**.
- A can of green paint is 1.5 liters. So she bought 1.5 • 2 = **3 liters**.

Guided Practice

Use the table from Example 2 in Exercises 5–7.

5. Eduardo bought one can of yellow paint and three liters of blue paint. How much did he spend? **$95**

6. Lamarr bought 2 cans of green paint and some yellow paint. He spent $165. How many liters of yellow paint did he buy? **6 liters.**

7. Amber spent $120. She bought twice as much red paint as blue paint. How many cans of red paint did she buy? **4 cans.**

Answers Should Always Have the Correct Units

When you work out the answer to a problem, you need to think about the right **units** to use.

If you apply the **same operations** to the units as you do to the numbers, you'll find out what units your answer should have.

Check it out:
When you're writing units, remember that km/hour means the same as (km ÷ hours), and person-days means the same as (persons × days).

Example 3

Laura drives her car 150 km in 2 hours. Use the formula speed = distance ÷ time to calculate her average speed.

Solution

speed = distance ÷ time
speed = 150 ÷ 2 = **75**
Now do the same operations to the units of the numbers:
km ÷ hours = **km/hour**.
So the average speed of the car is **75 km/hour**.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
Give an equation and have partners or groups write a word problem that matches the equation. Then ask groups to present their problems to the class. Ask the class to identify exactly what the problem is asking them to find, and to check their solution for reasonableness.

You can do this with any calculation to find the **correct units** for the answer.

Example 4

The power consumption of a computer is 0.5 kilowatts.
If the computer is running for 4 hours, how much energy will it use?
Use the equation: Power Consumption • Time Used = Energy Used.

Solution

First do the numerical calculation.
Power Consumption • Time Used = Energy Used
0.5 • 4 = 2

Then work out the units.
kilowatts • hours = kilowatt-hours

The computer will use 2 kilowatt-hours of energy.

Check it out:
A kilowatt-hour is a measure of energy consumption.

Guided Practice

Say what units the answers will have in Exercises 8–11.

8. 40 miles ÷ 2 hours = 20 ? miles/hour
9. 5 newtons • 3 meters = 15 ? newton-meters
10. 6 persons • 4 days = 24 ? person-days
11. $25 ÷ 5 hours = 5 ? $/hour

Check it out:
You can use the / symbol to mean "divided by" when you are writing units.

Independent Practice

1. The sale bin at a music store has CDs for $4 each. Eric buys four CDs and some posters, and uses a coupon for $2 off his purchase. He pays $26. How many posters did he buy? Say what information is missing from the question that you would need to solve the problem.
You need to know the price of a poster.

2. Liz meets Ana to go ice-skating at 7 p.m. Admission is $8 and coffee costs $1.50. Liz has $14 and wants to buy some $2 bottles of water for her and Ana to drink afterwards. Calculate how many bottles of water Liz can buy. What information are you given that isn't relevant ?
Number of bottles = (14 – 8) ÷ 2 = 3. You don't need to know who Liz meets, when they meet, or the price of a cup of coffee.

3. Sean has $60 to buy books for math club. A book costs $9.95. He orders them on a Monday. Shipping costs $10 an order. How many books could he buy? What information are you given that isn't relevant?
Number of books = (60 – 10) ÷ 9.95 = 5.025. He could buy 5 books. You don't need to know what he gets them for or when he orders them.

Say what units the answers will have in Exercises 4–7.

4. 4 persons • 4 hours = 16 ? person-hours
5. 100 trees ÷ 10 acres = 10 ? trees/acre
6. 6 meters • 7 meters = 42 ? meters • meters, or meter^2
7. 21 meters/second ÷ 7 seconds = 3 ? meters/second/second, or meters/second^2

Now try these:
Lesson 1.2.7 additional questions — p433

Round Up

When you're solving a math problem, you need to be able to pick out the important information. Then you can use the relevant bits to write an equation and find the solution. Always remember to check what units your answer needs to be written in too.

2 Teach (cont)

Additional examples
1) A slug moves 6 cm in 3 minutes. Its speed is calculated by dividing 6 cm by 3 minutes. Give the speed with its units.

2 cm/minute

2) It takes 300 person-hours to build a cabin. How long will to take 15 people working together? Check the units that you give with your answer.

20 hours

Guided practice
Level 1: q8–10
Level 2: q8–11
Level 3: q8–11

Independent practice
Level 1: q1, 4–7
Level 2: q1–2, 4–7
Level 3: q1–7

Additional questions
Level 1: p433 q1–2, 4
Level 2: p433 q1–6
Level 3: p433 q1–8

3 Homework

Homework Book — Lesson 1.2.7
Level 1: q1–3, 6, 8, 9
Level 2: q2–7, 9–11
Level 3: q2–11

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
• Worksheet 26 — Solving Linear Equations
• Worksheet 29 — Rates

Solutions
For worked solutions see the Solution Guide

Lesson 1.3.1 Inequalities

In this Lesson, students review the meaning of the symbols < and >, and are introduced to the symbols ≤ and ≥. They learn to plot simple inequality statements on number lines, using either open or closed circles to show if a value is included in the solution set.

Previous Study: As early as grade 1, students used the symbols <, =, and > to compare numbers. In grade 4, students used these symbols to compare fractions and decimals.

Future Study: In Chapter 4 students will solve two-step inequalities in one variable. They will also write simple systems of inequalities. Later, in Algebra 1, they will solve inequalities involving absolute values.

1 Get started

Resources:
- index cards showing inequality phrases

Warm-up questions:
- Lesson 1.3.1 sheet

2 Teach

Universal access

The notation can be introduced by saying that the symbols are like an alligator's mouth — and the alligator always wants to eat the biggest "meal" possible (the greater number).

Universal access

This activity helps students learn the meanings of the inequality symbols and become more comfortable with their use.

Give each student a set of index cards with the whole numbers from 0 to 9 them. Write incomplete inequalities on the board and have students hold up a card to represent a number that would make the inequality true.
For example, if you write '__ < 3' on the board, students could hold up the cards "0," "1," or "2."

Then complete the inequality using one of the possible answers, and have a student read out the statement, first from left to right, then from right to left. For example, "1 is less than 3," then "3 is greater than 1."

Additional examples

Put the correct symbols in each of these statements:
1) 4 __ 6 <
2) 8 __ 7 >
3) 8 __ 8 =
4) If $x \le 7$, what can you say about x?
x could be less than 7, or it could be 7.

Guided practice

Level 1: q1–4
Level 2: q1–4
Level 3: q1–4

Lesson 1.3.1

Section 1.3 Inequalities

California Standards:

Algebra and Functions 1.4
Use algebraic terminology (e.g., variable, equation, term, coefficient, **inequality**, expression, constant**) correctly.**

Algebra and Functions 1.5
Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

What it means for you:
You'll learn what an inequality is, and how to show one on a number line.

Key words:
- inequality
- greater than
- less than
- equal to

Inequalities are a lot like equations. But where an equation has an equals sign, an inequality has an inequality symbol. It tells you that the two sides may not be equal or are not equal — that's why it's different from an equation.

An Inequality Does Not Have to Balance

In the last Section you saw that an **equation** is a **balanced** math sentence. The expressions on each side of the equals sign are **equal in value**. An **inequality** is a math sentence that **doesn't have to be balanced**. The expression on one side **does not** have to have the **same value** as the expression on the other.

$x \le 5$, $10 > 3y$, $4h \ge 19$, and $k < 5$ are all inequalities.

Inequalities are made up of **two expressions** that are separated by one of the four **inequality symbols**:

The Inequality Symbols

<	means	"Less than."
>	means	"Greater than."
≤	means	"Less than or equal to."
≥	means	"Greater than or equal to."

The **symbol** that you use explains how **the two expressions relate** to each other.

So **2 < 10** means "**two is less than ten**"
and $x \ge 5$ means "**x is greater than or equal to five.**"

The **smaller end** of the symbol always points to the **smaller number**. So $x < 2$ and $2 > x$ are telling you the same thing — that the variable **x is a number less than 2**.

Guided Practice

Fill in the blanks in the statements in Exercises 1–4.

1. If $a < b$ then b__a. >
2. If $m \ge n$ then n__m. ≤
3. If $c > d$ then d__c. <
4. If $j \le k$ then k__j. ≥

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Have some or all of the students arrange themselves in a row across the front of the room by age. Any students who share the same birthdate can stand in front of each other. Ask, for example, "How could we write mathematically…" 1) "Jose and Deandre are the same age?" (J = D) 2) "Allegra is younger than Jose?" (A < J) 3) "Myesha is older than Jose?" (M > J).

- **English Language Learners**
  Phrases such as "6 less than 10" have been used previously to indicate subtraction. Where inequalities are concerned the words "less than" represent a symbol. Be aware that this often creates confusion for English language learners when translating verbal statements to inequalities. Give lots of examples involving "less than" to reinforce this meaning.

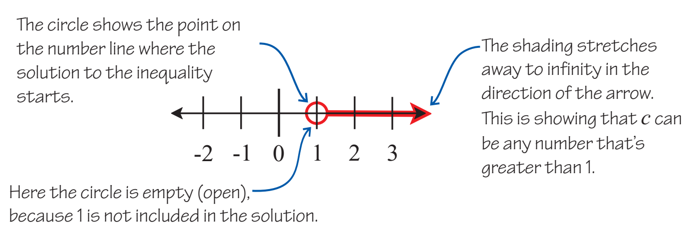
Plot the Solutions to an Inequality on a Number Line

An inequality has an **infinite number** of solutions. When you solve an inequality you are describing a group or **set of solutions**.

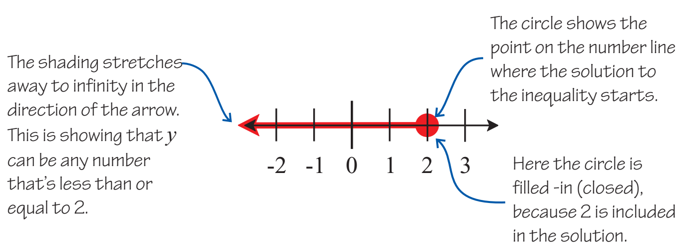
For example, for the inequality $x < 9$, **any** number that is **less than 9** is a solution of the inequality. The solutions are **not** limited to just whole numbers or positive numbers.

All the possible solutions of an inequality can be shown on a **number line**.

The following number line shows the solution of the inequality $c > 1$. The set of possible solutions for c is all the numbers greater than 1.



The number line below shows the solution of the inequality $y \le 2$. The set of possible solutions for y is 2 and all the numbers less than 2.



To plot an inequality on a number line:

1) Draw a circle on the number line around the point where the set of solutions starts. It should be a **closed circle** if the point is **included** in the solution set, but an **open circle** if it **isn't**.
2) Draw a **ray** along the number line in the direction of the numbers in the solution set. Add an **arrowhead** at the end to show its direction (and that it goes on forever).

Check it out:
A number line is a line that represents every number, with the numbers increasing in value along the line from left to right. To give the line a scale, some numbers are shown as labeled points spaced out evenly along the line.

Don't forget:
It's important to remember that there are an infinite amount of decimal numbers between each labeled point in the solution set. All of these are solutions too.

Don't forget:
A ray is just a straight line that begins at a point and goes on forever in one direction.

2 Teach (cont)

Concept question
"x is an integer greater than 3. Write four different statements using each of the inequality symbols to represent this relationship."

$x > 3$, $x \ge 4$, $3 < x$, $4 \le x$

Universal access
Using a number line, write inequality statements using random integers, mixed numbers, and decimal numbers. Model the language alternatives available to describe the statements.

Additional examples
Decide whether the circle would be open or closed if these inequalities were plotted on a number line:

1) $k \le -4$ **Closed**

2) $f > -3$ **Open**

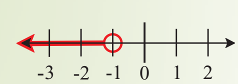
3) $s < 0$ **Open**

4) $c \ge -1$ **Closed**

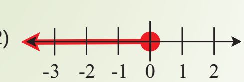
Concept question
"How does the number of solutions in an inequality differ from the number of solutions in a linear equation such as $2x + 1 = 5$?"

An inequality has an infinite number of solutions. A linear equation has at most one solution.
(Note that sometimes linear equations don't have any solutions — for instance, the equation $x + 2 = x + 3$.)

Additional examples
The solution sets of some inequalities are shown below. Give the inequalities.

1)


$x < -1$

2)


$x \le 0$

- **Advanced Learners**
  Ask students to write inequality statements to represent real-life situations. For instance, a school rule might state that only students in eighth grade or above can leave the school grounds in the lunch period. Provide them with the opportunity to research and represent other facts in similar ways. For example, mature ponies are 14.2 hands high or smaller at the shoulder (pony height ≤ 14.2 hands), while mature horses are taller than 14.2 hands at the shoulder (horse height > 14.2 hands).

2 Teach (cont)

Math background

Inequalities are used in establishing age limits. For example:

"You must be 16 or older to drive."

"You must be 18 or older to vote."

"Children's meal for those 12 and under."

"Senior discount for those 60 and older."

The students' attention will be drawn to real-life applications of inequalities such as these in the next Lesson.

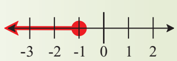
Additional examples

Plot the following inequalities on number lines:

1) $a < 0$

2) $b > 0$

3) $c \le -1$



4) $d \ge -1$

Guided practice

Level 1: q5–8
Level 2: q5–8
Level 3: q5–8

Independent practice

Level 1: q1–8
Level 2: q1–11
Level 3: q1–11

Additional questions

Level 1: p434 q1–6
Level 2: p434 q1–11
Level 3: p434 q1–14

3 Homework

Homework Book — Lesson 1.3.1

Level 1: q1, 2, 4a–c, 5a–b
Level 2: q1–5
Level 3: q1–6

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
- Worksheet 27 — Inequalities

Don't forget:

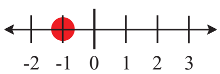
If the sign is ≥ or ≤ then you should use a closed circle to show that the start point is included in the solution.
If the sign is > or < then you should use an open circle to show that the start point isn't included in the solution.

Example 1

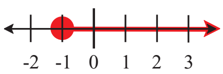
Plot the solution to the inequality $y \ge -1$ on a number line.

Solution

First place a closed circle at –1 on the number line to show that –1 is included in the solution set.



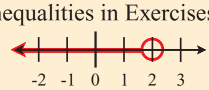
y is greater than or equal to –1, so add a ray with an arrowhead pointing along the number line to the right of the circle.



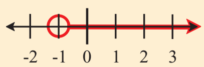
Guided Practice

Plot the inequalities in Exercises 5–8 on the number line.

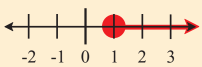
5. $h < 2$



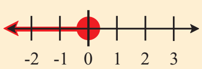
6. $-1 < a$



7. $k \ge 1$



8. $t \le 0$



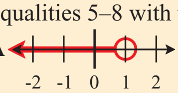
Independent Practice

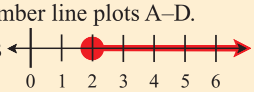
In Exercises 1–4, give a number that is part of the solution set of the inequality.

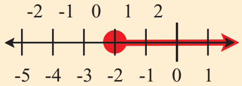
1. $m < -3$ **Any number less than –3.** **2.** $k \le 12$ **12, or a number less than 12.**
3. $w \ge 9$ **9, or a number greater than 9.** **4.** $h > -9$ **Any number greater than –9.**

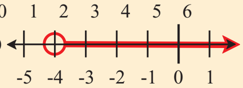
Match the inequalities 5–8 with the number line plots A–D.

5. $x > -4$
6. $x \ge 2$
7. $x < 1$
8. $x \ge -2$

A 

B 

C 

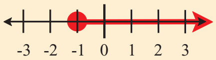
D 

5. = D
6. = B
7. = A
8. = C

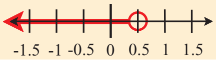
9. Explain why the solution of an inequality must be graphed on a number line and not listed. **An inequality has an infinite number of solutions. It would not be possible to list them all.**

Plot the inequalities in Exercises 10–11 on number lines.

10. $-1 \le x$



11. $x < 0.5$



Round Up

An inequality is like an unbalanced equation — its two expressions can have different values. You can show all the possible solutions of an inequality by graphing it on a number line. Don't forget the four symbols — you'll need them to write and solve inequalities later.

Now try these:

Lesson 1.3.1 additional questions — p434

Solutions

For worked solutions see the Solution Guide

Lesson 1.3.2 Writing Inequalities

In this Lesson, students translate verbal descriptions into algebraic statements with inequality symbols. They also practice using inequalities to represent real-life situations.

Previous Study: Last Lesson, students reviewed the symbols < and >, and were introduced to the symbols ≤ and ≥. They also learned to show inequalities on a number line.

Future Study: In Chapter 4 students will solve two-step inequalities in one variable. They will also write simple systems of inequalities. Later, in Algebra 1, they will solve inequalities involving absolute values.

Lesson 1.3.2 Writing Inequalities

California Standards:
Algebra and Functions 1.1
Use variables and appropriate operations to write an expression, an equation, **an inequality**, or a system of equations or inequalities **that represents a verbal description** (e.g., three less than a number, half as large as area A).

What it means for you:
You'll learn how to turn a word problem into an inequality.

Key words:
- inequality
- under
- over
- minimum
- maximum

In Lesson 1.2.2 you saw how to write an equation from a word problem. To write an inequality you use exactly the same process — but this time instead of joining the two expressions with an equals sign, you use an inequality symbol.

You Need to Spot Which Symbol is Being Described

To write an **inequality** you write two **expressions** that have (or can have) **different values** and join them with one of the four **inequality symbols**. You need to be able to recognize phrases that **describe** the four symbols.

> means "greater than" or "more than" or "over."
< means "less than" or "under."
≥ means "greater than or equal to" or "a minimum of" or "at least."
≤ means "less than or equal to" or "a maximum of" or "no more than."

Example 1

Write an inequality to describe the sentence,
"Four times a number, y, is less than 27."

Solution

The phrase "**is less than**" is represented by the **less than symbol**, <.
It separates the **expressions** that make up the two sides of the **inequality**.

One expression is "**Four times a number, y.**"
This turns into the expression **$4y$**.

The other expression is a number, **27**.

So the sentence, "Four times a number, y, is less than 27," turns into the inequality $4y < 27$.

Don't forget:
The inequality $27 > 4y$ means exactly the same thing as $4y < 27$.

Example 2

Write an inequality to describe the sentence,
"A number, h, increased by two is at least 16."

Solution

The phrase "**at least**" is represented by the **greater than or equal to symbol**, ≥.

One expression is "**A number, h, increased by two**." This turns into the expression **$h + 2$**.

The other expression is a number, **16**.

So the sentence, "A number, h, increased by two is at least 16," turns into the inequality $h + 2 \geq 16$.

Don't forget:
The inequality $16 \leq h + 2$ means exactly the same thing as $h + 2 \geq 16$.

1 Get started

Resources:
Teacher Resources CD-ROM
- Number Line

Warm-up questions:
- Lesson 1.3.2 sheet

2 Teach

Concept question
"Explain how the phrase 'is more than' differs from the phrase 'is a minimum of' when used in inequalities."

"Is more than" means that the number used in the situation or statement isn't included. The word "minimum" means that the number in the situation or statement is the smallest possible solution to the problem.

Additional examples
Translate these sentences into inequalities:

1) Seven less than a number, m, is greater than 40.
$m - 7 > 40$

2) Ten times a number, n, is less than or equal to 16.
$10n \leq 16$

3) Thirteen is less than eight more than a number, k.
$13 < k + 8$

4) The reciprocal of w is greater than or equal to nine. $\frac{1}{w} \geq 9$

- **Strategic Learners**
  Prepare number line templates for students to use. When they are writing inequalities to represent a situation, ask them to think of values that satisfy the situation, and mark these on the number line. They should then consider which number all the values that they've marked are above or below. The activity below for English language learners may be useful for strategic learners too.

- **English Language Learners**
  Using a number line, play "I'm thinking of a number." Ask questions that are phrased in terms of "greater than" and "less than." For instance, "Is the number greater than or equal to 6?" Have students write the inequalities. Repeat this as a partner activity.

2 Teach (cont)

Guided practice

Level 1: q1–3
Level 2: q1–4
Level 3: q1–5

Universal access

Introduce the writing of inequalities by posting different signs around the room. For example, "You must be at least 48 inches tall to ride this ride," "Maximum capacity 100 people," or "Play park for kids under 10 only." Have students walk around the room and write down numbers that will satisfy the requirements.

Once this exercise is completed, ask questions about what will work. For instance, "Can I be 47 and seven-eighths inches tall and go on the ride?", or, "Can I ride if I am exactly 48 inches tall?"

Additional example

Phil is seven years younger than his brother Nathan. Phil can drive, so he must be at least 16 years old. Write an inequality that could be used to determine Nathan's age.

Let Nathan's age = n
Phil's age = $n - 7$
$n - 7 \geq 16$
(or $n \geq 23$)

Common error

Many students have difficulty selecting the correct inequality symbol when translating situations into inequalities. For example, a > symbol is used when a situation requires a ≥ symbol.

Have students check the number in the situation to see if it is included in the solution. For example, in the situation, "Toy suitable for children 3 and up," would the toy be suitable for a child of 3? This questioning method helps students select the correct symbol.

Guided Practice

Write an inequality to describe each of the sentences in Exercises 1–5.

1. A number, x, increased by five is more than 12. **$x + 5 > 12$**

2. Twice a number, k, is greater than or equal to two. **$2k \geq 2$**

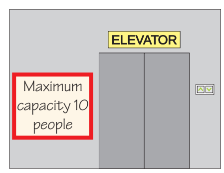
3. Fifteen decreased by a number, g, is no more than six. **$15 - g \leq 6$**

4. A number, p, divided by two, is under four. **$p \div 2 < 4$**

5. Negative two is less than the sum of a number, m, and five. **$-2 < m + 5$**

Inequalities are Often Used in Real-Life Situations

You'll come across lots of inequality phrases in real life.



"Number of people" ≤ 10

"Height" > 4 feet

"Age" ≤ 8

In math you might be asked to write an **inequality** to represent the information given in a **word problem**.
Writing inequalities from word problems is a lot like writing equations from word problems. You need to spot key information and use it to write expressions — but you also need to work out which **inequality symbol** to use.

Example 3

Your local conservation group runs a junior award program. To get a gold award you must complete a minimum of 50 hours' conservation work. You have already done 17 hours. Write an inequality to represent the additional amount of work you need to do to gain your gold award.

Solution

First define a variable: **let the additional amount of hours you need to complete = H**.

The amount of hours you need to complete to get your award is 50.
So one expression is just **50**.

The other expression describes the number of hours you have already done plus the additional number you need to spend — the variable H.
So the expression is **$17 + H$**.

The phrase "**minimum**" tells you that the number of hours you complete has to be **greater than or equal to** 50. So the inequality is **$17 + H \geq 50$**.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
Extend the basic idea of inequalities to two-part inequalities. For instance, positive integers less than 7 can be represented by the inequality "$0 < x < 7$." Ask students to devise situations that can be represented by two-part inequalities.

Guided Practice

Write an inequality to describe each of the situations in Exercises 6–9.

6. Lauren is three years younger than her friend Gabriela, who is k years old. Lauren is under 20. $k - 3 < 20$

7. Erin has visited b states. Kieran has visited two more states than Erin, and figures out that he has visited at least 28. $b + 2 \geq 28$

8. The number of boys enrolled at a university is half the number of girls, g, who are enrolled. The number of boys enrolled is more than 2000. $g \div 2 > 2000$

9. Pedro has set aside a maximum of $100 in order to buy gifts for his family. He wants to spend the same amount, $$x$, on each of his 3 family members. $3x \leq 100$

Don't forget:

You can write all of these inequalities in the reverse direction just by changing the sign you use.
For example: $a > b$ is the same as $b < a$.

Independent Practice

Write an inequality to describe each of the sentences in Exercises 1–5.

1. A number, m, decreased by seven is less than 16. $m - 7 < 16$
2. Nine more than a number, d, is at least 11. $d + 9 \geq 11$
3. The product of ten and a number, j, is a maximum of six. $10j \leq 6$
4. Four is more than a number, y, divided by five. $4 > y \div 5$
5. The sum of a number, f, and six is less than negative one. $f + 6 < -1$

6. Explain whether the two statements "six more than a number is at least four" and "six more than a number is more than four" mean the same. They are not the same. The first statement includes the number four, the second does not.

Write an inequality to describe each of the situations in Exercises 7–9.

7. Alex and Mallory both spent time cleaning the house. Alex spent y minutes cleaning. Mallory spent 15 minutes less than Alex, but over 55 minutes, cleaning. $y - 15 > 55$

8. Rebecca has a maximum of $40 to spend on her cat. She buys a collar for $17 and then spends $$d$ on cat food. $17 + d \leq 40$

9. Alejandra's collection of baseball cards is twice the size of Jordan's. Alejandra has collected at least 2000 cards, and Jordan has collected c baseball cards. $2c \geq 2000$

For each sentence in Exercises 10–12 say which inequality symbol would be used.

10. Maximum weight on this bridge is six tons. $\leq$ or less than or equal to.
11. The play park is for people under ten years old only. $<$ or less than.
12. This toy is for children aged three years and up. $\geq$ or greater than or equal to.

Now try these:

Lesson 1.3.2 additional questions — p434

Round Up

When you turn a word problem into an inequality, the key thing is to figure out which of the four inequality symbols is being described. Then just write out the two expressions and join them with the correct symbol. You'll see how to solve inequalities like the ones you've written in Chapter 4.

2 Teach (cont)

Guided practice

Level 1: q6–7
Level 2: q6–8
Level 3: q6–9

Additional example

Rachel and Justin have both spent time studying for their economics test. Rachel studied 10 minutes longer than Justin. Rachel spent no more than 45 minutes studying for the test.

Write an inequality that could be used to determine the number of minutes Justin studied.

Let Justin's studying time = j
Rachel's studying time = $j + 10$
$j + 10 \leq 45$
(or $j \leq 35$)

Independent practice

Level 1: q1–6, 10–12
Level 2: q1–7, 10–12
Level 3: q1–12

Additional questions

Level 1: p434 q1–6
Level 2: p434 q1–9
Level 3: p434 q1–11

3 Homework

Homework Book — Lesson 1.3.2

Level 1: q1–6
Level 2: q1–11
Level 3: q1–11

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 27 — Inequalities

Solutions

For worked solutions see the Solution Guide

Lesson 1.3.3

Two-Step Inequalities

In this Lesson, students use their understanding of inequalities and the order of operations to write two-step inequalities. After writing two-step inequalities to represent verbal descriptions, students will move on to writing them in response to word problems.

Previous Study: Over the last two Lessons, students have written one-step inequalities in response to word problems. They have also shown inequalities on a number line.

Future Study: In Chapter 4 students will solve two-step inequalities in one variable. They will also write simple systems of inequalities. Later, in Algebra 1, they will solve inequalities involving absolute values.

1 Get started

Resources:
- individual whiteboards

Warm-up questions:
- Lesson 1.3.3 sheet

2 Teach

Concept question

"Place parentheses in the inequalities below so that they are true."

1) $4 \cdot 2 + 3 \geq 20$

$4 \cdot (2 + 3) \geq 20$

2) $6 + 12 \div 6 < 2 \cdot 8 - 9$

$(6 + 12) \div 6 < 2 \cdot 8 - 9$

Universal access

Writing inequalities can be approached by a four-step method. Using this method forces students to read the situation at least three times.

Step 1: Read description and look for operational keywords (for example, add, subtract, multiply, divide).

Step 2: Read description again and look for inequality symbol keywords (such as, less than, maximum).

Step 3: Read description and define variables.

Step 4: Write inequality.

Lesson 1.3.3

Two-Step Inequalities

California Standards:

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression, an equation, **an inequality**, or a system of equations or inequalities **that represents a verbal description** (e.g., three less than a number, half as large as area A).

What it means for you:

You'll learn how to turn a word problem into a two-step inequality.

Key words:
- inequality
- under
- over
- minimum
- maximum

Don't forget:

The four inequality symbols:
$>$ is greater than.
$<$ is less than.
$\geq$ is greater than or equal to.
$\leq$ is less than or equal to.

An inequality with two different operations in it is called a two-step inequality. To write a two-step inequality, follow the same steps that you learned in the last Lesson. The only difference this time will be that one of your expressions could have two operations in it.

Two-Step Inequalities Have Two Operations

A **two-step inequality** is one that involves **two different operations**.

inequality symbol

$$4 \div x + 9 > 10$$

first operation second operation

It has the same structure as a **two-step equation**, but with an **inequality symbol** instead of an equals sign.

Writing a Two-Step Inequality

Writing a **two-step inequality** is like writing a one-step inequality. You still need to write out your two expressions and join them with the correct **inequality symbol** — but now one of the expressions will contain **two operations** instead of one. That also means that you need to remember to use **PEMDAS** — the order of operations.

Example 1

Write an inequality to describe the sentence,
"Six more than the product of four and a number, h, is under 42."

Solution

The phrase "**is under**" represents the **less than** symbol. It also separates the two **expressions** that make up the two sides of the inequality.

One expression is, "**Six more than the product of four and a number, h.**" This tells you to multiply four by h and add six to the product. It turns into the expression $4h + 6$.

The other expression is the number **42**.

So the sentence, "Six more than the product of four and a number, h, is under 42," turns into the inequality $4h + 6 < 42$.

- **Strategic Learners**
  Play "I'm thinking of a number" using individual whiteboards to practice translating two-step inequality statements. For example, say, "I'm thinking of a number. If you double it and add four, the sum will be less than 10." Students write "$2n + 4 < 10$."

- **English Language Learners**
  Provide students with tables of the keywords for operations, and also the meanings of the inequality symbols. Encourage the students to annotate the tables with their own notes and their own examples.

2 Teach (cont)

Guided practice
Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Additional examples
1) A handyman quoted that the cost for installing a bathroom would be at most $4000. Materials would cost $1500 and the rate for labor would be $35 an hour. Write an inequality that could be used to determine the maximum amount of time required to install the bathroom.

Let the number of hours = h
$1500 + 35h \le 4000$

2) A length of fabric is to be cut into three equal lengths to make curtains. Each curtain requires a 3-inch seam allowance. Each finished curtain is to be at least 32 inches long. Write an inequality that could be used to determine the minimum length of the original piece of fabric.

Let length of fabric = l
$\frac{l}{3} - 3 \ge 32$

Guided Practice

Write an inequality to describe each of the sentences in Exercises 1–6.

1. Five increased by the product of ten and a number, m, is more than 11. **$5 + 10m > 11$**

2. Two plus the result of dividing a number, k, by six is at least five. **$2 + \frac{k}{6} \ge 5$**

3. One subtracted from the product of a number, y, and nine is less than or equal to 33. **$9y - 1 \le 33$**

4. Ten subtracted from half of a number, t, is under –1. **$\frac{t}{2} - 10 < -1$**

5. A third of a number number, r, plus nine, is no greater than –4. **$\frac{r}{3} + 9 \le -4$**

6. Double the sum of a number, x, and two is less than 20. **$2(x + 2) < 20$**

An Inequality Can Describe a Word Problem

To write a two-step inequality from a word problem, just follow the same rules as for a one-step inequality:

- Identify the important information you have been given.
- Spot which operation phrases are being used.
- Work out what the two expressions are.
- Join them using the correct inequality symbol.

Example 2

Hector needs to save at least $250 to buy a new bicycle. He already has $80, and receives $10 each week for mowing the neighbor's lawn. Write an inequality to represent the number of weeks that Hector will need to save for.

Solution

First define a variable: **let the number of weeks Hector needs to save for = w**.

The minimum amount of money Hector needs to save is $250. So one expression is just **250**.

The other expression describes the amount of money he will have after w weeks. This will be the $80 he already has plus the number of weeks multiplied by the $10 he earns each week. So the expression is **$80 + 10w$**.

The phrase "**at least**" is telling you that the amount Hector needs to save has to be **greater than or equal to** 250. So the inequality is **$80 + 10w \ge 250$**.

Don't forget:
To make sure your expression is right you can check it with a simple number using mental math.
For example: after 2 weeks Hector will have $80 plus two times $10. This is $100.
Then put $w = 2$ into your expression. $80 + 10(2) = 100$.
So your expression is likely to be right.

Solutions
For worked solutions see the Solution Guide

● **Advanced Learners**
Ask students to think of other real-life situations that can be represented using two-step inequalities. They should write out the situations carefully, together with the corresponding inequalities. They can then exchange their written situations with a partner, and should write inequalities to match each other's descriptions. Finally, encourage them to compare the inequalities they've written and discuss any differences that exist between them (it's often possible to write inequalities in different ways).

2 Teach (cont)

Guided practice
Level 1: q7–8
Level 2: q7–9
Level 3: q7–10

Concept question
"Do the following inequalities mean the same thing? Explain your answer."

$x > y + 2$ and $y < x - 2$

Yes. Both inequalities are telling you that x is bigger than y by more than two.

Independent practice
Level 1: q1–5
Level 2: q1–6
Level 3: q1–7

Additional questions
Level 1: p434 q1–4
Level 2: p434 q1–6
Level 3: p434 q1–7

3 Homework

Homework Book — Lesson 1.3.3
Level 1: q1–4, 6, 7
Level 2: q1–8
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 27 — Inequalities

Guided Practice

Write an inequality to describe each of the following situations.

7. Mrs. Clark parks by a meter that charges $2 for the first hour and $0.50 for each additional hour parked. She spends no more than $10, and parks for the first hour and *h* additional hours. **$2 + 0.5h \le 10$**

8. Luis collects seashells. He has four boxes, each containing *s* shells. He gives 40 shells to Jon, and still has more than 200 shells in his collection. **$4s - 40 > 200$**

9. Marcia is buying new shirts that cost $15 each for *x* people in her Little League team. She has a coupon for $12 off her order, and a maximum of $150 to spend. **$15x - 12 \le 150$**

10. Daniel's teacher tells him that to be considered low-fat, a meal must contain less than three grams of fat. Daniel prepares a low-fat breakfast of yogurt topped with pumpkin seeds for a school project. A cup of yogurt contains *y* grams of fat. Daniel uses half a cup, and tops it with pumpkin seeds containing a total of 1 gram of fat. **$(y \div 2) + 1 < 3$**

Independent Practice

Write an inequality to describe each of the sentences in Exercises 1–4.

1. Twenty more than twice a number, *f*, is less than 35. **$2f + 20 < 35$**

2. Fifty subtracted from a quarter of a number, *n*, is at least 77. **$(n \div 4) - 50 \ge 77$**

3. Eight increased by the product of four and a number, *d*, is no more than 13. **$8 + 4d \le 13$**

4. Negative eighteen is more than a number, *a*, divided by 41, minus six. **$-18 > (a \div 41) - 6$**

Now try these:
Lesson 1.3.3 additional questions — p434

5. Vanessa has ordered a meal that costs under $12. She is having a baked potato, costing $7, and *d* salads costing $2 each. Write an inequality to describe this information. **$7 + 2d < 12$**

6. Filipa and her four friends are looking for a house to rent for a vacation. The price of an airplane ticket is $200, and they will split the cost of the house rental, $*r*. Filipa has budgeted $600 for the airplane ticket and house. Write an inequality to determine the maximum rental price that the house can be. **$(r \div 5) + 200 \le 600$**

7. Tom and his two friends run a babysitting service that makes $*p* a month income. Each month they spend $20 to advertise, then split the remaining money evenly. Tom wants to earn at least $80 a month. Write an inequality to describe how much income the service must make each month for this to happen. **$(p - 20) \div 3 \ge 80$**

Round Up

Writing a two-step inequality is just like writing a one-step inequality. You still need to look for the key information in the question, identify the operation phrases, and spot which inequality symbol is needed. But this time one of the expressions might contain two operations. You'll see how to solve two-step inequalities in Chapter 4.

Solutions
For worked solutions see the Solution Guide

Chapter 1

Investigation — Which Phone Deal is Best?

Purpose of the Investigation

This Investigation provides students with the opportunity to write expressions and evaluate them in order to solve a real-life problem. Students use a variety of methods to identify the best cell phone plan for different circumstances. The Extensions involve students writing inequalities to represent a problem.

Chapter 1 Investigation
Which Phone Deal is Best?

You can write expressions to model real-life situations. In this Investigation, you'll write expressions to represent different cell phone plans, and by evaluating your expressions you'll find out which is the best value plan for different users.

Two phone companies are offering **different family plan deals** to attract new customers.

Company A	Company B
\$30 a month for 500 minutes \$0.02 for each additional minute	\$10 a month for 500 minutes \$0.04 for each additional minute

Part 1:
Write **expressions** for Companies A and B that could be used to represent the **price of one month's phone bill**.

Which company offers the **better plan** for a family using **1000 minutes** a month?

Part 2:
How many minutes does a family have to talk so that Company A offers a **better deal** than Company B?

Things to think about:

- How can you **compare** the prices of both companies for different numbers of minutes?
- The **basic price** for Company B's plan is **\$20 more** than Company A's plan. Thinking just about cost, why would a family choose Company A's plan instead of Company B's?

Extensions

1) Write an inequality that could be used to calculate the number of minutes a family could talk with Company A if they want to spend under \$35 a month.
2) The Sutro family uses Company A and talks an average of 800 minutes a month. How much will they save over a year by switching to Company B?

Open-ended Extensions

1) Is it possible for the price of Company A's plan to be double the price of Company B's plan? Assume calls are charged to the nearest minute. Make an organized list or table to compare them.
2) Company C wants to charge a flat per minute fee and have their price lie between Company A's and Company B's prices when between 500 and 750 minutes are used. What per minute fees could Company C charge to accomplish this goal?

Round Up

A number that can change is called a variable and is represented with a letter. In the cell phone plans, the variable was the number of minutes used. By evaluating expressions with different values for the variable, you can find the prices when different numbers of minutes are used.

Strategic & EL Learners

Ask strategic learners to start by calculating the cost of Company B's plan when 600, 700, 800, 900 and 1000 minutes are used. Ask them to explain what they are doing in each calculation. This will help them to link their calculation to an algebraic expression.

Using this approach with EL learners will allow you to check their understanding of the problem, and of terms such as "additional."

Investigation Notes on p53 B-C

Investigation — Which Phone Deal is Best?

Mathematical Background

Algebraic expressions can be used to model real-life situations. When a quantity can have a range of different values, it is known as a variable. It is assigned a letter, which can be substituted with different values and the expression evaluated.

For example, in the expression, "$6x + 2$," x is the variable. You can substitute it for any value and find the value of the expression. If $x = 3$, the value of the expression will be: $(6 \times 3) + 2 = 18 + 2 = 20$.

Inequality symbols can be used to show that an expression is greater than, or less than another value.
For example, $6x + 2 > 16$, means $6x + 2$ is greater than 16. And $6x + 2 < 19$, means $6x + 2$ is less than 19.
Alternatively, the signs $\geq$ and $\leq$ can be used to indicate that an expression is "greater than or equal to" something, or "less than or equal to" something.

Approaching the Investigation

Part 1

This part asks students to write expressions to represent the prices of one month's phone bill for two different plans.
Each plan consists of a price for the first 500 minutes, and then a price for each additional minute used above this.
So: monthly bill = cost for first 500 minutes + cost of additional minutes.
The "additional minutes" is the number after the first 500. So if the total number of minutes used is x,
then the **number of additional minutes = x – 500**. (This only works when more than 500 minutes are used.)

For Company A the cost is \$30 a month for 500 minutes, then \$0.02 for each additional minute.
So, for **x > 500**, monthly bill = $\$30 + \$0.02(x - 500)$
$= \$30 + \$0.02x - \$10$
$= \mathbf{\$20 + \$0.02x}$
For **$0 \leq x \leq 500$, monthly bill = \$30**

For Company B the cost is \$10 a month for 500 minutes, then \$0.04 for each additional minute.
So, for **x > 500**, monthly bill = $\$10 + \$0.04(x - 500)$
$= \$10 + \$0.04x - \$20$
$= -\$10 + \$0.04x$
$= \mathbf{\$0.04x - \$10}$
For **$0 \leq x \leq 500$, monthly bill = \$30**

(Students might use a variable to represent the number of additional minutes instead.
For example, let y = number of additional minutes
For Company A: monthly bill = $\$30 + \$0.02y$, and for Company B: monthly bill = $\$10 + \$0.04y$
To use these formulas, you have to first subtract 500 from the total number of minutes. If less than 500 minutes are used, then $y = 0$.)

To find which is the best value for a family using 1000 minutes, the students need to substitute 1000 for the variable representing the total number of minutes in both expressions for $x > 500$. They can then evaluate the expressions to find out the cost of each plan, and see which is lower.

Company A: Monthly bill $= \$20 + \$0.02x$
$= \$20 + (\$0.02 \times 1000)$
$= \$20 + \$20 = \$40$

Company B: Monthly bill $= \$0.04x - \$10$
$= (\$0.04 \times 1000) - \$10$
$= \$40 - \$10 = \$30$

For 1000 minutes, **Company B** has the lower price.

Part 2

This asks students how many minutes a family must talk so that Company A offers the better deal. Students may choose to do this by trial and error.
For example, they may calculate the cost of each plan for various numbers of minutes.

This approach shows that Company B's plan starts off the least expensive, because it has a lower fixed fee for the first 500 calls. But its price rises more quickly because its additional minutes are more expensive. The price is the same for both companies if 1500 minutes are used. For anything above this, Company A is less expensive.

So, a family has to talk for more than 1500 minutes for Company A's plan to be the better deal.

Minutes used	Price with Company A (\$)	Price with Company B (\$)
500	30	10
600	32	14
700	34	18
800	36	22
900	38	26
1000	40	30
1100	42	34
1200	44	38
1300	46	42
1400	48	46
1500	50	50
1600	52	54

Investigation — Which Phone Deal is Best?

Extensions

1) A family's monthly phone bill with Company A is: **$20 + 0.02x$**, where x is the number of minutes used (and $x > 500$).
If the family wanted their phone bill to be less than $35 a month, then **$20 + 0.02x$ < $35, for $x > 500$**.
For $0 \leq x \leq 500$, monthly bill = $30, which is less than $35.

2) To find out the amount of money that the Sutro family would save each year if they switched to Company B's plan, you can calculate the difference in the monthly costs for 800 minutes, and then multiply it by 12, to find the saving over one year.

Company A: Monthly bill = $20 + 0.02$x$
= $20 + (0.02 × 800)
= $36

Company B: Monthly bill = 0.04x$ – $10
= ($0.04 × 800) – $10
= $22

The monthly saving with Company B is $36 – $22 = $14.
Over a year, the saving would be $14 × 12 = **$168**.

Open-Ended Extensions

1) The first Open-ended Extension essentially ask students whether there is a whole number of minutes that will make one plan double the price of the other. Looking at the table on the right, you can see that Company A's price is more than half of Company B's price for 600 minutes, but less than half for 700 minutes. This means that it'd be exactly double, somewhere in between 600 and 700 minutes.

Minutes used	Price with Company A ($)	Price with Company B ($)
500	30 (more than double B's price)	10
600	**32 (more than double B's price)**	**14**
700	**34 (less than double B's price)**	**18**
800	36 (less than double B's price)	22

Students can try different numbers of minutes to see if Company A's price is ever double Company B's. Each try narrows the possible numbers of minutes.

Minutes used	Price with Company A ($)	Price with Company B ($)
650	33 (more than double B's price)	16
675	33.50 (less than double B's price)	17

Company A's price would be double Company B's between 650 and 675 minutes.

Minutes used	Price with Company A ($)	Price with Company B ($)
660	33.20 (more than double B's price)	16.40
670	33.40 (less than double B's price)	16.80

Company A's price would be double Company B's between 660 and 670 minutes.

Minutes used	Price with Company A ($)	Price with Company B ($)
665	33.30 (more than double B's price)	16.60
666	33.32 (more than double B's price)	16.64
667	33.34 (less than double B's price)	16.68

Company A's price would be double Company B's between 666 and 667 minutes.

There is no whole number of minutes that falls between 666 and 667, so Company A's price is never double Company B's.

Advanced learners may be interested to see how this could be solved algebraically.
If Company A's monthly bill is double Company B's, then:
$20 + 0.02$x$ = 2(0.04x$ – $10)
⇒ $20 + 0.02$x$ = 0.08x$ – $20
⇒ $20 + $20 = 0.08x$ – 0.02x
⇒ $40 = 0.06x$
⇒ x = $40 ÷ $0.06 = 666.67 minutes

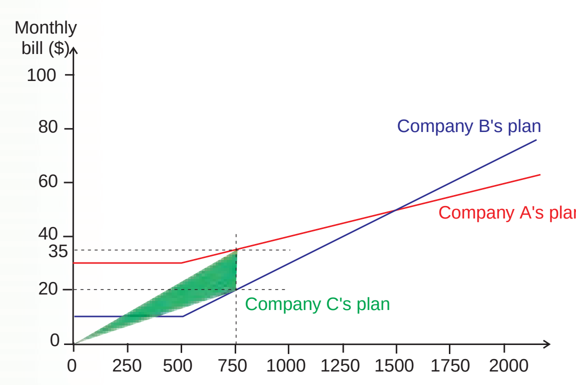
This isn't a whole number, so Company A's bill can **never** be double Company B's, if only whole minutes are billed.

2) This extension asks students to propose a per minute price for Company C's plan, so that its monthly bill lies between Company A's and Company B's when between 500 and 750 minutes are used.

This can be done by trial and error, or alternatively, by drawing a graph. The green area on the graph shows the region where Company C's bills must lie. They must pass though the origin, because the bill will be $0 when no minutes are used.

The cost of using 750 minutes must be between $20 and $35.
So the per minute cost must be between 20 ÷ 750 = $0.0267, and, 35 ÷ 750 = $0.0467.

If the per minute rate is to be a whole-cent, it must be **$0.03 or $0.04 per minute**.



Chapter 2

Rational and Irrational Numbers

How Chapter 2 fits into the K-12 curriculum

Section 2.1 — Rational Numbers		
Section 2.1 covers Number Sense 1.3, 1.4, 1.5 **Objective: To understand rational numbers and to convert terminating and repeating decimals to fractions**		
Previous Study Since grade 3 students have learned how integers, fractions and decimals are related. In grade 6 they learned the decimal equivalents of common fractions.	**This Section** Students are introduced to the concept of rational numbers. They then go on to convert terminating decimals to fractions, and finally to convert repeating decimals to fractions too.	**Future Study** In Algebra I students will be expected to be proficient at handling repeating decimals and converting between decimals and fractions in order to simplify calculations.

Section 2.2 — Absolute Value		
Section 2.2 covers Number Sense 2.5, Algebra and Functions 1.1 **Objective: To understand the concept of absolute value and to use absolute values to model real-life situations**		
Previous Study In grade 4 students learned to distinguish between positive and negative numbers. In grades 5 and 6 students learned to use negative numbers in calculations.	**This Section** Students are introduced to the concept of absolute value and learn how to apply it to positive and negative numbers. They then solve real-life problems that involve evaluating absolute values.	**Future Study** In Algebra I students will write and solve equations and inequalities that use absolute values. They will also write and solve absolute value equations based on word problems.

Section 2.3 — Operations on Rational Numbers		
Section 2.3 covers Number Sense 1.1, 1.2, 2.2 **Objective: To add, subtract, multiply, and divide integers and fractions**		
Previous Study In grade 5, students first learned how to add, subtract, multiply, and divide integers, decimals, and fractions. In grade 6 they learned to simplify fractions.	**This Section** Students add, subtract, multiply, and divide integers and simple decimals, and then apply similar techniques to fractions. They then cover common denominators and simplifying.	**Future Study** In Algebra I students will add, subtract, multiply, and divide expressions containing all types of rational numbers.

Section 2.4 — More Operations on Rational Numbers		
Section 2.4 covers Number Sense 1.2, 2.2, Mathematical Reasoning 2.2 **Objective: To multiply and divide decimals and to solve problems involving fractions and decimals**		
Previous Study In grade 5 students solved simple multiplication and division problems involving decimals. In grade 6 they saw how to add, subtract, multiply, and divide fractions and decimals.	**This Section** Students review the order of operations in relation to fractions. They then develop their understand of multiplication and division of decimals, and apply these skills to more difficult problems.	**Future Study** In Algebra I students will solve multistep problems containing rational numbers in any form, including decimals and fractions.

Section 2.5 — Basic Powers		
Section 2.5 covers Number Sense 1.1, 1.2, Algebra and Functions 2.1 **Objective: To understand powers and to apply the order of operations correctly to problems involving powers**		
Previous Study In grade 5 students learned how to multiply a series of integers together. At grade 6 they learned how to find area and volume, which introduced the concept of square and cubic units.	**This Section** Students are introduced to powers as repeated multiplications. They then raise fractions to powers, and find the area of regular polygons using powers.	**Future Study** In Algebra I, students will be expected to know and use all the exponent rules, and will apply these rules to all rational numbers.

Section 2.6 — Irrational Numbers and Square Roots		
Section 2.6 covers Number Sense 1.4, 2.4, Mathematical Reasoning 2.7 **Objective: To understand perfect squares and their roots, and to calculate and estimate irrational roots**		
Previous Study In grade 4 students learned how to find the area of a square and related the area to the side length. Earlier in this grade students learned about rational numbers.	**This Section** Students learn about perfect squares and what square roots are. They then learn about irrational numbers, and learn that square roots of non-perfect squares are irrational.	**Future Study** In Algebra I students will learn how to take roots, and how to work with this in expressions. They will use the techniques they learn to solve quadratic equations.

Pacing Guide – Chapter 2

40- to 50-Minute Class Periods

If your class periods are 40-50 minutes, we recommend allowing **29 days** for teaching Chapter 2.
As well as the **23 days of basic teaching**, you have **6 days** remaining to allocate 6 of the 9 optional activities (to be delivered at any appropriate point during the Chapter).

The table shows the 23 teaching days as well as all of the **optional days** you may choose for Chapter 2, in the order we recommend.

Day	Lesson	Description
Section 2.1 — Rational Numbers		
1	2.1.1	Rational Numbers
2	2.1.2	Converting Terminating Decimals to Fractions
3	2.1.3	Converting Repeating Decimals to Fractions
Optional		*Assessment Test — Section 2.1*
Section 2.2 — Absolute Value		
4	2.2.1	Absolute Value
5	2.2.2	Using Absolute Value
Optional		*Assessment Test — Section 2.2*
Section 2.3 — Operations on Rational Numbers		
6	2.3.1	Adding and Subtracting Integers and Decimals
7	2.3.2	Multiplying and Dividing Integers
8	2.3.3	Multiplying Fractions
9	2.3.4	Dividing Fractions
10	2.3.5	Common Denominators
11	2.3.6	Adding and Subtracting Fractions
12	2.3.7	Adding and Subtracting Mixed Numbers
Optional		*Assessment Test — Section 2.3*
Section 2.4 — More Operations on Rational Numbers		
13	2.4.1	Further Operations With Fractions
14	2.4.2	Multiplying and Dividing Decimals
15	2.4.3	Operations With Fractions and Decimals
16	2.4.4	Problems Involving Fractions and Decimals
Optional		*Assessment Test — Section 2.4*
Section 2.5 — Basic Powers		
Optional		*Exploration — Basic Powers*
17	2.5.1	Powers of Integers
18	2.5.2	Powers of Rational Numbers
19	2.5.3	Uses of Powers
20	2.5.4	More on the Order of Operations
Optional		*Assessment Test — Section 2.5*
Section 2.6 — Irrational Numbers and Square Roots		
Optional		*Exploration — The Side of a Square*
21	2.6.1	Perfect Squares and Their Roots
22	2.6.2	Irrational Numbers
23	2.6.3	Estimating Irrational Roots
Optional		*Assessment Test — Section 2.6*
Chapter Investigation		
Optional		*Investigation — Designing a Deck*

Accelerating and Decelerating

- To **accelerate** Chapter 2, allocate fewer than 6 days to the optional material. This will give you extra days to allocate to other Chapters.
  Note that you may use the remaining optional days at the end of the 160-day course.
- To **decelerate** Chapter 2, consider allocating more than 6 days to the optional Assessment Tests, Section Explorations, or Chapter Investigation, or spend longer teaching some Lessons. Also consider preparing students for difficult Lessons by reviewing previous coverage of math topics on related Skills Review Worksheets.
  Note that decelerating Chapter 2 will result in fewer days being available for teaching other Chapters.

90-Minute Class Periods

If you are following a block schedule with 90-minute class periods, we recommend allowing **14.5 days** for teaching Chapter 2.
The basic teaching material will take up **11.5 days**, and you can allocate the remaining **3 days** to the **optional material**.
To accelerate or decelerate a block schedule, follow the same advice as given above.

Lesson 2.1.1

Rational Numbers

In this Lesson students are introduced to the concept of rational numbers and learn that all integers, terminating decimals, and repeating decimals are rational. This leads on to learning how to convert fractions to decimals.

Previous Study: Students have studied the relationship between integers, fractions, and decimals since grade 3. In grade 5 students calculated the decimal equivalents of common fractions.

Future Study: Later in Chapter 2 students will multiply and divide with fractions and decimals.

Lesson 2.1.1

California Standards:

Number Sense 1.3
Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

Number Sense 1.4
Differentiate between rational and irrational numbers.

Number Sense 1.5
Know that every rational number is either a terminating or a repeating decimal and be able to convert terminating decimals into reduced fractions.

What it means for you:
You'll meet rational numbers and see which kinds of numbers that you've already met fall into this category.

Key words:
- rational number
- irrational number
- fraction
- decimal
- terminating
- repeating

Don't forget:
A fraction can be thought of as a division. So $\frac{5}{1}$ says the same thing as 5 ÷ 1, and when you divide a number by 1, it doesn't change.

Check it out:
In the next Lesson you'll see how to turn terminating decimals into fractions.

Section 2.1
Rational Numbers

Pretty much all the numbers you've met so far are rational — positive and negative integers and fractions are all rational, as are most decimals. The only decimals that aren't rational are the ones that go on and on forever, without having a repeating pattern of digits.

All Rational Numbers Can Be Written as Fractions

You get all sorts of numbers in the **rational set**, for example 1.05, 0.3333..., $\frac{1}{2}$, and 6 are all rational. Rational numbers have all got one thing in common — they can each be written as a **simple fraction**, with an **integer on the top and a nonzero integer on the bottom**.

In formal math:

A rational number is a number that can be written as $\frac{a}{b}$, where both a and b are integers (and b is not equal to 0).

There are basically **three types** of rational numbers.

All Integers are Rational

The **integers** are the numbers in the set {..., –4, –3, –2, –1, 0, 1, 2, 3, 4, ...}.

Any integer can be written as **a fraction over 1**.

So integers are all **rational**. For example: $7 = \frac{7}{1}$ $\quad -18 = \frac{-18}{1}$

Example 1

Show that 5 is a rational number.

Solution

5 can be written as $\frac{5}{1}$. This fits the above definition, so 5 must be rational.

All Terminating Decimals are Rational

Numbers like 1.2, 5.689, –3.72, and –0.69245 are known as **terminating decimals** — they all have definite ends.

All terminating decimals can be converted to fractions of the form $\frac{a}{b}$, where a and b are both integers. So all terminating decimals are **rational**.

For example, 1.2 is equivalent to $\frac{6}{5}$, 0.125 is equivalent to $\frac{1}{8}$, and 0.75 is equivalent to $\frac{3}{4}$.

1 Get started

Resources:
- large copies of the diagram in the Universal access section of p56
- a set of cards with numbers from the different rational number sets on them.

Warm-up questions:
- Lesson 2.1.1 sheet

2 Teach

Math background
Students should be familiar with the different types of rational numbers (whole numbers, integers, fractions, and decimals) from earlier grades.

If any are not, then review the necessary vocabulary. The Universal access activity on the next page is particularly suitable for this.

Additional examples
1) Show that 1 is rational. $\frac{1}{1}$
2) Show that –7 is rational. $\frac{-7}{1}$
3) Show that 217 is rational. $\frac{217}{1}$

Concept question
"If x is an integer, what fraction has the same value as x?"
$\frac{x}{1}$

Common error
Students tend to assume that if a decimal is very long, such as 1.682947276325, then it must be irrational. Remind students that it doesn't matter after how many digits a decimal stops, if it does stop at some point then it's a terminating decimal.

- **Strategic Learners**
Provide students with a list of common fractions that when converted to decimals include some integers, some terminating decimals (like one-half or one-quarter), and some simple repeating decimals (like one-third or one-ninth). Have them divide each fraction to find its corresponding decimal. Remind students that, since all the numbers started off being written as fractions, they must all be rational.

- **English Language Learners**
Ask students to create a vocabulary list of the names of the different sorts of number that are rational (integers, fractions, terminating decimals, repeating decimals) in English and in their own first language. Have them write three examples of each type of number by each entry in the list. Then have them write each number again in the form $\frac{a}{b}$ to show that they are all rational.

2 Teach (cont)

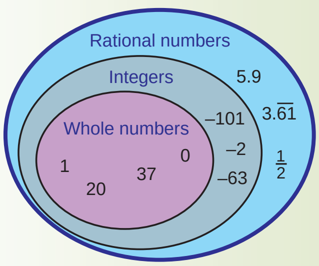
Concept question

"How long would it take you to write a repeating decimal in full?"

It would take you forever.

Universal access

Split the class into groups. Give each group a larger version of the diagram below without any numbers on it, and a set of cards with the numbers on them.



Have the students put the numbers into the correct rings. Remind them that all the numbers they have put inside the outer blue ring are rational numbers.

Guided practice

Level 1: q1–4, 7
Level 2: q1–7
Level 3: q1–8

Universal access

Give students a list of common fractions and a list of their decimal equivalents. Ask them to match up each fraction with the decimal of the same value. Then get them to use long division to find the decimal equivalent of each fraction, showing that the answer is the same.

All Repeating Decimals are Rational

Check it out:
In Lesson 2.1.3 you'll see how to turn repeating decimals into fractions.

0.09090909... is a **repeating decimal**. It will go on forever repeating the same digits (09) over and over again.

Repeating decimals can always be converted to the form $\frac{a}{b}$ where *a* and *b* are both integers — so they are always **rational**.

$0.0909090909... = \frac{1}{11}$

Other examples of repeating decimals are:

$0.33333... \left(\frac{1}{3}\right)$, and $0.045045045... \left(\frac{5}{111}\right)$.

The usual way to show that decimals are **repeating** is to put a small **bar** above them. The bar should cover one complete set of the **repeated digits**, so $0.\overline{15}$ means 0.151515..., but $0.\overline{151}$ means 0.151151151...

Never-Ending, Nonrepeating Decimals are Irrational

Check it out:
Irrational numbers are covered in more detail in Section 2.6.

A number that **cannot** be written in the form $\frac{a}{b}$ where *a* and *b* are both **integers** is an **irrational number**.

Irrational numbers are always **nonrepeating**, **nonterminating** decimals. π is an irrational number:

$\pi = 3.141592653... \leftarrow$ *goes on forever, never repeats*

Guided Practice

Show that the numbers in Exercises 1–6 are rational.

1. 2 **2 can be written as $\frac{2}{1}$**
2. –8 **–8 can be written as $\frac{-8}{1}$**
3. 0.5 **0.5 can be written as $\frac{1}{2}$**
4. 0.25 **0.25 can be written as $\frac{1}{4}$**
5. –0.1 **–0.1 can be written as $\frac{-1}{10}$**
6. 1.5 **1.5 can be written as $\frac{3}{2}$**

7. Luis does a complicated calculation and his 10-digit calculator screen shows the result 1.123456789. Can you say whether the answer of Luis's calculation is rational? **See below**

8. Is 2π rational? **No — π is irrational, so 2π is too.**

Fractions Can Be Converted into Decimals by Division

All **integers**, **terminating decimals**, and **repeating decimals** are rational, so they can be written as fractions. The **opposite** is also true — **every fraction** can be converted into an integer, a terminating decimal, or a repeating decimal.

Check it out:
b can't be zero — you can't divide by zero; it's undefined.

$\frac{a}{b}$ can be read as the instruction "***a* divided by *b***."

If you **divide** the integer *a* by the integer *b* you'll end up with an **integer**, a **repeating decimal**, or a **terminating decimal**.

Solutions

For worked solutions see the Solution Guide

7. The digits don't repeat, but the calculator screen may have cut some off — so it isn't clear whether this result is rational.

- **Advanced Learners**
  Challenge students to find eight or more fractions that turn into terminating decimals and eight or more that turn into repeating decimals, without using a calculator. Ask them to explain the strategies they used to find them. Ask them to try to find rules to tell you if a fraction will turn into a terminating decimal or a repeating decimal. For example, fractions with the denominator 2 are always terminating decimals, and 1 divided by any multiple of 3 will always be a repeating decimal.

2 Teach (cont)

A Remainder of Zero Means a Terminating Decimal

When you're dividing the numerator of a fraction by the denominator, you might get to a point where you have **no remainder** left — that means that it's a **terminating decimal**.

Don't forget:
When you add 0s to the end of a number after a decimal point, as with the 7 in Example 2, the value of the number doesn't change.
So 7.0000 = 7.0 = 7.

Math background
Students should know how to use long division, learned at grade 5, to convert a fraction to a decimal.

Example 2

Convert $\frac{7}{8}$ into a decimal.

Solution

Divide 7 by 8.

$$\begin{array}{r} 0.875 \\ 8\overline{)7.0000} \\ \underline{6\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 00 \end{array}$$

no remainder left, so this is a terminating decimal

So, $\frac{7}{8}$ as a decimal is **0.875**.

Guided Practice

Convert the fractions given in Exercises 9–12 into decimals without using a calculator.

9. $\frac{3}{6}$ 0.5 **10.** $\frac{4}{5}$ 0.8 **11.** $\frac{6}{4}$ 1.5 **12.** $\frac{5}{32}$ 0.15625

Guided practice
Level 1: q9–11
Level 2: q9–12
Level 3: q9–12

A Repeated Remainder Means a Repeating Decimal

If you get a **remainder** during long division that you've **had before**, then you have a **repeating decimal**.

$$\begin{array}{r} 0.13 \\ 15\overline{)2.000} \\ \underline{1\,5} \\ 50 \\ \underline{45} \\ 50 \end{array}$$

repeated remainder

Don't forget:
The quotient is what you get when you divide one number by another.

The repeating digits are **only** the ones that you worked out since the last time you saw the same remainder.

→ In this example you've had a remainder of **50** before. Since the last time you had this remainder, you've found the digit **3**. That means **3 is the repeating part of the decimal**.

repeating digit

$$\begin{array}{r} 0.13 \\ 15\overline{)2.000} \\ \underline{1\,5} \\ 50 \\ \underline{45} \\ 50 \end{array}$$

repeated remainder

So $\frac{2}{15}$ = 0.1333... with the 3 **repeating forever**.

Which you can write as $\frac{2}{15} = 0.1\overline{3}$.

Universal access
Write a fraction, mixed number, or decimal on the board (for example, 1.24). Have students place the number on a number line and write the two integers that it would go between (for example 1 < 1.24 < 2). This helps students grasp the idea that all fractions and decimals can be placed on a number line.

Common error
Students often include all the digits of their division in the repeating part of the answer. For example they write $\frac{2}{15} = 0.\overline{13}$. Emphasize that the bar only goes over the actual numbers that are repeated.

Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Additional examples

For each of the fractions below, identify how many digits form the repeating pattern.

a) $\frac{1}{3}$ **1 digit**

b) $\frac{1}{6}$ **1 digit**

c) $\frac{1}{27}$ **3 digits**

d) $\frac{3}{22}$ **2 digits**

Guided practice

Level 1: q13–15
Level 2: q13–16
Level 3: q13–16

Concept question

"If a decimal number goes on and on forever, does that mean it isn't a rational number?"

No — repeating decimals go on forever, but since they can be converted to fractions, they are rational.

Independent practice

Level 1: q1–5, 8, 9
Level 2: q1–10
Level 3: q1–13

Additional questions

Level 1: p435 q1–2, 5–6, 11–12
Level 2: p435 q1–7, 9-12
Level 3: p435 q1–14

3 Homework

Homework Book — Lesson 2.1.1

Level 1: q1, 2, 3a, 4a, 6
Level 2: q1–8
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:

• Worksheet 15 — Rational and Irrational Numbers

Example 3

Convert $\frac{5}{22}$ into a decimal.

Solution

Divide 5 by 22.

repeating digits	0.227
	$22\overline{)5.000}$
	44
repeated remainder	60
	44
	160
	154
repeated remainder	60

So, $\frac{5}{22}$ as a decimal is $\mathbf{0.2\overline{27}}$.

Guided Practice

Convert the fractions given in Exercises 13–16 into decimals, without using a calculator.

13. $\frac{5}{27}$ $0.\overline{185}$ **14.** $\frac{6}{41}$ $0.\overline{14634}$ **15.** $\frac{15}{7}$ $2.\overline{142857}$ **16.** $\frac{7}{12}$ $0.58\overline{3}$

Independent Practice

1. Read statements a) and b). Only one of them is true. Which one? How do you know?
a) All integers are rational numbers.
b) All rational numbers are integers.
See below

Show that the numbers in Exercises 2–7 are rational.

2. 4 — 4 can be written as $\frac{4}{1}$
3. 1 — 1 can be written as $\frac{1}{1}$
4. –2 — –2 can be written as $\frac{-2}{1}$
5. 0.2 — 0.2 can be written as $\frac{1}{5}$
6. 1.25 — 1.25 can be written as $\frac{5}{4}$
7. $-0.\overline{3}$ — $-0.\overline{3}$ can be written as $\frac{-1}{3}$

Convert the fractions given in Exercises 8–13 to decimals without using a calculator. Say whether they are terminating or repeating decimals.

8. $\frac{1}{9}$ $0.\overline{1}$, repeating **9.** $\frac{8}{5}$ 1.6, terminating **10.** $\frac{11}{16}$ 0.6875, terminating

11. $\frac{5}{11}$ $0.\overline{45}$, repeating **12.** $\frac{15}{8}$ 1.875, terminating **13.** $\frac{5}{22}$ $0.2\overline{27}$, repeating

Now try these:
Lesson 2.1.1 additional questions — p435

Round Up

Rational numbers can all be written as fractions, where the top and bottom numbers are integers, and the bottom number isn't zero. You already know how to write integers as fractions, and you'll see how to convert terminating decimals and repeating decimals to fractions in the next two Lessons.

Solutions

For worked solutions see the Solution Guide

1. Statement a) is true. All integers can be written as fractions in the form $\frac{a}{b}$, so are rational. Statement b) is false because there are lots of rational numbers that aren't integers.

Lesson 2.1.2

Converting Terminating Decimals to Fractions

In this Lesson students learn how to turn terminating decimals into fractions, and how to reduce a fraction to its simplest form. This includes turning decimals larger than 1 into improper fractions.

Previous Study: In grade 4 students learned how to read decimals and compare their values using the place-value system. They also learned the decimal equivalents of simple fractions.

Future Study: This Lesson introduces simplifying fractions, which is an important skill whenever students tackle topics involving fractions, both later in grade 7 and throughout Algebra I.

Lesson 2.1.2

California Standards:

Number Sense 1.5

Know that every rational number is either a terminating or a repeating decimal and **be able to convert terminating decimals into reduced fractions.**

What it means for you:

You'll see how to change terminating decimals into fractions that have the same value.

Key words:

- fraction
- decimal
- terminating

Don't forget:

You can ignore any extra 0s at the end of the decimal, because they don't change its value.

For example:

$0.270000 = 0.27 = \frac{27}{100}$.

Converting Terminating Decimals to Fractions

*This Lesson is a bit like the opposite of the last Lesson — you'll be taking **decimals** and finding their **equivalent fractions**. This is how you can show that they're definitely **rational numbers**.*

Decimals Can Be Turned into Fractions

If you read **decimals** using the **place-value** system, then it's more straightforward to convert them into **fractions**. For example, a number like 0.15 is said "fifteen-hundredths," so it turns into the fraction $\frac{15}{100}$.

You need to remember the **value** of **each position** after the **decimal point**:

decimal point

0.1234

tenths

hundredths

thousandths

ten-thousandths

Then when you are reading a decimal number, look at the **position** of the **last digit**. For example: 0.1 is one-tenth, which is the fraction $\frac{1}{10}$. 0.01 is one-hundredth, which is the fraction $\frac{1}{100}$.

Example 1

Convert 0.27 into a fraction.

Solution

0.27 is twenty-seven hundredths, so it is $\frac{27}{100}$.

Example 2

Convert 0.3497 into a fraction.

Solution

0.3497 is 3497 ten-thousandths, so it is $\frac{3497}{10,000}$.

Guided Practice

Convert the decimals in Exercises 1–12 into fractions without using a calculator.

1. 0.1 $\frac{1}{10}$ **2.** 0.23 $\frac{23}{100}$ **3.** 0.17 $\frac{17}{100}$ **4.** –0.87 $\frac{-87}{100}$

5. 0.7 $\frac{7}{10}$ **6.** 0.35 $\frac{35}{100}$ **7.** 0.174 $\frac{174}{1000}$ **8.** –0.364 $\frac{-364}{1000}$

9. 0.127 $\frac{127}{1000}$ **10.** 0.9827 $\frac{9827}{10,000}$ **11.** 0.5212 $\frac{5212}{10,000}$ **12.** –0.4454 $\frac{-4454}{10,000}$

1 Get started

Resources:

- individual whiteboards and pens

Teacher Resources CD-ROM

- Fraction Tokens

Warm-up questions:

- Lesson 2.1.2 sheet

2 Teach

Math background

From grade 1 onwards students become familiar with using the place-value system for whole numbers. From grade 4 onwards they begin to use it with decimals.

Universal access

Read decimal numbers using the place-value system (for example, "*eight-tenths*" or "*forty-nine hundredths*") and have students write the numbers on individual whiteboards as you read them.

Additional examples

1) Convert 0.3 into a fraction. $\frac{3}{10}$

2) Convert 0.07 into a fraction. $\frac{7}{100}$

3) Convert 0.009 into a fraction. $\frac{9}{1000}$

Concept question

"What is the number one ten-thousandth written as a decimal?"

0.0001

Guided practice

Level 1: q1–6

Level 2: q1–8

Level 3: q1–12

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  Have students begin by applying the step-by-step method in Example 3 to decimals whose fractional equivalents they already know, such as 0.1, 0.25, 0.5. This allows them to gain confidence with the method before applying it to "new" decimals.

- **English Language Learners**
  Provide each student with a worked example and a list of key terms, such as numerator, denominator, greatest common factor, mixed number, simplest form. Have them link each term to the correct part of the work. Take extra time to clearly explain acronyms, like GCF, so everyone has a clear understanding of what they mean.

2 Teach (cont)

Universal access

Give students a paired list of numbers. For example:

1	4
8	2
5	10
12	9

Have the students find all the factors of each number, and underline the common factors of each pair of numbers. Ask them to circle the GCF of each pair in red.

1	1	4	1, 2, 4
1, 2, 4, 8	8	2	1, 2
1, 5	5	10	1, 2, 5, 10
1, 2, 3, 4, 6, 12	12	9	1, 3, 9

Then get them to reduce the fractions $\frac{1}{4}$, $\frac{8}{2}$, $\frac{5}{10}$, and $\frac{12}{9}$ to their simplest forms using the method in Example 3.

Common error

Students sometimes divide only the numerator by the GCF. Remind them that they need to do the same thing to the top and bottom of the fraction in order to create an equivalent fraction.

Concept question

"What would you divide the numerator and denominator of $\frac{3}{9}$ by to reduce it to its simplest form?" 3

Guided practice

Level 1: q13–17
Level 2: q13–18, 21
Level 3: q13–21

Some Fractions Can Be Made Simpler

Don't forget:
Dividing the top and bottom of a fraction by the same thing doesn't change the value of the fraction.

When you convert decimals to fractions this way, you'll often get fractions that **aren't in their simplest form**. For instance, $\frac{5}{10}$ could be written more simply as $\frac{1}{2}$, and $\frac{75}{100}$ could be written more simply as $\frac{3}{4}$.
If an answer is a fraction, you should usually give it in its simplest form.

This is how to reduce a fraction to its simplest form:

1) Find the **biggest number** that will divide into both the **numerator** and the **denominator** without leaving any **remainder**.
   This number is called the **greatest common factor**, or **GCF**.
2) Then **divide** both the numerator and the denominator by the **GCF**.

Example 3

Convert 0.12 into a fraction.

Solution

- 0.12 is twelve-hundredths. As a fraction it is $\frac{12}{100}$.
- The factors of 12 are 1, 2, 3, 4, 6, and 12. The biggest of these that also divides into 100 leaving no remainder is 4.
  So the **greatest common factor** of 12 and 100 is 4.
- Divide both the numerator and denominator by 4.
  $\frac{12 \div 4}{100 \div 4} = \frac{3}{25}$. **So 0.12 as a fraction in its simplest form is $\frac{3}{25}$.**

If the greatest common factor is **1** then the fraction is already in its **simplest form**.

Example 4

Convert 0.7 into a fraction.

Solution

- 0.7 is seven-tenths so, it is $\frac{7}{10}$.
- The greatest common factor of 7 and 10 is 1, so this fraction is **already** in its simplest form.

Guided Practice

Convert the decimals in Exercises 13–20 into fractions and then simplify them if possible.

13. 0.25 $\frac{1}{4}$ **14.** 0.65 $\frac{13}{20}$ **15.** –0.02 $\frac{-1}{50}$ **16.** 0.256 $\frac{32}{125}$

17. 0.0175 $\frac{7}{400}$ **18.** –0.84 $\frac{-21}{25}$ **19.** 0.267 $\frac{267}{1000}$ **20.** 0.866 $\frac{433}{500}$

21. Priscilla measures a paper clip. She decides that it is six-eighths of an inch long. Otis measures the same paper clip with a different ruler and says it is twelve-sixteenths of an inch long. How can their different answers be explained? **$\frac{6}{8}$ is a simpler form of $\frac{12}{16}$. Both answers are the same.**

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  For an extra challenge, give students some problems where a decimal and a fraction have been combined in one number, for instance $\frac{2.5}{20}$. Ask them to rewrite the expression correctly as a fraction involving only integers, and then as a single decimal.

Don't forget:

A proper fraction is a fraction whose numerator is smaller than its denominator.

For example: $\frac{1}{2}$ and $\frac{9}{10}$.

An improper fraction is a fraction whose numerator is equal to or larger than its denominator.

For example: $\frac{3}{2}$ and $\frac{27}{4}$.

Don't forget:

A mixed number is a number made up of an integer and a fraction. For example: $1\frac{1}{2}$, $2\frac{2}{3}$, and $10\frac{4}{5}$.

Decimals Greater than 1 Become Improper Fractions

When you convert a **decimal** number **greater than 1** into a **fraction** it's probably easier to change it into a **mixed number** first.

Then you can change the mixed number into an **improper fraction**.

Example 5

Convert 13.7 into a fraction.

Solution

- Convert 0.7 first — this becomes $\frac{7}{10}$.
- Add on the 13. This can be written as $13\frac{7}{10}$. A mixed number.
- Now turn $13\frac{7}{10}$ into an improper fraction.

13 whole units are equivalent to $\frac{13}{1} \bullet \frac{10}{10} = \frac{130}{10}$. So add $\frac{7}{10}$ to this:

$$\left(\frac{13}{1} \bullet \frac{10}{10}\right) + \frac{7}{10} = \frac{130}{10} + \frac{7}{10} = \frac{137}{10}$$

A quicker way of doing this is: $\frac{(13 \bullet 10) + 7}{10} = \frac{137}{10}$

Guided Practice

Convert the decimals in Exercises 22–33 into fractions without using a calculator.

22. 4.3 $\frac{43}{10}$	**23.** –1.03 $\frac{-103}{100}$	**24.** 15.98 $\frac{799}{50}$	**25.** –1.7 $\frac{-17}{10}$
26. 9.7 $\frac{97}{10}$	**27.** –4.5 $\frac{-9}{2}$	**28.** 12.904 $\frac{1613}{125}$	**29.** –13.142 $\frac{-6571}{500}$
30. –8.217 $\frac{-8217}{1000}$	**31.** 0.3627 $\frac{3627}{10,000}$	**32.** 1.8028 $\frac{4507}{2500}$	**33.** 4.1234 $\frac{20,617}{5000}$

Independent Practice

Convert the decimals given in Exercises 1–20 to fractions in their simplest form.

1. 0.3 $\frac{3}{10}$	**2.** 0.2 $\frac{1}{5}$	**3.** 0.4 $\frac{2}{5}$	**4.** 0.30 $\frac{3}{10}$
5. 0.26 $\frac{13}{50}$	**6.** 0.18 $\frac{9}{50}$	**7.** –0.34 $\frac{-17}{50}$	**8.** –1.34 $\frac{-67}{50}$
9. 0.234 $\frac{117}{500}$	**10.** 2.234 $\frac{1117}{500}$	**11.** 9.140 $\frac{457}{50}$	**12.** 3.655 $\frac{731}{200}$
13. –0.121 $\frac{-121}{1000}$	**14.** –0.655 $\frac{-131}{200}$	**15.** –10.760 $\frac{-269}{25}$	**16.** 5.001 $\frac{5001}{1000}$
17. 0.2985 $\frac{597}{2000}$	**18.** 2.3222 $\frac{11,611}{5000}$	**19.** –9.3452 $\frac{-23,363}{2500}$	**20.** –0.2400 $\frac{-6}{25}$

Now try these:

Lesson 2.1.2 additional questions — p435

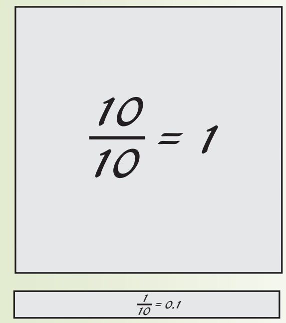
Round Up

The important thing when converting a decimal to a fraction is to think about the place value of the last digit. Then read the decimal and turn it into a fraction. If the decimal is greater than 1, ignore the whole number until you get the decimal part figured out. Take your time, do each step carefully, and you should be OK.

2 Teach (cont)

Universal access

Give everyone five big "1" fraction tokens and ten small "0.1" fraction tokens from the **Teacher Resources CD-ROM**.



Say a number to one decimal place. It should be greater than 1 but less than 6; for example, 1.4 or 5.6. Ask everyone to make the number using their tokens. Then ask the students to convert the decimal to a mixed number by counting the ones and tenths. Then ask them to change it to an improper fraction by counting the tenths.

Explain how counting up the tenths is really the same as the method in Example 5.

Guided practice

Level 1: q22–25
Level 2: q22–29
Level 3: q22–33

Independent practice

Level 1: q1–7
Level 2: q1–14
Level 3: q1–20

Additional questions

Level 1: p435 q1–2, 4, 7–8
Level 2: p435 q1–5, 7–11
Level 3: p435 q1–11

3 Homework

Homework Book — Lesson 2.1.2

Level 1: q1, 2, 4a–b, 5–7
Level 2: q1–11
Level 3: q1–11

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:

- Worksheet 2 — Simplifying Fractions
- Worksheet 3 — Converting Fractions and Decimals
- Worksheet 15 — Rational and Irrational Numbers

Solutions

For worked solutions see the Solution Guide

Lesson 2.1.3

Converting Repeating Decimals to Fractions

In this Lesson students learn how to turn repeating decimals into fractions. They apply the methods they learned in Lesson 2.1.2 to reduce the resulting fractions to their simplest form.

Previous Study: In grades 5 and 6 students learned the decimal equivalents of common fractions. This provided their first experiences of repeating decimals, like one-third.

Future Study: In Algebra I students will be expected to be proficient at handling repeating decimals and converting between decimals and fractions in order to simplify calculations.

1 Get started

Resources:
- calculators

Warm-up questions:
- Lesson 2.1.3 sheet

2 Teach

Common error

Students sometimes have difficulty understanding that even if a number goes on forever it can still be subtracted.

Give them a series of subtractions where the number of digits after the decimal point increases by one each time, for example: 3.3 – 0.3
3.33 – 0.33 etc.

When they are happy with this, give them the subtraction $3.\bar{3} - 0.\bar{3}$. Point out that it follows the same pattern as the rest of the subtractions.

Universal access

As an introduction, have students use calculators to convert the fractions below to decimals. Get them to record the decimal conversions in a table and state whether each decimal terminates or repeats. If a decimal repeats, ask them to identify the length of the repeating pattern.

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$$

Math background

Example 2 involves setting up a variable equation. To keep the equation balanced you must perform the same operations on both sides.

Additional example

Convert $0.\bar{8}$ to a fraction. $\frac{8}{9}$

Lesson 2.1.3

Converting Repeating Decimals to Fractions

California Standards:

Number Sense 1.5
Know that every rational number is either a terminating or a repeating decimal and be able to convert terminating **decimals into reduced fractions.**

What it means for you:
You'll see how to change repeating decimals into fractions that have the same value.

Key words:
- fraction
- decimal
- repeating

You've seen how to convert a terminating decimal into a fraction. But repeating decimals are also rational numbers, so they can be represented as fractions too. That's what this Lesson is all about — taking a repeating decimal and finding a fraction with the same value.

Repeating Decimals Can Be "Subtracted Away"

Look at the decimal 0.33333..., or $0.\bar{3}$.

If you **multiply it by 10**, you get 3.33333..., or $3.\bar{3}$.

In both these numbers, the digits **after** the decimal point are the **same**. So if you **subtract** one from the other, the decimal part of the number "disappears."

Don't forget: To multiply a decimal by 10, move the decimal point one place to the right.
So 0.3333... × 10 = 3.3333...

Example 1

Find $3.\bar{3} - 0.\bar{3}$.

Solution

The digits after the decimal point in both these numbers are the same, since $0.\bar{3} = 0.3333...$ and $3.\bar{3} = 3.3333...$

So when you subtract the numbers, the result has no digits after the decimal point.

$$\begin{array}{r} 3.3333... \\ -0.3333... \\ \hline 3.0000... \end{array} \quad \text{or} \quad \begin{array}{r} 3.\bar{3} \\ -0.\bar{3} \\ \hline 3.0 \end{array}$$

So $3.\bar{3} - 0.\bar{3} = 3$.

This idea of getting repeating decimals to "disappear" by subtracting is used when you convert a **repeating decimal** to a fraction.

Example 2

If $x = 0.\bar{3}$, find: (i) $10x$, and (ii) $9x$.
Use your results to write x as a fraction in its simplest form.

Solution

(i) $10x = 10 \times 0.\bar{3} = 3.\bar{3}$.
(ii) $9x = 10x - x = 3.\bar{3} - 0.\bar{3} = 3$ (from Example 1 above).

You now know that $9x = 3$.
So you can divide both sides by 9 to find x as a fraction:
$x = \frac{3}{9}$, which can be simplified to $x = \frac{1}{3}$.

Don't forget:
The greatest common factor of 3 and 9 is 3.
So: $\frac{3}{9} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}$

- **Strategic Learners**
  Begin by working through the conversion process step by step with a fraction the students already know the decimal equivalent of. Discuss with the students why each step is included. Have the students write the process in numbered steps and write in their own words beside each step why they need to do it. They can then refer to their own instruction sheet when they do a conversion.

- **English Language Learners**
  Discuss the meanings of the words “terminate” and “repeat” with the students. Emphasize to them that to terminate means to stop. Remind them that to repeat means to do the same thing over and over again — in the context of decimals, the repeating part goes on forever.

✓ Guided Practice

In Exercises 1–3, use $x = 0.\overline{4}$.

1. Find $10x$. $4.\overline{4}$

2. Use your answer to Exercise 1 to find $9x$. 4

3. Write x as a fraction in its simplest form. $\frac{4}{9}$

In Exercises 4–6, use $y = 1.\overline{2}$.

4. Find $10y$. $12.\overline{2}$

5. Use your answer to Exercise 4 to find $9y$. 11

6. Write y as a fraction in its simplest form. $\frac{11}{9}$

Convert the numbers in Exercises 7–9 to fractions.

7. $2.\overline{5}$ $\frac{23}{9}$ **8.** $4.\overline{1}$ $\frac{37}{9}$ **9.** $-2.\overline{5}$ $-\frac{23}{9}$

You May Need to Multiply by 100 or 1000 or 10,000...

If **two** digits are repeated forever, then multiply by **100** before subtracting.

Example 3

Convert $0.\overline{23}$ to a fraction.

Solution

Call the number x.

There are **two** repeating digits in x, so you need to multiply by **100** before subtracting.

$100x = 23.\overline{23}$

Now **subtract**: $100x - x = 23.\overline{23} - 0.\overline{23} = 23$.

So $\mathbf{99x = 23}$, which means that $x = \frac{23}{99}$.

Don't forget:
This means that
$0.\overline{23} = \frac{23}{99}$

If **three** digits are repeated forever, then multiply by **1000**, and so on.

Example 4

Convert $1.\overline{728}$ to a fraction in its simplest form.

Solution

Call the fraction y.

There are **three** repeating digits in y, so you need to multiply by **1000** before subtracting.

$1000y = 1728.\overline{728}$

Now **subtract**: $1000y - y = 1728.\overline{728} - 1.\overline{728} = 1727$.

So $\mathbf{999y = 1727}$, which means that $y = \frac{1727}{999}$.

Check it out:
In fact, you multiply by 10^n, where n is the number of repeating digits.
So where there's one repeating digit, you multiply by $10^1 = 10$.
Where there are two repeating digits, you multiply by $10^2 = 10 \times 10 = 100$, and so on.

2 Teach (cont)

Guided practice

Level 1: q1–3, 7
Level 2: q1–7
Level 3: q1–9

Math background

When you come across a repeating decimal as part of a calculation, it's usually much easier to handle if you convert it to a fraction first.
For example,
$22 \times \frac{1}{11}$ is a much easier calculation to do than $22 \times 0.\overline{09}$.

Concept question

“What would the decimal $0.\overline{n}$ be if you converted it to a fraction?” $\frac{n}{9}$

Additional examples

Convert the following repeating decimals to fractions:

1) $0.\overline{1}$ $\frac{1}{9}$
2) $0.\overline{01}$ $\frac{1}{99}$
3) $0.\overline{001}$ $\frac{1}{999}$

Solutions

For worked solutions see the Solution Guide

Advanced Learners
Before introducing the method for converting repeating decimals to fractions, provide an opportunity for students to discover the patterns themselves. Give students a list of repeating decimals where the number of digits in the repeat period increases by one each time, such as $0.\overline{4}$, $0.\overline{01}$, $0.\overline{551}$, etc. Also, give them the unsimplified equivalent fractions. Ask them to find any patterns they can connecting the number of digits in the repeat period of the decimal to the number of digits in the numerator or denominator of the fraction.

2 Teach (cont)

Guided practice
Level 1: q10–13
Level 2: q10–14
Level 3: q10–15

Common error
Students often just subtract the repeated part from both numbers and forget to make sure that they are subtracting numbers of equal place value. For example, they may write:
$34.\overline{3} - 3.4\overline{3} = 34$

Remind them that it's a good idea to always do the decimal subtraction at the side of their work in column form as a check. That way they can be sure that they've lined up the decimal points of the two numbers correctly.

Guided practice
Level 1: q16–18
Level 2: q16–18
Level 3: q16–18

Independent practice
Level 1: q1–4
Level 2: q1–6
Level 3: q1–9

Additional questions
Level 1: p435 q1–3, 7–9
Level 2: p435 q1–6, 10–12
Level 3: p435 q4–12

3 Homework

Homework Book — Lesson 2.1.3
Level 1: q1–4, 5a
Level 2: q3–10
Level 3: q3–11

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 2 — Simplifying Fractions
- Worksheet 15 — Rational and Irrational Numbers

Guided Practice

For Exercises 10–15, write each repeating decimal as a fraction in its simplest form.

10. $0.\overline{09}$ $\frac{1}{11}$ **11.** $0.\overline{18}$ $\frac{2}{11}$ **12.** $0.\overline{909}$ $\frac{101}{111}$
13. $0.\overline{123}$ $\frac{41}{333}$ **14.** $2.\overline{12}$ $\frac{70}{33}$ **15.** $0.\overline{1234}$ $\frac{1234}{9999}$

The Numerator and Denominator Must Be Integers

You **won't** always get a **whole number** as the result of the subtraction. If this happens, you may need to **multiply** the numerator and denominator of the fraction to make sure they are both integers.

Check it out:
Make the repeating digits line up to make the subtraction easier (and remember that the decimal points have to line up too).
So write $10x$ as $34.3\overline{3}$ rather than $34.\overline{3}$.
Then the subtraction becomes:
$$\begin{array}{r} 34.3\overline{3} \\ -3.4\overline{3} \\ \hline 30.9 \end{array}$$
Which is an easier subtraction to do than $34.\overline{3} - 3.4\overline{3}$.

Example 5

Convert $3.4\overline{3}$ to a fraction.

Solution

Call the number x.
There is **one** repeating digit in x, so multiply by **10**.
$10x = 34.3\overline{3}$ (using $34.3\overline{3}$ rather than $34.\overline{3}$ makes the subtraction easier).

Subtract as usual: $10x - x = 34.3\overline{3} - 3.4\overline{3} = 30.9$.

So $\mathbf{9x = 30.9}$, which means that $x = \frac{30.9}{9}$.

But the numerator here **isn't an integer**, so multiply the numerator and denominator by 10 to get an equivalent fraction of the same value.

$$x = \frac{30.9 \times 10}{9 \times 10} = \frac{309}{90}, \text{ or more simply, } \mathbf{x = \frac{103}{30}}$$

Guided Practice

For Exercises 16–18, write each repeating decimal as a fraction in its simplest form.

16. $1.1\overline{2}$ $\frac{101}{90}$ **17.** $2.3\overline{34}$ $\frac{2311}{990}$ **18.** $0.54\overline{321}$ $\frac{18,089}{33,300}$

Now try these:
Lesson 2.1.3 additional questions — p435

Independent Practice

Convert the numbers in Exercises 1–9 to fractions.
Give your answers in their simplest form.

1. $0.\overline{8}$ $\frac{8}{9}$ **2.** $0.\overline{7}$ $\frac{7}{9}$ **3.** $1.\overline{1}$ $\frac{10}{9}$
4. $0.\overline{26}$ $\frac{26}{99}$ **5.** $4.\overline{87}$ $\frac{161}{33}$ **6.** $0.\overline{246}$ $\frac{82}{333}$
7. $0.\overline{142857}$ $\frac{1}{7}$ **8.** $3.\overline{142857}$ $\frac{22}{7}$ **9.** $10.0\overline{1}$ $\frac{901}{90}$

Round Up

This is a really handy 3-step method — (i) multiply by 10, 100, 1000, or whatever, (ii) subtract the original number, and (iii) divide to form your fraction.

Solutions
For worked solutions see the Solution Guide

Lesson 2.2.1

Absolute Value

In this Lesson, students are introduced to the concept of absolute value and learn how to apply it to negative and positive numbers. This leads to evaluating simple expressions containing absolute value terms.

Previous Study: In grade 4 students learned to distinguish between positive and negative numbers. In grades 5 and 6 students learned to use negative numbers in calculations.

Future Study: In Algebra I students will solve equations and inequalities that use absolute values.

Lesson 2.2.1

California Standards:

Number Sense 2.5

Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

What it means for you:

You'll learn how to find the absolute value of a number, and use it in calculations.

Key words:

- absolute value
- distance
- opposite

Check it out:

Because you want to know the distance from the origin but don't care about direction, the absolute value of a number is always just the number without its original sign.

Check it out:

A number and its opposite (additive inverse) always have the same absolute value.
For example:
| 2 | = | –2 | = 2

Section 2.2
Absolute Value

You can think of the number "–5" as having two parts — a negative sign that tells you it's less than zero, and "5," which tells you its size, or how far from zero it is. The absolute value of a number is just its size — it's not affected by whether it's greater or less than zero.

Absolute Value is Distance from Zero

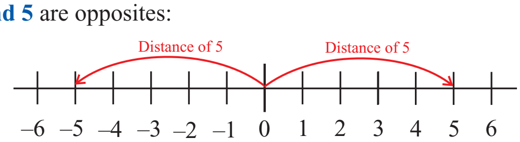
The absolute value of a number is its distance from 0 on the number line.

The **absolute value** of a number is **never negative** — that's because the absolute value describes **how far** the number is from **zero** on the number line. It doesn't matter if the number is to the **left** or to the **right** of zero — the distance **can't** be negative.

Opposites Have the Same Absolute Value

Opposites are numbers that are the **same distance from 0**, but going in **opposite directions**. Opposites have the same **absolute value**.

–5 and 5 are opposites:



So they each have an **absolute value of 5**.

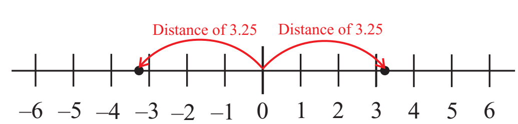
A set of **bars**, | |, are used to represent **absolute value**.
So the expression |–10| means "the absolute value of negative ten."

Example 1

What is |3.25|? What is |–3.25|?

Solution

3.25 and –3.25 are opposites. They're the same distance from 0, so they have the same absolute value.



So, **|3.25| = |–3.25| = 3.25**

1 Get started

Resources:
- large number line (see below)
- meter ruler
- weather forecast map
- grid paper

Warm-up questions:
- Lesson 2.2.1 sheet

2 Teach

Concept question

"What's the absolute value of 0?"
0

Universal access

Place a large number line on the floor, with 0 in the center of the room. Mark positive integers on the line at 1 m intervals to the right, and negative integers at 1 m intervals to the left. Have a student stand on any number, and another use a meter ruler to measure how far they are from 0.

Repeat this several times, with some students standing to the left of 0 and some to the right of 0. Write the measurements on the board. Remind everyone that the measurement only shows how far each person was from 0 — and as you were only interested in the distance away from 0 that they were, it didn't matter if they stood to its left or to its right.

Common error

When asked to find the absolute value of a number, some students will change the sign instead of discarding it. For example, they may say that if |–2| = 2, then |2| = –2.

Remind students that a number's absolute value is its distance from 0, and this distance can't be negative. Absolute value means ignoring the sign, not changing the sign.

Additional examples

Find:
a) |5| **5**
b) |–5| **5**

- **Strategic Learners**
  Have students make their own timeline with their date of birth as 0 and a scale marked in years. They should extend it at least 15 years before and after their birth. Have them mark on some events that happened before their birth, and some that happened after it.
  For each event ask them to write underneath how long before or after their birth it happened. They can mark events to the nearest year.

- **English Language Learners**
  Bring in an old winter weather forecast map that covers the whole USA. Make sure it has both positive and negative temperatures on it. (Alternatively, stick temperature labels on a normal map.) Pick two places on the map and ask students to write down which place is predicted to be coldest and which has a temperature that is furthest from 0 °C. Repeat the exercise, discussing with them how they are figuring out their answers.

2 Teach (cont)

Guided practice
Level 1: q1–4, 9–10
Level 2: q1–6, 9–12
Level 3: q1–12

Universal access
Give students a simple real-life word problem to think about, such as:

"*The temperature in Burney is 3 °F different from the temperature in Altura. It is 0 °F in Burney. What could the temperature be in Altura*?"
3 °F or –3 °F

When they have found the two possible answers, get them to draw a number line to show their answer. Have them pick a letter to stand for the temperature in Altura (for example, T), and help them to write an absolute value equation to represent it. **$|T| = 3$**

Show the students that they have in effect worked through Example 2.

Concept question
"If $|a| = 2$, does $|2| = a$?"
No — as $|a| = 2$, a can be either 2 or –2. If $a = –2$ then $|2|$ does not have the same value as a.

Guided practice
Level 1: q13–16
Level 2: q13–16
Level 3: q13–16

Math background
Absolute value bars are a grouping symbol. According to the order of operations they are evaluated first, like parentheses.

Guided Practice

Find the values of the expressions in Exercises 1–8.

1. $|12|$ **12** **2.** $|-9|$ **9** **3.** $|16|$ **16** **4.** $|-1|$ **1**

5. $|1.7|$ **1.7** **6.** $|-3.2|$ **3.2** **7.** $\left|-\frac{1}{2}\right|$ $\frac{1}{2}$ **8.** $|0|$ **0**

In Exercises 9–12, say which is bigger.

9. $|17|$ or $|16|$ **$|17|$** **10.** $|-2|$ or $|-5|$ **$|-5|$**

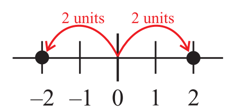
11. $|-9|$ or $|8|$ **$|-9|$** **12.** $|-1|$ or $|1|$ **They are the same size.**

Check it out:
Find the value of each absolute value expression, then compare them.

Absolute Value Equations Often Have Two Solutions

Think about the equation $|x| = 2$. The absolute value of x is 2, so you know that x is 2 units away from 0 on the number line, but you don't know in which direction. x could be 2, but it could also be –2.

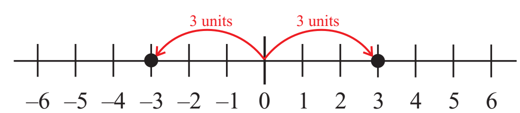
You can show the two possibilities like this:



Example 2

Solve $|z| = 3$.

Solution
z can be either 3 or –3.



Guided Practice

Give the solutions to the equations in Exercises 13–16.

13. $|a| = 1$ **a = 1 or –1** **14.** $|r| = 4$ **r = 4 or –4** **15.** $|q| = 6$ **q = 6 or –6** **16.** $|g| = 7$ **g = 7 or –7**

Treat Absolute Value Signs Like Parentheses

You should treat **absolute value bars** like **parentheses** when you're deciding what **order** to do the **operations** in. Work out what's inside them first, then take the absolute value of that.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  The concept of loci is an extension of the idea of absolute value. Give each student a piece of grid paper and tell them that the scale is 1 meter to a square. Have them mark a dot on the page. Ask them to figure out where a person could stand if they wanted to be exactly 5 m from that point. After thinking about it they should realize that the answer is a circle with a 5 m radius centered on the dot. Extend this to include questions relating to standing a precise distance from a line segment, a line, or two points.

Example 3

What is the value of $|7 - 3| + |4 - 6|$?

Solution

$|7 - 3| + |4 - 6|$

$= |4| + |-2|$ — **Simplify whatever is inside the absolute value signs**

$= 4 + 2$ — **Find the absolute values**

$= 6$ — **Simplify the expression**

Guided Practice

Evaluate the expressions in Exercises 17–22.

17. $|1 - 3| - |2 + 2|$ **–2** **18.** $|2 - 7| + |0 - 6|$ **11** **19.** $-|5 - 6|$ **–1**

20. $|-8| \times |2 - 3|$ **8** **21.** $2 \times |4 - 6|$ **4** **22.** $|7 - 2| \div |1 - 6|$ **1**

Check it out:
The absolute value bars only affect whatever's inside them. So the value of the whole expression can be negative. For example $-|a + 7|$ will be negative, unless $a = -7$.

Independent Practice

Evaluate the expressions in Exercises 1–4?

1. $|-45|$ **45** **2.** $|6|$ **6** **3.** $|-0.6|$ **0.6** **4.** $\left|\frac{5}{6}\right|$ $\frac{5}{6}$

5. Let x and y be two integers. The absolute value of y is larger than the absolute value of x. Which of the two integers is further from 0? ***y***

Show the solutions of the equations in Exercises 6–13 on number lines.

6. $|u| = 3$ **7.** $|d| = 9$ **8.** $|x| = 5$ **9.** $|w| = 15$

10. $|y| = 4$ **11.** $|v| = 1$ **12.** $|k| = 16$ **13.** $|z| = 7$

see below

In Exercises 14–19, say which is bigger.

14. $|-6|$ or $|-1|$ **$|-6|$** **15.** $|3|$ or $|-5|$ **$|-5|$** **16.** $|2 - 2|$ or $|5 - 8|$ **$|5 - 8|$**

17. $|6 - 8|$ or $|2 - 1|$ **$|6 - 8|$** **18.** $|3 - 2|$ or $|-5|$ **$|-5|$** **19.** $|11 + 1|$ or $|-2 - 8|$ **$|11 + 1|$**

Evaluate the expressions in Exercises 20–25.

20. $|3 - 5| + |2 - 5|$ **5** **21.** $|0 + 5| + |0 - 5|$ **10** **22.** $|5 - 10| - |0 - 2|$ **3**

23. $|-1| \times |3 - 3|$ **0** **24.** $8 \times |1 - 4|$ **24** **25.** $|2 - 8| \div |4 - 1|$ **2**

26. What is the sum of two different numbers that have the same absolute value? Explain your answer. **0. For two different numbers to have the same absolute value they must be "opposites," e.g. 3 and –3.**

27. Is it always true that $|y| < 2y$ when y is an integer?
No. If y is negative or zero it isn't true.

Now try these:
Lesson 2.2.1 additional questions — p436

Round Up

The absolute value of a number is its distance from zero on a number line. Absolute values are always positive. So if a number has a negative sign, get rid of it; if it doesn't, then leave it alone. If you see absolute value bars in an expression, work out what's between them first — just like parentheses.

2 Teach (cont)

Universal access

Put students in teams of three.
Give each team a three-part absolute value expression to evaluate.
Give each part a different color.
For example:

$|2 - 5| + |3 + 2| - |-2 \times 3|$ $3 + 5 - 6 = 2$

Each student in a team must find the value of one of the different colored parts. Then they can work together to find the value of the whole expression.

Have a race to see which team can find the answer first. Repeat a few times and mix up the teams if necessary until everyone has been on a winning team at least once.

Now have the students work out a problem individually. Remind them to break it down into simpler parts and work it out a bit at a time.

Guided practice

Level 1: q17–19
Level 2: q17–21
Level 3: q17–22

Independent practice

Level 1: q1–2, 6–8, 14–15, 20–22
Level 2: q1–16, 20–23, 26
Level 3: q1–27

Additional questions

Level 1: p436 q1–4, 7–8, 10–11, 14
Level 2: p436 q1–8, 12–16
Level 3: p436 q5–6, 9–13, 16–18

3 Homework

Homework Book — Lesson 2.2.1

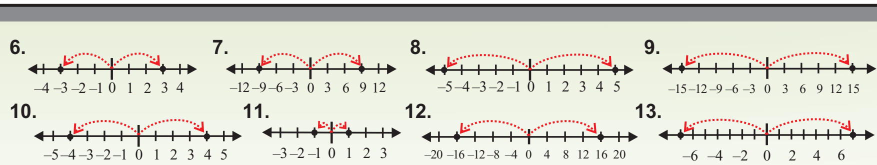
Level 1: q1–3, 4a–b, 5, 8
Level 2: q3–12
Level 3: q3–12

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
- Worksheet 4 — Closeness and Absolute Value

Solutions
For worked solutions see the Solution Guide



Lesson 2.2.2

Using Absolute Value

In this Lesson students see how absolute value is used in real life, for example, in stating manufacturing tolerances. They learn how to solve problems that involve writing and evaluating absolute value expressions.

Previous Study: In grade 5 students used negative numbers in simple calculations. They extended this in grade 6, and also wrote mathematical expressions from word problems.

Future Study: In Algebra I students will write and solve equations and inequalities that use absolute values. They will also write and solve absolute value equations based on word problems.

1 Get started

Resources:
- large number line (see below)
- meter ruler
- grid paper
- colored paper circles
- aquarium/large bowl
- selection of waterproof objects, for example, plastic bottle, tin can, pebble.
- homemade cookies
- thermometer

Warm-up questions:
- Lesson 2.2.2 sheet

2 Teach

Concept question

"If we started in the same place, and I walked a mile to the west while you walked a mile to the east, how far apart would we be?" **2 miles**

Universal access

Place a large number line on the floor, with 0 in the center of the room. Mark positive integers on the line at 1 m intervals to the right, and negative integers at 1 m intervals to the left.

This time, have two students stand on any two numbers. Get another student to measure how far apart they are using a meter ruler. Repeat this several times, writing the answers on the board as you go.

Show everyone that you get the same answers if you use the numbers marked on the line to work out how far apart each pair were using the absolute value subtraction method in Example 1.

Common error

Students sometimes forget to take the absolute value of a number after completing the subtraction step, and give a negative answer. Remind them that absolute values are always positive — so all their answers should be too.

Guided practice

Level 1: q1–4, 9
Level 2: q1–6, 9
Level 3: q1–9

Lesson 2.2.2

Using Absolute Value

California Standards:

Number Sense 2.5
Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

Algebra and Functions 1.1
Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities **that represents a verbal description** (e.g., three less than a number, half as large as area A).

What it means for you:
You'll use absolute value to find the difference between two numbers and see how absolute values apply to real-life situations.

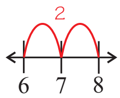
Key words:
- absolute value
- comparison
- difference

Check it out:
If $|a - b|$ is small then a and b are close to each other.
If $|a - b|$ is large then a and b are far away from each other.
For example, –1 and –3 are closer together than 6 and –6, so you'd expect $|(-1) - (-3)|$ to be smaller than $|6 - (-6)|$.

*You use **absolute value** a lot in real life — often without even thinking about it. For example, if the temperature falls from 3 °C to –3 °C you might use it to find the **overall change**. This Lesson looks at some of the ways that absolute value can apply to everyday situations.*

Absolute Values Help Find Distances Between Numbers

To find **the distance between two numbers** on the number line you could count the number of spaces between them.



Using **subtraction** is a quicker way — just subtract the lesser number from the greater. 8 – 6 = 2, so 8 and 6 are 2 units apart.

If you did the subtraction the other way around you'd get a **negative** number — and distances **can't** be negative. But if you use absolute value bars you can do the subtraction in **either order** and you'll always get a **positive value** for the distance.

$|6 - 8| = |-2| = 2$ and $|8 - 6| = 2$

For any numbers a and b:
The distance between a and b on the number line is $|a - b|$.

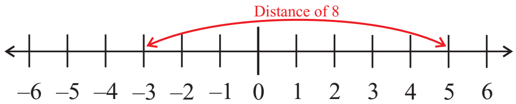
This is particularly useful when you're finding the distance between a **positive and negative number**.

Example 1

What is the distance between –3 and 5?

Solution

The distance between –3 and 5 is $|-3 - 5| = |-8| =$ **8**.



Guided Practice

Find the distance between the numbers given in each of Exercises 1–8.

1. 1, 5 **4** **2.** –3, –8 **5** **3.** 6, –9 **15** **4.** –1, 10 **11**
5. 3, –5 **8** **6.** 5, –1 **6** **7.** –1.2, 2.3 **3.5** **8.** –0.3, 2.7 **3**

9. At 1 p.m., Amanda was 8 miles east of her home. She then traveled in a straight line west until she was 6 miles west of her home. How many miles did she travel? **14 miles**

Solutions
For worked solutions see the Solution Guide

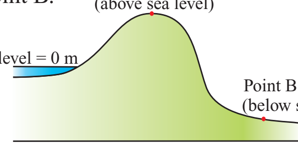
- **Strategic Learners**
  Give everyone a piece of grid paper. Get them to mark a point and label it **HOME**. Then map a trip: each square on the paper represents 1 block. Call out directions, for example: “Go east 4 blocks to the movie theater, then west 3 blocks to the store, then east 4 blocks to the park.” Have everyone work out how many blocks they traveled altogether (11 in the example). Then have them compare this to how far they are away from **HOME** at the end of the trip (this is their absolute value away from **HOME**, which is 5 in the example).
- **English Language Learners**
  Put a large number line up on the wall, with 0 in the middle. Place some different colored circles at various points on the number line. Ask students questions about the distance between the circles, such as, “*How far is the blue circle from the red circle*?” Emphasize **distance** as a key word for students to remember — have them write it at the side of their page in English and their own first language.

Absolute Values are Used to Compare Things

You can use absolute values to **compare** numbers when it doesn’t matter which side of a fixed point they are.

Example 2

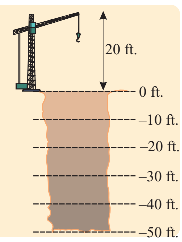
Find how far point A is above point B.



Solution
It doesn’t matter that B is **below** sea level and A is **above**.
It’s the **distance** between them that’s important.

You can find this by working out **|50 m – (–35 m)| = |85 m| = 85 m**.
You’d get the same answer if you did the subtraction the other way around:
|–35 m – 50 m| = |–85 m| = 85 m.

Guided Practice



10. A miner digs the shaft shown on the right. What distance was he from the top of the crane when he finished digging? **70 ft**

11. The top of Mount Whitney is 14,505 ft above sea level. The bottom of Death Valley is 282 ft below sea level. How much higher is the top of Mount Whitney than the bottom of Death Valley? **14,787 ft**

Absolute Values Can Describe Limits

Another use of absolute values is to describe the **acceptable limits** of a **measurement**. It might not be important whether something is above or below a set value, but **how far** above or below it is.

Example 3

The average temperature of the human body is 98.6 °F, but in a healthy person it can be up to 1.4 °F higher or lower. The difference between a person’s temperature, x, and the average healthy temperature can be found using the expression $|\ 98.6 - x\ |$.

Aaron is feeling unwell so measures his temperature. It is 100.2 °F. Is Aaron’s temperature within the healthy range?

Solution
The difference between Aaron’s temperature and the average healthy temperature is **| 98.6 – 100.2 | = | –1.6 | = 1.6 °F**.
Aaron’s temperature is outside of the normal healthy range.

Check it out:
If your body temperature goes too far from the normal value in either direction it can be dangerous, so this is an important use of absolute value.

2 Teach (cont)

Universal access
Fill an aquarium or large bowl half full of water. Put in a range of objects: some that protrude above the water level and some that are completely underwater. Label the waterline 0.

Have students use a ruler to measure how far above or below 0 the tops of the objects are, and write the results on the board.

Now ask them to use the measurements to find the distance between the tops of different objects. For example, “*How far is the top of the can from the top of the pebble*?” Remind them that they are using absolute values — in calculating the distances between objects it doesn’t matter whether they are above or below the waterline.

Concept question
“If my brother is 2 years older than me, and my sister is 3 years younger than me, what is the age difference between my brother and my sister?”

5 years

Guided practice
Level 1: q10–11
Level 2: q10–11
Level 3: q10–11

Additional example
A household refrigerator should be within 2 °C of a temperature of 4 °C.

The difference between a refrigerator’s temperature, t, and the standard temperature can be found using the expression $|\ 4 - t\ |$.

Are the following refrigerators at an appropriate temperature?
Refrigerator A: 4 °C **Yes**
Refrigerator B: 7 °C **No**
Refrigerator C: 2 °C **Yes**
Refrigerator D: 0 °C **No**

Solutions
For worked solutions see the Solution Guide

● **Advanced Learners**
Divide the students into groups. Give each group a thermometer. Get them to measure and record the temperature at 4 or 5 fixed points around the school, such as the classroom, the library, the schoolyard, the hall, and the gym. First have them find the difference in temperature between their own classroom and each of the other points. Then tell them that standard room temperature is 20 °C. Have them use their results to check whether all the indoor areas they tested are within 2 °C of standard room temperature.

2 Teach (cont)

Universal access
Bring in some circular homemade cookies all with a diameter of around the same size, say 7 cm. Make sure that there is some variability in size between the cookies. Alternatively, use card circles to represent the cookies.

Explain that you baked them for a bake sale, and you only want to sell those cookies that are 7 cm in diameter to within 0.5 cm each way. Have pupils measure and record the diameter of the cookies. Ask them to sort the cookies into two piles: those you can sell and those you can't.

Application
All factories use tolerance limits as a quality control check during the manufacturing process.

Guided practice
Level 1: q12–13
Level 2: q12–13
Level 3: q12–13

Additional example
A factory makes 30 mm long nails. They have a length tolerance limit of 0.05 mm. Would nails with the dimensions below be sold or rejected?
a) 30 mm **sold**
b) 30.05 mm **sold**
c) 29.94 mm **rejected**

Write an expression using absolute value that you could use to check if the length of a nail, n, is within the tolerance limit. **$|30 - n|$ or $|n - 30|$**

Independent practice
Level 1: q1–5
Level 2: q1–6
Level 3: q1–7

Additional questions
Level 1: p436 q1–8, 10
Level 2: p436 q1–11
Level 3: p436 q1–12

3 Homework

Homework Book — Lesson 2.2.2
Level 1: q1–4, 6
Level 2: q2–7
Level 3: q2–9

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 4 — Closeness and Absolute Value

Check it out:
Lots of manufacturers have tolerance limits for measurements. A tolerance limit is how different in size something is allowed to be from the size it should be. For example, size 6 knitting needles should have a diameter of 4 mm. Only very small differences from this size, such as 0.004 mm, will be allowed.

Example 4

A factory manufactures wheels that it advertises as no more than 1 inch away from 30 inches in diameter. They use the expression $|30 - d|$ to test whether wheels are within the advertised size (where d is the diameter).

Apply the expression to wheels of diameters 31, 29, and 35 inches, and say whether they meet the advertised standard.

Solution
• Wheel of diameter 31 inches: **$|30 - 31| = |-1| = 1$ inch**.
This wheel is within the standard.

• Wheel of diameter 29 inches: **$|30 - 29| = |1| = 1$ inch**.
This wheel is within the standard.

• Wheel of diameter 35 inches: **$|30 - 35| = |-5| = 5$ inches**.
This wheel is not within the standard.

Guided Practice

12. The height of a cupboard door should be no more than 0.05 cm away from 40 cm. The expression $|40 - h|$ is used to check whether a door of height h cm fits the size requirement. If a door measures 40.049 cm, is it within the correct range? **$|40 - 40.049| = 0.049$. It is within the correct range.**

13. Ms. Valesquez's car needs a tire pressure, p, of 30 psi. It should be within 0.5 psi of the recommended value. She uses the expression $|30 - p|$ to test whether the pressure is acceptable. Is a pressure of 29.4 psi acceptable? **$|30 - 29.4| = 0.6$. This isn't an acceptable pressure.**

Independent Practice

Find the distance between the numbers given in each of Exercises 1–4.
1. 9, –9 **18** **2.** 3, –4 **7** **3.** 5, 6 **1** **4.** –32, –52 **20**

5. The table below shows the temperature at different times of day. How much did the temperature change by between 7 a.m. and 8 a.m.?

Time	7 a.m.	8 a.m.
Temperature	–5 °C	1 °C

6 °C

Check it out:
It might help you to draw diagrams to go with Exercises 5 and 6.

6. A person stands on a pier fishing. The top of their rod is 20 feet above sea level. The line goes vertically down and hooks a fish 13 feet below sea level. How long is the line? **33 feet**

Now try these:
Lesson 2.2.2 additional questions — p436

7. Priscilla tries to keep the balance of her checking account, b, always less than \$50 away from \$200. She uses the expression $|200 - b|$ to check that it is within these limits. Is a balance of \$242.50 acceptable? **$|200 - 242.50| = 42.50$. This is an acceptable balance.**

Round Up
Absolute values are used to find distances between numbers. They're also useful when measurements are only allowed to be a certain distance away from a set value. In these situations, it doesn't matter if the numbers are above or below the set point — it's how far away they are that's important.

Solutions
For worked solutions see the Solution Guide

Lesson 2.3.1

Adding and Subtracting Integers and Decimals

In this Lesson, students review how to add and subtract integers using a number line and the column method. They then apply the same techniques to add and subtract decimals.

Previous Study: Students have studied addition and subtraction of integers since grade 1. They were introduced to addition and subtraction of decimals in grade 4.

Future Study: The ability to use addition and subtraction to manipulate integers and decimals will be crucial when solving equations throughout the rest of grade 7 and in Algebra I.

Lesson 2.3.1

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide **rational numbers** (**integers**, fractions, **and terminating decimals**) and take positive rational numbers to whole-number powers.

What it means for you:

You'll practice adding and subtracting integers on a number line, and then you'll go on to working with decimals.

Key words:

- integer
- decimal

Don't forget:

Integers are all the numbers that don't involve a decimal or a fraction. They can be positive or negative.

Section 2.3

Adding and Subtracting Integers and Decimals

You've had plenty of practice adding and subtracting integers in earlier grades — and you've probably done some adding and subtracting of decimals too. The same rules apply to decimals — you just have to be careful where you put the decimal point.

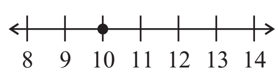
Add Integers Using a Number Line

A **number line** is a really good way to show integers in order. Adding and subtracting just involves **counting** left or right on the number line.

Example 1

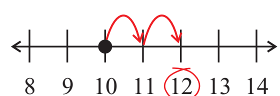
Calculate 10 + 2 using a number line.

Solution



Find the first number on the number line.

Count the number of positions given by the second number. Go **right** if it's a **positive** number.



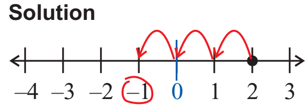
Then just **read off** the number you've ended up at. So 10 + 2 = **12**

If you're adding a **negative** number, you count to the **left**.

Example 2

Calculate 2 + (–3) using a number line.

Solution



So 2 + (–3) = **–1**

Subtractions can be turned into additions — then you can use the methods above for subtractions.

Subtracting a **positive** number is the same as **adding** a **negative** one. For example, **5 – 6 = 5 + (–6)**

And **subtracting** a **negative** number is the same as **adding** a **positive** one. For example, **5 – (–6) = 5 + 6**

Example 3

Calculate: (i) 2 – 3 (ii) 10 – (–2)

Solution

(i) 2 – 3 is the same as 2 + (–3). So 2 – 3 = **–1** (using Example 2).
(ii) 10 – (–2) equals 10 + 2. So 10 – (–2) = **12** (using Example 1).

1 Get started

Resources:

- large number line
- counters (red and blue)
- large 3-by-3 grid
- buttons labeled 1, 10, 100, and +
- Money Tiles (from **Teacher Resources CD-ROM**)
- homemade abacuses

Warm-up questions:

- Lesson 2.3.1 sheet

2 Teach

Universal access

Place a large number line in the middle of the classroom. Have students walk through addition and subtraction problems using the number line.

It may help to write a couple of rules on the board as reminders. For example, walk right to add a positive number or to subtract a negative number. Walk left to subtract a positive number or add a negative number.

Concept question

"If I was subtracting a negative number, would I move to the right or to the left on the number line?" **Right**

Common error

Students can be confused by the fact that positive numbers are usually shown without the + sign in front of them, even though negative numbers are always shown with a – sign.

Remind them that a number shown with no sign in front of it is positive, and that if it helps them they can include the positive sign. For example (+2) + (+4) = +6

- **Strategic Learners**
Give students some red counters and some blue counters. A red counter represents +1 and a blue counter represents –1. A pair of counters equals 0 and can be removed. Have them use the counters to work through some addition problems. For example: to do 3 + (–5) place 3 red counters and 5 blue counters on the table. 3 pairs can be removed, leaving 2 blue counters on the table. So 3 + (–5) = –2.

- **English Language Learners**
The vocabulary associated with addition and subtraction can be confusing when combined with the language of positive and negative numbers. Write some problems on the board and have students read them aloud, using "plus" or "minus" to refer to the operation used, and "positive" or "negative" to refer to the sign of each number. For example, they should read 1 – (–2) as "positive one minus negative two."

2 Teach (cont)

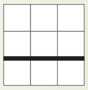
Guided practice
Level 1: q1–4
Level 2: q1–6
Level 3: q3–8

Additional examples
Use a decimal number line to work out these calculations:
1) 0.2 + 0.4 **0.6**
2) 0.2 + (–0.4) **–0.2**
3) 0.2 – 0.4 **–0.2**
4) 0.2 – (–0.4) **0.6**

Guided practice
Level 1: q9–12
Level 2: q9–15
Level 3: q9–17

Universal access
This activity can be used to illustrate the column method of addition. Give one student a button with a "**+**" symbol, and everyone else a button that says either **1**, **10**, or **100**.

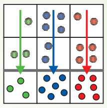
Put a large 3-by-3 grid on the floor, with a line separating off the bottom row of boxes like this:



Write an addition problem on the board, for example, 142 + 224. The top row of the grid represents the first number. Have two pupils with **1** buttons stand in the right-hand box, four with **10** buttons in the middle box, and one with a **100** button in the left-hand box. Repeat for the second row with the second number.

Get all the people in the right-hand column wearing **1**s to move down into the bottom-right square. Do the same with the **10**s and **100**s columns.

Now have "**+**" answer the problem by counting how many people are standing in each square in the bottom row, and writing it on the board.



142
+ 224
366

Don't forget:

$a - b \longrightarrow a + (-b)$
$a - (-b) \longrightarrow a + b$

Guided Practice

Use a number line to do the calculations in Exercises 1–8.

1. 10 + 3 **13** **2.** 6 + 1 **7** **3.** –3 + 5 **2** **4.** –9 – 7 **–16**
5. 7 + (–6) **1** **6.** 19 – (–5) **24** **7.** 3 + (–2) **1** **8.** –1 – (–9) **8**

Decimal Addition Needs a Decimal Number Line

Addition with **decimals** isn't any tougher than with integers — you've just got to remember to draw a number line that includes decimals.

Example 4

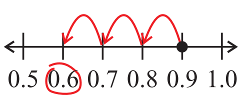
Calculate 0.9 – 0.3 using a number line.

Solution

0.9 – 0.3 is a subtraction. But because **subtracting a positive number** is the same as **adding a negative one**, it can be written as:

0.9 – 0.3 = 0.9 + (–0.3)

You're dealing with **decimals**, so you need a number line that shows **decimal values**. Find 0.9, and count 0.3 to the **left** because you're **adding a negative number**.



So 0.9 – 0.3 = **0.6**

Guided Practice

Use a number line to do the calculations in Exercises 9–17.

9. 0.6 + 0.4 **1** **10.** 0.1 + 0.8 **0.9** **11.** 1.3 + 0.7 **2**
12. 2.3 + (–0.4) **1.9** **13.** 3.1 – 0.7 **2.4** **14.** –0.7 – (–0.4) **–0.3**
15. –0.9 – 0.3 **–1.2** **16.** –1.2 – (–0.5) **–0.7** **17.** 0.3 – 0.5 **–0.2**

You Don't Always Have to Use a Number Line

It's hard to add big numbers on a number line, so you need to be able to add and subtract **without** a number line.

Example 5

Calculate 432 + 34 without using a number line.

Solution

You can break the calculation down into **hundreds**, **tens**, and **ones**.

432 + 34

Four hundreds, Three tens, Two ones; Three tens, Four ones

You need to add the ones of each number together, then the tens, and so on. Write the numbers on **top** of each other with the ones lined up.

  432
 +34

Work out the **sum** of the **ones** column first, then the **tens**, then the **hundreds**.

  432
 +34
  466

So 432 + 34 = **466**

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to research the Chinese or Japanese abacus, and its uses in the addition and subtraction of numbers. They could make a poster or give a brief presentation to the class about their research. Alternatively have them make their own abacuses to demonstrate to the rest of class. Have calculation races using the abacuses. Ask students to look at the history of abacuses in general — what were the earliest ones like?

The last example was easier than some calculations because all the column sums came to less than 10. If they go **over 10** then you have to **carry** the extra numbers to the next column.

Example 6

Calculate 567 + 125.

Solution

First you need to write the sum out **vertically** with the ones, the tens, and the hundreds lined up.

$$\begin{array}{r} 567 \\ +125 \\ \hline \end{array}$$

$$\begin{array}{r} 567 \\ +125 \\ \hline {}_1\ \ 2 \end{array}$$

10 is the same as 1 ten — so carry a 1 to the tens column

The ones calculation is 7 + 5 = 12. 12 is the same as saying **"one ten and two ones"** — so write 2 under the ones, and **carry** the 10 to the tens column.

When you add up the next column, remember to **add the 1** that you carried.

$$\begin{array}{r} 567 \\ +125 \\ \hline 692 \end{array}$$

So the tens column is now 6 + 2 + 1 = 9

So 567 + 125 = **692**

You can use a similar method for decimals, but there are a few things to remember. The digits after a decimal point show **parts of a whole** — so 24.56 means "2 tens, 4 ones, 5 tenths, and 6 hundredths."

24.560

two tens, four units, six hundredths, five tenths, zero thousandths

You can also add **extra zeros** onto the end of a decimal and it **doesn't change** the value.

> **Don't forget:**
> "Hundreds," "tens," "ones," "tenths," "hundredths," and so on are called place values.

You should **always** only add digits with the same place values. So when you're adding **decimals**, line the values up by the decimal point.

Example 7

Calculate 13.93 + 5.2.

Solution

Write the sum out **vertically** with the decimal points lined up.

$$\begin{array}{r} 13.93 \\ +\ 5.20 \\ \hline \end{array}$$

Make sure the decimal points are lined up

Add zeros to get the same number of decimal places

Then work out the **sum** of each **column** in turn, starting with the right-hand side. Don't forget the decimal point in the answer.

$$\begin{array}{r} 13.93 \\ +\ 5.20 \\ \hline 19.13 \end{array}$$

Carry numbers to the next column on the left just like before.

So 13.93 + 5.2 = **19.13**

Guided Practice

Calculate the following sums without using a number line.

18. 210 + 643 **853** **19.** 613 + 117 **730** **20.** 1264 + 527 **1791**
21. 33.7 + 12.4 **46.1** **22.** 14.8 + 16.2 **31** **23.** 55.82 + 34.81 **90.63**
24. 75.1 + 14.31 **89.41** **25.** 62.4 + 31.99 **94.39** **26.** 2.29 + 9.92 **12.21**

2 Teach (cont)

Universal access

Introduce the concept of decimal addition using money. Set a problem based around price. For example:

"*If I buy a notebook for $3.10 and a pen for $1.83, how much will I pay altogether*?"

Give each student a selection of Money Tiles (found on the **Teacher Resources CD-ROM**). Ask them to count out the right amount of money for each item. Then have them work out the answer by adding up the "coins" they have.

Connect this to the idea of the column format, for example, 1¢ is the same as one-hundredth, a dime is one-tenth, and $1 is a unit.

Have them write the same problem out in column format and work it through. They should get the same answer ($4.93).

Concept question

"What is 12 hundredths plus 12 thousandths?"

132 thousandths or 0.132

Common error

Students sometimes line up the numbers incorrectly when writing problems in column format; for example, left-aligning the numbers:

$$\begin{array}{l} 12.6 \\ +1.20 \\ \hline 2.46 \end{array} \quad ✗$$

Remind students that they need to line up digits that have the same place value. This means that the decimal points of the numbers should always be lined up too. For example:

$$\begin{array}{r} 12.6 \\ +\ 1.2 \\ \hline 13.8 \end{array} \quad ✓$$

Guided practice

Level 1: q18–21
Level 2: q18–24
Level 3: q18–26

Solutions

For worked solutions see the Solution Guide

2 Teach (cont)

Math background

The beginning of the Lesson covered the topic of adding negative numbers. An alternative way of thinking of subtraction is like this:

$$31 - 18 = (30 - 10) + (1 - 8) = 20 + (-7) = 13$$

Students may find this method easier when doing mental math.

Universal access

Use the context of money to approach subtraction. For example:

"*If I buy a \$2.14 bus ticket and pay with \$2.50, how much change will I receive*?"

Give each student a selection of money tiles (found on the **Teacher Resources CD-ROM**). Ask them to count out the larger amount of money (\$2.50). Then have them work out the answer by taking away from that pile the cost of the item (\$2.14). Some "coins" will have to be exchanged for "coins" of smaller value.

Again, connect this to the idea of the column format, and then have them write the problem out in column format and work it through. They should get the same answer (\$0.36).

Guided practice

Level 1: q27–31
Level 2: q27–33
Level 3: q27–35

Independent practice

Level 1: q1–9
Level 2: q1–10
Level 3: q1–13

Additional questions

Level 1: p436 q1–8, 13
Level 2: p436 q1–13
Level 3: p436 q5–18

3 Homework

Homework Book — Lesson 2.3.1

Level 1: q1–8, 11
Level 2: q3, 9, 11
Level 3: q3–13

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:

- Worksheet 5 — Addition and Subtraction

Some Subtractions Involve "Borrowing"

Doing column subtractions seems tough if the top number in a column is **smaller** than the number underneath — but there's a handy method.

Example 8

Calculate 30 – 18.

Solution

$$\begin{array}{r} 30 \\ -18 \\ \hline \end{array}$$

If you write this in columns, the upper number in the ones column is **smaller** than the number below.

You can **break down** 30 into different parts:

$$30 = 20 + 10$$

You could say this as "2 tens plus 1 ten" or you could say "2 tens plus 10 ones."

All you've done is separate one of the **tens** from the number.

You can show this in column notation:

$$\begin{array}{r} \overset{2}{\cancel{3}}\,\overset{10}{\cancel{0}} \\ -1\,8 \\ \hline 1\,2 \end{array}$$

"Borrow" 10 from the tens column, so now the column subtractions are 10 – 8 and 2 – 1.

So 30 – 18 = **12**

Example 9

Calculate 65.37 – 31.5.

Solution

Add a zero to make them the **same** number of decimal places:

$$\begin{array}{r} 65.37 \\ -31.50 \\ \hline \end{array}$$

If the top number in a column is smaller than the bottom one, use the same method of "**borrowing**" from the next column.

$$\begin{array}{r} 65.\overset{4}{\cancel{3}}\overset{13}{7} \\ -31.50 \\ \hline 33.87 \end{array}$$

You need to borrow from the ones column. A one is ten tenths — so the 3 in the tenths column becomes 13.

So 65.37 – 31.5 = **33.87**

Don't forget:

If you "borrow" a 10 when you're subtracting, remember that the number you borrowed from will now be 1 ten lower.

Guided Practice

Calculate the following sums without using a number line.

27. 50 – 26 **24** **28.** 62 – 18 **44** **29.** 76 – 49 **27**
30. 941 – 46 **895** **31.** 62.4 – 31.2 **31.2** **32.** 68.83 – 11.3 **57.53**
33. 29.42 – 13.18 **16.24** **34.** 46.11 – 21.95 **24.16** **35.** 42.38 – 36.45 **5.93**

Independent Practice

Draw number lines to answer Exercises 1–4.

1. 7 + 9 **16** **2.** 7 – (–9) **16** **3.** 9 + 7 **16** **4.** –9 – 7 **–16**

Calculate the following without using a number line.

5. 0.99 + 0.45 **1.44** **6.** 1.86 + 3.33 **5.19** **7.** 15.64 + 3.67 **19.31**
8. 45.64 + 13.88 **59.52** **9.** 164.31 + 251.3 **415.61** **10.** 32.12 – 12.1 **20.02**
11. 112.13 – 38.7 **73.43** **12.** 19.4 – 5.37 **14.03** **13.** 64.11 – 44.7 **19.41**

Now try these:

Lesson 2.3.1 additional questions — p436

Round Up

Adding and subtracting integers and decimals — it sounds like a lot to learn, but you use the same methods over and over. You'll need to add and subtract decimals when you're doing real-life math too.

Solutions

For worked solutions see the Solution Guide

Lesson 2.3.2

Multiplying and Dividing Integers

In this Lesson students review how a number line can be used to multiply and divide integers. This leads on to multiplication using the area method and the column method. A review of long division is also included.

Previous Study: Students have studied multiplication and division of integers since grade 2. They learned how to use the column format for multiplication and how to do long division at grade 5.

Future Study: Proficiency with the standard techniques for the multiplication and division of integers will be crucial when solving equations throughout the rest of grade 7 and in Algebra I.

Lesson 2.3.2

Multiplying and Dividing Integers

California Standards:

Number Sense 1.2

Add, subtract, **multiply, and divide rational numbers** (**integers**, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

What it means for you:

You'll practice multiplying and dividing integers on a number line, and then using other methods.

Key words:

- integer

Multiplying and dividing are important skills, both in math and real life. In this Lesson you'll see how multiplication and division can be modeled.

Picture Multiplication and Division on a Number Line

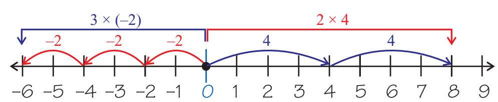
A number line is just a way of showing the **order** of numbers — so you can use it for any kind of calculation, including **multiplication** and **division**.

Multiplying by positive integers looks like a set of "hops" away from **zero**.

Example 1

Calculate using the number line: (i) 2 × 4, (ii) 3 × (–2)

Solution



(i) Multiplying 4 by 2 means you need to move 2 groups of 4 away from zero. So **2 × 4 = 8**

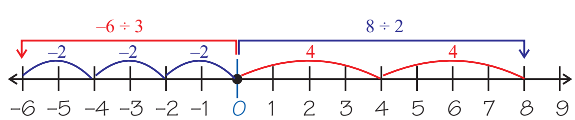
(ii) Multiplying –2 by 3 means you need to move 3 groups of –2 from zero. So **3 × (–2) = –6**

Dividing by a positive integer looks like you're breaking a number down into **equally sized** parts.

Example 2

Calculate using the number line: (i) 8 ÷ 2, (ii) –6 ÷ 3

Solution



(i) Dividing 8 by 2 means finding 8 on the number line, and then splitting the distance between 8 and 0 into 2 equal parts. So **8 ÷ 2 = 4**

(ii) Dividing –6 by 3 means you find –6 on the number line, and then split the distance between –6 and 0 into 3 equal parts. So **–6 ÷ 3 = –2**

Sometimes it's hard to tell whether your answer will be **positive or negative**. The table below shows what the sign of the answer will be:

positive ×/÷ positive = positive
positive ×/÷ negative = negative
negative ×/÷ positive = negative
negative ×/÷ negative = positive

So for example, 8 ÷ (–2) = –4 while –2 × (–4) = 8

Don't forget:

You learned about the rules for multiplying by positive and negative numbers in grade 6.

Don't forget:

The "sign" of a number just means whether it is positive or negative.

1 Get started

Resources:

- individual whiteboards and pens
- counters
- playing cards
- internet-enabled computers

Warm-up questions:

- Lesson 2.3.2 sheet

2 Teach

Common error

Students sometimes forget that the sign of the answer is important when you multiply. The Universal access activity below is useful for helping students remember to think about the sign.

Universal access

Help students to think about the sign of the answer by calling out multiplication and division questions and having them write down the sign of the answer only.

For example: "What will the sign of the answer be to:"

1. 15 × 23	**Positive**
2. –2 × 14	**Negative**
3. 122 ÷ –66	**Negative**
4. –17 × –17	**Positive**

Give each student an individual whiteboard to write their answers on, and get them to hold the board up after each question as a check.

Concept question

"If I divide an integer by another integer, will the absolute value of my answer be larger or smaller than the first integer?"

Smaller

- **Strategic Learners**
  Remind students that multiplication is the same as repeated addition. Give them a few simple problems to begin with, and give them some counters to group to help them find the answer. For example, 3 × 5 = 5 + 5 + 5, or 3 groups of 5. By putting the counters into 3 groups of 5 they should find that the answer is 15. Then have them do the same thing without counters.
- **English Language Learners**
  Begin with some simple multiplication questions, and go through them to make sure the students understand the concept and vocabulary fully. For example, give them the problem 21 × –5. Ask not only what the answer is but a series of additional questions like, "*What will the sign of the answer be*?," "*How many units, 10s, and 100s does your answer have*?," and "*How can you show your answer on a number line*?"

2 Teach (cont)

Guided practice
Level 1: q1–3, 7–9
Level 2: q1–4, 7–10
Level 3: q3–12

Math background
The area method uses the formula for the area of a rectangle:
Rectangle area = length × width

Concept question
"*If* $10x$ *= (10 × 10) + (10 × 5) then what is the value of* x?" **15**

Universal access
This game motivates students to practice multiplying numbers without a calculator. Each pair of students has a deck of playing cards, which they deal out between them.

Each player turns their top card over and both players try to be the first to multiply the numbers on the two cards together. A jack is worth 11, a queen 12, a king 13, and an ace 1. Allow them paper and a pencil to do their work, but no calculator.

The first player to give the correct answer wins the pair of cards. If they answer incorrectly, the other player gets the cards. The game is over when one player gets the whole deck.

For a quick version, award a point for a correct answer, the winner being the first to get to 10 points. For a tougher version, make your own cards with larger numbers on them.

Guided practice
Level 1: q13–16
Level 2: q13–18
Level 3: q13–20

Guided Practice

Use a number line to work out the problems given in Exercises 1–12.

1. 4 × 3 **12** **2.** 2 × (–5) **–10** **3.** –4 × 6 **–24** **4.** –4 × (–3) **12**
5. –6 × 4 **–24** **6.** –3 × (–3) **9** **7.** 20 ÷ 5 **4** **8.** 18 ÷ 6 **3**
9. 22 ÷ 2 **11** **10.** –15 ÷ 3 **–5** **11.** 16 ÷ (–4) **–4** **12.** –14 ÷ (–7) **2**

You Must Know How to Multiply Without a Calculator

You can only really use a number line for simple multiplications. Here are two good ways of multiplying any two numbers together:

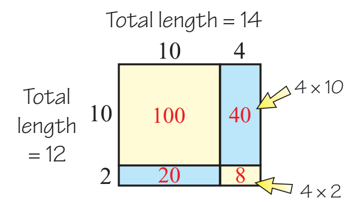
Example 3

Calculate 12 × 14.

Solution

Picture each number as the length of a side of a rectangle — but break each number down into tens and ones. Then you can do the multiplication "part by part."

The total area of the rectangle is 12 × 14, and you can see this equals 100 + 40 + 20 + 8 = **168**



Another method is to write the numbers on **top** of each other.

$$\begin{array}{r} 14 \\ \times\ 12 \\ \hline 28 \\ +\ 140 \\ \hline \mathbf{168} \end{array}$$

First write the numbers as a vertical calculation.
Multiply the top number by the **ones** digit of the bottom number. 2 × 14 = **28**
Then multiply the top number by the **tens** of the bottom number: 10 × 14 = **140**
Then add them together, just like in the "rectangle" method.

This is known as **long multiplication**.

Notice how the above two methods are **very similar**.

The first line of work in the long multiplication is the same as the area of the bottom part of the rectangle, and the second line of work in the long multiplication is the same as the area of the top part of the rectangle.

In both methods you then **add these together** to get the overall result.

Guided Practice

Use the methods in Example 3 for Exercises 13–20.

13. 12 × 13 **156** **14.** 22 × 16 **352** **15.** 32 × 18 **576** **16.** –14 × 37 **–518**
17. –46 × 42 **–1932** **18.** 25 × 58 **1450** **19.** 52 × 67 **3484** **20.** 85 × 95 **8075**

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Have students use the internet to research an alternative algorithm for multiplication — for example, the lattice method or Russian peasant multiplication. Alternatively you could research them yourself before class, and hand out a prepared instruction sheet on either method. Have students teach themselves the method and use it to solve some problems, checking their answers using long multiplication. Then get them to explain the method they studied to someone else.

Work Through Long Division from Left to Right

Long division is a good way of writing down and solving tricky division problems. It involves dividing big numbers bit by bit, by breaking them into collections of smaller numbers.

Check it out:
Long division means breaking a division down into small problems like figuring out that 9 × 5 goes into 46 (with 1 left over).
But, in Example 4, remember that the 4 is in the hundreds column and the 6 is in the tens column, so you're really working out that 90 × 5 goes into 460 (with 10 left over).
Long division is just a way of breaking up complicated division problems.

Example 4

Calculate 467 ÷ 5.

Solution

You need to divide the whole of 467 by 5, but you can work through it bit by bit.

The number you're dividing by goes on the left. Work from the left to find numbers that divide by 5.

Keep going until you've divided the whole number.

The answer to this division is **93 with remainder 2**.
The standard way of writing this is **467 = (93 × 5) + 2**.

$$\begin{array}{r} 9 \\ 5\overline{)467} \\ -45 \\ \hline 17 \end{array}$$

5 doesn't divide into 4, but it divides into 46 (this is actually 460). The maximum number of times 5 divides into 46 is 9, so put 9 in the tens column. You now know that the answer is in the 90s.

9 × 5 = 45, so put 45 underneath, lined up with 46.

Subtract 45 from 46, and bring the 7 down to find the amount left.

$$\begin{array}{r} 93 \\ 5\overline{)467} \\ -450 \\ \hline 17 \\ -15 \\ \hline 2 \end{array}$$

There's 17 remaining. Now divide 17 by 5. 5 goes into 17 a maximum of 3 times. Write this in the units column.

3 × 5 = 15, so put 15 underneath, lining up the units.

There's just 2 left. 5 won't go into 2, so it's a remainder.

$$\begin{array}{r} 93\text{ R2} \\ 5\overline{)467} \end{array}$$

Guided Practice

Write out and solve these calculations using long division.

21. 72 ÷ 6 **12** **22.** 104 ÷ 4 **26** **23.** 105 ÷ 7 **15**
24. 274 ÷ 13 **(21 × 13) + 1** **25.** 1955 ÷ 8 **(244 × 8) + 3** **26.** 5366 ÷ 13 **(412 × 13) + 10**

Independent Practice

Show the multiplications in Exercises 1–3 on a number line.
1. 3 × 1 **see below** **2.** 2 × 5 **see below** **3.** 4 × –2 **see below**

Use the area method to find the products in Exercises 4–6.
4. 13 × 15 **195** **5.** 12 × 65 **780** **6.** 33 × 56 **1848**

Find the products in Exercises 7–12.
7. 11 × 18 **198** **8.** 13 × 22 **286** **9.** 25 × 21 **525**
10. –33 × 12 **–396** **11.** 16 × –15 **–240** **12.** –23 × –51 **1173**

Find the quotients in Exercises 13–18.
13. 710 ÷ 5 **142** **14.** 138 ÷ 6 **23** **15.** 1248 ÷ –8 **–156**
16. 190 ÷ 3 **(63 × 3) + 1** **17.** 274 ÷ 4 **(68 × 4) + 2** **18.** 172 ÷ 5 **(34 × 5) + 2**

Check it out:
A quotient is what you get when you divide one number by another.
A product is what you get when you multiply numbers.

Now try these:
Lesson 2.3.2 additional questions — p437

Round Up

Multiplying and dividing large numbers without a calculator seems like quite a task. But if you break the numbers up into ones, tens, and hundreds, then they're much simpler to handle.

2 Teach (cont)

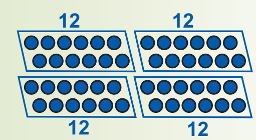
Universal access

If students find this concept tricky, give them some simpler problems to start with that don't involve remainders. Have them write out the problems as long divisions, but also give them some counters to use.

For example, "*What is 48 ÷ 4*?" They should write the problem out:

$$\begin{array}{r} 12 \\ 4\overline{)48} \\ 40 \\ \hline 8 \\ 8 \\ \hline 0 \end{array}$$

But they can also illustrate the division by counting out 48 counters and trying to split them into 4 equal piles:



This enables them to physically check their solutions.

Guided practice

Level 1: q21–23
Level 2: q21–25
Level 3: q21–26

Independent practice

Level 1: q1–2, 4, 7–9, 13–15
Level 2: q1–9, 13–15
Level 3: q1–18

Additional questions

Level 1: p437 q1–11
Level 2: p437 q4–14, q18–19
Level 3: p437 q7–19

3 Homework

Homework Book — Lesson 2.3.2

Level 1: q1–3, 5a–b, 6a–b, 7, 8
Level 2: q1–10
Level 3: q1–11

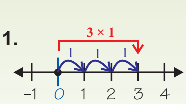
4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 6 — Multiplying and Dividing Integers

Solutions

For worked solutions see the Solution Guide

1.


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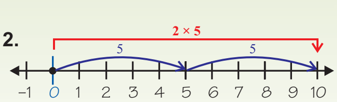

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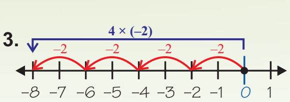


Lesson 2.3.3

Multiplying Fractions

In this Lesson, students develop their understanding of fraction multiplication by using area models. This leads to practicing multiplying fractions by other fractions, integers, and mixed numbers.

Previous Study: In grade 5 students learned how to multiply fractions and express them in their simplest form. This was also covered in Section 2.1.

Future Study: In Algebra I students will be expected to be able to multiply rational expressions and polynomials. They will solve equations and inequalities using these techniques later in grade 7 and in Algebra I.

1 Get started

Resources:
- squares of paper
- paper circles cut into 10 pieces
- counters

Warm-up questions:
- Lesson 2.3.3 sheet

2 Teach

Concept question

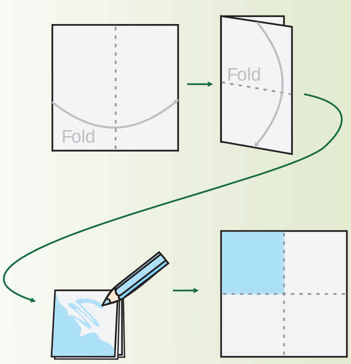
"If I multiply a positive integer by a positive proper fraction, will my answer be bigger or smaller than the original integer?"
Smaller

Universal access

Teach the area model by paper folding. Start with a simple example like $\frac{1}{2}\times\frac{1}{2}$.

Give everyone a square piece of paper. Ask them to fold it in half, and then in half again. Have them shade in the top side. Say that the shaded piece they have in front of them is half of a half. Now have them unfold the paper — they should be able to see that the area of the folded square was equal to $\frac{1}{4}$ of the original.

Write the multiplication on the board: $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$.



Guided practice
Level 1: q1–3
Level 2: q1–3
Level 3: q1–3

Lesson 2.3.3

Multiplying Fractions

California Standards:
Number Sense 1.2
Add, subtract, **multiply**, and divide **rational numbers** (integers, **fractions**, and terminating decimals) and take positive rational numbers to whole-number powers.

What it means for you:
You'll practice multiplying fractions, and you'll extend this to multiplying fractions by integers and mixed numbers.

Key words:
- area model
- mixed numbers

Check it out:
The answer here is correct, although it could be simplified. See the next page (or Section 2.1) for more information.

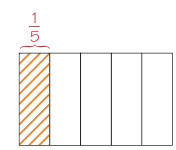
You've multiplied fractions in other grades — but it's still not an easy topic. In this Lesson you'll get plenty more practice at it.

Area Models Show Fraction Multiplication

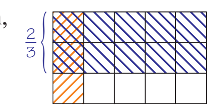
Multiplying a fraction by another fraction means working out **parts of a part**. For example, $\frac{1}{5}\times\frac{2}{3}$ means "**one-fifth of two-thirds**."

You can show this graphically — it's called an **area model**. You need to start by drawing a rectangle.

- Shade in $\frac{1}{5}$ of the rectangle in one direction:



- Then shade in $\frac{2}{3}$ of it in the other direction, using a different color:



The part showing $\frac{1}{5}\times\frac{2}{3}$ is the part that represents **one-fifth of two-thirds** — this is the part that's shaded in both colors.

There are 2 squares shaded out of a total of 15, so $\frac{1}{5}\times\frac{2}{3}=\frac{2}{15}$.

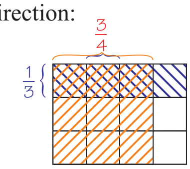
Example 1

Calculate $\frac{3}{4}\times\frac{1}{3}$ using the area model method.

Solution

You need to work out **three-quarters of one-third** — so shade in $\frac{3}{4}$ of the rectangle in one direction, and $\frac{1}{3}$ in the other direction:

There are 3 out of 12 squares shaded in both colors, so $\frac{3}{4}\times\frac{1}{3}=\frac{3}{12}$.



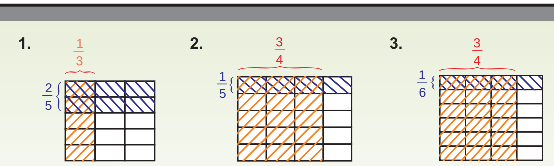
Guided Practice

Calculate these fraction multiplications by drawing area models:

1. $\frac{1}{3}\times\frac{2}{5}$ **$\frac{2}{15}$**
2. $\frac{3}{4}\times\frac{1}{5}$ **$\frac{3}{20}$**
3. $\frac{3}{4}\times\frac{1}{6}$ **$\frac{3}{24}$ or $\frac{1}{8}$**

See below for area models

Solutions
For worked solutions see the Solution Guide

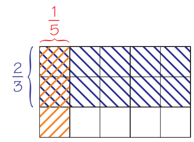


- **Strategic Learners**
Give each student a circle of paper cut into 10 equal wedges. Remind them that each wedge is $\frac{1}{10}$ of the whole. Tell them they are going to do the multiplication $\frac{9}{10} \times \frac{1}{3}$. Have them count out 9 wedges. This is $\frac{9}{10}$. Ask them to find $\frac{1}{3}$ of the pile of wedges – 3 wedges. As each wedge is $\frac{1}{10}$, they should be able to see that $\frac{9}{10} \times \frac{1}{3}$ is $\frac{3}{10}$. Now go over the same problem using the area method.
- **English Language Learners**
Ask everyone who plays music to stand. Write on the board how many this is as a fraction of the class. Ask everyone who plays music and sport to stand. Write this as a fraction of those who play music. Ask everyone to work out what fraction of the class play music and sport. As a check, have everyone who plays music and sport stand up and count how many this is compared to the number of people in the class.

You Can Multiply Fractions Without Drawing Diagrams

When you draw an area model, the **total number of squares** is always the same as the **product of the denominators** of the fractions you're multiplying.

You've already seen the area model for $\frac{1}{5} \times \frac{2}{3}$:



Multiply the denominators:
- The total number of squares is $5 \times 3 = 15$.

Also, the **number of squares shaded in both colors** is always the same as the **product of the numerators**.

Multiply the numerators:
- The total number of squares shaded in both colors is $1 \times 2 = 2$.

That means you can work the product out without drawing the area model.

Example 2

Calculate $\frac{3}{4} \times \frac{1}{3}$ without drawing a diagram.

Solution

Multiply the numerators: $3 \times 1 = 3$

Multiply the denominators: $4 \times 3 = 12$

Now write this as a fraction: $\frac{3}{12}$ ← numerator (3), ← denominator (12)

So $\frac{3}{4} \times \frac{1}{3} = \frac{3}{12}$.

The solution to Example 2 could be simplified a bit more.
Simplifying just means writing the solution using **smaller numbers**, but so that the fraction still means the same thing.

The numerator and denominator can both be divided by 3... $\frac{3}{12} \xrightarrow{\div 3} \frac{1}{4}$ ...so $\frac{1}{4}$ represents the same amount, but simplified.

That means you could write $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$.

Don't forget:
The top number in a fraction is called the numerator, and the bottom number is called the denominator.
$\frac{2}{5}$ 2 ← numerator, 5 ← denominator

Don't forget:
You can simplify fractions using the greatest common factor (GCF) — there's a lot more about this in Section 2.1.

Don't forget:
A fraction is in its simplest form if 1 is the only number that divides exactly into both the numerator and denominator.

Guided Practice

Calculate these fraction multiplications without drawing area models. Simplify your answer where possible.

4. $\frac{5}{6} \times \frac{1}{2}$ $\frac{5}{12}$ **5.** $\frac{2}{3} \times \frac{1}{3}$ $\frac{2}{9}$ **6.** $\frac{1}{3} \times \frac{6}{7}$ $\frac{2}{7}$

7. $\frac{3}{8} \times \frac{1}{4}$ $\frac{3}{32}$ **8.** $\frac{5}{7} \times \frac{3}{5}$ $\frac{3}{7}$ **9.** $\frac{11}{12} \times \frac{6}{7}$ $\frac{11}{14}$

2 Teach (cont)

Common error
Having learned cross-multiplication in grade 6, some students multiply the numerator of one fraction by the denominator of the other.

Emphasize that when you multiply two fractions together, you always multiply the numerators by each other, and the denominators by each other.

Additional examples
Calculate without drawing a diagram:

1) $\frac{1}{2} \times \frac{1}{4}$ $\frac{1}{8}$

2) $\frac{2}{3} \times \frac{1}{5}$ $\frac{2}{15}$

3) $\frac{1}{3} \times \frac{1}{3}$ $\frac{1}{9}$

Concept question
"If $\frac{1}{2} \times \frac{y}{7} = \frac{3}{14}$, what is y?" 3

Math background
To simplify a fraction means to reduce its numerator and denominator to the smallest integers possible.
It's most efficient to divide both numbers by the greatest common factor, or GCF. A full description of how to find the GCF is in Lesson 2.1.2.

Alternatively, students can just divide the numerator and denominator by any common fractions until the simplest form is reached.

Guided practice
Level 1: q4–6
Level 2: q4–8
Level 3: q4–9

Solutions
For worked solutions see the Solution Guide

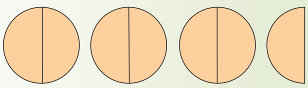
- **Advanced Learners**
  Teach students to cancel common factors before multiplying fractions — this eliminates the need to simplify the fraction later.
  For example: $\frac{12}{26}\times\frac{52}{7}=\frac{12\times52}{26\times7}=\frac{\cancel{2}\times6\times\cancel{13}\times4}{\cancel{2}\times\cancel{13}\times7}=\frac{6\times4}{7}=\frac{24}{7}$. Remind them that the calculation will be easiest if they deliberately try to break the numerator and denominator down to the same factors. In the example above, the 12 could have been split into 2 × 6 or 3 × 4, but 2 × 6 was chosen as 26 had been split into 2 × 13 — so a common factor was created.

2 Teach (cont)

Universal access

If students have difficulty converting mixed numbers to fractions, encourage them to use diagrams, for example:

$3\frac{1}{2}=$ seven halves $=\frac{7}{2}$



Relate mixed number problems to concrete items. For example, *"If 3 pizzas are all divided in half, you have six halves."*

Guided practice

Level 1: q10–12
Level 2: q10–13
Level 3: q10–15

Independent practice

Level 1: q1–3, 5
Level 2: q1–6
Level 3: q1–6

Additional questions

Level 1: p437 q1–4, q6–7
Level 2: p437 q2–8, q12
Level 3: p437 q3–12

3 Homework

Homework Book — Lesson 2.3.3

Level 1: q1–3, 5, 6a
Level 2: q1–8
Level 3: q2–10

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 2 — Simplifying Fractions
- Worksheet 9 — Multiplying Fractions with Integers
- Worksheet 10 — Multiplying Fractions by Fractions

Don't forget:
A mixed number contains an integer and a fraction.
For example, $3\frac{1}{2}$ is a mixed number.

Don't forget:
The whole number 3 is equivalent to $3\times\frac{2}{2}=\frac{6}{2}$.
$\frac{2}{2}$ is the same as one whole.

First Convert Whole or Mixed Numbers to Fractions

To multiply **fractions** by **mixed numbers**, you can just write out the mixed numbers as a **single fraction** and carry on multiplying as normal.
The same is true if you need to multiply a fraction by an **integer** — you can write the integer as a fraction and use the multiplication method from before.

Example 3

Calculate: (i) $3\frac{1}{2}\times\frac{1}{4}$, (ii) $\frac{4}{5}\times8$

Solution

(i) Convert $3\frac{1}{2}$ to a fraction: $3\frac{1}{2}=\frac{(3\times2)+1}{2}=\frac{7}{2}$

Then just multiply out the fractions as normal:

$3\frac{1}{2}\times\frac{1}{4}=\frac{7}{2}\times\frac{1}{4}=\frac{7}{8}$

(ii) The integer 8 can be written as $\frac{8}{1}$.

So you can multiply as normal: $\frac{4}{5}\times8=\frac{4}{5}\times\frac{8}{1}=\frac{32}{5}$

Guided Practice

Calculate the following, and simplify your solutions where possible.

10. $1\frac{1}{3}\times\frac{1}{5}$ $\frac{4}{15}$ **11.** $\frac{1}{3}\times2\frac{1}{3}$ $\frac{7}{9}$ **12.** $1\frac{2}{3}\times\frac{2}{3}$ $\frac{10}{9}$ or $1\frac{1}{9}$

13. $3\times\frac{2}{7}$ $\frac{6}{7}$ **14.** $1\frac{1}{4}\times\frac{1}{5}$ $\frac{1}{4}$ **15.** $1\frac{5}{7}\times1\frac{1}{2}$ $\frac{18}{7}$ or $2\frac{4}{7}$

Don't forget:
If you're multiplying negative numbers, remember the rules on p75 for working out the sign of the answer.

Don't forget:
You should usually write answers as mixed numbers instead of improper fractions — this keeps the numbers simpler.

Now try these:
Lesson 2.3.3 additional questions — p437

Independent Practice

Find the product and simplify each calculation in Exercises 1–3.

1. $\frac{2}{3}\times\frac{7}{10}$ $\frac{7}{15}$ **2.** $-\frac{4}{9}\times\frac{3}{15}$ $-\frac{4}{45}$ **3.** $-\frac{5}{12}\times2\frac{1}{3}$ $-\frac{35}{36}$

4. A positive whole number is multiplied by a positive fraction smaller than one. Explain how the size of the product compares to the original whole number. **The product will always be smaller than the original whole number.**

5. A rectangular patio measures $8\frac{1}{4}$ feet wide and $12\frac{1}{2}$ feet long. What is the area of the patio? **$103\frac{1}{8}$ square feet**

6. A recipe for 12 muffins calls for $3\frac{1}{4}$ cups of flour. How many cups of flour are needed to make 42 muffins? **$11\frac{3}{8}$ cups**

Round Up

Multiplying fractions is OK because you don't need to put each fraction over the same denominator. If you need to multiply by integers or mixed numbers, just turn them into fractions too.

Solutions
For worked solutions see the Solution Guide

Lesson 2.3.4

Dividing Fractions

In this Lesson students develop their understanding of division by fractions by modeling it on a number line. They then review how to divide fractions by multiplying by the inverse.

Previous Study: In grade 6 students learned how to simplify expressions involving the multiplication and division of fractions by other fractions, integers, and mixed numbers.

Future Study: In Algebra I students will be expected to be able to divide rational expressions and polynomials. They will solve equations and inequalities using these techniques later in grade 7 and in Algebra I.

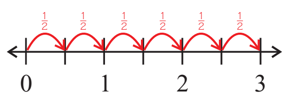
Lesson 2.3.4

Dividing Fractions

Dividing fractions is hard to grasp — but once you learn a couple of useful techniques it'll all seem a lot easier. The most important thing to learn is how to use reciprocals.

You Can Show Fraction Division on a Number Line

A problem like $3 \div \frac{1}{2}$ can be hard to imagine. In words, it means "work out how many halves are in three." You can show it on a number line.



You can see that $\frac{1}{2}$ fits into 3 six times.

So $3 \div \frac{1}{2} = 6$.

You can use a number line to show that the number of halves in any number will always be double that number.

For example, there are 8 halves in 4, so $4 \div \frac{1}{2} = 4 \times 2 = 8$.

Dividing is the Same as Multiplying by the Reciprocal

Multiplication and division are really closely linked. In fact, you can write any **division** problem as a **multiplication** problem using a **reciprocal**.

Dividing by a number is the same as multiplying by its reciprocal.

The **product** of a number and its **reciprocal** is **1**.

For example, the reciprocal of 2 is $\frac{1}{2}$. This means that dividing something by $\frac{1}{2}$ is the **same** as multiplying by 2.

Example 1

Calculate $3 \div \frac{1}{4}$.

Solution

The reciprocal of $\frac{1}{4}$ is 4.

So you can rewrite this as a multiplication: $3 \div \frac{1}{4} = 3 \times 4 = 12$.

So $3 \div \frac{1}{4} = 12$.

Guided Practice

Calculate these by converting each division problem into a multiplication problem. Give your solutions in their simplest form.

1. $3 \div \frac{1}{3}$ 9 **2.** $8 \div \frac{1}{4}$ 32 **3.** $8 \div \frac{2}{3}$ 12

4. $3 \div \frac{3}{5}$ 5 **5.** $\frac{5}{6} \div 5$ $\frac{1}{6}$ **6.** $\frac{11}{12} \div 11$ $\frac{1}{12}$

California Standard:
Number Sense 1.2
Add, subtract, multiply, and **divide rational numbers** (integers, **fractions**, and terminating decimals) and take positive rational numbers to whole-number powers.

What it means for you:
You'll learn about the reciprocal of a fraction and how to use it when dividing by fractions.

Key words:
- reciprocal
- divisor

Don't forget:
The reciprocal (or multiplicative inverse) of a number is what you have to multiply the number by to get 1. There's more about reciprocals in Lesson 1.1.4.

Don't forget:
Also, since the reciprocal of $\frac{1}{2}$ is 2, dividing something by 2 is the same as multiplying by $\frac{1}{2}$. There's more about this on the next page.

1 Get started

Resources:
- grid paper
- Money Tiles (found on **Teacher Resources CD-ROM**)

Warm-up questions:
- Lesson 2.3.4 sheet

2 Teach

Universal access
Have students think about what fraction division problems actually mean by rephrasing them. For example, get them to think of "$2 \div \frac{1}{2}$" as asking "*How many halves are there in 2?*"

Use real-life examples to help with the visualization process. For example, "*If I have 2 pizzas and I divide both of them in half, how many half-pizzas will I have?*"

Concept question
"*Is dividing by $\frac{3}{4}$ the same as multiplying by $\frac{4}{3}$?*"
Yes — because $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$.

Math background
A reciprocal is the same as a number's multiplicative inverse. Reciprocals were introduced in Section 1.1.

Guided practice
Level 1: q1–3
Level 2: q1–5
Level 3: q1–6

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Introduce fraction division using money. Put students into groups. Give each group some Money Tiles (from the **Teacher Resources CD-ROM**). Set some fraction problems based around coins, such as "*How many quarters are in $2?*" or "*How many dimes are in 2 quarters*?" Remind them that each coin is a fraction of $1 (for example 1 dime = $$\frac{1}{10}$). Have them rewrite each problem as a fraction division.
- **English Language Learners**
  Division is the inverse of multiplication. Give students some fraction division problems and have them check their answers by multiplying by the divisor to get the dividend (for example, $5 \div \frac{1}{4} = 20$ so $20 \times \frac{1}{4} = 5$). Have them work in pairs and take turns doing the multiplication and division steps of the process.

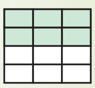
2 Teach (cont)

Universal access

Have students use the area method alongside their regular work to help them understand fraction ÷ fraction calculations.

For example:

$$\frac{2}{4} \div \frac{1}{12}$$
$$= \frac{2}{4} \times \frac{12}{1}$$
$$= \frac{24}{4} = 6$$



6 small squares have been shaded.

Concept question

"If I divide a positive number by a positive proper fraction, will my answer be bigger or smaller than my original number?"

Bigger

Guided practice

Level 1: q7–12
Level 2: q7–15
Level 3: q7–15

Don't forget:

$\frac{1}{9}$ and $\frac{9}{1}$ are reciprocals.
They multiply to give 1.
$\frac{1}{9} \times \frac{9}{1} = \frac{1 \times 9}{9 \times 1} = \frac{9}{9} = 1$

Don't forget:

To multiply two fractions, you first multiply the numerators to get the numerator of your answer. You then multiply the two denominators, and this becomes the denominator of your answer.
See the previous Lesson for more information.

Solve Fraction ÷ Fraction Using Reciprocals Too

You can turn **any** division into a **multiplication** using the **reciprocal** of the divisor (the thing you're dividing by).

It makes dividing fractions by fractions much easier than it seems at first.

Example 2

Calculate $\frac{2}{3} \div \frac{1}{9}$.

Solution

Dividing by a number is the same as multiplying by its reciprocal.

The reciprocal of $\frac{1}{9}$ is 9, or $\frac{9}{1}$.

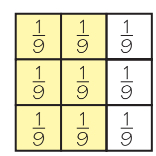
Fraction: $\frac{1}{9}$ Reciprocal: $\frac{9}{1}$

So you need to work out $\frac{2}{3} \times \frac{9}{1}$.

This is $\frac{2}{3} \times \frac{9}{1} = \frac{2 \times 9}{3 \times 1} = \frac{18}{3} = 6$

To convince yourself that $\frac{2}{3} \div \frac{1}{9}$ really does equal 6, remember that in words, the problem means, "how many ninths are in two-thirds?"

Look at the square on the right.
Two-thirds of it has been colored in.
The square has then been divided into ninths — there are **six ninths** in the colored two-thirds.
In other words, $\frac{2}{3} \div \frac{1}{9} = 6$.



Guided Practice

Find the reciprocal of the following fractions:

7. $\frac{1}{5}$ **5** **8.** $\frac{2}{3}$ $\frac{3}{2}$ **9.** $\frac{5}{7}$ $\frac{7}{5}$

Calculate Exercises 10–15 by converting each division problem into a multiplication problem. Give your solutions in their simplest form.

10. $\frac{1}{3} \div \frac{1}{5}$ $\frac{5}{3}$ **11.** $\frac{2}{5} \div \frac{1}{6}$ $\frac{12}{5}$ **12.** $\frac{1}{5} \div \frac{3}{4}$ $\frac{4}{15}$

13. $\frac{1}{2} \div \frac{3}{8}$ $\frac{4}{3}$ **14.** $\frac{1}{3} \div \frac{9}{1}$ $\frac{1}{27}$ **15.** $\frac{6}{7} \div \frac{1}{4}$ $\frac{24}{7}$

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to use algebra to explain why dividing by a number is the same as multiplying by its reciprocal.
  Tell the students that since $\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$ what they need to prove is that $\frac{a}{b}\div\frac{c}{d}=\frac{ad}{bc}$.
  Allow students to work in groups and to explain their thoughts to each other as they go along. One possible proof is shown in the box on the right.

$$\frac{a}{b}\div\frac{c}{d}=\frac{a/b}{c/d}$$
$$\frac{a/b}{c/d}\times\frac{b}{b}=\frac{a}{bc/d}$$
$$\frac{a}{bc/d}\times\frac{d}{d}=\frac{ad}{bc}$$

Convert Mixed Numbers into Fractions First

If you have a division problem involving mixed numbers, you need to write them out as **fractions** before you start to divide.

Example 3

Calculate $-4\frac{6}{7}\div 2\frac{1}{6}$.

Solution

First convert the mixed numbers to fractions.

$$-4\frac{6}{7}=-\frac{(4\times 7)+6}{7}=-\frac{34}{7}\text{ and }2\frac{1}{6}=\frac{(2\times 6)+1}{6}=\frac{13}{6}$$

Write the division as a multiplication using the reciprocal of the **divisor**. The divisor is the number that you're **dividing by**.

$$-4\frac{6}{7}\div 2\frac{1}{6}=-\frac{34}{7}\div\frac{13}{6}=-\frac{34}{7}\times\frac{6}{13}$$

Do the multiplication in the normal way.

$$-\frac{34}{7}\times\frac{6}{13}=-\frac{34\times 6}{7\times 13}=-\frac{204}{91}\text{, or }-2\frac{22}{91}$$

Don't forget:

1 can be written as $\frac{7}{7}$.

So 4 is equal to $4\times\frac{7}{7}$.

Guided Practice

Calculate the answers to the following divisions.
Write each solution as a mixed number or an integer.

16. $4\div 2\frac{1}{2}$ $1\frac{3}{5}$ **17.** $5\frac{1}{3}\div\frac{4}{9}$ 12 **18.** $-9\frac{1}{2}\div -3\frac{1}{8}$ $3\frac{1}{25}$

Independent Practice

Calculate the following.

1. $\frac{3}{4}\div 5$ $\frac{3}{20}$ **2.** $\frac{2}{3}\div 7$ $\frac{2}{21}$ **3.** $\frac{9}{13}\div 2$ $\frac{9}{26}$

4. $\frac{1}{2}\div\frac{1}{3}$ $\frac{3}{2}$ **5.** $\frac{2}{5}\div\frac{3}{4}$ $\frac{8}{15}$ **6.** $\frac{7}{9}\div\frac{2}{18}$ 7

7. $2\frac{1}{3}\div\frac{1}{2}$ $\frac{14}{3}$ **8.** $4\div\frac{5}{6}$ $\frac{24}{5}$ **9.** $8\frac{9}{10}\div 7\frac{1}{6}$ $\frac{267}{215}$

10. A one-pound bag of sugar is equal to $2\frac{2}{5}$ cups. A recipe for cornbread requires $\frac{2}{3}$ of a cup of sugar. How many batches of cornbread can be made with the bag of sugar? $3\frac{3}{5}$ batches

11. The area of a rectangular floor is $62\frac{3}{4}$ square feet. The length of the room is $10\frac{1}{4}$ feet. What is the width of the room? $6\frac{5}{41}$ feet

Now try these:

Lesson 2.3.4 additional questions — p437

Round Up

The most important thing to remember here is that you can convert any fraction division into a multiplication using the reciprocal. Remember that little fact, and you'll make your life a lot easier.

2 Teach (cont)

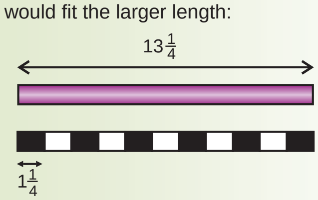
Universal access

Dividing mixed numbers can be a hard concept for students to understand. Help them by linking it to real-life problems. For example:

1. Ellen has $13\frac{1}{4}$ meters of ribbon. She gives everyone in her sewing club $1\frac{1}{4}$ meters for a project. What is the maximum number of people there could be in the sewing club? **10**

2. To buy 1 kg of oranges you would need 3 dollars and 1 quarter. If you had 9 dollars and 3 quarters, how many kilograms of oranges could you buy? **3**

Encourage students to draw pictures to help them solve problems. For example, for question 1 above, they could draw a picture of the length of ribbon, and then use a scale to work out how many of the smaller pieces would fit the larger length:



Guided practice

Level 1: q16–18
Level 2: q16–18
Level 3: q16–18

Independent practice

Level 1: q1–4
Level 2: q1–6, 10
Level 3: q1–11

Additional questions

Level 1: p437 q1–6, q13
Level 2: p437 q2–10, q14
Level 3: p437 q4–14

3 Homework

Homework Book — Lesson 2.3.4

Level 1: q1–4, 5a–c, 6, 8
Level 2: q3–13
Level 3: q3–13

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 2 — Simplifying Fractions
- Worksheet 10 — Multiplying Fractions by Fractions
- Worksheet 11 — Dividing Fractions

Solutions
For worked solutions see the Solution Guide

Lesson 2.3.5

Common Denominators

In this Lesson students learn how to write a number as the product of its prime factors, and how to find the least common multiple of two numbers. This leads to finding a common denominator for two fractions.

Previous Study: In grade 5, students first learned how to add and subtract fractions with different denominators. Earlier in this Section students learned how to reduce fractions to their simplest form.

Future Study: In Algebra I students will be required to add and subtract algebraic expressions containing all types of rational numbers, including fractions. They will learn to solve problems and equations using these techniques.

1 Get started

Resources:
- grid paper
- Number and Operator Tiles (see **Teacher Resources CD-ROM**)

Warm-up questions:
- Lesson 2.3.5 sheet

2 Teach

Math background

A prime number is a positive integer that has exactly two different positive integers that divide it evenly (the number itself and 1) — no more and no fewer. This means that 1 is not a prime number.

Remind students of this before beginning prime factorization — a similar but less formal definition is given in the student margin below.

Universal access

Write a number on the board. Have a student come and add a branch, writing a factor of the number at the bottom. Then have another student come up and add the matching branch. Carry on until the class has found all the number's prime factors.

When a prime number is reached, underline it in red to show that that's where the branch stops. When all the branches are complete, write the number as the product of its prime factors below the tree.

When you feel the class has the concept, ask them to underline primes as they write them.

Do one number in at least two different ways to show that although there may be many ways to split a number, you always end up with the same prime factors.

Guided practice

Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Lesson 2.3.5

Common Denominators

California Standards:

Number Sense 1.1
Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), **compare rational numbers in general.**

Number Sense 1.2
Add, subtract, multiply, and divide **rational numbers** (integers, **fractions**, and terminating decimals) and take positive rational numbers to whole-number powers.

Number Sense 2.2
Add and subtract fractions by using factoring to find common denominators.

What it means for you:
You'll learn the first step of adding and subtracting fractions — putting fractions over a common denominator. You'll also see how this allows you to compare fractions to find which is greater.

Key words:
- prime
- factorization
- least common multiple
- common denominator

Don't forget:
A number that has no factors except itself and 1 is called a prime number.

Don't forget:
Factors of a number are any whole numbers that divide exactly into it.

Adding and subtracting fractions isn't always straightforward. Before you can add or subtract fractions, they need to be over a common denominator, and one way to find a common denominator is to find the least common multiple (LCM) of the denominators.

Prime Factorization — the First Step to Finding an LCM

Finding the **prime factorization** of a number involves writing it as a **product of prime factors** (prime numbers multiplied together).

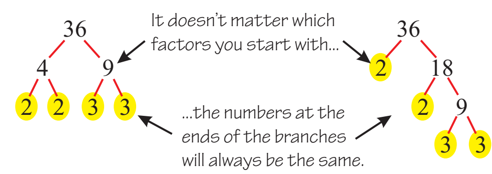
Example 1

Write 36 as a product of prime factors.

Solution

One way to do this is to look for **factors of 36**, then look for **factors of those factors**, and then look for **factors of those factors**, and so on.

You can arrange your factors in a "tree." Break down 36 into two factors and write them underneath. Then look for factors of your factors.



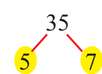
Each branch ends in a prime number — these can't be broken down further. So the prime factorization of 36 is **36 = 2 × 2 × 3 × 3**

Example 2

Write the following as products of prime factors: **a)** 35, **b)** 37

Solution

a)



5 and 7 are both prime, so the tree stops here. The prime factorization of 35 is **35 = 5 × 7**

b) 37

37 is prime, so the tree has no branches. 37 **doesn't factor**.

Guided Practice

Write the following as products of prime factors.

1. 15 3 × 5 **2.** 30 2 × 3 × 5 **3.** 17 17
4. 16 2 × 2 × 2 × 2 or 2^4 **5.** 24 2 × 2 × 2 × 3 or 2^3 × 3 **6.** 50 2 × 5 × 5 or 2 × 5^2

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Put students into groups. Have them list all the prime numbers under 10 (2, 3, 5, 7). Give each group a set of Number and Operator Tiles (found on the **Teacher Resources CD-ROM**) including three copies of each of these primes and three "×" tiles. Give them a list of target numbers to make using only these tiles. Make sure that all the targets can be made from the primes they have (for example, 6, 14, 27, 90, 210).

- **English Language Learners**
  Have students write their own fraction dictionary, including all the terms relating to fractions and fraction operations that they can think of, such as proper fraction, improper fraction, mixed number, numerator, denominator, reciprocal, GCF, LCM, prime factor, equivalent fraction. Have them write their own definition of each term, in English and their own first language, and give an example of each one beside it.

Don't forget:
Multiples are what you get in multiplication tables.
For example, the multiples of 3 are 3, 6, 9, 12, 15, 18... and so on.

Check it out:
The least common multiple is useful when finding a common denominator for two fractions.

The LCM is the Least Common Multiple

You can use prime factorizations to find the **least common multiple** of two numbers, which is the **smallest** number that **both** will **divide into evenly**.

Example 3

Find the least common multiple of 8 and 14.

Solution

There are two ways of doing this.

1. You can **start listing all the multiples** of each number, and the **first one** that they have **in common** is the least common multiple:
- **Multiples of 8**: 8, 16, 24, 32, 40, 48, 56, 64, 72...
- **Multiples of 14**: 14, 28, 42, 56...

So **the least common multiple is 56**.
This method is fine, but it can get a bit tedious when the LCM is high.

2. You can first write each number as **the product of its prime factors**.
8 = 2 × 2 × 2 and **14 = 2 × 7**.

8	2	2	2	
14	2			7

Then make **a table** showing these factorizations. Wherever possible, put each factor of the second number in a column with the same factor. Start a **new column** if the number's not already there — so in this example, start a new column for the 7.

8	2	2	2	
14	2			7
LCM	2	2	2	7

Now make a new row — **the LCM row**.
For each box in the LCM row, write down the number in the boxes above.

Finally, **multiply together** all the numbers in the LCM row to find the least common multiple.

So the least common multiple of 8 and 14 is **2 × 2 × 2 × 7 = 56**.

Example 4

Find the least common multiple of 10 and 15.

Solution

- Prime factorizations: **10 = 2 × 5** and **15 = 3 × 5**.
- Make a table:
- Multiply the numbers in the LCM row.

10	2	5	
15		5	3
LCM	2	5	3

So the least common multiple of 10 and 15 is **2 × 5 × 3 = 30**.

Guided Practice

Find the least common multiple of each pair of numbers in Exercises 7–15.

7. 6 and 15 **30** **8.** 10 and 12 **60** **9.** 8 and 9 **72**
10. 4 and 10 **20** **11.** 8 and 16 **16** **12.** 20 and 30 **60**
13. 13 and 7 **91** **14.** 9 and 81 **81** **15.** 42 and 30 **210**

2 Teach (cont)

Common error

Students sometimes confuse the least common multiple with the greatest common factor. Remind them of what the GCF means before beginning to study the LCM.

Universal access

If students are having trouble with the table method for finding the LCM, teach them the box method, which is more similar to a factor tree.

1) Draw a box, and write both numbers inside it. For example, 9 and 21.

[9 21]

2) Find a prime factor of both numbers. Write it to the left of the box.

3 [9 21]

3) Divide each number in the box by the number on the left, and write the answers underneath.

3 [9 21]
3 7

4) Both the numbers beneath the box are prime. Multiply all the numbers outside the box together and you get the LCM of 9 and 21: **3 × 3 × 7 = 63**

If after step 3 all the numbers outside the box aren't prime, continue finding common factors until they are all prime. For example, to find the LCM of 30 and 45:

3 [30 45]
10 15

Write a common prime factor of 10 and 15 at the side.

3 [30 45]
5 [10 15]
2 3

Extend the box to surround 10 and 15.

Once again, when only prime numbers are left outside the box, multiply them all together.
LCM of 30 and 45 = **2** × **3** × **3** × **5** = **90**

Guided practice

Level 1: q7–9
Level 2: q7–12
Level 3: q7–15

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Hot dogs are often sold in packs of 10, while hot dog buns are usually sold in packs of 8. Ask students to imagine they are planning a cookout — what math problems might they encounter related to the number of hot dogs and buns? For example, if you eat one hot dog in one bun, have you eaten a greater fraction of a pack of hot dogs or a pack of buns? What's the smallest number of hot dogs you can make using an exact number of packets of both? How about if you want to add a dill pickle to each one, and they come 16 to a jar? Have them prepare a poster on the topic.

2 Teach (cont)

Universal access

Use an area comparison example. Give students some grid paper, and have them mark out two 10-by-10 grids.

Ask them to shade $\frac{1}{5}$ of the 1st grid and $\frac{3}{10}$ of the 2nd. Can they tell which is the larger fraction just by looking at the grids?

Now have them work out which is larger by writing both fractions with a common denominator. They can do this by counting how many squares out of 100 they have colored in.

When they have done this, have them look at their fractions. Can they simplify them any further? Ask them to try the LCM method, to check that they get the same answer.

Concept question

"Which fraction is larger, $\frac{1}{2}$ or $\frac{2}{4}$?"

They are equivalent fractions — so they're the same size.

Guided practice

Level 1: q16–18
Level 2: q16–19
Level 3: q16–21

Independent practice

Level 1: q1–3, 7–9, 13
Level 2: q1–14
Level 3: q1–16

Additional questions

Level 1: p438 q1–4, 7–9, 13
Level 2: p438 q3–14, 17–18
Level 3: p438 q4–18

3 Homework

Homework Book — Lesson 2.3.5

Level 1: q1–3, 5a, 6a, 7a
Level 2: q2, 4–9
Level 3: q2, 4–9

Check it out:
You don't have to use the least common multiple here — any common multiple would be fine. For example, you could use 8 × 14 = 112. However, using the least common multiple will mean all the numbers will be smaller, which can be very useful.

Don't forget:
When you multiply both the numerator and denominator of a fraction by the same number, you don't change its value.
This is because you are really just multiplying the fraction by 1, since $\frac{7}{7}=\frac{4}{4}=1$.

Now try these:
Lesson 2.3.5 additional questions — p438

Compare Fractions Using a Common Denominator

Least common multiples can be used as **common denominators**.

Example 5

By using a common denominator, find which is greater, $\frac{5}{8}$ or $\frac{9}{14}$.

Solution

It's not easy to say which is the larger fraction when they have different denominators. You need to find fractions **equivalent** to each of these that have a **common denominator**.

You can use **any common multiple** of 8 and 14 as the common denominator. Here, we'll use the **least common multiple** (which is 56 — see Example 3). You need to decide how many 8s and how many 14s make 56 — and then **multiply the top and bottom** of each fraction by the right number: $\frac{5}{8}=\frac{5\times7}{8\times7}=\frac{35}{56}$ $\quad\frac{9}{14}=\frac{9\times4}{14\times4}=\frac{36}{56}$

You can now see that $\frac{9}{14}$ **is greater than** $\frac{5}{8}$, since $\frac{36}{56}$ is greater than $\frac{35}{56}$.

Guided Practice

In Exercises 16–21, put each pair of fractions over a common denominator to find which is the greater in each pair.

16. $\frac{1}{4}$ and $\frac{2}{10}$ $\frac{1}{4}$ **17.** $\frac{1}{3}$ and $\frac{2}{7}$ $\frac{1}{3}$ **18.** $\frac{3}{10}$ and $\frac{2}{7}$ $\frac{3}{10}$

19. $\frac{4}{9}$ and $\frac{5}{11}$ $\frac{5}{11}$ **20.** $\frac{11}{15}$ and $\frac{12}{17}$ $\frac{11}{15}$ **21.** $\frac{6}{9}$ and $\frac{2}{3}$ Equal

Independent Practice

Write the numbers in Exercises 1–6 as products of prime factors.

1. 21 3 × 7 **2.** 100 2 × 2 × 5 × 5 **3.** 32 2 × 2 × 2 × 2 × 2
4. 50 2 × 5 × 5 **5.** 49 7 × 7 **6.** 132 2 × 2 × 3 × 11

Find the least common multiple of each pair in Exercises 7–12.

7. 6 and 8 24 **8.** 10 and 25 50 **9.** 48 and 21 336
10. 32 and 50 800 **11.** 100 and 49 4900 **12.** 49 and 132 6468

Find the greater fraction in each pair in Exercises 13–15.

13. $\frac{10}{21}$ and $\frac{51}{100}$ $\frac{51}{100}$ **14.** $\frac{50}{132}$ and $\frac{13}{49}$ $\frac{50}{132}$ **15.** $\frac{33}{50}$ and $\frac{32}{49}$ $\frac{33}{50}$

16. Order these fractions from least to greatest: $\frac{2}{3},\frac{14}{17},\frac{13}{16},\frac{3}{4},\frac{16}{21}$
See below

Round Up

Finding common denominators is something you should get real comfortable with doing, because you need to do it a lot in math — in the next few Lessons, for example.

Solutions
For worked solutions see the Solution Guide

16. $\frac{2}{3},\frac{3}{4},\frac{16}{21},\frac{13}{16},\frac{14}{17}$

Lesson 2.3.6

Adding and Subtracting Fractions

In this Lesson, students practice adding and subtracting fractions. They apply the prime factorization method learned in the previous Lesson to find the least common multiple of the two denominators. They then use this as the common denominator of the fractions.

Previous Study: In grade 5 students learned to add and subtract simple fractions. In grade 6 they saw how to find the GCF and LCM of two numbers, and used these to find common denominators of fractions.

Future Study: In Algebra I students will be required to add and subtract expressions containing all types of rational numbers, including fractions. They will learn to solve problems and equations using these techniques.

Lesson 2.3.6

California Standards:

Number Sense 1.2
Add, subtract, multiply, and divide **rational numbers** (integers, **fractions**, and terminating decimals) and take positive rational numbers to whole-number powers.

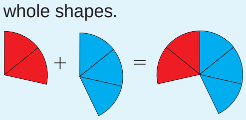
Number Sense 2.2
Add and subtract fractions by using factoring to find common denominators.

What it means for you:
You'll see how to add and subtract fractions.

Key words:
- denominator
- numerator
- common denominator

Don't forget:
The numerator is the top line of a fraction.
The denominator is the bottom line of a fraction.

Don't forget:
You can think of adding fractions as adding parts of whole shapes.



$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$

Adding and Subtracting Fractions

Adding and subtracting fractions can be quick, or it can be quite a long process — it all depends on whether the fractions already have a common denominator, or whether you have to find it first.

You Can Add Fractions with a Common Denominator

If fractions have a **common denominator** (their denominators are the **same**), adding them is fairly straightforward.

To find the **numerator of the sum**, you **add the numerators** of the individual fractions. The **denominator** stays the same.

Example 1

Find $\frac{2}{7} + \frac{3}{7}$.

Solution

These two fractions have a common denominator, 7.
So 7 will also be the **denominator of the sum**.

The **numerator** of the sum will be 2 + 3 = 5.

So $\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$.

You **subtract** fractions with a common denominator in exactly the same way.

Example 2

Find $\frac{7}{9} - \frac{2}{9}$.

Solution

The **denominator** of the result will be 9 (the fractions' common denominator). The **numerator** of the result will be 7 – 2 = 5.

So $\frac{7}{9} - \frac{2}{9} = \frac{7-2}{9} = \frac{5}{9}$.

Guided Practice

Find the sums and differences in Exercises 1–8.

1. $\frac{1}{5} + \frac{2}{5}$ $\frac{3}{5}$
2. $\frac{3}{11} + \frac{4}{11}$ $\frac{7}{11}$
3. $\frac{4}{5} - \frac{1}{5}$ $\frac{3}{5}$
4. $\frac{10}{21} - \frac{2}{21}$ $\frac{8}{21}$
5. $\frac{7}{15} + \frac{4}{15}$ $\frac{11}{15}$
6. $\frac{23}{50} + \frac{19}{50}$ $\frac{42}{50}$ or $\frac{21}{25}$
7. $\frac{9}{25} - \frac{7}{25}$ $\frac{2}{25}$
8. $\frac{5}{17} + \frac{1}{17}$ $\frac{6}{17}$

1 Get started

Resources:
- cardboard pizzas
- bags containing 20 marbles
- individual whiteboards and pens
- triangular grid paper
- Fraction Tiles (see **Teacher Resources CD-ROM**)

Warm-up questions:
- Lesson 2.3.6 sheet

2 Teach

Common error

Sometimes students try to add fractions with a common numerator rather than a common denominator.

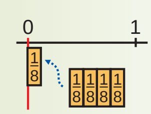
For example, $\frac{5}{2} + \frac{5}{6} = \frac{5}{8}$.

Remind them that the denominator is like the name of the fraction, and the numerator is there to tell you how many of that fraction you have.
To help them remember, have them write a few examples in the form:
2 fifths + 4 fifths = 6 fifths.

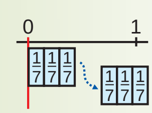
Universal access

Have students use Fraction Tiles (found on the **Teacher Resources CD-ROM**) to go through addition and subtraction problems as they do them. For example:

$\frac{1}{8} + \frac{4}{8} = \frac{5}{8}$



$\frac{6}{7} - \frac{3}{7} = \frac{3}{7}$



Concept question

"What is $\frac{3}{x} + \frac{2}{x}$?" $\frac{5}{x}$

Guided practice

Level 1: q1–4
Level 2: q1–6
Level 3: q1–8

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  Make seven pictures of pizzas out of card. Cut four into halves, two into quarters, and one into eighths (this is 24 pieces — make more if your class is larger). Mark each piece with the fraction it represents. Give everyone a piece. Pair up people with different size pieces. Have each pair work out what fraction of a pizza they have between them. Use this to introduce equivalent fractions with a common denominator.
- **English Language Learners**
  Use the "take notes, make notes" strategy: have students make a line down the page, then on the left-hand side of the page write down the method for adding fractions with different denominators as a series of steps. Then on the right-hand side, they should add their own notes or reminders to help them remember the process or clarify anything they found difficult.

2 Teach (cont)

Common error

Sometimes when presented with a fraction sum, students will try to add the numerator and the denominator, following the pattern for multiplying fractions that they learned earlier.

Remind them that this does not work when adding fractions.
Give them a simple illustration — for example, $\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$, not $\frac{2}{6}$.

Concept question

"The LCM of 2 and 4 is 4.
Give another number that you could use as a common denominator if you were subtracting a quarter from a half."

Although the LCM is 4, you could use any number that is a multiple of 4.

Universal access

Give students word problems involving simple fraction addition and subtraction. For example: "*I have 20 marbles in my collection. If I give $\frac{1}{5}$ of my collection away to my friend, and $\frac{1}{4}$ to my brother, what fraction of my original collection do I have left?*"

Put the students in small groups, and give each group a bag of 20 marbles, and a small whiteboard and pen. Have them work through the problem. They should try to use the methods in Example 3, writing down their work on the whiteboard as they go, but allow them to use the marbles to help with the math and to verify their answer. When everyone has finished, have them hold up their boards as a check.

You May Need to Find a Common Denominator First

Fractions with **unlike** denominators **cannot** be directly added or subtracted. You must first find **equivalent** fractions with a **common denominator**.

Example 3

Find $\frac{7}{9}+\frac{5}{6}$.

Solution

The denominators are different here. This means you need to find two fractions equivalent to them, but with a **common denominator**.

The common denominator can be **any** common multiple of 9 and 6.

- You could use 9 × 6 = **54** as your common denominator.
  Then the equivalent fractions will be:

  $$\frac{7}{9}\times\frac{6}{6}=\frac{7\times 6}{9\times 6}=\frac{42}{54} \qquad \frac{5}{6}\times\frac{9}{9}=\frac{5\times 9}{6\times 9}=\frac{45}{54}$$

  Now you can add these fractions: $\frac{42}{54}+\frac{45}{54}=\frac{87}{54}$, which you can simplify to $\frac{29}{18}$ by dividing the numerator and denominator by 3.
- Or you could find the **LCM** (least common multiple) using prime factorizations. Since $9 = 3^2$ and 6 = 2 × 3, the LCM is 2 × 3 × 3 = **18**.

  This time, the equivalent fractions are:

  $$\frac{7}{9}\times\frac{2}{2}=\frac{7\times 2}{9\times 2}=\frac{14}{18} \text{ and } \frac{5}{6}\times\frac{3}{3}=\frac{5\times 3}{6\times 3}=\frac{15}{18}.$$

  Now you can add these to get $\frac{14}{18}+\frac{15}{18}=\frac{29}{18}$.

You can use the **LCM** or **any other common multiple** as your common denominator — you'll end up with the **same answer**. But using the LCM means that the numbers in your fractions are smaller and easier to use.

Example 4

By putting both fractions over a common denominator, find $\frac{11}{15}-\frac{3}{20}$.

Solution

Use a table to find the LCM of 15 and 20
— this is is 5 × 3 × 2 × 2 = **60**.

15	5	3		
20	5		2	2
LCM	5	3	2	2

Find equivalent fractions with denominator 60:

$$\frac{11}{15}=\frac{11\times 4}{15\times 4}=\frac{44}{60} \text{ and } \frac{3}{20}=\frac{3\times 3}{20\times 3}=\frac{9}{60}$$

So rewriting the subtraction gives: $\frac{11}{15}-\frac{3}{20}=\frac{44}{60}-\frac{9}{60}=\frac{35}{60}$

This can be simplified to $\frac{35}{60}=\frac{7}{12}$.

Don't forget:

A fraction whose numerator and denominator are the same is equal to 1 — so multiplying another fraction by, say, $\frac{6}{6}$ or $\frac{9}{9}$ doesn't change its value. This is why you can multiply the numerator and denominator by the same amount, and leave the value of the fraction unchanged.

Don't forget:

To find an LCM, you can use a table like this one. (See Lesson 2.3.5 for more information.)

9		3	3
6	2	3	
LCM	2	3	3

Check it out:

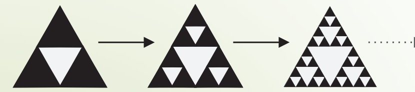
Again, any common multiple of 15 and 20 can be used as a common denominator — including their product 15 × 20 = 300.
But this will mean the numbers in the calculation will be bigger than necessary.
For example:

$$\frac{11}{15}-\frac{3}{20}=\frac{220}{300}-\frac{45}{300}=\frac{175}{300}=\frac{7}{12}$$

Check it out:

Even if you use the LCM as your common denominator, you may still be able to simplify the answer at the end.

- **Advanced Learners**
Introduce students to the Sierpinski triangle, illustrated right. Show them how to make the triangle, and let them make their own on triangular grid paper. Have them work out the fraction of the triangle that is NOT shaded in each step. Ask them to explain the pattern and use it to predict the fraction NOT shaded in future steps. The proportion not shaded can be expressed as a sum of a series of fractions. You add on one new term each time — the numerators are multiplied by 3, and the denominators by 4.



$\frac{1}{4}$ $\quad \frac{1}{4}+\frac{3}{16}$ $\quad \frac{1}{4}+\frac{3}{16}+\frac{9}{64}$

Guided Practice

Calculate the answers in Exercises 9–11.

9. $\frac{14}{15}-\frac{2}{5}$ $\frac{8}{15}$ **10.** $\frac{7}{10}+\frac{2}{3}$ $\frac{41}{30}$ **11.** $-\frac{2}{3}+\frac{4}{7}$ $-\frac{2}{21}$

Be Extra Careful if There are Negative Signs

As always in math, if there are **negative numbers** around, you have to be extra careful.

Example 5

Find $-\frac{3}{5}-\left(\frac{-2}{7}\right)$.

Solution

This looks tricky because of all the negative signs.
So take things slowly and carefully.

This sum can be rewritten as $\frac{-3}{5}+\frac{2}{7}$ — it means exactly the same.

The LCM of 5 and 7 is 5 × 7 = **35**.
So put both these fractions over a common denominator of 35.

$$\frac{-3}{5}=\frac{-3\times 7}{5\times 7}=\frac{-21}{35} \qquad \frac{2}{7}=\frac{2\times 5}{7\times 5}=\frac{10}{35}$$

Now you can add the two fractions in the same way as before.

$$\frac{-3}{5}-\left(\frac{-2}{7}\right)=\frac{-21}{35}+\frac{10}{35}=\frac{-21+10}{35}=\frac{-11}{35} \text{ or } -\frac{11}{35}$$

Don't forget:
Remember... subtracting a negative number is exactly the same as adding a positive number.

Don't forget:
You could also swap the terms around to form a subtraction:
$\frac{-3}{5}+\frac{2}{7}=\frac{2}{7}-\frac{3}{5}$

Don't forget:
$\frac{-11}{35}$, $-\frac{11}{35}$, and $\frac{11}{-35}$ are all equal.

Guided Practice

Calculate the answers in Exercises 12–14. Simplify your answers.

12. $-\frac{2}{3}-\left(-\frac{3}{8}\right)$ $-\frac{7}{24}$ **13.** $\frac{7}{6}-\left(-\frac{4}{9}\right)$ $\frac{29}{18}$ **14.** $-\frac{25}{48}-\left(-\frac{7}{16}\right)$ $-\frac{1}{12}$

Independent Practice

Calculate the following. Give all your answers in their simplest form.

1. $\frac{2}{7}+\frac{8}{7}$ $\frac{10}{7}$ **2.** $\frac{8}{5}-\frac{2}{5}$ $\frac{6}{5}$ **3.** $\frac{7}{9}-\frac{2}{9}$ $\frac{5}{9}$ **4.** $\frac{7}{16}-\frac{3}{8}$ $\frac{1}{16}$

5. $\frac{19}{30}+\frac{7}{20}$ $\frac{59}{60}$ **6.** $\frac{5}{7}+\frac{5}{8}$ $\frac{75}{56}$ **7.** $-\frac{3}{16}-\frac{5}{17}$ $-\frac{131}{272}$ **8.** $\frac{19}{20}+\left(-\frac{20}{21}\right)$ $-\frac{1}{420}$

Now try these:
Lesson 2.3.6 additional questions — p438

Round Up

Keep practicing this until it becomes routine... (i) find a common denominator, (ii) put both fractions over this denominator, (iii) do the addition or subtraction, (iv) simplify your answer if possible. You'll see this again next Lesson.

2 Teach (cont)

Guided practice
Level 1: q9–11
Level 2: q9–11
Level 3: q9–11

Concept question
"Which expression has the greater value: $\frac{1}{5}+\frac{1}{7}$ or $\frac{1}{5}-\left(-\frac{1}{7}\right)$?"

Both have the same value — adding a positive number is the same as subtracting its opposite.

Additional examples
Find:
1) $-\frac{1}{3}+\frac{1}{9}$ $\quad -\frac{2}{9}$
2) $\frac{1}{3}-\frac{1}{9}$ $\quad \frac{2}{9}$
3) $\frac{1}{3}+\left(-\frac{1}{9}\right)$ $\quad \frac{2}{9}$
4) $-\frac{1}{3}-\left(-\frac{1}{9}\right)$ $\quad -\frac{2}{9}$

Guided practice
Level 1: q12–13
Level 2: q12–13
Level 3: q12–14

Independent practice
Level 1: q1–4
Level 2: q1–6
Level 3: q1–8

Additional questions
Level 1: p438 q1–3, 10–12
Level 2: p438 q1–6, 10–12
Level 3: p438 q4–12

3 Homework

Homework Book — Lesson 2.3.6
Level 1: q1, 2, 4, 5, 8, 9
Level 2: q1–10
Level 3: q1–11

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 2 — Simplifying Fractions
- Worksheet 8 — Adding and Subtracting Fractions

Solutions
For worked solutions see the Solution Guide

Lesson 2.3.7 Adding and Subtracting Mixed Numbers

In this Lesson students learn how to add and subtract mixed numbers by first converting them to improper fractions. They then apply the strategies that they used for adding and subtracting proper fractions in the previous Lesson.

Previous Study: In grade 6 students learned how to find the GCF and LCM of two numbers, and used them to find common denominators of fractions. They applied these methods to proper and improper fractions.

Future Study: In Algebra I students will learn how to add and subtract algebraic expressions containing all types of rational numbers, including improper fractions and mixed numbers.

1 Get started

Resources:
- poster-size paper
- flour, salt, and water
- cups
- bowls

Warm-up questions:
- Lesson 2.3.7 sheet

2 Teach

Math background

A proper fraction has a numerator smaller than its denominator, for example, $\frac{1}{2}$.

An improper fraction has a numerator equal to or larger than its denominator, for example, $\frac{5}{2}$.

A mixed number is a combination of an integer and a proper fraction.

For example, $5\frac{1}{2}$

Universal access

Some students might prefer the following method for adding mixed numbers.

A mixed number is like an integer added to a fraction. For example:

$2\frac{1}{2} = 2 + \frac{1}{2}$.

So you can rewrite a mixed number sum with all the terms as fractions.

For example : $2\frac{1}{2} + 3\frac{1}{3} = \frac{2}{1} + \frac{1}{2} + \frac{3}{1} + \frac{1}{3}$

You can now find a common denominator for all these fractions.

$\frac{2}{1} + \frac{1}{2} + \frac{3}{1} + \frac{1}{3} = \frac{12}{6} + \frac{3}{6} + \frac{18}{6} + \frac{2}{6} = \frac{35}{6}$

Additional example

What is $1\frac{1}{2} + 2\frac{3}{6}$? 4

Lesson 2.3.7

Adding and Subtracting Mixed Numbers

California Standards:

Number Sense 1.2
Add, subtract, multiply, and divide **rational numbers** (integers, **fractions**, and terminating decimals) and take positive rational numbers to whole-number powers.

Number Sense 2.2
Add and subtract fractions by using factoring to find common denominators.

What it means for you:
You'll learn to add and subtract numbers like $2\frac{1}{2}$ and $3\frac{5}{16}$.

Key words:
- mixed number
- common denominator

Don't forget:
You can convert $2\frac{2}{3}$ to a fraction by remembering that:

(i) $2\frac{2}{3}$ means $(2 \times 1) + \frac{2}{3}$.

(ii) 1 can be written as $\frac{3}{3}$.

So $2\frac{2}{3} = \left(2 \times \frac{3}{3}\right) + \frac{2}{3}$
$= \frac{2 \times 3}{3} + \frac{2}{3}$
$= \frac{(2 \times 3) + 2}{3}$

This Lesson covers nothing new — it just combines things that you've learned before — converting mixed numbers to fractions, and adding and subtracting fractions. So if they seemed tricky the first time around, this is a good chance to get a bit more practice.

Adding Mixed Numbers — Convert to Fractions First

Mixed numbers are things like $4\frac{1}{2}$ and $-6\frac{7}{8}$, with both an integer and a fraction part.

It's easiest to add the numbers together if you **convert them to fractions first**. Once they're written as **improper fractions** you can add them by finding the **least common multiple** as before.

Example 1

Find $1\frac{1}{2} + 2\frac{2}{3}$.

Solution

Convert both the numbers to fractions.

$$1\frac{1}{2} = \frac{(1\times 2)+1}{2} = \frac{3}{2} \qquad 2\frac{2}{3} = \frac{(2\times 3)+2}{3} = \frac{8}{3}$$

Put both fractions over a common denominator of 2 × 3 = **6**.

$$\frac{3}{2} = \frac{3\times 3}{2\times 3} = \frac{9}{6} \qquad \frac{8}{3} = \frac{8\times 2}{3\times 2} = \frac{16}{6}$$

Do the addition: $1\frac{1}{2} + 2\frac{2}{3} = \frac{9}{6} + \frac{16}{6} = \frac{25}{6}$

Example 2

Find $4\frac{1}{2} - 6\frac{7}{8}$.

Solution

Convert both the numbers to fractions.

$$4\frac{1}{2} = \frac{(4\times 2)+1}{2} = \frac{9}{2} \qquad 6\frac{7}{8} = \frac{(6\times 8)+7}{8} = \frac{55}{8}$$

Put both fractions over a common denominator of **8**.

$\frac{9}{2} = \frac{9\times 4}{2\times 4} = \frac{36}{8}$, and $\frac{55}{8}$ is already over a denominator of 8.

Do the subtraction: $4\frac{1}{2} - 6\frac{7}{8} = \frac{36}{8} - \frac{55}{8} = \frac{36-55}{8} = -\frac{19}{8}$

- **Strategic Learners**
  Put the students into groups. Give each group a topic related to adding and subtracting fractions, such as adding fractions with the same denominator, adding fractions with different denominators, adding and subtracting mixed numbers, etc. Each group should then make a poster on their topic. Display the posters around the classroom. Have each group present their poster to the class as a topic review.

- **English Language Learners**
  Put students into groups. Give each group a word problem involving the addition or subtraction of mixed numbers. Have them underline the key numbers and instructions in the problem, write out the problem as a number sentence, find the answer, and check it for reasonableness. Then have each group explain to the class how they tackled their problem.

Guided Practice

For Exercises 1–6, give your answers as fractions.

1. $3\frac{3}{4}+2\frac{3}{4}$ $\frac{13}{2}$ **2.** $8\frac{1}{2}+1\frac{2}{3}$ $\frac{61}{6}$ **3.** $6\frac{2}{3}-\frac{1}{2}$ $\frac{37}{6}$

4. $2\frac{3}{4}-4\frac{3}{5}$ $-\frac{37}{20}$ **5.** $1\frac{1}{7}-2\frac{5}{8}$ $-\frac{83}{56}$ **6.** $4\frac{4}{5}-1\frac{1}{6}$ $\frac{109}{30}$

Take Extra Care With Negative Signs

If there are a lot of **negative signs**, you should take extra care.

Example 3

Find $-3\frac{2}{3}-\left(-2\frac{6}{7}\right)$.

Solution

Convert the mixed numbers to fractions.

$$-3\frac{2}{3}=-\left(\frac{(3\times3)+2}{3}\right)=-\frac{11}{3} \quad\text{and}\quad -2\frac{6}{7}=-\left(\frac{(2\times7)+6}{7}\right)=-\frac{20}{7}$$

Now find a common denominator — you can use 3 × 7 = **21**.

$$-\frac{11}{3}=-\frac{11\times7}{3\times7}=-\frac{77}{21} \quad\text{and}\quad -\frac{20}{7}=-\frac{20\times3}{7\times3}=-\frac{60}{21}$$

So the calculation becomes

$$-\frac{77}{21}-\left(-\frac{60}{21}\right)=-\frac{77}{21}+\frac{60}{21}=\frac{-77+60}{21}=-\frac{17}{21}$$

> **Don't forget:**
> Subtracting a negative number is the same as adding a positive number.

Also take extra care if the calculation has more than two terms — you have to make sure that all the fractions have the same denominator.

Example 4

Find $\frac{5}{3}-\frac{3}{5}+2\frac{2}{3}$.

Solution

Convert the mixed number to a fraction. $2\frac{2}{3}=\frac{(2\times3)+2}{3}=\frac{8}{3}$

Now find a common denominator — you can use 5 × 3 = **15**.

$$\frac{5}{3}=\frac{5\times5}{3\times5}=\frac{25}{15} \qquad \frac{3}{5}=\frac{3\times3}{5\times3}=\frac{9}{15} \qquad \frac{8}{3}=\frac{8\times5}{3\times5}=\frac{40}{15}$$

So the calculation becomes $\frac{25}{15}-\frac{9}{15}+\frac{40}{15}=\frac{25-9+40}{15}=\frac{56}{15}$

> **Don't forget:**
> Always do additions and subtractions from left to right.

2 Teach (cont)

Guided practice

Level 1: q1–3
Level 2: q1–5
Level 3: q1–6

Universal access

Put students into groups. Give each group the salt dough recipe below.

> Salt Dough Decorations
> 1 cup salt
> $2\frac{1}{4}$ cups flour
> $\frac{3}{4}$ cup water
>
> 1) Mix salt and flour.
> 2) Add water gradually until dough becomes elastic. Knead well.
> 3) Model into any shape you like.
> 4) Bake at 200 °C until hard right through.

Ask a series of questions about the recipe, such as, "*What is the total volume of the ingredients*?" **(4 cups)**, or "*If I made twice the recipe, what combined volume of flour and water would I need*?" **(6 cups)**, or "*If I measured out the flour and then removed one and a half cups, how much would I have left*?" **($\frac{3}{4}$ cup)**

Common error

When students convert mixed numbers in an expression to fractions, they often forget to give the fractions the correct signs when they rewrite the expression.

Have them color-code each bit of the expression from when they first write it out using mixed numbers until they rewrite it using improper fractions. For example: $-3\frac{2}{3}-\left(-2\frac{6}{7}\right)$

$$-3\frac{2}{3}=-\left(\frac{(3\times3)+2}{3}\right)=-\frac{11}{3} \qquad -2\frac{6}{7}=-\left(\frac{(2\times7)+6}{7}\right)=-\frac{20}{7}$$

$$-\frac{11}{3}=-\frac{11\times7}{3\times7}=-\frac{77}{21} \qquad -\frac{20}{7}=-\frac{20\times3}{7\times3}=-\frac{60}{21}$$

$$-\frac{77}{21}-\left(-\frac{60}{21}\right)=-\frac{77}{21}+\frac{60}{21}=\frac{-77+60}{21}=-\frac{17}{21}$$

Solutions

For worked solutions see the Solution Guide

Advanced Learners

Have students research recipes and find, or bring from home, two recipes that each use more than one mixed number measurement, for example $3\frac{1}{2}$ cups of flour. On the board, draw four different sizes of mixing bowls and write their volumes next to them. Have students work out which mixing bowl would be the best size to make their complete recipe they will need to find the total volume of their ingredients. Remind them that as the ingredients must be mixed, there must be some extra space left in the bowl.

2 Teach (cont)

Guided practice
Level 1: q7–8
Level 2: q7–9
Level 3: q7–9

Additional examples
Find:

1) $1\frac{1}{6}+2\frac{2}{4}$ $\frac{11}{3}$

2) $2\frac{1}{5}-1\frac{2}{6}$ $\frac{13}{15}$

3) $3\frac{3}{21}+6\frac{6}{7}$ 10

Concept question
"Is the fraction $\frac{3x}{6}$ in its simplest form?"

No — both the numerator and denominator have the factor 3. It can be simplified to $\frac{x}{2}$.

Guided practice
Level 1: q10–12
Level 2: q10–12
Level 3: q10–12

Independent practice
Level 1: q1–4
Level 2: q1–6
Level 3: q1–7

Additional questions
Level 1: p438 q1–6
Level 2: p438 q1–9, 13
Level 3: p438 q7–14

3 Homework

Homework Book — Lesson 2.3.7
Level 1: q1–6
Level 2: q1–10
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 2 — Simplifying Fractions
- Worksheet 8 — Adding and Subtracting Fractions

Guided Practice

Do each of the calculations in Exercises 7–9.
Give your answers as fractions.

7. $-4\frac{5}{6}-2\frac{2}{3}$ $-\frac{45}{6}$ **8.** $-5\frac{5}{6}-\left(-4\frac{11}{12}\right)$ $-\frac{11}{12}$ **9.** $2\frac{1}{2}-\frac{5}{6}+1\frac{1}{12}$ $\frac{33}{12}$

Simplify Your Answers if Possible

It's usually a good idea to **simplify** your answers if possible.

Example 5

Find $1\frac{2}{3}+2\frac{5}{6}$.

Solution

Convert the mixed numbers to fractions.

$$1\frac{2}{3}=\frac{(1\times 3)+2}{3}=\frac{5}{3} \quad \text{and} \quad 2\frac{5}{6}=\frac{(2\times 6)+5}{6}=\frac{17}{6}$$

You can use 6 as a common denominator.

Find a fraction equivalent to $\frac{5}{3}$ with a denominator of 6: $\frac{5}{3}=\frac{5\times 2}{3\times 2}=\frac{10}{6}$.

Do the addition: $1\frac{2}{3}+2\frac{5}{6}=\frac{10}{6}+\frac{17}{6}=\frac{10+17}{6}=\frac{27}{6}$

Simplify this to get your final answer: $\frac{27}{6}=\frac{9}{2}$ **Divide the numerator and denominator by 3**

Don't forget:
To simplify a fraction you divide the numerator and denominator by the same number (a common factor).

Guided Practice

For Exercises 10–12, give your answers in their simplest form.

10. $2\frac{3}{8}+3\frac{1}{8}$ $\frac{11}{2}$ **11.** $3\frac{5}{16}-1\frac{1}{16}$ $\frac{9}{4}$ **12.** $1\frac{3}{8}+2\frac{5}{10}$ $\frac{31}{8}$

Independent Practice

Calculate the following. Give all your answers in their simplest form.

1. $2\frac{2}{3}+3\frac{1}{3}$ 6 **2.** $3\frac{1}{4}-1\frac{1}{2}$ $\frac{7}{4}$ **3.** $2\frac{2}{7}-1\frac{4}{5}$ $\frac{17}{35}$ **4.** $2\frac{3}{5}-9\frac{4}{15}$ $-\frac{20}{3}$

5. $5\frac{7}{8}-12\frac{3}{16}$ $-\frac{101}{16}$ **6.** $8\frac{1}{4}-1\frac{7}{11}$ $\frac{291}{44}$ **7.** $2\frac{3}{4}+3\frac{2}{3}-5\frac{3}{8}$ $\frac{25}{24}$

Now try these:
Lesson 2.3.7 additional questions — p438

Round Up

You can hopefully see how you can use the same old routine over and over...
Convert mixed numbers to fractions, find a common denominator, do the calculation.

Solutions
For worked solutions see the Solution Guide

Lesson 2.4.1

Further Operations with Fractions

This Lesson brings together everything students have learned about calculating with fractions. They review the order of operations and apply it to problems involving fractions, mixed numbers, grouping symbols, and all four operations.

Previous Study: At grade 6 students learned how to add, subtract, multiply, and divide using fractions. They also learned about the order of operations. These things have been reviewed earlier in grade 7.

Future Study: In Algebra I students will learn to evaluate expressions containing fractions and using all four basic operations. They will solve problems and equations using these techniques.

Lesson 2.4.1

California Standards:

Number Sense 1.2
Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

Number Sense 2.2
Add and subtract fractions by using factoring to find common denominators.

What it means for you:
You'll see how doing complex calculations involving fractions uses exactly the same ideas you've seen earlier in this Section.

Key words:
- order of operations
- common denominator

Don't forget:
LCM means least common multiple — see Lesson 2.3.5.

Don't forget:
You can use PEMDAS to help you remember the correct order of operations. Parentheses, Exponents, Multiplication and Division, Addition, and Subtraction.

Don't forget:
Dividing by a fraction is the same as multiplying by its reciprocal.

Don't forget:
A raised dot · means multiplication.
So $2\frac{4}{5}\cdot 2 = 2\frac{4}{5}\times 2$

Section 2.4

Further Operations with Fractions

The problems start to get more and more complicated now. There are all kinds of calculations in this Lesson. But you just have to remember what you've learned before, and use it carefully.

Order of Operations: Multiplication Before Addition

Math problems with fractions can involve **combinations** of operations — for example, you might need to do a multiplication and addition.

Example 1

Calculate $\frac{2}{5} + \frac{5}{2} \times \frac{7}{3}$.

Solution

Remember — if there are no parentheses, you do multiplication **before** addition. So this calculation becomes

$$\frac{5\times 7}{2\times 3} + \frac{2}{5} = \frac{35}{6} + \frac{2}{5}$$

The LCM of 6 and 5 is 6 × 5 = 30. Use this as the common denominator.

$$\frac{35}{6} = \frac{35\times 5}{6\times 5} = \frac{175}{30} \text{ and } \frac{2}{5} = \frac{2\times 6}{5\times 6} = \frac{12}{30}$$

Find the result.

$$\frac{175}{30} + \frac{12}{30} = \frac{175+12}{30} = \frac{187}{30}$$

Example 2

Find $\frac{30}{45} \div \frac{2}{3} - 1\frac{3}{4}$.

Solution

Do the division first. $\frac{30}{45} \div \frac{2}{3} = \frac{30}{45} \times \frac{3}{2} = \frac{30\times 3}{45\times 2} = \frac{90}{90} = 1$

Then the subtraction. $1 - 1\frac{3}{4} = \frac{4}{4} - \frac{7}{4} = \frac{4-7}{4} = -\frac{3}{4}$

Guided Practice

Calculate the following. Simplify your answers where possible.

1. $\frac{1}{2} + \frac{3}{2} \times \frac{5}{6}$ $\frac{7}{4}$ **2.** $4\frac{3}{4} \div 2 - \frac{1}{8}$ $\frac{9}{4}$ **3.** $1\frac{7}{8}\cdot 2 + 2\frac{4}{5}\cdot 2$ $\frac{187}{20}$

1 Get started

Resources:
- small whiteboards and pens
- recipes

Warm-up questions:
- Lesson 2.4.1 sheet

2 Teach

Common error

Students may attempt to evaluate expressions by doing the operations in order from left to right, because they have generalized the concept that we read everything from left to right.

Review the order of operations, and write PEMDAS or GEMA on the board as a memory prompt.

Encourage students to rewrite expressions using parentheses to emphasize the order of operations and make expressions less ambiguous.
For example, they could write the first step of Example 1 as:

$$\frac{2}{5} + \frac{5}{2} \times \frac{7}{3} = \frac{2}{5} + \left(\frac{5}{2} \times \frac{7}{3}\right)$$

Additional examples

Find:

1) $1\frac{1}{6} - \frac{2}{3} \times \frac{1}{5}$ $\frac{31}{30}$

2) $\frac{2}{5} + 2\frac{1}{4} \div \frac{4}{2}$ $\frac{61}{40}$

3) $\frac{1}{2} \div \frac{3}{4} + 1\frac{5}{8}$ $\frac{11}{8}$

4) $4\frac{1}{5} \div \frac{2}{7} \times \frac{1}{9}$ $\frac{49}{30}$

Guided practice
Level 1: q1–2
Level 2: q1–3
Level 3: q1–3

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
Have a fraction race. Split the class into teams of four. Give each team a whiteboard and pen. Call out a fraction expression containing three operations. The first team member must write it down. Then they pass it on to the second member, who works out the first operation. As the board is passed on the problem is solved. The last person stands up when they finish. Award a point to the first team to answer correctly.

- **English Language Learners**
Review words and symbols students might encounter in problems that indicate "operations" (add, subtract, multiply, divide, sum, difference, product, quotient). Teach a color-based traffic-light strategy for students to go through when evaluating an expression: RED — look at what operations are involved in the problem. YELLOW — work out what order to do them in. GREEN — evaluate the expression.

2 Teach (cont)

Universal access

Students may have difficulty working out which operation to do first in a complicated expression that contains lots of fractions and mixed numbers.

If they are finding it difficult, give them a "coding" system: suggest that they write the expression out again using a variable to stand in for each fraction or number.

For example: $\dfrac{2\frac{1}{2}}{\frac{2}{3}} - \dfrac{\frac{5}{4}}{5\frac{3}{11}}$

Let:

$a = 2\frac{1}{2}$ $\quad b = \frac{2}{3}$ $\quad c = \frac{5}{4}$ $\quad d = 5\frac{3}{11}$

Now the expression is: $\frac{a}{b} - \frac{c}{d}$

This makes it simpler to work out the correct order. The first thing to do is calculate $\frac{a}{b}$, and $\frac{c}{d}$.

Concept question

"In the expression $\left(1\frac{1}{4}+3\frac{2}{3}\right)\times 2\frac{1}{5}$, which operation should you do first?"

$1\frac{1}{4}+3\frac{2}{3}$ — because it is inside parentheses.

Remember PEMDAS with Really Complex Expressions

With calculations that **look complicated**, take things **slowly**.

Example 3

Calculate $\left(4\frac{2}{3} + \frac{1}{2}\right) \times \frac{1}{6}$.

Solution

First convert the mixed number to a fraction so that you can do the **calculation** more easily.

$$4\frac{2}{3} = \frac{(4\times 3)+2}{3} = \mathbf{\frac{14}{3}}$$

Then calculate the **sum inside the parentheses** using **6 as the common denominator**.

$$\left(\frac{14}{3} + \frac{1}{2}\right) \times \frac{1}{6} = \left(\frac{28}{6} + \frac{3}{6}\right) \times \frac{1}{6}$$
$$= \left(\frac{31}{6}\right) \times \frac{1}{6}$$

Finally, do the **multiplication**.

$$\frac{31}{6} \times \frac{1}{6} = \mathbf{\frac{31}{36}}$$

Example 4

Calculate $\left(3\frac{3}{4}+\frac{1}{4}\right)-3\cdot\frac{1}{4}$

Solution

First turn the mixed number into a fraction. $3\frac{3}{4} = \frac{(3\times 4)+3}{4} = \mathbf{\frac{15}{4}}$

Evaluate the expression in the **parentheses**. $3\frac{3}{4}+\frac{1}{4} = \frac{15}{4}+\frac{1}{4} = \frac{16}{4} = \mathbf{4}$

Do the **multiplication**. $3\cdot\frac{1}{4} = \mathbf{\frac{3}{4}}$

Now you have: $\left(3\frac{3}{4} + \frac{1}{4}\right) - 3\cdot\frac{1}{4} = 4 - \frac{3}{4}$

Finally, do the **subtraction**. $= \frac{16}{4} - \frac{3}{4} = \mathbf{\frac{13}{4}}$

Don't forget:
It's a good idea to simplify fractions whenever you can — don't wait until you've got your final answer.
This way, you're making the numbers smaller and easier to use in the rest of the calculation.

- **Advanced Learners**

Give everyone a recipe involving fraction measurements of ingredients, for example, $3\frac{1}{2}$ cups of flour. Ask them to change the recipe so that they can make enough to feed the class. For example, if the recipe is for $1\frac{1}{2}$ dozen muffins, and there are 24 students in the class, they should work out how many times the recipe they need. Then have them work out how much of each ingredient this is. Then have them consider how the calculation would change if every two people in the class could eat three muffins between them.

Example 5

Calculate $\dfrac{\frac{1}{2}}{3\frac{1}{3}}\times\left(1+4\frac{1}{3}\right)$

Solution

Parentheses first:

To add the numbers, put them over a common denominator of 3.

$$1+4\frac{1}{3}=\frac{3}{3}+\frac{(4\times3)+1}{3}=\frac{3}{3}+\frac{13}{3}=\frac{16}{3}$$

The **division** is next, but you'll need to convert $3\frac{1}{3}$ to $\frac{10}{3}$ first.

$$\frac{\frac{1}{2}}{3\frac{1}{3}}=\frac{1}{2}\div\frac{10}{3}=\frac{1}{2}\times\frac{3}{10}=\frac{1\times3}{2\times10}=\frac{3}{20}$$

You can now do the **multiplication**.

$$\frac{\frac{1}{2}}{3\frac{1}{3}}\times\left(1+4\frac{1}{3}\right)=\frac{3}{20}\times\frac{16}{3}=\frac{3\times16}{20\times3}=\frac{48}{60}$$

The final step is to simplify the answer: $\frac{48}{60}=\frac{4}{5}$

Don't forget:

Do divisions and multiplications from left to right.

Don't forget:

$3\frac{1}{3}=\frac{(3\times3)+1}{3}=\frac{10}{3}$

Guided Practice

Calculate the expressions in Exercises 4–5.
Give your answers in their simplest form.

4. $\frac{1}{6}\cdot\frac{2}{7}-\frac{5}{7}\cdot\frac{1}{2}+\frac{7}{12}\cdot\frac{8}{7}$ $\frac{15}{42}$

5. $\dfrac{3\frac{1}{8}}{\frac{1}{4}}+\dfrac{\frac{5}{8}}{\frac{1}{2}}$ $\frac{55}{4}$

Independent Practice

Calculate the following. Give all your answers in their simplest form.

1. $\frac{3}{4}\cdot\frac{1}{6}+\frac{5}{12}$ $\frac{13}{24}$

2. $\frac{8}{15}-\frac{3}{5}\cdot\frac{1}{3}+\frac{27}{30}$ $\frac{37}{30}$

3. $\frac{5}{12}\cdot\frac{9}{2}+\frac{3}{4}\cdot\frac{5}{3}-\frac{5}{8}\cdot\frac{1}{3}$ $\frac{35}{12}$

4. $\dfrac{\frac{3}{5}+\frac{2}{15}}{\frac{2}{3}}+2\frac{2}{3}-\frac{1}{2}$ $\frac{49}{15}$

Now try these:

Lesson 2.4.1 additional questions — p439

Round Up

Some of the expressions in this Lesson were really complicated. When you have a tricky expression to evaluate, you just need to take your time, remember PEMDAS, and work through it very carefully.

Solutions

For worked solutions see the Solution Guide

2 Teach (cont)

Additional examples

Find:

1) $\left(1\frac{1}{2}-\frac{1}{2}\right)\times\frac{1}{2}$ **$\frac{1}{2}$**

2) $\dfrac{\frac{1}{2}}{\frac{1}{2}}+\frac{1}{2}$ **$\frac{3}{2}$**

3) $\frac{1}{2}\times\frac{1}{2}-\frac{1}{2}\div\frac{1}{2}+\frac{1}{2}$ **$-\frac{1}{4}$**

4) $\dfrac{1\frac{1}{2}}{\frac{1}{2}}\times\dfrac{\frac{1}{2}+\frac{1}{2}}{\frac{1}{2}}$ **6**

Guided practice

Level 1: q4–5
Level 2: q4–5
Level 3: q4–5

Independent practice

Level 1: q1–3
Level 2: q1–3
Level 3: q1–4

Additional questions

Level 1: p439 q1–6
Level 2: p439 q1–9, 13–14
Level 3: p439 q4–14

3 Homework

Homework Book — Lesson 2.4.1

Level 1: q1–6
Level 2: q2–9
Level 3: q2–11

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:

- Worksheet 8 — Adding and Subtracting Fractions
- Worksheet 10 — Multiplying Fractions by Fractions
- Worksheet 11 — Dividing Fractions
- Worksheet 21 — Order of Operations

Lesson 2.4.2 Multiplying and Dividing Decimals

In this Lesson students develop their understanding of decimal multiplication by using area models. They then learn how to multiply and divide decimals by converting them to fractions, and also by operating on integers and moving the decimal point.

Previous Study: In grade 5 students learned how to solve simple multiplication and division problems involving decimals. In grade 6 they saw how to compare the value of two decimals.

Future Study: Throughout the rest of grade 7 and through Algebra I students will need to be able to evaluate decimal expressions involving all four basic operations.

1 Get started

Resources:
- cake diagram (see below)
- scissors
- counters

Teacher Resources CD-ROM
- Number and Operator Tiles
- Money Tiles (Coins and Bills)

Warm-up questions:
- Lesson 2.4.2 sheet

2 Teach

Concept question

"When you multiply a positive integer by a decimal between 0 and 1, will your answer be bigger or smaller than the original integer?" **Smaller**

Universal access

Rational decimals between 0 and 1 have proper fraction equivalents. So when you multiply one by another, you are finding a part of the first one.

Help students grasp this using a square cake picture. Draw a picture of a square cake, with a 10 × 10 grid on it:

Tell them that 0.5 of the cake is yours: how much is that? When you have discussed this, cut the cake in half. Put one half aside and keep the other — this is 0.5 of the cake.

Say that you are giving away 0.2 of your piece to a friend. Ask how much of your piece 0.2 is. Let them look at it and count squares if they need to. They should realize it is one fifth of your piece, or 10 squares. Cut this amount off your piece.

Lay the three pieces on the table. Make them into a whole cake again. Ask what part of the cake the small bit is in decimal terms — 10 squares, or 0.1.

Explain that they have just multiplied two decimals together — write on the board the calculation 0.5 × 0.2 = 0.1.

Guided practice

Level 1: q1–3
Level 2: q1–3
Level 3: q1–3

Lesson 2.4.2

Multiplying and Dividing Decimals

California Standards:

Number Sense 1.2
Add, subtract, **multiply, and divide rational numbers** (integers, fractions, and **terminating decimals**) and take positive rational numbers to whole-number powers.

What it means for you:
You'll practice multiplying and dividing decimals.

Key words:
- integer
- decimal

The trickiest thing about multiplying and dividing decimals is working out where the decimal point should go. Once you understand the method, it's a lot more straightforward though.

Modeling Decimal Multiplication

You can use an **area model** to show what happens when you multiply two decimal numbers.

Example 1

Calculate 0.2 × 0.6.

Solution

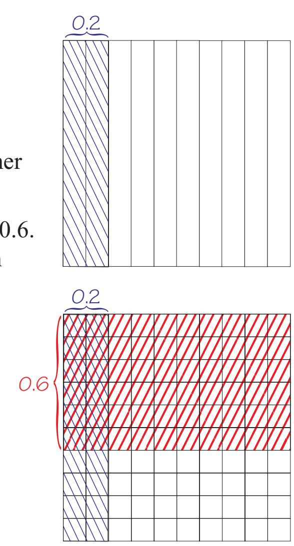
0.2 is two-tenths — that's how much of the square on the right is shaded.

0.6 is six-tenths — this is shaded in the other direction on the diagram below.

The part that's been **shaded twice** is 0.2 × 0.6. The square's now divided into **100**, so each small square is 1 ÷ 100 = **0.01**.

There are **12 squares** that have been shaded twice and each represents 0.01.

So 0.2 × 0.6 = **0.12**



Check it out:
This is really similar to the fraction multiplication area model in Lesson 2.3.3. You would have shaded exactly the same areas if you were multiplying $\frac{1}{5}$ and $\frac{3}{5}$. These fractions are equivalent to the decimals 0.2 and 0.6.

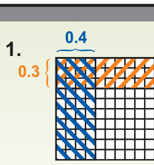
Check it out:
In Example 1, multiplying two decimals each with 1 decimal place gives you a solution with 2 decimal places. In fact, you always count the decimal places in both factors to find out how many decimal places the solution has.

Guided Practice

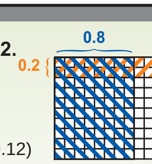
Use an area model to show the following multiplications.

1. 0.4 × 0.3 see below **2.** 0.8 × 0.2 see below **3.** 0.1 × 0.9 see below

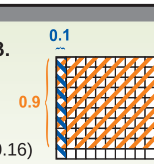
Solutions
For worked solutions see the Solution Guide

1. 

(= 0.12)

2. 

(= 0.16)

3. 

(= 0.09)

- **Strategic Learners**
  Review place values of decimals and how this relates to multiplying and dividing by 10. Use warm-up questions such as 16 ÷ __ = 1.6 to familiarize students with these concepts. Also, review the relationship between fractions and division.

- **English Language Learners**
  Put everyone in pairs. Give each pair a set of Money Tiles (see **Teacher Resources CD-ROM**). Ask them to write a decimal multiplication question and a decimal division question using the Tiles, such as, "*I bought four pens. Each one cost $0.55. How much did I pay in total?*" and "*I have $3. If it's all in dimes, how many coins do I have*?" Then have each pair present a question for the class to solve.

Multiplying Decimals

Multiplying decimals is just like multiplying integers — only you have to make sure you put the **decimal point** in the correct place.

You can **rewrite** a decimal multiplication as a **fraction calculation**.

Example 2

Calculate: 2×1.6

Solution

$$2 \times 1.6 = 2 \times \frac{16}{10}$$ ← Write the decimal as a fraction...

$$= \frac{2 \times 16}{10}$$ ← ...which can be rewritten like this.

$$= \frac{32}{10} = 32 \div 10 = \mathbf{3.2}$$ ← Now divide by the 10 to get the decimal answer.

This works if **both** numbers are decimals too:

Example 3

Calculate: 2.1×0.04

Solution

$$2.1 \times 0.04 = \frac{21}{10} \times \frac{4}{100}$$ ← Write both decimals as fractions.

$$= \frac{21 \times 4}{10 \times 100}$$ ← Multiply them.

$$= \frac{84}{1000} = 84 \div 1000 = 0.084$$ ← Now divide by the 1000 to get the decimal answer.

Check it out:

Another way of multiplying decimals is to just ignore the decimal point, do an integer multiplication, then figure out where the decimal point goes.

For example:
2.1 × 0.04
Ignore the decimal points and do 21 × 4 = 84.

The factors had a total of 3 decimal places:
2.1 (1 decimal place) ×
0.04 (2 decimal places)

So you know the product must have 3 decimal places, so move the decimal point 3 places to the left:

84. ⇒ 0.084

Always Check the Position of Your Decimal Point

You can check you have the **correct number of decimal places** in the product by making sure it's the same as the **total** number of decimal places **in the factors**.

For example, $2.\underbrace{1}_{1\text{ digit}} \times 3.\underbrace{67}_{2\text{ digits}} = 7.\underbrace{707}_{3\text{ digits}}$

1 digit + 2 digits = 3 digits

Watch out for calculations that give decimals with **zeros at the end** — you have to count the final zeros. For example, multiplying 1.5 by 1.2 gives **1.80**. You must count up the decimal places before rewriting this as 1.8.

Guided Practice

Write out and solve these calculations.

4. 1.2 × (–1.1) **–1.32**
5. 4.5 × 5.9 **26.55**
6. 1.6 × (–8.2) **–13.12**
7. 6.31 × 6.4 **40.384**
8. (–2.77) × (–7.3) **20.221**
9. 9.1 × 2.44 **22.204**

Don't forget:

Multiplying a positive by a negative gives a negative — see page 75 for more.

2 Teach (cont)

Math background

The process for converting decimals to fractions is covered in Section 2.1.

Multiplying fractions is covered in Lesson 2.3.3.

Additional examples

Calculate:
1) 3 × 1.4 **4.2**
2) 3.1 × 1.4 **4.34**
3) 3.15 × 1.42 **4.473**

Concept question

"*If I evaluated 1.2345 × 6.76767, how many decimal places would my answer have?*" **9**

Guided practice

Level 1: q4–6
Level 2: q4–7
Level 3: q4–9

Solutions

For worked solutions see the Solution Guide

Advanced Learners
Before teaching students the methods, give them a series of decimal multiplication and division calculations to consider, for example, 0.5 ÷ 2.5 = 0.2; 0.05 ÷ 2.5 = 0.02; 0.005 ÷ 2.5 = 0.002. Ask students to look for patterns in their answers, and use any patterns that they find to devise their own algorithms for multiplication and division of decimals.

2 Teach (cont)

Universal access

To familiarize students with the method of decimal division shown in Example 5, give everyone some number tiles and 1 division operator tile (from the Number and Operator Tiles on the **Teacher Resources CD-ROM**) and also 2 counters to represent decimal points.

Write some decimal division problems on the board. Then they can:

1) Lay out the question using the tiles.

0 ● 1 2 ÷ 0 ● 4

2) Count how many tiles are after the decimal point (counter) in each number. Write these numbers down.

2 places 0 ● 1 2 ÷ 0 ● 4 1 place

3) Remove both decimal points and do the integer division that remains.

0 1 2 ÷ 0 4

4) Lay out the answer using the tiles.

3 ●

5) Move the decimal point as many places left as there were digits after the decimal point in the first number, then as many right as there were in the second, adding zeros where necessary.

0 0 ● 3

Guided practice

Level 1: q10–12
Level 2: q10–15
Level 3: q10–18

Independent practice

Level 1: q1, 4–5, 7
Level 2: q1, 4–8
Level 3: q1–9

Additional questions

Level 1: p439 q1–4, 6–7, 12–13
Level 2: p439 q2–10, 12–14
Level 3: p439 q3–14

3 Homework

Homework Book — Lesson 2.4.2

Level 1: q1–3, 6–9
Level 2: q3–10
Level 3: q4–11

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 12 — Multiplying and Dividing Decimals

Dividing Decimals — Take Care with the Decimal Point

You can think about **decimal division** in a similar way.

Example 4

Calculate: 46.5 ÷ 0.05

Solution

$$46.5 \div 0.05 = \frac{465}{10} \div \frac{5}{100}$$ **Write the decimals as fractions**

$$= \frac{465}{10} \times \frac{100}{5}$$ **To divide by a fraction, multiply by its reciprocal**

$$= \frac{4650}{5} = 4650 \div 5 = \mathbf{930}$$

Don't forget:
You can do integer division using long division.
Check back to Lesson 2.3.2.

Another way is to ignore the decimal points and just use **integer division**. Then you have to find the correct position for the **decimal point**:

- Count the number of digits after the decimal point in the **first** number and move the decimal point this many places to the **left**.
- Count the number of digits after the decimal point in the **second** number and move the decimal point this many places to the **right**.

Check it out:
You could also do the division **46.5 ÷ 0.05** in Example 4 by doing integer division, then working out the decimal places: **465 ÷ 5 = 93**.
There's **1 decimal place** in 46.5, and **2 decimal places** in 0.05. So move **1 decimal places** to the left, and **2 to the right**. So the decimal point moves one place right overall: **46.5 ÷ 0.05 = 930**

Example 5

If 5 ÷ 25 = 0.2, calculate 0.005 ÷ 2.5.

Solution

0.005 is the first number — it has **3 decimal places**.
2.5 is the second number — it has **1 decimal place**.
So you have to move **3 places to the left**, and then **1 to the right**.

0.0 0 0.2 ⇒ 0.0.0 0 2 ⇒ 0.005 ÷ 2.5 = **0.002**

Check it out:
0.5 is 10 times smaller than 5. This means that the result of dividing something by 0.5 will be 10 times bigger than the result of dividing the same thing by 5.
To make a result 10 times bigger, you move the decimal point one place to the right.

Guided Practice

Solve these divisions.

10. 273 ÷ 1.3 **210** **11.** 195.2 ÷ 8 **24.4** **12.** 53.56 ÷ 1.3 **41.2**
13. 1.56 ÷ 3 **0.52** **14.** 1.44 ÷ 3 **0.48** **15.** 6.55 ÷ 5 **1.31**
16. 3.84 ÷ 1.2 **3.2** **17.** 4.64 ÷ 1.6 **2.9** **18.** 33.58 ÷ 2.3 **14.6**

Independent Practice

In Exercises 1–6, find the product of each pair of numbers.

1. 1.03 × 0.5 **0.515** **2.** 4.781 × –6.0 **–28.686** **3.** –3.11 × (–9.14) **28.4254**
4. 7.2 × 0.6 **4.32** **5.** 1.04 × 4 **4.16** **6.** 10.5 × 0.07 **0.735**

Find the quotient in Exercises 7–9.

7. –7.25 ÷ 0.25 **–29** **8.** –16.8 ÷ (–0.04) **420** **9.** 12.48 ÷ –1.2 **–10.4**

Now try these:
Lesson 2.4.2 additional questions — p439

Round Up

Wow — that was a lot of information about multiplying and dividing. But don't panic — when you're multiplying and dividing with decimals, you just need to take it slow, working bit by bit.

Solutions
For worked solutions see the Solution Guide

Lesson 2.4.3

Operations with Fractions and Decimals

This Lesson gives students the chance to practice the skills they have developed in the previous Lessons by performing calculations involving both fractions and decimals, and all four operations.

Previous Study: In grades 5 and 6 students learned how to evaluate expressions containing either fractions or decimals and all four operations.

Future Study: In Algebra I students will evaluate and simplify expressions containing all four basic operations and rational numbers in any form, including decimals and fractions.

Lesson 2.4.3

Operations with Fractions and Decimals

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide rational numbers (integers, **fractions, and terminating decimals**) and take positive rational numbers to whole-number powers.

What it means for you:

You'll practice adding, subtracting, multiplying, and dividing fractions and decimals together.

Key words:

- fraction
- decimal
- multiply
- divide

You've probably now seen nearly all the ideas, techniques, and tricks you'll ever need for almost any kind of number problem. Now you're going to put them all into action...

Calculations Can Involve Both Fractions and Decimals

To do a calculation involving **a fraction and a decimal**, you either have to make them **both fractions** or **both decimals**.

For example, if you needed to find $0.3 \times \frac{3}{5}$, you could...

- Convert 0.3 to a fraction $\left(\frac{3}{10}\right)$ and find $\frac{3}{10} \times \frac{3}{5} = \mathbf{\frac{9}{50}}$.
- Convert $\frac{3}{5}$ to a decimal (0.6) and find $0.3 \times 0.6 = \mathbf{0.18}$.

Both methods are correct, and they both give the same answer (you can check by calculating **9 ÷ 50**, which gives **0.18**).

Example 1

Calculate $0.25 \times \frac{4}{5}$ by:

a) converting 0.25 to a fraction,

b) converting $\frac{4}{5}$ to a decimal.

Solution

a) 0.25 is $\frac{25}{100} = \frac{1}{4}$. So $0.25 \times \frac{4}{5} = \frac{1}{4} \times \frac{4}{5} = \frac{4}{20} = \mathbf{\frac{1}{5}}$

b) $\frac{4}{5}$ is $4 \div 5 = 0.8$. So $0.25 \times \frac{4}{5} = 0.25 \times 0.8 = \mathbf{0.2}$

The best idea is to choose the method that makes the calculation **easiest**.

Example 2

Calculate $\frac{1}{3} - 0.5$.

Solution

If you convert $\frac{1}{3}$ to a decimal, you get 0.333333333..., which is difficult to subtract 0.5 from. So it's best to use **fractions** here.

So $\frac{1}{3} - 0.5 = \frac{1}{3} - \frac{1}{2} = \frac{2}{6} - \frac{3}{6} = \mathbf{-\frac{1}{6}}$

Don't forget:

0.3333... can also be written as $0.\overline{3}$. (But that doesn't make it any easier to subtract 0.5 from.)

1 Get started

Resources:

- sets of fraction and decimal snap cards (see page 100)
- grid paper

Warm-up questions:

- Lesson 2.4.3 sheet

2 Teach

Universal access

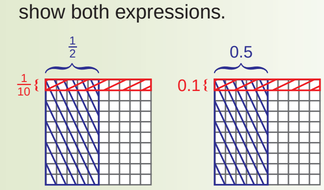
Show students that it doesn't matter whether you convert both numbers to fractions, or both to decimals. Give them a simple "fraction × decimal" multiplication question to do — for example, $\frac{1}{2} \times 0.1$.

Have them rewrite the question in both "fraction × fraction" and "decimal × decimal" form, for example:

$\frac{1}{2} \times 0.1 = \frac{1}{2} \times \frac{1}{10}$ and

$\frac{1}{2} \times 0.1 = 0.5 \times 0.1$.

Now ask them to draw two 10 × 10 grids of equal size on grid paper. Have them use the area method to show both expressions.



They should be able to see that they have colored the same area of each grid. In this case, they have colored 5 squares out of 100 in both grids — so both expressions are equal to 0.05 or $\frac{1}{20}$.

Math background

Converting terminating and repeating decimals to fractions is covered in Section 2.1.

- **Strategic Learners**
  Treating multiplication by a fraction as a two-step process was introduced in Lesson 1.2.5, on solving two-step equations. Do a review exercise based on this. Put a "fraction × decimal" expression on the board. Have students rewrite it in as many different ways as they can. For example, $\frac{2}{3} \times 4.7 = (2 \div 3) \times 4.7 = (2 \times 4.7) \div 3 = (4.7 \div 3) \times 2$. Have students check the answers are really the same using a calculator.
- **English Language Learners**
  Play snap with "fraction × decimal" expressions and "× and ÷" expressions. Make a set of 30 cards. On 15 put "fraction × decimal" expressions in red, for example, $\frac{4}{5} \times 2.3$, $\frac{2}{7} \times 9.1$. On the others put equivalent "× and ÷" expressions for each one in blue, for example, $(4 \times 2.3) \div 5$, $(9.1 \div 7) \times 2$. Put students in small groups. Have them play snap with the shuffled cards. The first to see a snap gets a point.

2 Teach (cont)

Guided practice

Level 1: q1–3, 7
Level 2: q1–5, 7–8
Level 3: q1–9

Concept question

"Which expression has the larger value: (3 ÷ 5) × 2, or (3 × 2) ÷ 5?"

Both expressions have the same value.

Additional examples

Calculate:

1) $\frac{1}{2} \times 0.25$ — $\frac{1}{8}$

2) $\frac{1}{4} \times 0.5$ — $\frac{1}{8}$

3) $\frac{1}{2} \div 0.25$ — 2

Guided Practice

For Exercises 1–6, find the answers to the calculations by:
(i) converting all the numbers to fractions,
(ii) converting all numbers to decimals.
Check that both your answers for each exercise are equivalent.

1. $\frac{1}{2}+0.2$ $\frac{7}{10}$ or 0.7 **2.** $\frac{1}{4}-0.2$ $\frac{1}{20}$ or 0.05 **3.** $\frac{1}{4}+0.4$ $\frac{13}{20}$ or 0.65

4. $\frac{1}{3}\times 0.3$ $\frac{1}{10}$ or 0.1 **5.** $\frac{1}{6}\times 1.2$ $\frac{1}{5}$ or 0.2 **6.** $\frac{1}{10}\times 2.5$ $\frac{1}{4}$ or 0.25

For Exercises 7–9, use the easiest method to do each calculation.

7. $\frac{1}{7}+\frac{3}{98}$ $\frac{17}{98}$ **8.** $0.15 \div \frac{1}{2}$ $\frac{3}{10}$ or 0.3 **9.** $0.3-\left(\frac{1}{3}\times 4.5\right)$ $-\frac{6}{5}$ or -1.2

You Can Think of Fraction Multiplication Another Way

There's another useful way to think about multiplying by fractions.

When you multiply by the fraction $\frac{3}{5}$, it is the same as either:
(i) **multiplying by 3** and then **dividing by 5**, or
(ii) **dividing by 5** and then **multiplying by 3**.

> **Check it out:**
> Multiplying by $\frac{1}{5}$ is the same as dividing by 5.

Example 3

Calculate $6.93\times\frac{2}{3}$.

Solution

This would be difficult to do using decimals (6.93 × 0.6666...).

And 6.93 doesn't convert to an "easy" fraction $\left(\frac{693}{100}\right)$.

But multiplying by $\frac{2}{3}$ means **dividing by 3** and then **multiplying by 2**.

This is much quicker.

Dividing by 3 gives: $6.93 \div 3 = \mathbf{2.31}$
Then multiplying by 2 gives: $2.31 \times 2 = \mathbf{4.62}$

So, $6.93\times\frac{2}{3} = \mathbf{4.62}$

Example 4

Calculate $\frac{2}{7}\times 1.5$.

Solution

You're multiplying 1.5 by $\frac{2}{7}$, so **multiply it by 2**, then **divide by 7**.

So $1.5 \times 2 = \mathbf{3}$, and then $3 \div 7 = \frac{3}{7}$. So, $\frac{2}{7}\times 1.5 = \frac{3}{7}$

> **Check it out:**
> You can do the multiplication and division in either order — so choose whichever will be easier.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Expressions with fractions and decimals can be rewritten in many different ways. For example, you can convert fractions to decimals, and vice versa, or rewrite fractions as divisions (for example $\frac{1}{2} = 1 \div 2$). You can change division to multiplication by a fraction, or multiplication to division by a fraction. Challenge students to rewrite expressions in as many ways as they can. Ask them to think about why this is a useful skill — sometimes an expression is easier to evaluate when written in a different format.

Example 5

Calculate $\frac{1}{3}(6.35 - 3.02)$.

Solution

As always, work out the parentheses first.
$6.35 - 3.02 = \mathbf{3.33}$.

Then you can divide by 3.
$3.33 \div 3 = \mathbf{1.11}$

Don't forget:
Multiplying by $\frac{1}{3}$ is the same as dividing by 3.

Guided Practice

Calculate the value of the expressions in Exercises 10–14.

10. $\frac{3}{4} \times 0.24$ **0.18** **11.** $\frac{7}{15} \times 1.5$ **$\frac{7}{10}$ or 0.7** **12.** $0.9 \times \frac{8}{9}$ **$\frac{4}{5}$ or 0.8**

13. $3.5 \times \left(\frac{3}{5} - \frac{1}{5}\right)$ **$\frac{7}{5}$ or 1.4** **14.** $\frac{5}{9} \times (8.03 + 0.97)$ **5**

Independent Practice

Calculate the following.

1. $\frac{1}{2} \times 0.8$ **$\frac{2}{5}$ or 0.4** **2.** $\frac{1}{3} \times 1.2$ **$\frac{2}{5}$ or 0.4** **3.** $\frac{1}{2} + 0.25$ **$\frac{3}{4}$ or 0.75**

4. $1.25 + \frac{7}{4}$ **3** **5.** $0.8 - \frac{1}{5}$ **$\frac{3}{5}$ or 0.6** **6.** $-1.3 - \frac{3}{10}$ **$-\frac{8}{5}$ or -1.6**

7. $2.5 \times \frac{2}{5}$ **1** **8.** $1.3 \times \frac{10}{13}$ **1** **9.** $2.7 \div \frac{27}{13}$ **1.3 or $\frac{13}{10}$**

10. $(4.57 + 3.53) \times \frac{1}{9}$ **$\frac{9}{10}$ or 0.9** **11.** $(-5.36 - 1.64) \times \frac{2}{7}$ **-2**

12. $(2.89 - 18.89) \div \frac{8}{5}$ **-10** **13.** $\dfrac{(5.67 + 12.53)}{\frac{2}{3}}$ **27.3**

Don't forget:
Dividing by $\frac{27}{13}$ is the same as multiplying by its reciprocal, $\frac{13}{27}$.

Now try these:
Lesson 2.4.3 additional questions — p439

Round Up

The theme that's been running through the last few Lessons is this — although questions might look complicated, you just need to take things real slow, and use all those things you learned earlier in the Section. Don't try to hurry — that makes you more likely to make a mistake.

Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Concept question

"Does the expression $\frac{1}{3}(6.35 - 3.02)$ have the same value as the expression $0.\overline{3}\left(\frac{635}{100} - \frac{302}{100}\right)$?"

Yes – the numbers have been converted to their equivalent decimals or fractions.

Guided practice
Level 1: q10–12
Level 2: q10–13
Level 3: q10–14

Independent practice
Level 1: q1–5
Level 2: q1–9
Level 3: q1–13

Additional questions
Level 1: p439 q1–4
Level 2: p439 q1–6, 10–11
Level 3: p439 q1–11

3 Homework

Homework Book — Lesson 2.4.3
Level 1: q1–4, 6, 7
Level 2: q1, 4–9
Level 3: q1, 4–11

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 8 — Adding and Subtracting Fractions
- Worksheet 10 — Multiplying Fractions by Fractions
- Worksheet 11 — Dividing Fractions
- Worksheet 12 — Multiplying and Dividing Decimals

Lesson 2.4.4 Problems Involving Fractions and Decimals

This Lesson brings all the concepts and skills developed earlier in this Section together, and provides students with the opportunity to apply what they have learned to real-life problems involving fractions and decimals.

Previous Study: In grades 5 and 6 students learned how to evaluate expressions containing fractions or decimals, and how to construct math expressions from word problems.

Future Study: In Algebra I students will solve multistep problems containing rational numbers in any form, including decimals and fractions.

1 Get started

Resources:
- meter ruler

Warm-up questions:
- Lesson 2.4.4 sheet

2 Teach

Common error

Students very often translate real-life questions into math language, and then treat them as abstract problems, forgetting about the original context. This leads to them not including units, or forgetting to check that their answer is reasonable in the context of the question.

Providing a checklist, like the one suggested in Lesson 1.2.6, can help avoid this.

Universal access

Put students into small groups. Have each group write their own real-life word problem involving fractions and decimals.

If they are struggling for ideas you may like to write some broad scenarios on the board, such as finding the area of a room, or calculating ingredient weights in a recipe, or sharing something between a group of friends.

When all the groups have finished, have each swap questions with another and attempt to answer each other's questions.

Then have a class discussion — which questions were easiest or hardest to answer, and why? Did the groups use any particular strategies to help them answer their question?

Guided practice

Level 1: q1–2
Level 2: q1–2
Level 3: q1–2

Lesson 2.4.4

Problems Involving Fractions and Decimals

California Standards:

Number Sense 1.2
Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

Mathematical Reasoning 2.2
Apply strategies and results from simpler problems to more complex problems.

What it means for you:
You'll use ideas from previous Lessons in this Section to solve some real-life problems.

Key words:
- word problem
- units

This is the last Lesson in this Section. And again, it's all about using the skills you've already learned. But this time before you can do the math, you have to write the problem down in math language using a description of a real-life situation.

First Write Real-Life Problems as Math

Example 1

The length of a rectangular room is $54\frac{1}{3}$ feet, while its width is 33.3 feet. What is the area of the room?

Solution

You find the area of a rectangle by multiplying its length by its width.

So the area of this room is given by $54\frac{1}{3} \times 33.3$.

Now you need to work through all the steps from the previous Lessons.

$$54\frac{1}{3}\times 33.3 = \frac{163}{3}\times 33.3 \quad \text{Convert mixed number to a fraction}$$
$$= (33.3 \div 3)\times 163 \quad \text{Rewrite as "÷ then ×"}$$
$$= 11.1\times 163 \quad \text{Do the division}$$

You could use a calculator here, but you don't really need to.
To do this multiplication without a calculator, you can rewrite it using the ideas in Section 2.3.2.

$$11.1 \times 163 = (10 \times 163) + (1 \times 163) + (0.1 \times 163)$$
$$= 1630 + 163 + 16.3$$
$$= 1809.3$$

Check it out:
Very often, writing your answer in a sensible way just means adding units.
You should also always check that it's realistic — see Chapter 1 for more information.

With real-life problems you must always think about what your answer means, and then write it in a sensible way. Here, you need to add **units**.

So the area of the room is **1809.3 square feet**.

Guided Practice

1. A rectangular dance floor is 28.6 feet wide and $15\frac{1}{2}$ feet long. What is the area of the dance floor? 443.3 square feet

2. A rectangular playing field is $16\frac{2}{3}$ yards wide and 30.9 yards long. What is the area of the playing field? 515 square yards

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Have students think about, write, and solve a real-life math problem. For example, say you are putting up a class display. Explain that you want to cover the board with sheets of paper that measure, for example, 0.25 m by 0.2 m. Give them a meter ruler and one sheet of paper. Challenge them to find out how many sheets are needed. Have them write it out as a word problem, and show how they solved it.

- **English Language Learners**
  Help students to develop a strategy for solving word problems. Put students in pairs and give each pair a word problem to look at. They could underline important words in red, and circle key numbers in green. They should draw a diagram were appropriate. Let them try solving the problem using these strategies, then ask them to check their answer for reasonableness by looking back at the problem.

Drawing a Diagram Can Make Things Clearer

Sometimes it's not **doing** the math that's the hardest thing in a problem. It's working out **what math to do** in the first place.

Example 2

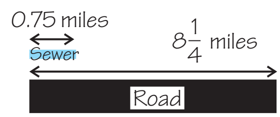
Some public sewer lines are being installed along $8\frac{1}{4}$ miles of road.

The supervisor says they will be able to complete 0.75 of a mile a day.

How long will the project take?

Solution

If you can't see how to answer a question, draw a picture.



You need to find out how many times 0.75 goes into $8\frac{1}{4}$.

In math language, this is a division — so you need to solve $\mathbf{8\frac{1}{4} \div 0.75}$.

Now that the problem is written in math language, you can use the techniques from previous Lessons.

$$8\frac{1}{4} \div 0.75 = 8\frac{1}{4} \div \frac{3}{4} \quad \textbf{Convert the decimal to a fraction}$$
$$= \frac{33}{4} \div \frac{3}{4} \quad \textbf{Convert the mixed number to a fraction}$$
$$= \frac{33}{4} \times \frac{4}{3} \quad \textbf{Rewrite the division as a multiplication}$$
$$= \frac{33 \times 4}{4 \times 3} \quad \textbf{Multiply the fractions}$$
$$= \frac{33}{3} = \mathbf{11} \quad \textbf{Simplify}$$

Don't forget: about **units**...

The project will take **11 days**.

Don't forget:
Your diagram doesn't have to be accurate — just good enough to get an idea of what you need to do.

Check it out:
You could work out the fraction here as $\frac{132}{12}$, and then simplify it. But here, the factor of 4 in the numerator and the factor of 4 in the denominator have been canceled straightaway.

$$\frac{33 \times \cancel{4}}{\cancel{4} \times 3}$$

Guided Practice

3. A piece of rope 17.25 feet long must be divided up into smaller lengths of $\frac{3}{4}$ of a foot long.

How many lengths can be made from this piece of rope? **23 lengths**

4. A birthday cake needs $2\frac{1}{2}$ cups of raisins. However, because a big party is planned, several cakes are made by increasing the amount of all the ingredients used. In fact, $8\frac{3}{4}$ cups of raisins are used.

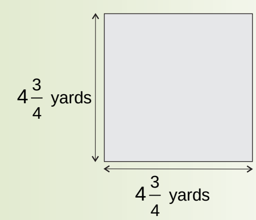
How many cakes is this enough raisins for? **$3\frac{1}{2}$ cakes**

2 Teach (cont)

Universal access

Put a real-life math question on the board. For example: *"You are carpeting a room. It is square, and has a side length of $4\frac{3}{4}$ yards. You can buy enough carpet to cover an area of 1 square yard for \$15.65. How much will it cost you to carpet the whole room?"*

Ask for ideas about the diagram you might draw to help answer the question. Draw any ideas you are given up on the board. When everyone is happy, have them use the picture to answer the question.
For example:



$$\text{Area} = 4\frac{3}{4} \times 4\frac{3}{4} = \frac{19}{4} \times \frac{19}{4} = \frac{361}{16}\ \text{yd}^2$$

$$\text{Cost} = \frac{361}{16} \times 15.65 = 353.10$$

So, assuming you can buy the exact amount of carpet, the cost will be \$353.10.

Common error

Students often make errors when answering word problems due to not reading the question carefully enough. For example, given the question, "*If 1 lb of apples costs \$1.27, how much would 1.5 kg cost?,"* many will multiply \$1.27 by 1.5 without realizing that the units are different.

Encourage students to read a word problem right through, then read it again making a note of important facts. Only having done this should they begin trying to solve it.

Guided practice

Level 1: q3–4
Level 2: q3–4
Level 3: q3–4

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Present students with a situation to model. For example, say that the classroom floor is going to be retiled, and the school wants to know how many tiles to buy, and how much they will cost. Give them a choice of two sizes of tile (noting that they can't mix tiles), with prices, each being sold in packs of 10. Challenge them to find the most cost-efficient way of tiling the floor. This will require them to measure the room, develop a model, and do complex calculations that are likely to involve fractions and decimals.

2 Teach (cont)

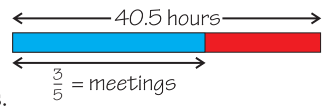
Example 3

Aisha worked 40.5 hours in one particular week. Three-fifths of these hours she was in meetings, while the rest of the time was spent traveling. How many hours did Aisha spend traveling during the week?

Solution

Again, a diagram might help.

You need to work out how many hours the red part of the bar represents.



If the blue part of the bar is $\frac{3}{5}$ of the total hours, then the red part must represent: $1-\frac{3}{5}=\frac{2}{5}$

So you need to work out $\frac{2}{5}$ of 40.5 hours — this can be written as a multiplication: $40.5\times\frac{2}{5}$

Now you can do the math.

$$40.5\times\frac{2}{5}=(40.5\div 5)\times 2$$ **Rewrite the fraction multiplication as a division, followed by a multiplication**

$$=8.1\times 2$$ **Do the division**

$$=16.2$$ **Do the multiplication**

Now say what your answer means.
Aisha spent **16.2 hours** traveling during the week.

Check it out:

Here, you could do the math as fractions...

$$40.5\times\frac{2}{5}=\frac{81}{2}\times\frac{2}{5}$$
$$=\frac{81}{5}$$
$$=16\frac{1}{5}$$

...or decimals...
40.5 × 0.4 = 16.2
You get the same answer whatever method you use.

Guided Practice

5. A small town has a sports field with a total area of 1533.75 square yards. One third of this area is used only by parents with small children, while the rest can be used by anyone.

How many square yards are for anyone's use? **1022.5 square yards**

Independent Practice

1. The floor area of a room is $99\frac{15}{16}$ square feet. The length of the room is $10\frac{1}{4}$ feet. What is the width of the room? **9.75 (= $9\frac{3}{4}$) feet**

2. Approximately $\frac{1}{5}$ of the students in a high school take an advanced placement class. Of those students it is thought 50% will receive a scholarship for college. What fraction of the students in the high school are expected to receive a scholarship? $\frac{1}{10}$

Now try these:

Lesson 2.4.4 additional questions — p440

Round Up

That's the end of a Section containing a lot of little bits of information. An important thing to remember from it all is that you must work through complex problems slowly and carefully.

Guided practice
Level 1: q5
Level 2: q5
Level 3: q5

Independent practice
Level 1: q1–2
Level 2: q1–2
Level 3: q1–2

Additional questions
Level 1: p440 q1–2, 5–6
Level 2: p440 q1–6
Level 3: p440 q1–7

3 Homework

Homework Book — Lesson 2.4.4
Level 1: q1–4, 6, 8
Level 2: q1–9
Level 3: q1–11

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 8 — Adding and Subtracting Fractions
- Worksheet 10 — Multiplying Fractions by Fractions
- Worksheet 11 — Dividing Fractions
- Worksheet 12 — Multiplying and Dividing Decimals

Solutions
For worked solutions see the Solution Guide

Section 2.5 Exploration — Basic Powers

Purpose of the Exploration

The goal of this Exploration is to have students physically produce the powers of small whole numbers. Students will see the exponential growth effect by repeatedly folding a piece of paper. The end result is that students will see how powers are the same as multiplying a number by itself.

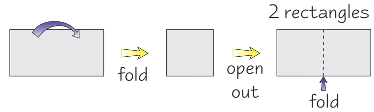
Resources

• sheets of paper (the thinner and longer the better)
• rulers

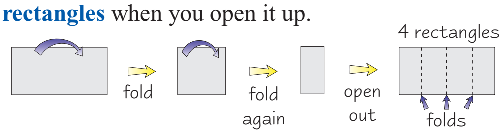
Section 2.5 introduction — an exploration into: Basic Powers

You can find the powers of numbers by repeatedly folding a piece of paper. Each extra fold you make produces more rectangles. You'll see how the numbers of rectangles produced are powers.

Take a sheet of paper and fold it into **two equal sections**. When you open it up, you'll see there are **two rectangles**.



If you fold the paper into two again, then fold it once more, you'll have **four rectangles** when you open it up.



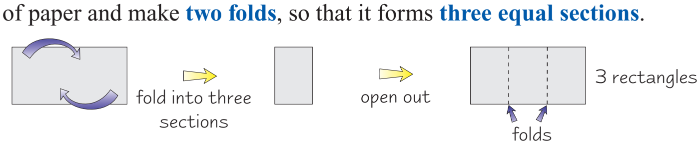
The number of rectangles produced by each fold represents the powers of 2.

$2^0 = 1$ $\quad 2^1 = 2$ $\quad 2^2 = 4$

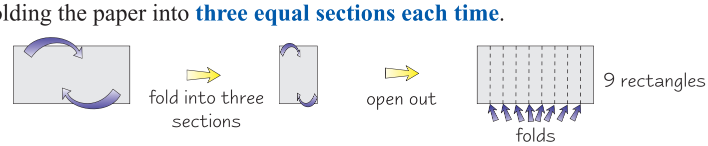
Exercises

1. Continue to fold the paper into two equal sections each time. Write the total number of rectangles produced after the given number of folds.
   a. 3 8 **b.** 4 16 **c.** 5 32
2. Write your answers to **1a**, **b**, and **c** as powers of 2. $8 = 2^3$ $\quad 16 = 2^4$ $\quad 32 = 2^5$

Now take a sheet of paper and make **two folds**, so that it forms **three equal sections**.



Repeat this by folding the paper into **three equal sections each time**.



Exercises

3. Fold a piece of paper repeatedly into three, in the way described above. What is the total number of rectangles produced from the:
   A long piece of paper makes this part easier.
   a. second set of folds? 9 **b.** third set of folds? 27 **c.** fourth set of folds? 81
4. Write your answers to **3a**, **b**, and **c** as powers of 3. $9 = 3^2$ $\quad 27 = 3^3$ $\quad 81 = 3^4$
5. With a new piece of paper, experiment with this process to make 25 rectangles.
   Fold a piece of paper into 5, then into 5 again ($5^2 = 25$).
6. Is it possible to make 10 rectangles of equal size by using the process above? **No — you have to fold it into 10 rectangles at the first stage. 10 can't be written as a power with a whole number exponent.**

Round Up

When you repeatedly fold a piece of paper into two, you repeatedly multiply the number of rectangles by two. And when you repeatedly fold a piece of paper into three, you repeatedly multiply the number of rectangles by three. This gives you the powers of 2 and the powers of 3 — because powers are produced by repeated multiplication.

Section 2.5 Exploration — *Basic Powers* 105

Strategic & EL Learners

Strategic learners may have difficulty making equally spaced folds. A good fold in Exercise 1 can be accomplished by matching up the corners of the paper and pushing down gently. For Exercises 3 and 4, it may be necessary to use a pencil and ruler to mark where to fold the sheet.

EL learners may be unfamiliar with words like "crease." Explain that to crease is to make a folded edge of the paper.

Concept question

"Do you think that you'd be able to continue to fold the paper for much longer?"

No — each fold means there are twice as many rectangles. These get smaller and smaller, and the fold gets thicker and thicker.

Common error

Students may encounter difficulty with the folding process. This activity does require a certain level of manual dexterity. Students who rush and make sloppy folds will not have a product that is easy to interpret. Remind students that they should make a light fold and check it before making a heavy crease.

Math background

When you multiply a number repeatedly, the product grows very quickly. This activity develops students' understanding of this concept.

Lesson 2.5.1

Powers of Integers

In this Lesson students are introduced to powers as repeated multiplications. They learn how to write powers in base and exponent form.

Previous Study: In grade 5 students learned how to multiply a series of integers together. In grade 4 they learned how to find areas, and in grade 5 volume, so they are familiar with square and cubic units.

Future Study: In Algebra I students raise numbers to fractional powers and equate this process to finding roots. They learn and use all the exponent rules.

1 Get started

Resources:
- individual whiteboards and pens
- empty egg cartons
- rice

Warm-up questions:
- Lesson 2.5.1 sheet

2 Teach

Universal access

Explain to everyone that a power is just an easier way to write a repeated multiplication. Connect this to the idea that, in the same way, a multiplication is just an easier way to write a repeated addition.

For example: $2 + 2 + 2 + 2 = 2 \times 4$
$2 \times 2 \times 2 \times 2 = 2^4$

Common error

When a power is written in base and exponent form, students often have trouble remembering what the parts mean.

Consider $2^3 = 2 \times 2 \times 2 = 8$

They sometimes mix up the two parts: $2^3 = 3 \times 3 = 9$

Or they sometimes multiply the base and exponent together: $2^3 = 2 \times 3 = 6$

Lesson 2.5.1

Section 2.5
Powers of Integers

California Standards:

Number Sense 1.2
Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and **take positive rational numbers to whole-number powers.**

Algebra and Functions 2.1
Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

What it means for you:
You'll learn how to write repeated multiplications in a shorter form.

Key words:
- power
- base
- exponent
- factor

Check it out:
Raising something to the second power is called squaring it. Raising something to the third power is called cubing it. There's more on this in Lesson 2.5.3.

Don't forget:
A factor is one of the terms in a multiplication expression.

Check it out:
You can describe a number as being "raised to" or "taken to" a power. So 2^4 could be read as, "two to the fourth power," or "two raised to the fourth power," or "two taken to the fourth power."

A power is just the product that you get when you repeatedly multiply a number by itself, like 2 • 2, or 3 • 3 • 3. Repeated multiplication expressions can be very long. So there's a special system you can use for writing out powers in a shorter way — and that's what this Lesson is about.

A Power is a Repeated Multiplication

A **power** is a **product** that results from **repeatedly multiplying** a number by itself. For example:

$2 \cdot 2 = 4$, or "two to the second power."
$2 \cdot 2 \cdot 2 = 8$, or "two to the third power."
$2 \cdot 2 \cdot 2 \cdot 2 = 16$, or "two to the fourth power."

So **4**, **8**, and **16** are all **powers of 2**.

You Can Write a Power as a Base and an Exponent

If every time you used a **repeated multiplication** you wrote it out in full, it would make your work very complicated. So there's a **shorter way** to write them. For example:

$$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$$

2 is the **base** — it's **the number that's being multiplied**

This is the **exponent** — it tells you **how many times** the base number is a **factor** in the multiplication expression.

For example, the expression $10 \cdot 10$ can be written in this form — **the base is 10** and since 10 occurs twice, **the exponent is 2**.

base → 10^2 ← exponent

You can rewrite any repeated multiplication in this form. So any number, x, to the **nth power** can be written as:

base → x^n ← exponent

- **Strategic Learners**
  Have everyone close their books. Put a repeated multiplication on the board — for example, 2 × 2 × 2. Ask students to write the expression in base and exponent form (2^3). Tell them to circle the exponent in red and the base in blue (2^3). Ask them to write down how they would read it aloud ("two to the third power," or "two cubed"). Lastly, have them evaluate the power (8). Check their answers, and identify any problem areas.

- **English Language Learners**
  Practice reading powers out loud to reinforce their meaning. Write powers up on the board, and have the class read them aloud. For example, 5^3 should be read as "five to the third power", or "five cubed", and 2^6 as "two to the sixth power." Then have everyone copy down the power, and write it out on individual whiteboards as a repeated multiplication. Have students hold their boards up as an answer check.

Example 1

Write the expression 3 • 3 • 3 • 3 • 3 in base and exponent form.

Solution

The number that is being multiplied is 3. So the base is 3.

3 occurs as a factor five times in the multiplication expression. So the exponent is 5.

So 3 • 3 • 3 • 3 • 3 = 3^5.

If a number has an **exponent of 1** then it occurs only once in the expanded multiplication expression. So any number to the **power 1** is just the **number itself**. For example:

$$5^1 = 5,$$
$$137^1 = 137,$$
$$x^1 = x.$$

Guided Practice

Write each of the expressions in Exercises 1–8 as a power in base and exponent form.

1. 8 • 8 8^2
2. 2 • 2 • 2 2^3
3. 7 • 7 • 7 • 7 • 7 7^5
4. 5 5^1
5. 9 • 9 • 9 • 9 9^4
6. 4 • 4 • 4 • 4 • 4 • 4 • 4 4^7
7. –5 • –5 $(-5)^2$
8. –8 • –8 • –8 • –8 • –8 $(-8)^5$

Check it out:

When you write a negative number raised to a power, you need to put parentheses around the number.
For example, $(-2)^4$ tells you to raise negative two to the fourth power — it has a value of 16.

Evaluate a Power by Doing the Multiplication

Evaluating a power means working out its value. Just write it out in its **expanded form** — then treat it as any other multiplication calculation.

Example 2

Evaluate 5^4.

Solution

5^4 means "four copies of the number five multiplied together."
5^4 = 5 • 5 • 5 • 5
5^4 = 625.

2 Teach (cont)

Universal access

You could introduce the idea of powers of 2 with a doubling exercise.

Put students into small groups. Give each group some grains of rice and an empty 6-egg cartons. Have them put 2 grains in the first compartment of the carton.

Ask them to fill the carton by putting double the number of grains in each compartment than were in the previous one. The compartments should contain 2, 4, 8, 16, 32, and 64 grains respectively.

Have them write down how many grains are in each compartment of the carton using only the number 2 and the multiplication symbol.

Then ask them what pattern the numbers follow. Challenge them to write the number of grains in each compartment as a power of 2.

Guided practice

Level 1: q1–4
Level 2: q1–6
Level 3: q1–8

Concept question

"Do the expressions 2^3 and 3^2 have the same value?"

No — 2^3 = 2 × 2 × 2 = 8, 3^2 = 3 × 3 = 9

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Give students this puzzle: "A man who pleased a king by playing chess with him was offered a choice of two rewards. He could have 10 million gold coins at once, or he could have gold coins placed on the squares of his chessboard: 2 on square 1, 4 on square 2, 8 on square 3, etc., so that each square had twice as many as the one before until all 64 were covered." Have them work out which reward is greater, and how many coins will be on square 64. **The second reward is greater. It is a doubling situation — the number of coins on a square is 2 raised to the power of the square number (2^1, 2^2, 2^3 etc up to 2^{64}). So the number of coins on square 64 = 2^{64}, which on its own is much larger than 10 million.**

2 Teach (cont)

Concept question

"If you evaluated the power $(-40)^{28}$, would the answer be positive or negative?"

Positive — the base is raised to an even power.

Guided practice

Level 1: q9–12
Level 2: q9–14
Level 3: q9–16

Independent practice

Level 1: 1–3, 7–10
Level 2: q1–13
Level 3: q1–14

Additional questions

Level 1: p440 q1–6, 10–12, 14–17
Level 2: p440 q1–17
Level 3: p440 q1–17

3 Homework

Homework Book — Lesson 2.5.1

Level 1: q1–7
Level 2: q1–9
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
- Worksheet 13 — Powers

Check it out:

A negative number raised to an odd power always gives a negative answer. For example:
$(-2)^3 = -2 \cdot -2 \cdot -2 = -8$

A negative number raised to an even power always gives a positive answer. For example:
$(-2)^4 = -2 \cdot -2 \cdot -2 \cdot -2$
$= (-2 \cdot -2) \cdot (-2 \cdot -2)$
$= 4 \cdot 4 = 16$

Example 3

Evaluate $(-2)^2$.

Solution

$(-2)^2$ means "two copies of the number negative two multiplied together."
$(-2)^2 = -2 \cdot -2$
$(-2)^2 = 4$.

Guided Practice

Evaluate the exponential expressions in Exercises 9–16.

9. 10^2 **100**
10. 5^3 **125**
11. 7^1 **7**
12. 3^6 **729**
13. 47^1 **47**
14. $(-15)^1$ **–15**
15. $(-3)^2$ **9**
16. $(-4)^3$ **–64**

Independent Practice

Write each of the expressions in Exercises 1–6 in base and exponent form.

1. $4 \cdot 4 \cdot 4$ **4^3**
2. $9 \cdot 9$ **9^2**
3. 8 **8^1**
4. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ **5^6**
5. $-4 \cdot -4 \cdot -4$ **$(-4)^3$**
6. $-3 \cdot -3 \cdot -3 \cdot -3$ **$(-3)^4$**

7. Kiera and 11 of her friends are handing out fliers for a school fund-raiser. Each person hands out fliers to 12 people. How many people receive a flier? **$12^2 = 144$**

Evaluate the exponential expressions in Exercises 8–13.

8. 15^2 **225**
9. 4^3 **64**
10. 8^1 **8**
11. 1^8 **1**
12. $(-5)^3$ **–125**
13. $(-5)^4$ **625**

14. A single yeast cell is placed on a nutrient medium. This cell will divide into two cells after one hour. These two cells will then divide to form four cells after another hour. The process continues indefinitely.
a) How many yeast cells will be present after 1 hour, 2 hours, and 6 hours? **1 hour = 2 cells, 2 hours = 4 cells, 6 hours = 64 cells**
b) Write exponential expressions with two as the base to describe the number of yeast cells that will be present after 1 hour, 2 hours, and 6 hours. **1 hour = 2^1 cells, 2 hours = 2^2 cells, 6 hours = 2^6 cells**
c) How many hours will it take for the yeast population to reach 256? **8 hours.**

Now try these:

Lesson 2.5.1 additional questions — p440

Round Up

If you need to use a repeated multiplication, it's useful to have a shorter way of writing it. That's why bases and exponents come in really handy when you're writing out powers of numbers. You'll see lots of powers used in expressions, equations, and formulas. For example, the formula for the area of a circle is πr^2 where r is the radius. So it's important you know what they mean.

Solutions
For worked solutions see the Solution Guide

Lesson 2.5.2

Powers of Rational Numbers

In this lesson students learn how to raise fractions to powers by applying the exponent to both the numerator and denominator. They also practice raising decimals to powers by converting them to fractions first.

Previous Study: In grade 5 students learned how to multiply decimals and fractions. In the previous Lesson they learned how to raise whole numbers to powers.

Future Study: In Algebra I students will be expected to know and use all the exponent rules. They will apply the exponent rules to all rational numbers.

Lesson 2.5.2

Powers of Rational Numbers

California Standards:

Number Sense 1.2

Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and **take positive rational numbers to whole-number powers**.

Algebra and Functions 2.1

Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

What it means for you:

You'll learn how to take fractions and decimals to powers.

Key words:

- power
- exponent
- base
- decimal
- fraction

In just the same way that you can raise whole numbers to powers, you can also raise fractions and decimals to powers.

You Can Raise a Fraction to a Power

A **fraction raised to a power** means exactly the same as a whole number raised to a power — repeated multiplication. But now the **complete fraction** is the **base**.

$$\underbrace{\left(\frac{2}{3}\right)}_{\text{base}}{}^{\overbrace{2}^{\text{exponent}}}$$

This expression means $\frac{2}{3} \cdot \frac{2}{3}$.

When you raise a fraction to a power, you are raising the **numerator** and the **denominator separately** to the **same power**. For example:

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$$

This makes **evaluating** the fraction easier. You can **evaluate** the **numerator** and the **denominator separately**.

Example 1

Evaluate $\left(\frac{1}{4}\right)^3$.

Solution

$$\left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3}$$

$1^3 = 1 \cdot 1 \cdot 1 = 1$

$4^3 = 4 \cdot 4 \cdot 4 = 64$

Raise both the numerator and the denominator to the third power.

$$\frac{1^3}{4^3} = \frac{1}{64}$$

Example 2

Evaluate $\left(-\frac{2}{5}\right)^2$.

Solution

$$\left(-\frac{2}{5}\right)^2 = \frac{(-2)^2}{5^2}$$

$(-2)^2 = -2 \cdot -2 = 4$

$5^2 = 5 \cdot 5 = 25$

Raise both the numerator and the denominator to the second power.

$$\frac{(-2)^2}{5^2} = \frac{4}{25}$$

Check it out:

If you are raising a negative fraction to a power, just keep the minus sign with the numerator all the way through your work.

For example:

$$\left(-\frac{3}{4}\right)^4 = \frac{(-3)^4}{4^4}$$

1 Get started

Resources:

- grid paper
- scissors
- card squares marked with 8 × 8 grids

Warm-up questions:

- Lesson 2.5.2 sheet

2 Teach

Common error

Students sometimes forget to raise both the numerator and denominator to the power and just raise the numerator — for example:

$$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4}$$

Remind them to apply the power to **both** the numerator **and** denominator.

Give an example showing the right method, and what happens if you use the wrong method, using an integer written as a fraction.

For example: $\frac{4}{2} = 2$.

$2^2 = 2 \times 2 = 4$

$$\left(\frac{4}{2}\right)^2 = \frac{4^2}{2^2} = \frac{4 \times 4}{2 \times 2} = \frac{16}{4} = 4$$

$$\frac{4^2}{2} = \frac{16}{2} = 8$$

Universal access

It's often useful for students to write out fractions raised to powers as repeated multiplications. For example:

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{2^3}{3^3} = \frac{8}{27}$$

Remind them that when they multiply two proper fractions, they are finding a part of a part. So the product will be smaller than the original fractions.

- **Strategic Learners**
  Give everyone scissors and a card square marked with an 8 × 8 grid. Have them cut the square in half. Ask what fraction of the square each part is. Then have them halve the $\frac{1}{2}$. Ask what fraction of the square each new small part is. Repeat until the smallest part is 1 unit, or $\frac{1}{64}$ of the square. Have students write the fractions as powers of $\frac{1}{2}$. Point out that the denominators are a series they know — powers of 2.
- **English Language Learners**
  Do a brief review of the vocabulary associated with the Lesson. Cover rational numbers, proper and improper fractions, decimals, powers, bases, and exponents. Have each student write down their own definition of each of these in their notebooks, along with their own examples and notes.

2 Teach (cont)

Concept question

"If I raise a positive proper fraction to a positive whole number power, will my answer be bigger or smaller than the base?"

Smaller

Guided practice

Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Universal access

The other way of raising a decimal to a power is to:

1) Drop the decimal point.
2) Raise the resulting integer to the given power.
3) Move the decimal point backwards the same number of decimal points that were in all the factors together.

For example: $0.12 \times 0.12 = (0.12)^2$
1) 012^2
2) $12^2 = 144$
3) The factors (0.12 × 0.12) have a total of 4 decimal places in them (two in each factor). So the answer is 0.0144.

Math background

The method described in the Universal access box above works because you can rewrite the expression using powers of 10:

$$\begin{aligned} 0.12 \times 0.12 &= 12 \times 10^{-2} \times 12 \times 10^{-2} \\ &= 12 \times 12 \times 10^{-2} \times 10^{-2} \\ &= 12^2 \times 10^{-4} \\ &= 144 \times 10^{-4} = 0.0144 \end{aligned}$$

Guided Practice

Evaluate the exponential expressions in Exercises 1–6.

1. $\left(\frac{1}{2}\right)^2$ $\frac{1}{4}$

2. $\left(\frac{1}{2}\right)^4$ $\frac{1}{16}$

3. $\left(\frac{5}{3}\right)^1$ $\frac{5}{3}$

4. $\left(\frac{9}{7}\right)^3$ $\frac{729}{343}$

5. $\left(-\frac{3}{10}\right)^2$ $\frac{9}{100}$

6. $\left(-\frac{3}{10}\right)^3$ $-\frac{27}{1000}$

You Can Raise a Decimal to a Power

A **decimal** raised to a power means exactly the same as a whole number raised to a power — it's a repeated multiplication. The decimal is the **base**.

exponent
base $\rightarrow 0.24^2$

This expression is the same as saying 0.24 • 0.24.

When you evaluate a decimal raised to a power, you **multiply the decimal by itself** the specified number of times. The tricky thing when you're multiplying decimals is to get the decimal point in the right place — you saw how to do this in Section 2.4.

Example 3

Evaluate $(0.3)^3$.

Solution

The multiplication you are doing here is $(0.3)^3 = 0.3 \cdot 0.3 \cdot 0.3$.

$$0.3 \times 0.3 \times 0.3 = \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} \quad \text{write the decimals as fractions}$$

$$= \frac{3 \times 3 \times 3}{10 \times 10 \times 10} \quad \text{multiply the fractions}$$

$$= \frac{27}{1000} = 27 \div 1000 = 0.027$$

So, $(0.3)^3 = 0.027$

Check it out:

To check the number of decimal places your answer should have, just count the number of decimal places in all of the factors and add them together. For example, in the expression 0.3 • 0.3 • 0.3, you have three factors, and each contains one decimal place. So your answer should contain three decimal places, which 0.027 does.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
On the board write the numbers $(-10)^0$, $(-10)^1$, $(-10)^2$, $-(10)^3$, $(-10)^4$, $(-10)^5$. Have the students evaluate the powers. Then ask them to look at their answers and see what patterns exist in the differences between the numbers in the series. Ask students to investigate the patterns in the differences between other series of powers with negative bases, such as $(-3)^0$, $(-3)^1$, $(-3)^2$, $(-3)^3$, $(-3)^4$, $(-3)^5$. Ask them if they can come up with some general rules to describe any patterns they find.
The first difference is always the base number less 1 (so, –11 in the first example). Each subsequent difference is multiplied by the base number (so the differences in the first example are –11, 110, –1100, 11,000).

Check it out:
When you put the decimal point back into your answer, put in 0s to fill up any places between the decimal point and the numerical part of the answer.
For example, here the numerical part of your answer is 529, but you need four decimal places in your answer. So use a 0 before the numerical part — so the answer is 0.0529.

Example 4

Evaluate $(0.23)^2$.

Solution

$0.23 \times 0.23 = \frac{23}{100} \times \frac{23}{100}$ **write the decimals as fractions**

$= \frac{529}{10{,}000} = 529 \div 10{,}000 = \mathbf{0.0529}$ **multiply**

Guided Practice

Evaluate the exponential expressions in Exercises 7–12.

7. $(0.5)^2$ **0.25**
8. $(0.2)^3$ **0.008**
9. $(0.78)^1$ **0.78**
10. $(0.12)^2$ **0.0144**
11. $(0.15)^3$ **0.003375**
12. $(0.08)^2$ **0.0064**

Independent Practice

Write each of the expressions in Exercises 1–4 in base and exponent form.

1. $\frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2}$ $\left(\frac{1}{2}\right)^3$
2. $0.25 \bullet 0.25$ $(0.25)^2$
3. $\frac{1}{7} \bullet \frac{1}{7} \bullet \frac{1}{7} \bullet \frac{1}{7}$ $\left(\frac{1}{7}\right)^4$
4. $-\frac{1}{4} \bullet -\frac{1}{4} \bullet -\frac{1}{4}$ $\left(-\frac{1}{4}\right)^3$

Evaluate the exponential expressions in Exercises 5–10.

5. $\left(\frac{1}{9}\right)^2$ $\frac{1}{81}$
6. $\left(\frac{1}{4}\right)^3$ $\frac{1}{64}$
7. $\left(\frac{2}{3}\right)^3$ $\frac{8}{27}$
8. $\left(\frac{9}{8}\right)^1$ $\frac{9}{8}$
9. $\left(\frac{1}{5}\right)^5$ $\frac{1}{3125}$
10. $\left(-\frac{2}{5}\right)^3$ $-\frac{8}{125}$

11. Mark is feeding chickens. He divides 135 g of corn into thirds. Each portion is then divided into thirds again to give small portions. What fraction of the original amount is in each small portion? How much does each small portion weigh? $\frac{1}{9}$, **15 g.**

Evaluate the exponential expressions in Exercises 12–17.

12. $(0.4)^2$ **0.16**
13. $(0.1)^4$ **0.0001**
14. $(0.21)^2$ **0.0441**
15. $(0.97)^1$ **0.97**
16. $(0.02)^2$ **0.0004**
17. $(0.25)^3$ **0.015625**

Now try these:
Lesson 2.5.2 additional questions — p440

Round Up

To raise a fraction to a power, you raise the numerator and the denominator separately to the same power. To raise a decimal to a power, you use the decimal as the base and raise it to a power as you would a whole number — just by multiplying it by itself the correct number of times.

2 Teach (cont)

Concept question
"If I raise the decimal 0.01 to the power 12, how many decimal places will my answer have?"
24

Guided practice
Level 1: q7–8
Level 2: q7–10
Level 3: q7–12

Independent practice
Level 1: q1–2, 5–6, 12–13
Level 2: q1–3, 5–8, 11–15
Level 3: q1–17

Additional questions
Level 1: p440 q1–3, 6, 10–11
Level 2: p440 q1–6, 10–12, 13
Level 3: p440 q4–15

3 Homework

Homework Book — Lesson 2.5.2
Level 1: q1–4, 7
Level 2: q1–9
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 13 — Powers

Solutions
For worked solutions see the Solution Guide

Lesson 2.5.3

Uses of Powers

This Lesson links the second and third powers to finding areas and volumes. Another important real-life use of powers is then covered — using scientific notation to write large numbers efficiently.

Previous Study: In grade 4 students learned how to find areas of squares and rectangles, and in grade 5 volumes of cubes and cuboids, so they are familiar with square and cubic units.

Future Study: Later in grade 7 students will find areas of polygons and volumes of polyhedra. In Chapter 5 they will develop an understanding of negative powers and use them in representing very small numbers.

1 Get started

Resources:

- grid paper (with 1 cm × 1 cm squares)
- rulers
- cards with scientific notation expressions on them (see page 114)
- internet-enabled computers
- tracing paper squares of different sizes (such as 2 cm × 2 cm, 3 cm × 3 cm, 5 cm × 5 cm)
- centimeter cubes

Warm-up questions:

- Lesson 2.5.3 sheet

2 Teach

Universal access

This activity emphasizes the link between the area of a square and its side length to the power of 2.

Give students a selection of tracing paper squares with side lengths a whole number of centimeters. They should measure the side of each square and raise it to the power of 2. They should record this in a table, like the one below:

side length (cm)	(side length)^2 (cm^2)	Number of small squares (area)
2	$2^2 = 4$	4

They should then lay the tracing paper square over grid paper (with 1 cm × 1 cm squares) and count the number of squares it covers. They should put this number in the last column of the table.

Additional examples

Find the areas of the squares with side lengths:

1) 1 cm **1 cm²**
2) 5 yards **25 yards²**
3) 10 units **100 units²**

Lesson 2.5.3

Uses of Powers

California Standards:

Number Sense 1.1
Read, write, and compare rational numbers in scientific notation (positive and negative **powers of 10),** compare rational numbers in general.

Number Sense 1.2
Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and **take positive rational numbers to whole-number powers.**

What it means for you:
You'll see how you can use exponents to work out areas of squares and volumes of cubes, and learn about a shorter way to write very large numbers.

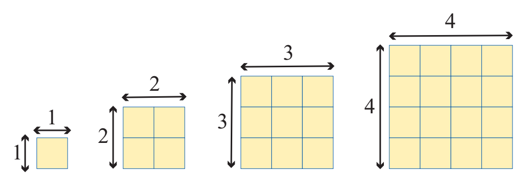
Key words:
- squared
- cubed
- scientific notation

*You'll come across powers a lot both in math and real-life situations. That's because you use them to work out **areas** and **volumes**. They're also handy when you need to write out a very **big number** — you can use powers to write these numbers in a **shorter form**.*

Exponents are Used in Some Formulas

Exponents are used in the **formulas** for the **areas** of **squares and circles**. In this Lesson you'll see how **exponents** are used in finding the **area** of a **square**. In the next Chapter you'll use a formula to find the area of a circle.

The formula for the area of a square is **Area** $= s \cdot s = s^2$, where s represents the **side length** of the **square**.



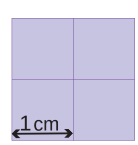
$1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 16$

When you find the area of a square, the side length is used as a **factor twice** in the multiplication. So raising a number to the **second power** is called **squaring** it.

Check it out:
The areas of all shapes, not just squares, are measured in square units. These could be cm², m², inches², feet², or even miles².
You'll learn more about this in Chapter 3.

Example 1

Find the area of the square shown below.



Solution

Each small square is **1 cm wide**. So the side length of the whole square is **2 cm**. The **area** of the whole square is **2 cm • 2 cm = 4 cm²**.

You can see that this is true, because it is made up of **four** smaller **1 cm² squares**.

- **Strategic Learners**
  The Universal access activity on the previous page is particularly suitable for helping strategic learners develop an understanding of area. The Universal access activity on this page helps them to develop an understanding of volume formulas in a similar way.

- **English Language Learners**
  Review the terms "area" and "volume." For area, have students count the number of grid squares inside a shape, and also apply the formulas for the areas of squares and rectangles. For volume, have students fill boxes with unit cubes. Review with them the units that can be used for area and volume — area units are squared lengths, volume units are cubed lengths.

Don't forget:
When you're working out the units that go with a calculation, use the unit analysis method you saw in Chapter 1 (p42). Just apply the same operations to the units as you did to the numbers.

Example 2

A square has a side length of 11 inches. Find its area.

Solution

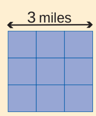
Area = (side length)^2
Area = $11^2 = 11 \cdot 11 = 121$

Units: inches • inches = in^2

Area = 121 in^2

Guided Practice

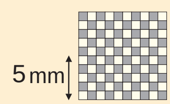
Find the areas of the squares in Exercises 1–4.

1. 

9 miles^2

2. Square of side length 6 feet. 36 feet^2

3. Square of side length 3.5 m. 12.25 m^2

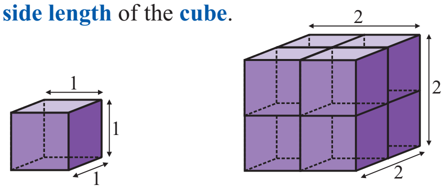
4. 

100 mm^2

Exponents are Used to Find Volumes of Some Solids

Exponents are also used in **formulas** to work out **volumes** of **solids**, like cubes, spheres, and prisms.

The formula for the volume of a cube is **Volume** $= s \cdot s \cdot s = s^3$, where s represents the **side length** of the **cube**.



$1^3 = 1$ $2^3 = 8$

When you find the volume of a cube, the side length is used as a **factor three times** in the multiplication. So raising a number to the **third power** is called **cubing** it.

Check it out:
The volumes of all solids, not just cubes, are measured in cubed units. For example, the units could be cm^3, m^3, inches^3, feet^3, or even miles^3. You'll learn more about this in Chapter 7.

Example 3

A cube has a side length of 5 cm. Find its volume.

Solution

Volume = (side length)^3
Volume = $5 \cdot 5 \cdot 5 = 5^3 = 125$

Units: cm • cm • cm = cm^3

Volume = 125 cm^3.

2 Teach (cont)

Concept question
"There is a unit of length called a rod. If I measure the side length of a square field in rods, then use my measurement to find the field's area, what will the units of the area be?"
rods^2

Guided practice
Level 1: q1–2
Level 2: q1–3
Level 3: q1–4

Universal access
In pairs, ask students to build different-size cubes using cm cubes. They should measure the side lengths of their cubes and raise them to the power of 3. They should then link this to the number of cm cubes that each large cube is built from. They can record their findings in a table similar to the one on the previous page.

Concept question
"Why does the formula Volume = s^3 work for cubes?"

The length, width, and height of a cube are all equal. The base of a cube is made up of s square units. This is multiplied by the height, s units, to produce the volume (s cubed units).

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Logarithms are a commonly used way of expressing powers that students will encounter in later grades. Put students into small groups, and have them use the internet to research what logarithms are and how they work. Then get them to turn their research into a group poster. Ask them to include a section on one real-life use of logarithms — for example, the pH scale used for describing acidity, the decibel system used for expressing the loudness of a sound, or the Richter scale, which describes the magnitude of an earthquake.

2 Teach (cont)

Guided practice
Level 1: q5–6
Level 2: q5–7
Level 3: q5–8

Universal access
Cut small rectangles out of two different colors of card.

On one color, write large numbers. On the other color write their equivalents in scientific notation. For example:

41,000	4.1×10^4
29,000,000	2.9×10^7

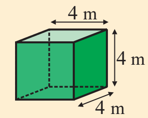
Make sure that there is one card for each student. Give a card to everyone and ask each student to find the person whose card represents the same number as theirs.

Math background
Scientific notation is also used to express very small numbers, such as the weight of an atom, or the lengths of some electromagnetic waves. This requires the use of negative exponents and is covered in Chapter 5.

Universal access
Have students suggest examples of times when you might use very big numbers — for example, when expressing the distances between planets, or very high speeds, such as the speed of light.

Guided Practice

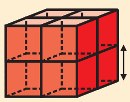
Find the volumes of the cubes in Exercises 5–8.

5. 

$64\ m^3$

6. Cube of side length 5 feet. $125\ feet^3$

7. Cube of side length 7.5 mm. $421.875\ mm^3$

8. 

$8\ in^3$

Use Scientific Notation to Write Big Numbers

Sometimes in math and science you'll need to deal with numbers that are **very big**, like 570,000,000. To avoid having to write numbers like this out in full every time, you can rewrite them as a **product of two factors**.

For instance: $570{,}000{,}000 = 5.7 \times 100{,}000{,}000$

The second factor is a **power of ten**. You can write it in **base and exponent** form.

$5.7 \times 100{,}000{,}000 = 5.7 \times 10^8$.

So 5.7×10^8 is 570,000,000 written in **scientific notation**.

Check it out:
To work out what power of ten the second factor is, just count the zeros in it.
For example, 10 is 10^1, 1000 is 10^3, and 10,000,000 is 10^7.

Scientific Notation
To write a number in scientific notation turn it into two factors:
→ the first factor must be a number that's at least one but less than ten.
→ the second factor must be a power of 10 written in exponent form.

Example 4

Write the number 128,000,000,000 in scientific notation.

Solution

$128{,}000{,}000{,}000 = 1.28 \times 100{,}000{,}000{,}000$ Split the number into a decimal between 1 and 10 and a power of ten.

$= \mathbf{1.28 \times 10^{11}}$ Write the number as a product of the two factors.

Solutions
For worked solutions see the Solution Guide

Check it out:

If the number you were putting into scientific notation was 51,473,582, then you would probably round it before putting it into scientific notation. You'll see more about how to round numbers in Chapter 8.

Example 5

The number 5.1×10^7 is written in scientific notation. Write it out in full.

Solution

$5.1 \times 10^7 = 5.1 \times 10{,}000{,}000$

$= 51{,}000{,}000$

Write out the power of ten as a factor in full.

Multiply the two together: move the decimal point as many places to the right as there are zeros in the power of ten.

Guided Practice

Write the numbers in Exercises 9–12 in scientific notation.

9. 6,700,000 6.7×10^6
10. 32,800 3.28×10^4
11. –270,000 -2.7×10^5
12. 1,040,000,000 1.04×10^9

Write out the numbers in Exercises 13–16 in full.

13. 3.1×10^3 3100
14. 8.14×10^6 8,140,000
15. -5.05×10^7 –50,500,000
16. 9.091×10^9 9,091,000,000

Independent Practice

Find the areas of the squares in Exercises 1–4.

1. Square of side length 2 cm. 4 cm^2
2. Square of side length 8 yd. 64 yd^2
3. Square of side length 13 m. 169 m^2
4. Square of side length 5.5 ft. 30.25 ft^2

5. Maria is painting a wall that is 8 feet high and 8 feet wide. She has to apply two coats of paint. Each paint can will cover 32 $feet^2$. How many cans of paint does she need? 4 cans

Find the volumes of the cubes in Exercises 6–9.

6. Cube of side length 3 ft. 27 ft^3
7. Cube of side length 6 yd. 216 yd^3
8. Cube of side length 9 cm. 729 cm^3
9. Cube of side length 1.5 in. 3.375 in^3

10. Tyreese is tidying up his little sister's toys. Her building blocks are small cubes, each with a side length of 3 cm. They completely fill a storage box that is a cube with a side length of 15 cm. How many blocks does Tyreese's sister have? 125 blocks

Write the numbers in Exercises 11–14 in scientific notation.

11. 21,000 2.1×10^4
12. –51,900,000 -5.19×10^7
13. 42,820,000 4.282×10^7
14. 31,420,000,000,000 3.142×10^{13}

Write out the numbers in Exercises 15–18 in full.

15. 8.4×10^5 840,000
16. 2.05×10^8 205,000,000
17. -9.1×10^4 –91,000
18. 3.0146×10^{10} 30,146,000,000

19. In 2006 the population of the USA was approximately 299,000,000. Of those 152,000,000 were female. How many were male? Write your answer in scientific notation. 1.47×10^8

Now try these:

Lesson 2.5.3 additional questions — p441

Round Up

When you're finding the area of a square or the volume of a cube, your calculation will always involve powers. That's because the formulas for both the area of a square and the volume of a cube involve repeated multiplication of the side length. Powers also come in useful for writing very large numbers in a shorter form — that's what scientific notation is for.

2 Teach (cont)

Math background

Scientific notation requires students to have a working knowledge of multiplying by powers of 10.

Guided practice

Level 1: q9–10, 13–14
Level 2: q9–11, 13–15
Level 3: q9–16

Independent practice

Level 1: q1–4, 6–10, 11–12, 15–16
Level 2: q1–18
Level 3: q1–19

Additional questions

Level 1: p441 q1–6, 9–10
Level 2: p441 q1–11
Level 3: p441 q1–11

3 Homework

Homework Book — Lesson 2.5.3

Level 1: q1, 2a–b, 3–7
Level 2: q1–9
Level 3: q2–11

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 13 — Powers

Solutions

For worked solutions see the Solution Guide

Lesson 2.5.4

More on the Order of Operations

This Lesson reviews the order of operations. This time students evaluate expressions that also include rational numbers raised to positive whole number powers.

Previous Study: At grade 6, and earlier in grade 7, students learned to apply the order of operations to evaluate expressions containing the four operations and parentheses.

Future Study: In Algebra I, students will solve multistep problems, including equations and inequalities derived from word problems, and be able to justify each step of their work.

1 Get started

Warm-up questions:
- Lesson 2.5.4 sheet

2 Teach

Universal access

Play the number target game. Write a target number and some other integers on the board. Students have to use the integers, any of the four operations, and any grouping symbols they like to make the target number. Each integer can be used only once. Accept any valid solution, as long as it follows the order of operations.

For example: Target number: 8; Integers: 2, 2, 6
Example solution: $2 \times (6 - 2) = 8$

Whoever comes up with a valid solution first gets 3 points, but they must show you all of their work. Everyone who finds a valid solution gets 1 point — students can check one another's work here.

Once you have played a couple of rounds, introduce a twist: give a few integers and one power, written in base and exponent form.

For example: Target number: 36; Integers/powers: 5, 4, 2^3
Example solution: $2^3 \times 5 - 4 = 36$

Common error

Students working with the PEMDAS mnemonic sometimes forget that Multiplication and Division, and Addition and Subtraction, have equal priority, and must be done from left to right.

To help with this they could be introduced to GEMA (see Lesson 1.1.3).

Guided practice

Level 1: q1–2
Level 2: q1–4
Level 3: q1–6

Lesson 2.5.4

More on the Order of Operations

California Standards:
Number Sense 1.2
Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

What it means for you:
You'll learn how to use the PEMDAS rules with expressions that have decimals, fractions, and exponents.

Key words:
- PEMDAS
- operations
- exponent

In Chapter One you saw how the order of operations rules help you to figure out which operation you need to do first in a calculation. This Lesson will review what the order is, and give you practice at applying it to expressions with exponents in them.

PEMDAS Tells You What Order to Follow

When you come across an **expression** that contains **multiple operations**, the **PEMDAS rule** will help you to work out which one to do **first**. For example:

Parentheses	$4 + 6 \cdot (2 + 4)^2 - 10 \div 2$
Exponents	$= 4 + 6 \cdot (6)^2 - 10 \div 2$
Multiplication and Division	$= 4 + 6 \cdot 36 - 10 \div 2$
Addition and Subtraction	$= 4 + 216 - 5$
	$= 215$

Multiplication and division have **equal priority** in PEMDAS. You work them out from **left to right**.

Addition and subtraction have **equal priority** too — work them out from **left to right**.

Check it out:
If you see parentheses with an exponent, the exponent applies to the whole expression inside them.
So $(1 + 2)^2$ is $(1 + 2) \cdot (1 + 2)$.
Everything in the parentheses is the repeated factor in the multiplication. So to follow the PEMDAS rules you need to simplify the contents of the parentheses first, and then apply the exponent to the result.
So $(1 + 2)^2 = 3^2$

Example 1

Evaluate the expression $5^2 - 16 \div 2^3 \cdot (3 + 2)$.

Solution

$5^2 - 16 \div 2^3 \cdot (3 + 2)$	
$= 5^2 - 16 \div 2^3 \cdot 5$	**Do the addition in the parentheses**
$= 25 - 16 \div 8 \cdot 5$	**Then evaluate the two exponents**
$= 25 - 2 \cdot 5$	**Next it's multiplication and division — do the division first, as it comes first, then do the multiplication**
$= 25 - 10$	
$= 15$	**Finally do the subtraction**

Guided Practice

Evaluate the expressions in Exercises 1–6.

1. $6 - 10 \cdot 3^2$ –84
2. $(5 - 3)^3 + 4^3 \div 8$ 16
3. $2^4 + (3 \cdot 2 - 10)^2$ 32
4. $5 + 6^4 \div (6 - 2)^1$ 329
5. $(36 \div 12 - 2^4)^2$ 169
6. $(10 \cdot 2 - 5)^2 - (4 \div 2)^3 \cdot 3$ 201

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Have students write expressions that use all parts of the PEMDAS or GEMA mnemonic, and a fraction or decimal. Then have everyone swap problems. Ask them to write the relevant part of the mnemonic above each bit of the expression, like the example shown on the right. They should then evaluate it using the funnel method (see Universal access, below).

E A M G M
E AS MD P MD
$0.1^2 + \frac{1}{2} \times (5 - 2) \div 2$

- **English Language Learners**
  Revisit the vocabulary associated with the order of operations (parentheses, grouping symbols, exponents, multiplication, division, addition, subtraction, PEMDAS, and GEMA). Write a complex expression on the board, for example, $\frac{3^2 \times (14 - 6) + 2}{13 - 3 \times 2^3}$. Have everyone work with a partner to identify the various operations in the problem, and the correct order in which to do them. Solve and check solutions.

Check it out:

- When you multiply a negative number by a negative number, the result is positive.
- When you multiply a positive number and a negative number, the result is negative.
- So if you raise a negative number to an even power, the result will be positive.
- But if you raise a negative number to an odd power, the result will be negative.
- For example:

$(-2)^3 = -2 \cdot -2 \cdot -2$
$= 4 \cdot -2 = -8$

$(-2)^4 = -2 \cdot -2 \cdot -2 \cdot -2$
$= 4 \cdot 4 = 16$

Check it out:

If you have parentheses inside parentheses, for example, (3 + (4 + 2)), you should start with the innermost parentheses and work outward.

Take Care with Expressions That Have Negative Signs

When an expression contains a combination of negative numbers and exponents, you need to think carefully about what it means. For example:

$$-(2^2) = -(2 \cdot 2) = -4$$

$$(-2)^2 = -2 \cdot -2 = 4$$

Example 2

Evaluate the expression $(-(3^2) \cdot 5) + (-3)^2$.

Solution

$(-(3^2) \cdot 5) + (-3)^2$
$= (-9 \cdot 5) + (-3)^2$ **Evaluate the exponent in the inner parentheses**
$= -45 + (-3)^2$ **Do the multiplication in the parentheses**
$= -45 + 9$ **Evaluate the exponent**
$= -36$ **Finally do the addition**

Guided Practice

Evaluate the expressions in Exercises 7–12.

7. $-(4^2)$ **–16**
8. $(-4)^2$ **16**
9. $-(2^2) \cdot 5 + 1$ **–19**
10. $(-4)^2 \div 2 - 4$ **4**
11. $10 + (2 \cdot -(5^2)) + (-7)^2$ **9**
12. $12 + (-(2^2) + (-2)^2) \div 2$ **12**

The Order Applies to Decimals and Fractions Too

When you're working out a problem involving decimals or fractions you follow the same order of operations.

Example 3

Evaluate the expression $\left(\frac{1}{2}\right)^4 + \frac{1}{16} \cdot (10 - 7)^2$.

Solution

$\left(\frac{1}{2}\right)^4 + \frac{1}{16} \cdot (10 - 7)^2$

$= \left(\frac{1}{2}\right)^4 + \frac{1}{16} \cdot 3^2$ **Do the subtraction in the parentheses**

$= \frac{1}{16} + \frac{1}{16} \cdot 9$ **Then evaluate the two exponents**

$= \frac{1}{16} + \frac{9}{16}$ **Perform the multiplication**

$= \frac{10}{16} = \frac{5}{8}$ **Finally do the addition and simplify**

2 Teach (cont)

Concept questions

"Is $-(202^{56})$ negative or positive?"

Negative. The negative sign is applied to the expression in the parentheses, which is positive.

"Is $(-202)^{56}$ negative or positive?"

Positive — the exponent is even.

"Is $(-202)^{57}$ negative or positive?"

Negative — the base is negative and the exponent is odd.

Universal access

Remind students to use the "funnel method" when evaluating complex expressions. Each step is written out vertically until the expression is funneled down to one result. For example:

$(-(3^2) \cdot 5) + (-3)^2$
$= (-9 \cdot 5) + (-3)^2$
$= -45 + (-3)^2$
$= -45 + 9$
$= -36$

Guided practice

Level 1: q7–8
Level 2: q7–10
Level 3: q7–12

Universal access

Play the number target game again (see previous page).

Play the game in the same way as before, only this time include some fractions and decimals, both as numbers to use and as targets.

For example: Target number: 0.5
Numbers: 2, 5, 8
Example solution: 5 ÷ (8 + 2) = 0.5

Once again, after a couple of rounds begin to include some simple fractions and decimals raised to powers.

For example: Target number: 0

Numbers: 1, 4, $\left(\frac{1}{2}\right)^2$

Example solution: $\left(\frac{1}{2}\right)^2 \times 4 - 1 = 0$

Solutions

For worked solutions see the Solution Guide

Advanced Learners
The Universal access "target number" activity on the previous page can be made more challenging by including more fractions, decimals, and powers. The exponents can also be given separately from the base numbers, so that students have to experiment with different combinations. Students can make up their own problems and share them with other students.

2 Teach (cont)

Additional example

Evaluate the expression

$\left(\frac{1}{2}+\frac{1}{2}\right)\times\frac{1}{2}-\left(\frac{1}{2}\right)^2$ $\frac{1}{4}$

Guided practice

Level 1: q13–16
Level 2: q13–18
Level 3: q13–20

Independent practice

Level 1: q1–2, 8–9
Level 2: q1–4, 7–11
Level 3: q1–14

Additional questions

Level 1: p441 q1–5, 13–14, 16–18
Level 2: p441 q1–10, 13–18
Level 3: p441 q1–18

3 Homework

Homework Book — Lesson 2.5.4

Level 1: q1, 3, 4, 5a–b, 6a–b, 8
Level 2: q2–12
Level 3: q2–12

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:

- Worksheet 13 — Powers
- Worksheet 21 — Order of Operations

Example 4

Evaluate the expression $0.25 + 7.75 \div 3.1 - (0.3)^4$.

Solution

$0.25 + 7.75 \div 3.1 - (0.3)^4$	
$= 0.25 + 7.75 \div 3.1 - 0.0081$	**Evaluate the exponent**
$= 0.25 + 2.5 - 0.0081$	**Then perform the division**
$= 2.75 - 0.0081$	**Do the addition first, as it comes first**
$= 2.7419$	**Finally do the subtraction**

Guided Practice

Evaluate the expressions in Exercises 13–20.

13. $\left(\frac{1}{2}+\frac{3}{2}\right)^3 - \frac{1}{8}$ $\frac{63}{8}$

14. $\left(\frac{1}{4}\right)^2 \bullet 3 + 4 \div \frac{1}{2} - 2$ $\frac{99}{16}$ or $6\frac{3}{16}$

15. $0.1 + (0.25)^2 - 0.2 \div 2$ **0.0625**

16. $(0.72 + 0.08) \div 16 + (0.4)^2$ **0.21**

17. $\left(\frac{3}{4}\div\frac{1}{2}\right)^2 + \left(\frac{2}{3}\bullet\frac{1}{2}\right)^3$ $\frac{247}{108}$

18. $0.5 \bullet (1 + 0.25)^2 + 1.2$ **1.98125**

19. $2 \bullet \left(\frac{1}{2}\right)^2 + (5 \div 10)^2 \bullet 4$ $\frac{3}{2}$

20. $(5 \bullet 0.1 + 0.2) \bullet \left(\frac{1}{5}\right)^2$ **0.028** or $\frac{7}{250}$

Don't forget:
If your calculation involves a mixture of fractions and decimals, convert everything to either fractions or decimals first.
For a reminder of how to do this, see Section 2.1.

Independent Practice

Evaluate the expressions in Exercises 1–6.

1. $\frac{12+2^3}{5}$ **4**

2. $(4^2 - 2^3) \div 2^2 + 8^1$ **10**

3. $(10 + 2^4 \bullet 3) + (5^2 - 15)^2$ **158**

4. $-3^3 \bullet 2^2 + 9$ **–99**

5. $(-6)^3 \bullet 3 - 12^2$ **–792**

6. $(4^3 - 3^4)^2 \div (17)^2$ **1**

7. In the expression $(x - y^2 \bullet z)^6$, x, y, and z stand for whole numbers. If you evaluate it, will the expression have a positive or a negative value? (The expression is not equal to zero.) Explain your answer. **see left**

Positive: when you raise any number, positive or negative, to an even power, the result is always positive.

Evaluate the expressions in Exercises 8–13.

8. $\left(\frac{1}{3}\right)^2 + 2 \bullet \frac{2}{27}$ $\frac{7}{27}$

9. $(0.5)^2 + 0.8 \div (0.1)^3$ **800.25**

10. $\left(\frac{2}{3}\div\frac{4}{5}\right)^2 + \left(\frac{1}{6}\right)^2 \bullet 4$ $\frac{29}{36}$

11. $(0.5 + 1.8)^2 \bullet 1.5 + 0.065$ **8**

12. $\left(0.5 \bullet \frac{6}{8}\right)^2 - \left(\frac{1}{4}\right)^3$ **0.125** or $\frac{1}{8}$

13. $(0.2 \bullet 4 - 0.3)^2 + \left(\frac{1}{2}\right)^3 \bullet 2$ **0.5** or $\frac{1}{2}$

14. Lakesha is making bread. She has $\frac{5}{4}$ lb of flour, which she splits into two equal piles. Then she splits each of these in half again. She adds three of the resulting piles to her mixture. How much flour has she added to her mixture? Give your answer as a fraction. $\frac{15}{16}$ **lb**

Now try these:
Lesson 2.5.4 additional questions — p441

Round Up

When you have an expression containing exponents, you must follow the order of operations to evaluate it. You use the same order with expressions that contain fractions and decimals too.

Solutions
For worked solutions see the Solution Guide

Exploration — The Side of a Square

Purpose of the Exploration

The goal of the Exploration is to have students see the meanings of perfect squares and square roots. Often students confuse a square root with dividing a number by two. This activity helps them see the connection between a square root and the lengths of the sides of a square.

Section 2.6 introduction — an exploration into: The Side of a Square

*A **perfect square** has sides whose lengths are whole numbers. You'll be given square tiles and be asked to construct larger squares with particular **areas** — you'll be able to produce some of the larger squares, but not others. The lengths of the sides of the squares are the **square roots** of the areas. You'll see that the areas of some squares have **whole number square roots**, but others don't.*

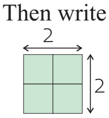
Each small square has an area of **1 square unit**.

Example

Make a square with an area of 4 square units. Then write down the square root of 4.

Solution

With 4 square tiles:



This a **perfect square** — it's got an area of **4 square units** and sides of **2 units**.
So, **2 is the square root of 4**.

Exercises

1. Use the tiles to make squares with the given areas.
   When you have made a square, write the lengths of the sides.
   a. 9 square units sides = 3 units **b.** 16 square units sides = 4 units **c.** 25 square units sides = 5 units **d.** 36 square units sides = 6 units
2. What are the square root of the following? Use your answers to Exercise 1 to help you.
   a. 9 3 **b.** 16 4 **c.** 25 5 **d.** 36 6

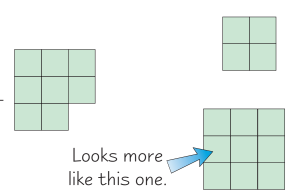
You can use the tiles to **estimate** the square root of a number that is **not** perfect square.

Example

Use tiles to estimate the square root of 8.

Solution

This is the **closest shape** you can make to a square using 8 tiles —
It's bigger than a 2 by 2 square, but smaller than a 3 by 3 square.
As it's closer to a 3 by 3 square, you can estimate that the square root of 8 is **about 3**.



Exercises

3. Construct a figure that is as close to a square as possible.
   Use this to estimate the square roots of these numbers.
   a. 5 **b.** 14 **c.** 22 see below

Round Up

*Some numbers are **perfect squares** — like 4, 9, 16, 25. These numbers have square roots that are **whole numbers**. If you make a square with a perfect square area, its sides will be whole numbers.*

Resources

• 40 square tiles per student (Tiles can be made from a heavy card stock or regular printer paper.)

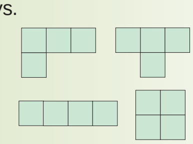
Strategic & EL Learners

Strategic learners may benefit from having a grid system laid out for them. With a grid system they can lay out the tiles in an ordered way. This will help them make squares more readily.

EL learners may have difficulty with the word "estimate." Inform these students that they are trying to make a good guess given the figure they have made.

Universal access

Start the activity by showing that the tiles can be arranged several different ways.



Remind students that a square has sides of equal length.
A rectangle is not considered a square.

Common error

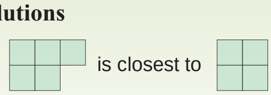
Students may encounter difficulty making a square when given a larger area. The areas that will not produce a square will be difficult, especially for the square root of 22 in Exercise 3c. Students will also make rectangles and believe they have found the square roots. Remind students that all the sides of a square are the same length. They could also be encouraged to first make a 2 × 2 square, then extend it to 3 × 3, then to 4 × 4, and so on, until they run out of tiles.

Math background

Students should understand the concept of area and the properties of a square. Students should also have an understanding of what it means to estimate.

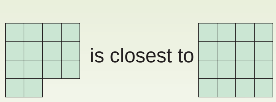
Solutions

a.



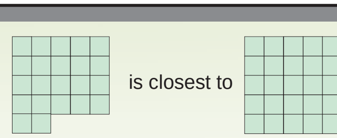
square root of 8 ≈ **2**

b.



square root of 14 ≈ **4**

c.



square root of 22 ≈ **5**

Lesson 2.6.1

Perfect Squares and Their Roots

In this Lesson students learn what perfect square numbers are and what square roots are. They discover that all positive numbers have two square roots, but that the square root symbol indicates just the positive root.

Previous Study: In grade 4 students learned how to find the area of a square by using the formula $A = s \times s$, linking the area of a square and its side length.

Future Study: In Algebra I students will learn how to take roots of algebraic expressions, and raise numbers to any fractional power. They will use these skills to solve quadratic equations.

1 Get started

Resources:
- grid paper
- sets of small square tiles (these can be made from grid paper)
- calculators

Warm-up questions:
- Lesson 2.6.1 sheet

2 Teach

Universal access

Give everyone a piece of grid paper. Have them draw a square with a side length of 6 units. Ask them to find the area of the big square by counting how many little squares it contains. It should contain 36 small squares. So 36 is a perfect square number.

Now ask the students to draw another square with a side length of 4.5 units, and to find its area by counting the small squares again (20.25 units^2). This time it's harder to count the square units inside. As the side length is not a whole number, the area isn't a perfect square number.

Concept question

"I have 20 small squares. Can I arrange them to make one large square using all the tiles?"

No — 20 is not a perfect square number.

Guided practice

Level 1: q1–2
Level 2: q1–4
Level 3: q1–6

Lesson 2.6.1

California Standards:

Number Sense 2.4

Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why.

What it means for you:

You'll see what a square root is and how to find the square root of a square number.

Key words:
- perfect square
- square root
- positive root
- negative root

Section 2.6

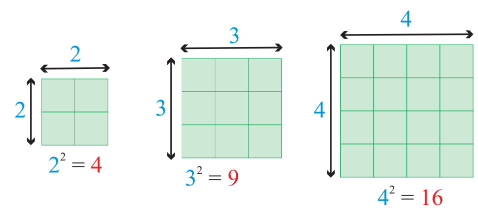
Perfect Squares and Their Roots

If you multiply the side length of a square by itself you get the area of the square. You can do the opposite too — find the side length of the square from the area. That's called finding the square root.

The Square of an Integer is a Perfect Square Number

Raising a number to the power two is called **squaring** it. That's because you find the area of a square by multiplying the side length by itself.

So, the **area of a square = $s \cdot s = s^2$**, where s is the side length.



All the numbers in **red** are the squares of the numbers in **blue**.

The square of an **integer** is called a **perfect square**.
Perfect squares are always integers too.

$1 \cdot 1 = \mathbf{1}$
$2 \cdot 2 = \mathbf{4}$
$4 \cdot 4 = \mathbf{16}$ — These numbers **are** perfect squares

$3.5 \cdot 3.5 = \mathbf{12.25}$
$5.1 \cdot 5.1 = \mathbf{26.01}$ — These numbers **aren't** perfect squares

Example 1

Is the number 81 a perfect square?

Solution

$9 \cdot 9 = 81$

As 9 is an integer, **81 is a perfect square**.

Guided Practice

Give the square of each of the numbers in Exercises 1–6.

1. 4   16   **2.** 7   49
3. 12   144   **4.** 1   1
5. –2   4   **6.** –12   144

Solutions

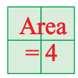
For worked solutions see the Solution Guide

- **Strategic Learners**
  Split everyone into groups. Give each group 25 small tiles. Ask them to experiment with arranging the tiles — starting with 1 tile, and working up to 25, and ask them what numbers of tiles can be put into square arrays. Have them record the numbers of tiles they find that can be arranged in squares, and write the side length by each. By the end of the activity they will have found the first 5 perfect square numbers.

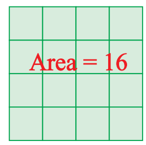
- **English Language Learners**
  The strategic learner activity described above is also useful for English language learners. This provides the opportunity to emphasize the relevant language. After completing the activity, students should write a table of the perfect squares and their roots. They should also write a list of the nonperfect square numbers they found, which are all the other numbers.

The Opposite of Squaring is Finding the Square Root

You might **know the area** of a square and **want to know the side length**. You find the side length of a square by finding the **square root**.



2 • 2 = 4
Side length is 2
2 is the square root of 4



4 • 4 = 16
Side length is 4
4 is the square root of 16

> So, to find the square root of a square number, you have to **find the number that multiplied by itself gives the square number**.

For example: 5 • 5 = 25. 25 is a square number. 5 is a **square root** of 25.

The symbol $\sqrt{}$ is used to show a square root.

So you can say that $\sqrt{25} = 5$.

Unless the number you're finding the square root of is a perfect square, the square root will be a **decimal** — and may well be **irrational**. (There's more on this in Lesson 2.6.2.)

Check it out:

Your calculator should have a square root button on it that looks like this: √.

To find the square root of 2 on your calculator press:

√ 2 =.

On some types of calculators you have to press: 2 √

All Positive Numbers Have Two Square Roots

Every positive number has **one positive**, and **one negative**, square root. This is because **4 • 4 = 16 and –4 • –4 = 16** — so the square root of 16 could be **either** 4 or –4.

A square root symbol, $\sqrt{}$, by itself means just "**the positive square root**."

$\sqrt{16} = 4$ — this is the **positive square root** of 16.

$-\sqrt{16} = -4$ — this is the **negative square root** of 16.

Check it out:

You can't find the square root of a negative number.
To square a number you have to multiply it by itself.
- If you multiply a positive number by a positive number the result is always positive.
- If you multiply a negative number by a negative number the result is also positive.

So if you multiply any number by itself the answer is always positive.

✓ Guided Practice

Evaluate the square roots in Exercises 7–14.

7. $\sqrt{36}$ 6 **8.** $-\sqrt{64}$ –8
9. $\sqrt{100}$ 10 **10.** $-\sqrt{144}$ –12
11. $\sqrt{121}$ 11 **12.** $-\sqrt{169}$ –13
13. $\sqrt{1}$ 1 **14.** $\sqrt{400}$ 20

Check it out:

These numbers are all perfect square numbers. So their square roots will all be whole numbers.

2 Teach (cont)

Universal access

Split the students into small groups. Give each group a set of tiles. The number of tiles should be a square number — for example, 25, 36, 49, etc.

Ask the students to work out the square root of the number of tiles that they have.

They should realize that if you lay the tiles out in a square, the side length of the square is equal to the square root.

Math background

Remind students of the rules of multiplying together negative and positive numbers:

Positive × Positive = Positive
Negative × Negative = Positive
Positive × Negative = Negative
Negative × Positive = Negative

Remind them that there are two ways to get a positive answer when you multiply two factors — multiplying together two positives or two negatives. That's why every number has two possible square roots.

Math background

It may be worth emphasizing that there are no real square roots of negative numbers. In Algebra II students will meet imaginary numbers, which have squares that are negative real numbers.

Common error

If asked in words to give the square root of a number, students often forget to include the negative answer.

Remind them that if a question doesn't clearly state that just the positive or the negative root is required, they should always give both.

Guided practice

Level 1: q7–10
Level 2: q7–12
Level 3: q7–14

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to write a list of all the perfect square numbers between 1 and 200 on their own, and then exchange it with someone else to check they have them all. Have them think about what strategy they used to find the numbers. Now ask them to look at the list: can they see any patterns in the numbers themselves, or in the differences between the numbers? **(For example, the differences between consecutive perfect squares are consecutive odd numbers.)** Have them try to explain any patterns they find. Also, ask students to think about whether 0 is a perfect square number. It is considered to be so mathematically — do they think this is right? Or does it depend on how we define what a perfect square number is?

2 Teach (cont)

Universal access

Have everyone find the value of $\sqrt{16}$ using a calculator. They should all get the answer 4.

Now have everyone find the value of $16^{\frac{1}{2}}$. They should get the answer 4 again. This helps to reinforce the idea that these operations have the same effect.

Math background

When a number that is raised to a power is raised to another power, you multiply the exponents together to find the new power.

$(x^{\frac{1}{2}})^2 = x^{\frac{1}{2}\times 2} = x^1 = x$

This means that when you square a number that is raised to the power $\frac{1}{2}$, you get the number itself — so raising a number to the power $\frac{1}{2}$ must give a square root of the number.

Guided practice

Level 1: q15–18
Level 2: q15–18
Level 3: q15–18

Independent practice

Level 1: q1–4, 7, 9–12
Level 2: q1–4, 8–14, 17–18
Level 3: q1–19

Additional questions

Level 1: p441 q1–9
Level 2: p441 q4–15
Level 3: p441 q4–15

3 Homework

Homework Book — Lesson 2.6.1

Level 1: q1–4, 6, 7
Level 2: q2–10
Level 3: q2–11

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
- Worksheet 14 — Squares and Square Roots

Writing Square Roots with Fractional Exponents

In Section 2.4 you saw that an **exponent** means a **repeated multiplication**. When you **square** a number it is repeated as a factor **two times** in the multiplication expression. So you can write $x \cdot x = x^2$.

Taking the **square root** of a number is the **reverse process** to squaring it. Because it **undoes** squaring we can also write $\sqrt{x}$ as $x^{\frac{1}{2}}$.

So, $\sqrt{9} = 9^{\frac{1}{2}}$ and $\sqrt{16} = 16^{\frac{1}{2}}$.

Check it out:
If you multiply a square root by itself you get back to the original number.
$\sqrt{x} \cdot \sqrt{x} = x.$

Guided Practice

Evaluate the expressions in Exercises 15–18.

15. $4^{\frac{1}{2}}$ 2 **16.** $-25^{\frac{1}{2}}$ –5
17. $49^{\frac{1}{2}}$ 7 **18.** $-100^{\frac{1}{2}}$ –10

Don't forget:
$-100^{\frac{1}{2}}$ means $-(100^{\frac{1}{2}})$. It can't be $(-100)^{\frac{1}{2}}$ because that would be the square root of a negative number.

Independent Practice

Give the square of each of the numbers in Exercises 1–6.

1. 6 36 **2.** 11 121
3. 16 256 **4.** –10 100
5. –13 169 **6.** –15 225

7. Marissa is making patterns with small square mosaic tiles. She has 50 tiles. Can she arrange them to make one larger square, using all the tiles? Explain your answer. No – 50 is not a perfect square number, so the tiles cannot be arranged to make a square.

8. This year's senior class will have 225 students graduating. The faculty wants the chairs to be arranged in the form of a square. How many chairs should be put in each row? 15 chairs

Evaluate the square roots in Exercises 9–16.

9. $\sqrt{25}$ 5 **10.** $-\sqrt{25}$ –5
11. $\sqrt{64}$ 8 **12.** $-\sqrt{49}$ –7
13. $\sqrt{9}$ 3 **14.** $\sqrt{289}$ 17
15. $361^{\frac{1}{2}}$ 19 **16.** $-49^{\frac{1}{2}}$ –7

17. Give the square roots of 16. 4 and –4

18. Give the square roots of 81. 9 and –9

19. A square deck has an area of 81 feet². Paul is planning to enlarge the deck by increasing the length of each side by 2 feet. How much will the area of the deck increase by? 40 feet²

Now try these:
Lesson 2.6.1 additional questions — p441

Round Up

If you multiply any integer by itself you will get a perfect square number. The factor that you multiply by itself to get a square number is called its square root. Every square number has one positive and one negative square root. And don't forget — negative numbers don't have square roots.

Solutions

For worked solutions see the Solution Guide

Lesson 2.6.2 Irrational Numbers

In Section 2.1 students were introduced to the concept of rational numbers and learned to identify them. In this Lesson, they take a closer look at the numbers that didn't fit into this category — the irrational numbers.

Previous Study: In grade 6 students learned about the constant π and used it to find the circumference and area of a circle. Earlier in grade 7 they learned about rational numbers and how they are defined.

Future Study: In Algebra I students will be expected to be familiar with irrational numbers, including π and square roots of nonperfect squares, and to use them in expressions and equations.

Lesson 2.6.2

Irrational Numbers

California Standards:
Number Sense 1.4
Differentiate between rational and irrational numbers.

What it means for you:
You'll learn what irrational numbers are, and how they're different from rational numbers.

Key words:
- rational
- irrational
- integer
- terminating decimal
- repeating decimal

If you find the square root of 2 on your calculator, you get a number that fills the display, and none of the digits repeat. In this Lesson you'll learn what makes numbers like that special.

Rational Numbers Can Be Written as Fractions

In Section 2.1 you saw that any number that can be written in the form $\frac{a}{b}$ where a and b are both integers, and b isn't 0, is called a **rational number**.

For example:

$2 \xrightarrow{\text{can be written as}} \frac{2}{1}$

$3.7 \xrightarrow{\text{can be written as}} \frac{37}{10}$

$0.\overline{81} \xrightarrow{\text{can be written as}} \frac{9}{11}$

You can write all of these as fractions as described above, so they are all rational numbers.

All **fractions**, **integers**, **terminating decimals**, and **repeating decimals** are **rational numbers**. You can add **square roots of perfect squares** to that list too, because they are always **integers**.

Guided Practice

Prove that the numbers in Exercises 1–4 are rational by writing each one as a fraction in its simplest form.

1. 6 $\frac{6}{1}$ **2.** 0.8 $\frac{4}{5}$

3. $0.\overline{3}$ $\frac{1}{3}$ **4.** $\sqrt{16}$ $\frac{4}{1}$

Check it out:
You could never write an irrational number out in full because it would go on forever.

Irrational Numbers Can't Be Written as Fractions

- Any number that **can't** be written as a ratio of two integers is called an **irrational number**.
- Irrational numbers are **nonterminating, nonrepeating decimals**.

0.1234567891011121314151617181920 21...

5.1211211121111211111211111121111111 2...

Neither of these decimals terminate or have repeating patterns of digits. They're both irrational numbers.

Check it out:
In Chapter 3, you'll use π to calculate the circumference of a circle. You can never calculate the circumference exactly, because π goes on forever.

The most famous irrational number is **π** — you can't write π as a fraction.

π starts **3.14159265358979323846264338327 95...**

The value of π has been calculated to over a million decimal places so far — it **never ends** and **never repeats**.

1 Get started

Resources:
- pieces of colored card
- large copies of the diagram at the top of page 124 (for the Strategic Learners activity), and a set of number cards
- Number cards for the Universal access game on page 124

Warm-up questions:
- Lesson 2.6.2 sheet

2 Teach

Additional examples
Say whether the following numbers are rational or not:

1) 5 **Rational**
2) $\frac{7}{9}$ **Rational**
3) 0.123 **Rational**
4) π **Not rational**
5) $0.\overline{52}$ **Rational**

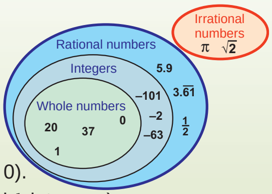
Guided practice
Level 1: q1–4
Level 2: q1–4
Level 3: q1–4

Math background
π is the ratio of the circumference of a circle to its diameter.

Students will have used π at grade 6 to find the circumference and area of a circle. This will be reviewed in Chapter 3.

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
To reinforce the idea that rational and irrational numbers form two distinct sets, repeat the number sorting activity from Lesson 2.1.1. Add an orange ring labeled "irrational numbers," as shown in the diagram on the right. Add extra cards to the pack with π and $\sqrt{2}$ on. Have students put the cards in the correct rings.



- **English Language Learners**
Emphasize the connection between the word "ratio" and the word "rational". Rational numbers are those that can be written as a ratio of two integers. This means they can be written in the form $\frac{a}{b}$ where a and b are integers (and b is not 0). Irrational numbers are just those that can't be written as a ratio — there is no way to write them in the form $\frac{a}{b}$ (with a and b integers.)

2 Teach (cont)

Universal access

Give each member of the class a card with a number written on it.

The numbers should be a mixture of positive and negative integers, terminating decimals, repeating decimals, fractions, square roots of perfect squares, and irrational numbers, such as π and the square roots of nonperfect squares.

Have the students stand in a line, going in order according to their numbers, across the room. Then tell them to step forward if their number meets a certain criterion that you call out, such as, "can be written as a fraction," "is irrational," "is odd."

Guided practice

Level 1: q5–10
Level 2: q5–10
Level 3: q5–10

Common error

Students may assume that because they can't see an obvious repeat in a decimal, it is irrational — or that because their calculator screen only shows a certain number of digits, that the decimal terminates and is therefore rational.

Remind them that calculators cut off answers at the end of the screen, and they can't be sure that a decimal is irrational just by looking at the start of it — the repeat period may be too large for all the repeating digits to fit on the screen.

The square roots of **perfect squares** are **integers** — so they're **rational**.

> The square root of **any integer** other than a perfect square is **irrational**.

$\sqrt{1} = 1, \sqrt{2}, \sqrt{3}, \sqrt{4} = 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9} = 3, \sqrt{10} \ldots$

These numbers are rational — These numbers are irrational

Guided Practice

Classify the numbers in Exercises 5–10 as rational or irrational.

5. $\frac{7}{9}$ Rational — **6.** π Irrational

7. 5 Rational — **8.** $\sqrt{100}$ Rational

9. $1.2\overline{543}$ Rational — **10.** $\sqrt{14}$ Irrational

Check it out:
The "repeat period" of any decimal is the length of the bit of it that repeats. So the repeat period of $0.\overline{3}$ is one digit — the digit 3, the repeat period of $2.\overline{945}$ is three digits — the group of digits 945, and the repeat period of $5.273\overline{16}$ is two digits — the pair of digits, 16.

Check it out:
You know that $1 \div 7$ must be a rational number because it can be expressed as $\frac{1}{7}$.

Some Decimals Have a Long Repeat Period

Sometimes it might not be obvious straightaway whether a number is **rational** or **irrational**. Some decimals that have a **large repeat period** may look as if they are irrational but are actually rational.

For example, when you **divide 1 by 7** on your calculator you get a decimal number. From the number on your calculator display, you can't tell if that decimal ever **ends or repeats**.

$$1 \div 7 = 0.1428571\ldots$$

To show that the decimal does in fact **repeat**, work out $1 \div 7$ using **long division**:

```
    0.14285714...
 7 )1.00000000...
    7
    30
    28
     20
     14
      60
      56
       40
       35
        50
        49
         10
         07
          30
          28
```

The repeat period starts here. So you can write this decimal as $0.\overline{142857}$

now the same cycle begins again — this is a repeating decimal

You can see that the same long division cycle **begins again**.
This means the decimal does **repeat**.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Have students do some research into the history of π, its discovery, and its uses in mathematics. Ask them to investigate how approximations of π have increased in accuracy over the years, from the earliest rough estimations to the vast number of decimal places that have been calculated today. Have them prepare a report, poster, or presentation of their findings.

It's really hard to prove a number's **irrational** — because it could just be a decimal with a **really really long** repeat pattern.
The only way you can know that a number is irrational is if it has a pattern that you know will never repeat — there's an endless set of irrational numbers like this to choose from:

→ They might have a pattern of digits that you could generate with a formula. $\underline{0.1248163264128256512...}$

This pattern of digits represents the powers of 2: 2^0, 2^1, 2^2, 2^3, 2^4, etc. It goes on forever without repeating.

→ They could have any pattern of digits where each time the pattern is repeated the number of copies of each digit increases.
$\underline{0.121122111222111122222...}$

In this pattern the numbers 1 and 2 alternate, but each time they are repeated the number of copies of the digits increases by 1.

Check it out:
A bar above some digits in a decimal number shows that these digits are repeated over and over again.

Guided Practice

Classify the numbers in Exercises 11–15 as rational or irrational.

11. 0.369121518... where the pattern of digits are the multiples of 3 **Irrational**

12. $3.12\overline{2468}$ **Rational**

13. 0.2647931834 **Rational**

14. $0.1411441114441\overline{4}$ **Rational**

15. 0.123112233111222... **Irrational**

Independent Practice

Prove that the numbers in Exercises 1–4 are rational by expressing them as fractions in their simplest form.

1. 14 $\frac{14}{1}$

2. $\sqrt{121}$ $\frac{11}{1}$

3. 2.6 $\frac{13}{5}$

4. $1.\overline{6}$ $\frac{5}{3}$

5. Read statements a) and b). Only one of them is true. Which one? How do you know?
a) All fractions can be written as decimals.
b) All decimals can be written as fractions.
Only statement a) is true. You can write any fraction as a decimal, but irrational numbers are decimals, and can't be written as fractions.

Now try these:
Lesson 2.6.2 additional questions — p442

Classify the numbers in Exercises 6–13 as rational or irrational.

6. 10 **Rational**

7. $\frac{14}{5}$ **Rational**

8. 0.497623 **Rational**

9. 3π **Irrational**

10. $9.12958725364\overline{8}$ **Rational**

11. $\sqrt{27}$ **Irrational**

12. $\sqrt{225}$ **Rational**

13. 22.343344333444... **Irrational**

14. Write any four irrational numbers between zero and five.
Accept any numbers that are nonrepeating, nonterminating decimals between 0 and 5, including π and any non-perfect square root from $\sqrt{2}$ to $\sqrt{24}$.

Round Up

Irrational numbers can't be written as fractions where the numerators and denominators are both integers. Irrational numbers are always nonterminating and nonrepeating decimals.

2 Teach (cont)

Universal access
Let everyone have a go at writing the start of their own irrational number, using a pattern of digits that could go on forever. Have everyone write them on bright colored cards, and display them, in order, as an irrational number frieze, along with π, 2π, and some irrational square roots.

Concept question
"The number x is a nonterminating, nonrepeating decimal. Is x rational or irrational?"
Irrational

Guided practice
Level 1: q11–15
Level 2: q11–15
Level 3: q11–15

Independent practice
Level 1: q1, 3, 6–9
Level 2: q1–3, 5–11
Level 3: q1–14

Additional questions
Level 1: p442 q1–2, 6, 7–10, 16–22
Level 2: p442 q1–12, 16–22
Level 3: p442 q1–22

3 Homework

Homework Book — Lesson 2.6.2
Level 1: q1, 2a–b, 3a–b, 5, 7
Level 2: q1–5, 7, 8, 10–12
Level 3: q3–12

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 14 — Squares and Square Roots
- Worksheet 15 — Rational and Irrational Numbers

Solutions
For worked solutions see the Solution Guide

Lesson 2.6.3 Estimating Irrational Roots

In this Lesson students develop their understanding of the fact that square roots of nonperfect squares are irrational, and can't be written out in full. They practice finding approximations of square roots using calculators, and finding which integers a square root lies between.

Previous Study: In Lesson 2.6.1, students were introduced to the concept of perfect squares and their roots. In Lesson 2.6.2, they studied irrational numbers.

Future Study: In Algebra I students work with roots in expressions. They will use the techniques they learn to solve quadratic equations.

1 Get started

Resources:
- grid paper
- calculators
- individual whiteboards and pens

Teacher Resources CD-ROM
- Number Line

Warm-up questions:
- Lesson 2.6.3 sheet

2 Teach

Universal access

Give everyone a piece of grid paper. Have them draw out a 10 × 10 grid, and put the numbers 1 to 100 in the squares.

Ask them to color all the squares containing perfect square numbers red — these should be 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100.

Now have them color all the remaining squares blue.

All the squares they have colored red have square roots that are integers, and so are rational numbers. They can add the square roots to the grid.

All the squares they have colored blue are nonperfect square numbers — so their square roots are irrational. They can try calculating some of these square roots on a calculator.

Ask them to write this below their grid as a reminder.

Guided practice

Level 1: q1–2
Level 2: q1–4
Level 3: q1–6

Math background

Sometimes in math it is appropriate to leave the answer as a square root, and sometimes it is appropriate to give an approximate answer. There is guidance about when to do each of these things in Section 8.3.

Lesson 2.6.3

Estimating Irrational Roots

California Standard:

Number Sense 2.4
Use the inverse relationship between raising to a power and extracting the root of a perfect square integer; **for an integer that is not square, determine without a calculator the two integers between which its square root lies and explain why**.

Mathematical Reasoning 2.7
Indicate the relative advantages of exact and approximate solutions to problems and **give answers to a specified degree of accuracy.**

What it means for you:
You'll learn how to find the approximate square root of any number without using a calculator.

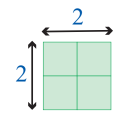
Key words:
- irrational
- perfect square
- square root

Don't forget:
Rounding a number makes it shorter and easier to work with. How much you round a number depends on how accurate you need to be.
For example, you might round 99.26 up to 100, down to 99, or up to 99.3 — depending on how accurate you want to be.
There are rules for rounding — they're explained in Chapter 8.

You saw in the last Lesson that all square roots of integers that aren't perfect squares are irrational numbers. That means that you could never write their exact decimal values, because the numbers would go on forever. But you can use an approximate value instead.

Square Roots of Nonperfect Squares are Irrational

Perfect square numbers have square roots that are **integers**.

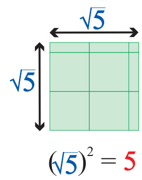


$2^2 = 4$

The area of this square is 4 units.
So its side length must be
$\sqrt{4}$ units = 2 units — which is **rational.**

Numbers that are **not perfect squares** still have square roots.

Square roots of integers that are **not perfect squares** are always **irrational numbers**.



$(\sqrt{5})^2 = 5$

The area of this square is 5 units.
So its side length must be $\sqrt{5}$ units
— which is **irrational.**

Guided Practice

Say whether each number in Exercises 1–6 is rational or irrational.

1. $\sqrt{9}$ Rational

2. $\sqrt{2}$ Irrational

3. $\sqrt{12}$ Irrational

4. $\sqrt{16}$ Rational

5. $\sqrt{169}$ Rational

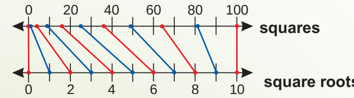
6. $\sqrt{140}$ Irrational

You Can Approximate Irrational Square Roots

If you are asked to give a **decimal** value for $\sqrt{2}$, or for any other irrational number, you would have to give an **approximation**. You could never give an exact answer because the exact answer goes on forever.

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Have students draw two number lines one above the other, one showing 0 to 100, and the other 0 to 10. Have them mark the perfect square numbers on the first line, and join them to their square roots on the second line. To find which integers a square root of a nonperfect square lies between, they can find it on the first line, and read down to the second.



- **English Language Learners**
  The Universal access activity on the previous page is useful for reinforcing the terms "perfect square," "nonperfect square," and "square root." The number line method used below on the student page presents estimating square roots in a visual way that English language learners may find useful.

2 Teach (cont)

Concept question

"When I find the value of $\sqrt{5}$ using my calculator, the screen says 2.236067977. Is this the actual value of $\sqrt{5}$?"

No — $\sqrt{5}$ is irrational, and written as a decimal it would go on forever. The calculator screen has truncated the answer. (It may also round the last digit, although in this case it makes no difference.)

Math background

In order to be able to say how accurate their answer is, students need to be familiar with the place-value system and decimal places.

There is more on this in Section 2.1 and Section 8.3.

Guided practice

Level 1: q7–8
Level 2: q7–10
Level 3: q7–12

Check it out:
Some calculators need you to press 2 √

Check it out:
The symbol "≈" means "is approximately equal to." You can use it in situations where you can't write the exact value of a number because it's irrational.

Check it out:
When you round a decimal answer you will usually be told how many decimal places to round it to. That's just how many digits there are after the decimal point.

If you're rounding a number to six decimal places, you have to look at the digit in the seventh decimal place. If it's 5 or more, then round the sixth digit up. If it's less than 5, then round the sixth digit down.

For example,

$\sqrt{28}$ = 5.2915026 22...

seventh digit

The seventh digit is more than 5, so the sixth digit is rounded up to 3.

So, $\sqrt{28}$ = 5.291503 to 6 d.p.

There's more on how and when to round numbers in Section 8.3.

Example 1

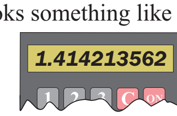
Find the approximate value of $\sqrt{2}$ using a calculator.

Solution

You should have a button on your calculator that has the square root symbol on it. It will look like this: √ .

To find the square root of 2, press the square root button, then the number 2, and then the equals button, like this: √ 2 = .

You will get an answer on the screen that looks something like this:



Even though the answer on your screen stops, it's not the exact answer. It's just an approximation based on how many digits can fit on the screen.

So you should write your answer like this:

$\sqrt{2}$ = 1.414213562 (to 9 decimal places) — *State the accuracy you've given the approximate answer to.*

Or like this: **$\sqrt{2} \approx 1.41$**

Then you've shown that you know it's an approximate answer.

Guided Practice

Use your calculator to approximate the square roots in Exercises 7–12. Give the values to six decimal places.

7. $\sqrt{5}$ 2.236068 **8.** $\sqrt{6}$ 2.449490

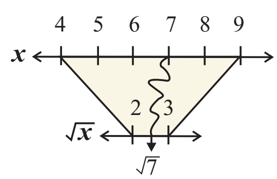
9. $\sqrt{10}$ 3.162278 **10.** $\sqrt{29}$ 5.385165

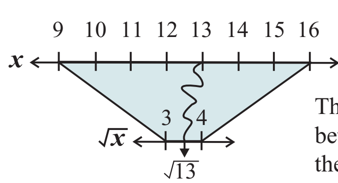
11. $\sqrt{47}$ 6.855655 **12.** $\sqrt{160}$ 12.649111

Estimate Square Roots Using a Number Line

Estimating the square root of a number without using a calculator involves working out which two **perfect square numbers** it lies between.

For example, the square roots of all the numbers between **4 and 9** lie between **2 and 3** on the number line.





The square roots of all the numbers between **9 and 16** lie between **3 and 4** on the number line.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Have students use the "guess, divide, average" method to find the square root of a number by hand. They start by guessing the square root of a number, and then dividing the number by their guess. Next have them find the average of their guess and the quotient — this is an estimate of the root. They should repeat these steps, using the estimate as a new guess. The accuracy will improve with each repetition. Have them use this method to find a square root to 3 decimal places, and compare it to the answer given by a calculator.

2 Teach (cont)

Universal access

Have a race to find which two integers the square root of a nonperfect square number lies between. This will give lots of practice at using the methods.

Give each member of the class a small whiteboard and call out a nonperfect square number. Everyone should try to work out which two integers its square root lies between and write it on their board.
Some students may find a number line useful for this activity.

Concept question

"The integer x lies between 49 and 64 on the number line. Is $\sqrt{x}$ rational? Which two integers will $\sqrt{x}$ lie between on the number line?"

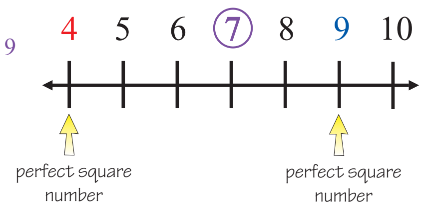
$\sqrt{x}$ is irrational. It will lie between 7 and 8 on the number line.

There are two steps to finding an approximation of the square root of a number.

For example: find the two numbers that $\sqrt{7}$ lies between on the number line.

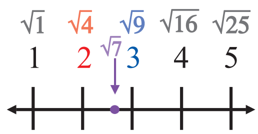
1) First find the two **perfect square numbers** that 7 lies **between** on the number line.

7 lies between 4 and 9



2) Find the **square roots of these two perfect square numbers**. The square root of 7 must be **between** these two square roots.

$\sqrt{4} = 2$, $\sqrt{9} = 3$.



So $\sqrt{7}$ must lie between **2 and 3**.

Example 2

Find the two numbers that $\sqrt{14}$ lies between on the number line.

Solution

First find the two **perfect squares** that 14 lies between on the number line.
14 lies between **9 and 16**.

$\sqrt{9} = 3$ and $\sqrt{16} = 4$.

So $\sqrt{14}$ lies between **3 and 4** on the number line.

Example 3

Find the numbers that $\sqrt{18}$ lies between on the number line.

Solution

First find the two perfect squares that 18 lies between on the number line.
18 lies between 16 and 25.

$\sqrt{16} = 4$ and $\sqrt{25} = 5$.

So $\sqrt{18}$ lies between **4 and 5** on the number line.

Guided Practice

In Exercises 13–20 find the whole numbers that the root lies between.

13. $\sqrt{5}$	2 and 3	**14.** $\sqrt{15}$	3 and 4
15. $\sqrt{24}$	4 and 5	**16.** $\sqrt{46}$	6 and 7
17. $\sqrt{68}$	8 and 9	**18.** $\sqrt{98}$	9 and 10
19. $\sqrt{125}$	11 and 12	**20.** $\sqrt{150}$	12 and 13

Independent Practice

Use your calculator to approximate the square roots in Exercises 1–4 to four decimal places.

1. $\sqrt{17}$	4.1231	**2.** $\sqrt{28}$	5.2915
3. $\sqrt{73}$	8.5440	**4.** $\sqrt{155}$	12.4499

In Exercises 5–10 say which two perfect square numbers the number lies between.

5. 3	1 and 4	**6.** 29	25 and 36
7. 50	49 and 64	**8.** 95	81 and 100
9. 125	121 and 144	**10.** 200	196 and 225

In Exercises 11–18 find the whole numbers that the root lies between.

11. $\sqrt{3}$	1 and 2	**12.** $\sqrt{13}$	3 and 4
13. $\sqrt{22}$	4 and 5	**14.** $\sqrt{33}$	5 and 6
15. $\sqrt{58}$	7 and 8	**16.** $\sqrt{93}$	9 and 10
17. $\sqrt{160}$	12 and 13	**18.** $\sqrt{216}$	14 and 15

19. A square has an area of 85 inches^2. What whole-inch measurements does the side length lie between? 9 and 10 inches

20. If $\sqrt{a} \approx 2.4$ then which two perfect squares does a lie between? 4 and 9

21. Latoya has a new office. It is a square room, with a floor area of 230 feet^2. She wants to fit a 15 ft desk area along one wall — will this fit along one of the sides? Explain your reasoning. Yes — each wall must be between 15 and 16 ft long.

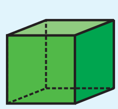
22. A math class is shown a cube made of card. Pupils are told that the total surface area of the cube is 90 cm^2. They are asked to guess the length of each side of the cube in centimeters. Peter guesses 10 centimeters, and John guesses 4 centimeters. Whose guess is the closer? John's — the length must be between 3 and 4 cm.

Round Up

You can never write out the exact value of the square root of a nonperfect square number — but you can use an approximation. To figure out which two integers a number's square root lies between, it's just a case of knowing which two perfect squares the number lies between, and finding their square roots.

Now try these:
Lesson 2.6.3 additional questions — p442

Don't forget:
A cube has 6 faces that are all identical squares.



Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Guided practice
Level 1: q13–16
Level 2: q13–18
Level 3: q13–20

Independent practice
Level 1: q1–2, 5–6, 11–14, 19
Level 2: q1–3, 5–8, 11–16, 19–21
Level 3: q1–22

Additional questions
Level 1: p442 q1–10, 13–14, 17–19
Level 2: p442 q1–19
Level 3: p442 q1–20

3 Homework

Homework Book — Lesson 2.6.3
Level 1: q1, 2, 3a, 4a, 5, 6, 10
Level 2: q1–5, 7, 9–12
Level 3: q2–12

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
• Worksheet 14 — Squares and Square Roots
• Worksheet 15 — Rational and Irrational Numbers

Chapter 2

Investigation — Designing a Deck

Purpose of the Investigation

This Investigation involves exploring the possible ways a family could increase the size of their deck so that it has a certain area. Students have to apply their knowledge of integer operations and square roots in area and perimeter calculations. The Investigation also provides a bridge between Chapter 2 and Chapter 3, which is about two-dimensional shapes.

Resources

- grid paper for each student
- inch squares

Strategic & EL Learners

Provide students with inch squares. This will help them to experiment with different deck designs. Three-inch-long strips could also be provided to make the Extensions more accessible to strategic learners.

Check that English language learners are familiar with the terms used, such as deck and railing.

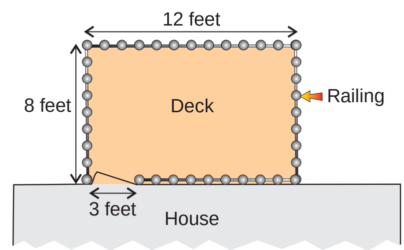
Investigation Notes on p130 B-C

Chapter 2 Investigation

Designing a Deck

You have to add, subtract, multiply, and divide numbers to solve lots of real-life problems. Being able to use powers and find square roots can sometimes come in useful too.

The Dedona family has a deck on the back of their house that they want to **make bigger**. The current deck is a **rectangle** with a length of 12 feet and width of 8 feet. They want the new deck to be **225 square feet** in area.



Part 1:
What would the dimensions of the enlarged deck be if it were in the shape of a square?

Part 2:
Make four different designs for additions to the deck that satisfy the Dedonas' area requirement.

Things to think about:

- The new deck **must** contain the original, rectangular deck. However, the new, enlarged deck **doesn't** have to be rectangular itself.

Extensions

The cost of railing is $18.95 for a 3-foot section. Railing is only sold in 3-foot sections.

1) What design for the new deck provides the required area with the least amount of railing?
2) How many sections of railing do they need to buy for the new deck?
   What is the total cost for the railing?

Open-ended Extensions

1) Propose a design that would satisfy the Dedonas' area requirements and cost the most money for railings. The narrowest any section of the deck can ever be is 1 foot.
   How would your solution change if the existing deck was torn down?
   How would it change if the 3-foot railing sections could not be cut?
2) Would a square design be the least expensive if there was no railing against the house?

Round Up

When you're solving real-life problems, you often have to combine all the operations. You use multiplication to find areas and addition to find the total length around the edge. And if you have a square with a set area, you can find its side length by finding the square root of the area.

Investigation — Designing a Deck

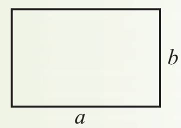
Mathematical Background

The **area** of a rectangle is calculated by multiplying the length by the width.

The **perimeter** of any shape is found by adding the lengths of the sides.

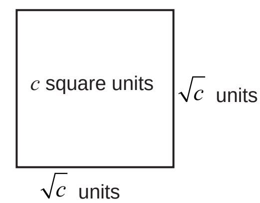
For example:
Area = length × width = $a \times b$

Perimeter = $a + b + a + b = 2a + 2b$



The **side lengths** of a square can be found by taking the square root of the area.

A **square** will always have a **smaller perimeter** than a rectangle of equal area.



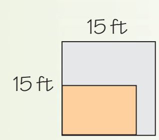
When doing real-life calculations, students will often have to round their answers. They have to look at the circumstances to determine if they should round up or down. This Investigation provides students with practice at this, as they are required to find the cost of railing to surround a deck, bearing in mind the railing is sold in 3-foot lengths.

Approaching the Investigation

Part 1:
Students should realise that the square root of 225 will be the side length of the square. They could find this on a calculator, or by a trial and error approach.

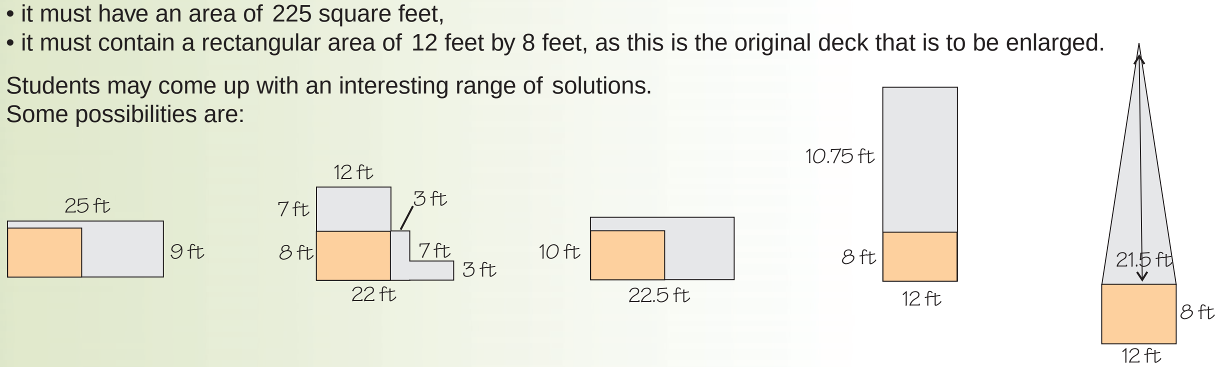
$\sqrt{225}$ = 15 feet

So the dimensions of a square deck, with an area of 225 square feet will be **15 feet by 15 feet**.



Part 2:
The only constraints on the size and shape of the new deck are:
• it must have an area of 225 square feet,
• it must contain a rectangular area of 12 feet by 8 feet, as this is the original deck that is to be enlarged.

Students may come up with an interesting range of solutions.
Some possibilities are:



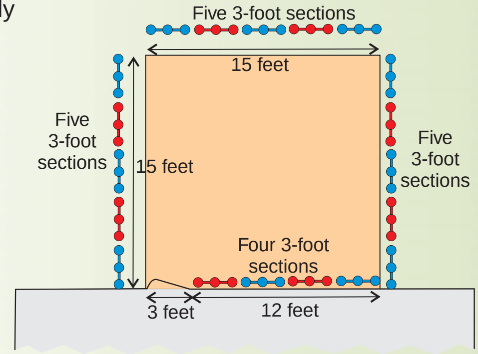
Extensions

1) Students should find the perimeters of the square deck from Part 1, and the other decks they designed in Part 2. They do this by adding up all the side lengths of their deck designs, and subtracting 3 feet for the width of the door. If students included any triangular parts in their designs, they should draw a scale diagram on grid paper to find any necessary lengths (the Pythagorean theorem isn't covered until Chapter 3).

   By comparing the shapes and perimeters, students should realise that the closer a deck is to being a square, the smaller the perimeter is.

2) Railing only comes in 3-foot sections. Luckily, the square deck with a 225 square foot area, can be enclosed exactly with 3 foot railings (and the doorway is conveniently 3 foot wide). So there is no wastage from buying 3-foot sections of railing. The diagram shows the number of sections needed for each side. In total this is: 5 + 5 + 5 + 4 = 19, so 19 3-foot sections of railing are needed.

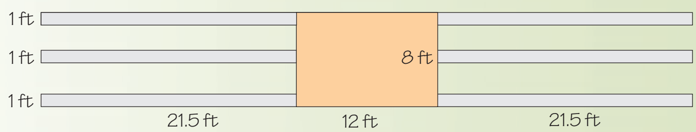
   Finding the total cost of the railing involves decimal multiplication. $18.95 × 19 = **$360.05**



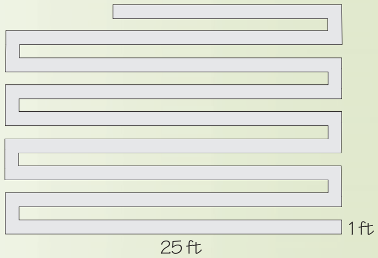
Open-Ended Extensions

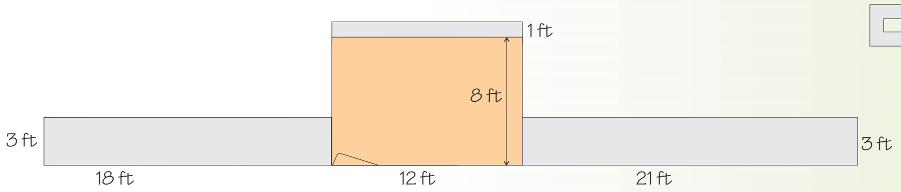
1) The largest perimeter would be the longest, narrowest shape. If the existing deck is kept, a structure like the one on the right could be built. Its perimeter is 12 + 12 + 5 + 5 + 6(21.5 + 21.5 + 1) = 298 feet.

   This would require 298 ÷ 3 = 99.33 3-foot sections of railing. As you can't buy a fraction of a section, you'd have to round up, and buy 100 sections. This would cost £18.95 × 100 = **$1895**.



   If students remove the constraint of their design including the existing deck, they may come up with a shape like the one on the right. Note that this has the same perimeter as a 225 ft by 1 ft rectangle. Alternatively, if they assume that the railing sections could not be cut, they may produce a design like the one below.





2) In this extension, students investigate the effect of not placing railings against the house. They are asked to find out whether the design that minimizes the cost of the railings is still a square. Assuming the original deck is kept, the length must be at least 12 feet and the width at least 8 feet. Students can experiment with varying these values.

   The square deck (15 ft by 15 ft) requires 45 feet of railing. This is 45 ÷ 3 = 15 sections. The table to the right shows that a rectangular deck with a length of between 16 and 28 feet inclusive requires slightly less railing than the square design. However, you can't buy a fraction of a section of railing, so to save any money, the length of railing required would have to be less than 42 feet (fourteen 3-foot sections). This does not happen.

   So there is no actual saving made by not using a square deck, but there are many other deck designs that would cost the same amount. (Don't forget, the length doesn't have to be an integer.)

Length, L (ft)	**Width, W (ft) 225 ÷ L**	**Railings, L + 2W (ft)**
12	18.75	49.50
13	17.31	47.62
14	16.07	46.14
15	15.00	45.00
16	14.06	44.13
17	13.24	43.47
18	12.50	43.00
19	11.84	42.68
20	11.25	42.50
21	10.71	42.43
22	10.23	42.45
23	9.78	42.57
24	9.38	42.75
25	9.00	43.00
26	8.65	43.31
27	8.33	43.67
28	8.04	44.07
29	7.76	44.52

This one can't be used, because the width is less than 8 ft.

Chapter 3

Two-Dimensional Figures

How Chapter 3 fits into the K-12 curriculum

Section 3.1 — Perimeter, Circumference, and Area		
Section 3.1 covers Measurement and Geometry 2.1, 2.2 **Objective: To calculate the perimeter, circumference, and area of regular polygons, circles, and complex shapes**		
Previous Study In grade 5 students used formulas to find the areas of triangles, parallelograms and rectangles, and in grade 6 were introduced to the formulas for area and circumference of a circle.	**This Section** Students calculate the perimeter and areas of regular polygons, and then calculate area and circumference of circles. They then apply these techniques to complex shapes.	**Future Study** In Section 7.1 of this course students will find the surface areas of solids by calculating the area of their nets. They will also find the volume and total edge length of complex 3-D figures.

Section 3.2 — The Coordinate Plane		
Section 3.2 covers Measurement and Geometry 3.2 **Objective: To plot points and draw figures in the coordinate plane**		
Previous Study In grade 4, students used a coordinate grid to represent points. In grade 5, they identified and graphed ordered pairs in the four quadrants of the coordinate plane.	**This Section** Students review plotting points, reading coordinates, and the four quadrants. They then plot figures and calculate lengths of sides of plotted figures.	**Future Study** Students going on to study Geometry will prove theorems by using coordinate geometry, for example, the midpoint of a segment.

Section 3.3 — The Pythagorean Theorem		
Section 3.3 covers Measurement and Geometry 3.2, 3.3 **Objective: To understand the Pythagorean Theorem and to apply the theorem to solve problems involving right triangles**		
Previous Study In Chapter 2 of this Program, students found powers of numbers, and also found square roots.	**This Section** Students learn to recognize right triangles and are introduced to the Pythagorean theorem. They then apply the theorem to problems involving right triangles.	**Future Study** Students going on to study Trigonometry, will prove that the identity $\cos^2(x) + \sin^2(x) = 1$ is equivalent to the Pythagorean theorem.

Section 3.4 — Comparing Figures		
Section 3.4 covers Measurement and Geometry 1.2, 2.0, 3.2, 3.4 **Objective: To reflect, translate, and apply scale factors to figures in the coordinate plane**		
Previous Study In grade 3 students measured lengths using appropriate tools and units. In grade 4 students identified figures with bilateral symmetry, and identified congruent figures.	**This Section** Students reflect and translate figures on coordinate axes, and then apply scale factors to simple figures to change their size. Finally, they learn the definitions of "congruent" and "similar".	**Future Study** In Geometry, students continue to reflect and translate figures in the coordinate plane. In Chapter 7 of this Program, they will apply scale factors to 3-D figures.

Section 3.5 — Constructions		
Section 3.5 covers Measurement and Geometry 3.1 **Objective: To construct circles and perpendicular bisectors, perpendiculars, altitudes, and angle bisectors**		
Previous Study In grade 5 students measured, identified, and drew angles, perpendicular and parallel lines, rectangles, and triangles using appropriate tools, such as rulers, compasses, and protractors.	**This Section** Students construct circles and perpendicular bisectors using compasses. They then go on to construct perpendiculars, altitudes, and angle bisectors.	**Future Study** Students going on to study Geometry will revisit constructions, and will extend their knowledge to the construction of lines parallel to a given line, through a point.

Section 3.6 — Conjectures and Generalizations		
Section 3.6 covers Measurement and Geometry 3.3, Algebra and Functions 1.1, Mathematical Reasoning 1.2, 2.2, 2.4, 3.3 **Objective: To make conjectures about geometrical patterns, and to write predictions algebraically**		
Previous Study During previous grades, students have been developing their ability to generalize results and apply them in other situations.	**This Section** Students learn to identify and write their own conjectures relating to geometrical patterns. They then learn how to generalize and to predict how patterns will continue.	**Future Study** In Algebra I, students determine whether algebraic statements are true sometimes, always, or never.

Pacing Guide – Chapter 3

40- to 50-Minute Class Periods

If your class periods are 40-50 minutes, we recommend allowing **30 days** for teaching Chapter 3.
As well as the **22 days of basic teaching**, you have **8 days** remaining to allocate 8 of the 11 optional activities (to be delivered at any appropriate point during the Chapter).

The table shows the 22 teaching days as well as all of the **optional days** you may choose for Chapter 3, in the order we recommend.

Day	Lesson	Description
Section 3.1 — Perimeter, Circumference, and Area		
Optional		*Exploration — Area and Perimeter Patterns*
1	3.1.1	Polygons and Perimeter
2	3.1.2	Areas of Polygons
3	3.1.3	Circles
4	3.1.4	Areas of Complex Shapes
5	3.1.5	More Complex Shapes
Optional		*Assessment Test — Section 3.1*
Section 3.2 — The Coordinate Plane		
Optional		*Exploration — Coordinate 4-in-a-Row*
6	3.2.1	Plotting Points
7	3.2.2	Drawing Shapes on the Coordinate Plane
Optional		*Assessment Test — Section 3.2*
Section 3.3 — The Pythagorean Theorem		
Optional		*Exploration — Measuring Right Triangles*
8	3.3.1	The Pythagorean Theorem
9	3.3.2	Using The Pythagorean Theorem
10	3.3.3	Applications of the Pythagorean Theorem
11	3.3.4	Pythagorean Triples & the Converse of the Theorem
Optional		*Assessment Test — Section 3.3*
Section 3.4 — Comparing Figures		
Optional		*Exploration — Transforming Shapes*
12	3.4.1	Reflections
13	3.4.2	Translations
14	3.4.3	Scale Factor
15	3.4.4	Scale Drawings
16	3.4.5	Perimeter, Area, and Scale
17	3.4.6	Congruence and Similarity
Optional		*Assessment Test — Section 3.4*
Section 3.5 — Constructions		
18	3.5.1	Constructing Circles
19	3.5.2	Constructing Perpendicular Bisectors
20	3.5.3	Perpendiculars, Altitudes, and Angle Bisectors
Optional		*Assessment Test — Section 3.5*
Section 1.1 — Sets and Expressions		
21	3.6.1	Geometrical Patterns and Conjectures
22	3.6.2	Expressions and Generalizations
Optional		*Assessment Test — Section 3.6*
Chapter Investigation		
Optional		*Investigation — Designing a House*

Accelerating and Decelerating

- To **accelerate** Chapter 3, allocate fewer than 8 days to the optional material. This will give you extra days to allocate to other Chapters. Note that you may use the remaining optional days at the end of the 160-day course.
- To **decelerate** Chapter 3, consider allocating more than 8 days to the optional Assessment Tests, Section Explorations, or Chapter Investigation, or spend longer teaching some Lessons. Also consider preparing students for difficult Lessons by reviewing previous coverage of math topics on related Skills Review Worksheets.
  Note that decelerating Chapter 3 will result in fewer days being available for teaching other Chapters.

90-Minute Class Periods

If you are following a block schedule with 90-minute class periods, we recommend allowing **15 days** for teaching Chapter 3.
The basic teaching material will take up **11 days**, and you can allocate the remaining **4 days** to the **optional material**.
To accelerate or decelerate a block schedule, follow the same advice as given above.

Exploration — Area and Perimeter Patterns

Purpose of the Exploration

The goal of the Exploration is to have students see how the perimeter of a shape with a fixed area changes as the dimensions of the shape change, and also how the area of a shape with a fixed perimeter changes as the dimensions change. Students will be encouraged to generalize this to draw figures with the maximum area or perimeter.

Resources

- grid paper
- rulers

Strategic & EL Learners

Strategic learners may benefit from having a multiplication table in front of them. A multiplication table serves as an easy reference for finding the dimensions for an area. This allows students to focus on the Exploration instead of on multiplication facts.

EL learners may have difficulty with the word “maximize.” Instruct students that to maximize is to make something as big as possible.

Math background

Students should understand the concepts of area and perimeter. Remind them that the area is the space inside a figure and the perimeter is the distance around the outside of the figure. It is not necessary that they know the area and perimeter formulas for this Exploration, but they must know how to calculate them by counting and adding squares.

Common error

Students often don't use the appropriate units for area and perimeter. Remind them that areas have squared units, whereas perimeters are lengths.

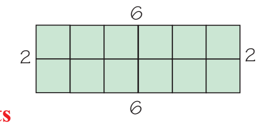
Section 3.1 introduction — an exploration into:
Area and Perimeter Patterns

You can draw shapes which have the same area, but different perimeters.
In this Exploration, you'll look at how to maximize the perimeter for a given area.
You'll also look at shapes that have the same perimeters, but different areas.

You can find the **area** of a shape by counting up the number of **unit squares**.
The **perimeter** is calculated by finding the **sum of all the side lengths**.

Example

Find the area and the perimeter of this shape.

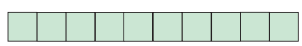


Solution

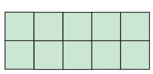
Area = **12 square units** Perimeter = 6 + 2 + 6 + 2 = **16 units**

When given a **set area**, you can draw shapes with **different perimeters** — like these:

These shapes both have an area of 10 square units.



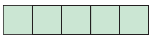
This one has a perimeter of 22 units...



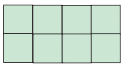
...but this one has a perimeter of 14 units.

And when given a **set perimeter**, you can draw shapes with **different areas** — like these:

These shapes both have a perimeter of 12 units.



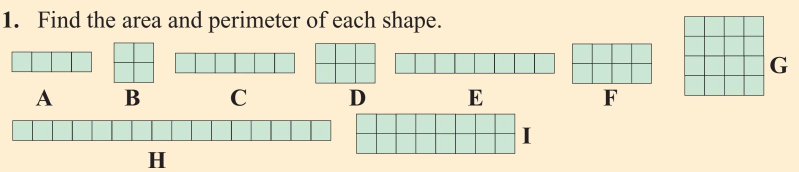
This one has an area of 5 square units...



...but this one has an area of 8 square units.

Exercises

1. Find the area and the perimeter of each shape.



2. Look at the areas and perimeters of the following sets of shapes. What do you notice about them? Which type of shape maximizes the perimeter in each set?
   A and B   C and D   E and F   G, H and I **They have the same areas but different perimeters. The longest, thinnest shapes have the greatest perimeters.**

Use only whole-number dimensions for Exercises 3–6.

3. Draw three different rectangles with areas of 12 square units that all have different perimeters. What are the dimensions of the rectangle that has the largest perimeter? **1 unit by 12 units. See below for possible shapes.**
4. Draw the rectangle with an area of 20 square units that has the largest perimeter possible. **see below**
5. Draw two rectangles that have different areas but both have perimeters of 14 units. **see below**
6. Draw a rectangle that has a perimeter of 20 units and has the largest area possible. **see below**

Round Up

A rectangle with a big difference between its length and width measurement will have a large perimeter for its area. It works the other way for maximizing the area of a rectangle with a fixed perimeter — the closer the shape is to a square, the bigger the area will be.

Solutions:

1. **A**: Area = 4 square units, Perimeter = 10 units

B: Area = 4 square units, Perimeter = 8 units

C: Area = 6 square units, Perimeter = 14 units

D: Area = 6 square units, Perimeter = 10 units

E: Area = 8 square units, Perimeter = 18 units

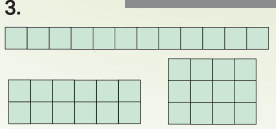
F: Area = 8 square units, Perimeter = 12 units

G: Area = 16 square units, Perimeter = 16 units

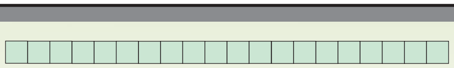
H: Area = 16 square units, Perimeter = 34 units

I: Area = 16 square units, Perimeter = 20 units

3.



4.



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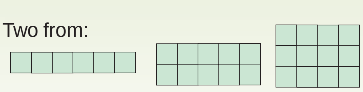

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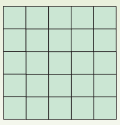


Lesson 3.1.1

Polygons and Perimeter

In preparation for the rest of the work in this Chapter, students review the names of different types of polygons, and learn how to tell if a polygon is regular. The meaning of perimeter is then reviewed and students practice using formulas for finding perimeters of shapes.

Previous Study: In grade 5 students used formulas to find the areas of triangles, parallelograms, and rectangles. In grade 4, they learned the definitions of different quadrilaterals.

Future Study: In Geometry, students will prove and use theorems involving the properties of quadrilaterals. In Chapter 7, they will calculate the total edge length of 3-D shapes.

Lesson 3.1.1

California Standards:

Measurement and Geometry 2.1
Use formulas routinely for finding the perimeter and area **of basic two-dimensional figures**, and the surface area and volume of basic three-dimensional figures, **including rectangles**, **parallelograms**, **trapezoids**, **squares**, **triangles**, circles, prisms, and cylinders.

What it means for you:

You'll learn about the names of different shapes and use formulas for finding the perimeters of some shapes.

Key words:

- polygon
- perimeter
- regular polygon
- irregular polygon
- quadrilateral
- parallelogram
- trapezoid

Don't forget:

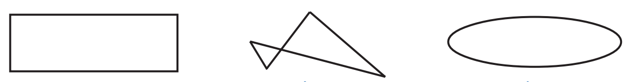
Dashes are used to show that certain lengths are equal. You might see single and double dashes. Sides with double dashes are the same length as each other, but not the same as those with single dashes.

Section 3.1
Polygons and Perimeter

*You're probably pretty familiar with a lot of **shapes** — this Lesson gives you a chance to brush up on their names, and shows you how you can use **formulas** to find the **distance around the outside** of some shapes.*

Polygons Have Straight Sides

Polygons are **flat shapes**. They're made from **straight line segments** that **never cross**. The line segments are joined **end to end**.



This **is** a polygon — This **isn't** a polygon (the lines cross) — This **isn't** a polygon (it's not made from straight lines)

The **name** of a polygon depends on how many **sides** it has.

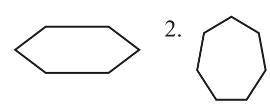
3 sides	Triangle
4 sides	Quadrilateral
5 sides	Pentagon
6 sides	Hexagon

7 sides	Heptagon
8 sides	Octagon
9 sides	Nonagon
10 sides	Decagon

Example 1

Identify each of the following shapes.

1. 2.

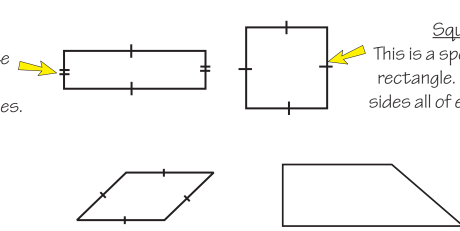


Solution

1. This shape has **6 sides**, so it's a **hexagon**.
2. This shape has **7 sides**, so it's a **heptagon**.

A Quadrilateral is a Polygon with Four Sides

A **quadrilateral** is any shape that has **four sides**.
You need to be able to name a few of them.



Rectangle
A rectangle has opposite sides of equal length. It also has four right angles.

Square
This is a special kind of rectangle. It has four sides all of equal length.

Parallelogram
Its opposite sides are parallel.

Rhombus
This is a parallelogram with all sides of the same length.

Trapezoid
This has only one pair of parallel sides.

1 Get started

Resources:

- polygons cut from heavy construction paper
- pattern blocks (construction paper polygons can be used instead)
- string
- rulers
- meter rulers/trundle wheels

Warm-up questions:

- Lesson 3.1.1 sheet

2 Teach

Concept question

"Can you find any hexagons around the room?"

Students should try to identify hexagons within the classroom.
This can be repeated with other shapes.

It's likely to be the surfaces of 3-D shapes that students pick out.
Make them aware that it is the flat surface they are referring to, not the 3-D object.

Universal access

Play a questioning game to identify shapes. Think of a shape and have students ask questions about it, such as, "Are all the angles equal?"
The questions can only be answered "yes" or "no." Keep going until the shape is identified. Students can repeat the game with a partner.

Math background

The definition of a trapezoid is a matter of some mathematical controversy — some books and mathematicians define trapezoids as having **at least** one pair of parallel sides. By this definition, parallelograms and rectangles are special cases of trapezoids.

- **Strategic Learners**
  The activity described in the Universal access section below is useful for reinforcing the concept of perimeter, and the fact that it is equivalent to the sum of the lengths of each side.

- **English Language Learners**
  Put a variety of polygons (pattern blocks or construction paper cutouts) in a paper bag. Students should work with partners, and one of them should reach into the bag and take a shape at random. They should then give as many names for it as they can (for example, regular polygon, hexagon) and count how many sides it has. Their partner should check and record their answers.

2 Teach (cont)

Guided practice
Level 1: q1–3
Level 2: q1–3
Level 3: q1–3

Universal access
Using polygons cut from heavy construction paper, ask students to measure the length of each side with a ruler. They should then lay a piece of string around the outside of the shape and measure the length of string needed.

They should write their findings in a table — prepare blank tables with the following column headings:
name of shape, number of sides, lengths of each side, total length of string going around the figure (the perimeter).

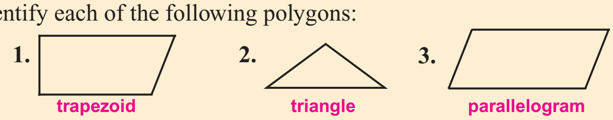
Universal access
If possible, go outside and measure the perimeter of various buildings and other things, such as the school playing field. Instill "measuring the distance around an object" as the way to find the perimeter.

Ask students to estimate the lengths and distances before measuring.

Guided practice
Level 1: q4–8
Level 2: q4–8
Level 3: q4–8

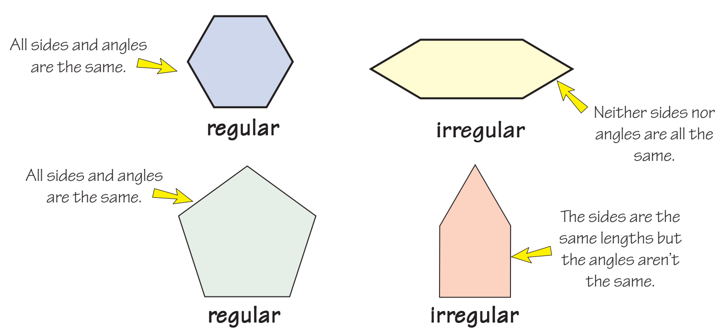
Guided Practice

Identify each of the following polygons:



Regular Polygons Have Equal Sides and Angles

Regular polygons have **equal angles**, and sides of **equal length**.
Irregular polygons don't have all sides and angles equal.



Check it out:
A rhombus has all sides of equal length, but the angles aren't all the same. So a rhombus is **always irregular**. A square, however, has all sides and all angles equal, so a square is **always regular**.

Example 2

Decide whether this polygon is regular or irregular.

Solution

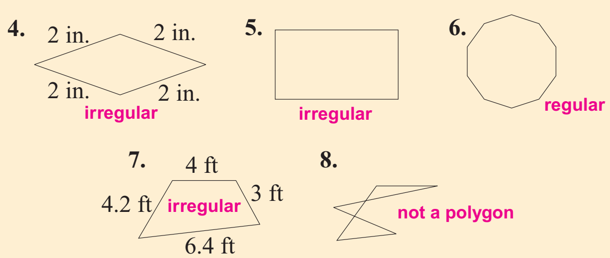
The shape has **all angles equal**.

But the lengths of the sides are **not** the same, so it is an **irregular polygon**.

Check it out:
Polygons occur a lot in everyday life — for example:

Guided Practice

Decide whether each of the following shapes is a regular polygon, an irregular polygon, or not a polygon at all.



Solutions
For worked solutions see the Solution Guide

Advanced Learners

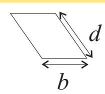
Ask students to investigate the perimeters of shapes made out of certain numbers of unit squares. They can draw the outline of the shapes on grid paper. Ask them to try to find a general rule for how the unit squares should be arranged to give the greatest and smallest possible perimeters. This can be repeated with equilateral triangles cut out of construction paper.

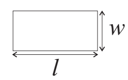
Perimeter is the Distance Around a Polygon

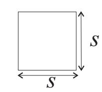
The **perimeter** is the **distance around the edge** of a shape.

You can find the perimeter by **adding** up the lengths of the sides of a polygon, but some **polygons** have a **formula** you can use to find the perimeter more quickly.

Parallelogram:	Rectangle:	Square:
$P = 2(b + d)$	$P = 2(l + w)$	$P = 4s$







Don't forget:

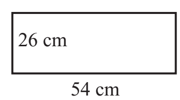
The "d" in the formula for the perimeter of a parallelogram stands for the **length of the diagonal**. Don't get this confused with the vertical height, which you'll use when you work out the area in the next Lesson.

Example 3

Find the perimeter of a rectangle of length 54 cm and width 26 cm.

Solution

Draw a diagram, and use the formula $P = 2(l + w)$.



Substitute the values for l and w, and **evaluate**.

$P = 2 \times (54 \text{ cm} + 26 \text{ cm}) = 2 \times 80 \text{ cm} = \mathbf{160 \text{ cm}}$.

Don't forget:

Perimeter is a distance and needs a unit. Check what unit the question contains and make sure your answer has the correct unit.

Guided Practice

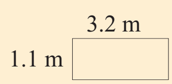
9. Find the length of the diagonal of a parallelogram that has a base of 4.6 in. and a perimeter of 14 in. 2.4 in.

10. Find the perimeter of a square of side 8.3 m. 33.2 m

Independent Practice

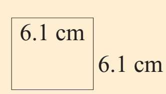
Find the perimeter of the figures in Exercises 1–4.

1.



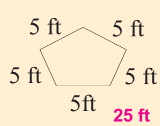
8.6 m

2.



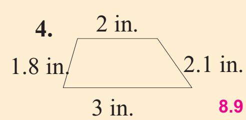
24.4 cm

3.



25 ft

4.



8.9 in.

5. Brandy wants to know how many pieces of wood she needs to mark out the boundary of her new house. How many pieces of wood will she need if each piece of wood is 50 in. long and her boundary is a square of side 650 in.? 52 pieces

Now try these:

Lesson 3.1.1 additional questions — p443

Round Up

There are a few formulas here that make it much quicker to do perimeter calculations. If you can't apply one of the formulas, remember that you can always just add up the lengths of the sides.

Solutions

For worked solutions see the Solution Guide

2 Teach (cont)

Concept question

"The perimeter of a square is 60 inches. What is the length of one of the sides?"

15 inches

Common error

Perimeter is a length measurement and needs an appropriate unit. Students often calculate the perimeter and leave off the unit. Emphasize the need to check that the correct unit is included with the answer. Make checking for units a step in the general problem-solving process (along with checking the answer is reasonable, etc.).

Guided practice

Level 1: q9–10
Level 2: q9–10
Level 3: q9–10

Independent practice

Level 1: q1–4
Level 2: q1–5
Level 3: q1–5

Additional questions

Level 1: p443 q1–6
Level 2: p443 q1–7
Level 3: p443 q1, 4–9

3 Homework

Homework Book — Lesson 3.1.1

Level 1: q1–5, 7a, 8,
Level 2: q1–8
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:

- Worksheet 31 — Perimeter
- Worksheet 34 — Polygons

Lesson 3.1.2

Areas of Polygons

In this Lesson, students use formulas to work out the areas of rectangles, squares, parallelograms, triangles, and trapezoids. They will also develop an understanding of how the area formulas for parallelograms, triangles, and trapezoids are derived.

Previous Study: In grade 5, students derived the formulas for the areas of triangles and parallelograms by comparing each with the formula for the area of a rectangle.

Future Study: Later in this Section, students use the area formulas of simple geometric shapes to find the areas of more complex shapes. They also use some of the formulas to calculate surface areas of solids.

1 Get started

Resources:
- grid paper
- construction paper
- scissors
- matching area-formula and shape cards
- home improvement information

Warm-up questions:
- Lesson 3.1.2 sheet

2 Teach

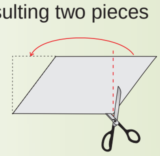
Universal access

Students can find areas of shapes by drawing the figures on grid paper and counting the number of square units. This technique works for rectangles and squares, and helps to reinforce the idea of area as a measurement in square units. Following this, the formulas for the areas of squares and rectangles can be introduced.

Next, introduce parallelograms and triangles on the grid paper. Students will see that the grid paper isn't adequate for finding the area. It is at this point that the formulas for parallelograms and triangles can be introduced.

Universal access

The parallelogram area formula is very similar to the rectangle area formula. Help students to understand why by getting them to cut through a line marking the vertical height of a parallelogram (made from construction paper). Then have them rearrange the resulting two pieces into a rectangle.



Guided practice

Level 1: q1–2
Level 2: q1–2
Level 3: q1–2

Lesson
3.1.2

Areas of Polygons

Area is the amount of space inside a shape. Like for perimeter, there are formulas for working out the areas of some polygons. You'll practice using some of them in this Lesson.

California Standards:

Measurement and Geometry 2.1
Use formulas routinely for finding the perimeter and **area of basic two-dimensional figures**, and the surface area and volume of basic three-dimensional figures, **including rectangles**, **parallelograms**, **trapezoids**, **squares**, **triangles**, circles, prisms, and cylinders.

What it means for you:
You'll use formulas to find the areas of regular shapes.

Key words:
- area
- triangle
- parallelogram
- trapezoid
- formula
- substitution

Check it out:
The area of a parallelogram is exactly the same as the area of a rectangle of the same base length and vertical height.

Check it out:
Remember — the height you use to calculate the area of a parallelogram is the vertical height and not the length of the side.

Don't forget:
Use dimensional analysis to make sure your answer has the correct units.
You're multiplying a length by a length — for example, meters × meters.
So areas should always be a square unit — such as square meters (m^2).

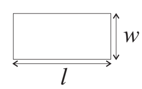
Area is the Amount of Space Inside a Shape

Area is the amount of **surface covered by a shape**.

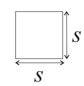
Parallelograms, **rectangles**, and **squares** all have useful formulas for finding their **areas**.

Triangles and other shapes can be a little more difficult, but there are formulas for those too — which we'll come to next.

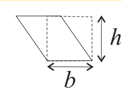
Rectangle: $A = lw$



Square: $A = s^2$

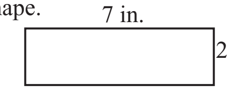


Parallelogram: $A = bh$



Example 1

Use a formula to evaluate the area of this shape.



Solution

Use the formula for the area of a rectangle.
Substitute in the values given in the question to evaluate the area.

$A = lw = 7 \text{ in.} \times 2 \text{ in.} = \mathbf{14 \text{ in}^2}$

You can also **rearrange** the formulas to find a **missing length**:

Example 2

Find the height of a parallelogram of area 42 cm^2 and base length 7 cm.

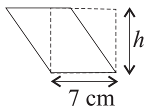
Solution

Rearrange the formula for the area of a parallelogram, and substitute.

$A = bh$

$\frac{A}{b} = \frac{b}{b}h = h$

$h = \frac{42}{7} = 6 \text{ cm}$



Guided Practice

1. Find the area of a square of side 2.4 m. 5.76 m^2

2. Find the length of a rectangle if it has area 30 in^2, and width 5 in. 6 in.

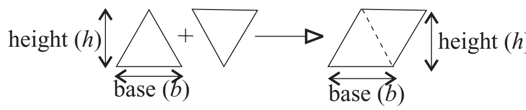
Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  The approach of drawing shapes on grid paper and counting unit squares to find their area (described on the previous page) is especially useful for strategic learners.

- **English Language Learners**
  Make matching card sets of area formulas and the names of common figures (for example, "A = ½ *bh*" on one card, and "Area of a triangle" on the matching card). Get students to play card matching games with a partner.

The Area of a Triangle is Half that of a Parallelogram

The area of a **triangle** is **half** the area of a **parallelogram** that has the **same base length** and **vertical height**.



$$\text{Area of triangle} = \frac{1}{2} \cdot \text{area of parallelogram}$$
$$= \frac{1}{2}(\text{base} \times \text{height})$$
$$= \frac{1}{2}bh$$

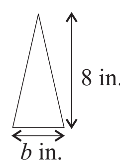
$$\boxed{A = \frac{1}{2}bh}$$

Don't forget:
The height you use for working out the area of a triangle is the **vertical height** and **not** the length of a side. This vertical height is often called the **altitude**.

Don't forget:
Multiplying by $\frac{1}{2}$ is the same as dividing by 2.

Example 3

Find the base length of the triangle opposite if it has an area of 20 in² and a height of 8 in.



Solution

Rearrange the formula for the area to give an expression for the base length of the triangle.

$$A = \frac{1}{2}bh$$

$2A = bh$ — **Multiply both sides by 2**

$\frac{2A}{h} = \frac{h}{h}b = b$. So $b = \frac{2A}{h}$ — **Divide both sides by the height (h)**

Now **substitute** in the values and **evaluate** to give the base length.
$b = (2 \times 20) \div 8 =$ **5 in.**

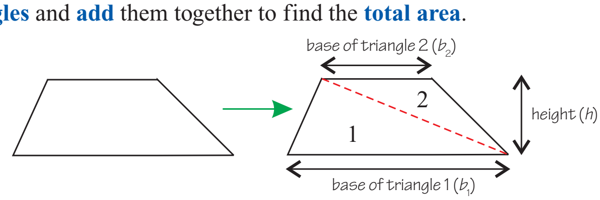
Guided Practice

3. Find the area of a triangle of base length 3 ft and height 4.5 ft. **6.75 ft²**

4. Find the base length of a triangle with height 50 m and area 400 m². **16 m**

Break a Trapezoid into Parts to Find its Area

The most straightforward way to find the area of a trapezoid is to **split it up** into **two triangles**. You then have to work out the **area of both triangles** and **add** them together to find the **total area**.



Notice that both triangles have the **same height** but **different bases**.

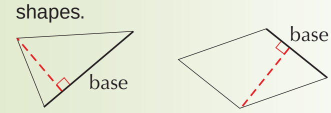
Check it out:
Splitting the trapezoid into two triangles isn't the only way — it's just the easiest. You will get the same answer if you split it into two triangles and a rectangle, for example.

2 Teach (cont)

Common error
Students will often use the slanted height to calculate the area of a parallelogram or triangle, rather than the vertical height at 90° to the base.

Give students practice at identifying the "vertical" heights of triangles and parallelograms that don't have horizontal bases.

For example mark the heights you'd use to calculate the area on these shapes.



Additional examples
Find the area of the given figure.

1) Parallelogram
Base length = 14 inches,
height = 6 inches

84 in²

2) Square
Side length = 11 meters

121 m²

3) Rectangle
Length = 15 inches, width = 3 inches

45 in²

4) Triangle
Base length = 18 cm, height = 7 cm

63 cm²

Guided practice
Level 1: q3–4
Level 2: q3–4
Level 3: q3–4

Solutions
For worked solutions see the Solution Guide

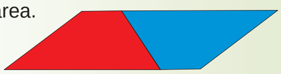
- **Advanced Learners**
  Ask students to think of examples of home improvement activities that require a knowledge of area. For example, painting a room, tiling a wall, laying a new floor, etc. Ask them to investigate and write up one home-improvement activity with a partner. For example, they should make a list of the dimensions they would need to measure. (Bring in samples with information on coverage, etc.)

2 Teach (cont)

Universal access

The derivation for the area formula of a trapezoid can be shown in a similar way to that of the triangle.

Take two copies of a trapezoid and rotate one through a half-turn. The two copies can then be lined up to form a parallelogram of double the area.



Students should then be able to see why the area formula for a trapezoid is half that of a parallelogram.

Concept question

"A trapezoid has been split into two triangles, A and B. The area of triangle A is 16 square meters. The area of triangle B is 24 square meters. What is the area of the trapezoid?"

16 m² + 24 m² = 40 m²

Guided practice

Level 1: q5–6
Level 2: q5–7
Level 3: q5–8

Independent practice

Level 1: q1–6
Level 2: q1–7
Level 3: q1–7

Additional questions

Level 1: p443 q1–7
Level 2: p443 q1–7
Level 3: p443 q1–7

3 Homework

Homework Book — Lesson 3.1.2

Level 1: q1, 2–4, 5, 7
Level 2: q1–8
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 33 — Area
- Worksheet 34 — Polygons

Check it out:
Taking out the common factor uses the distributive property that you learned about in Chapter 1 (see p8).

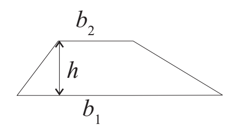
So, the area of the trapezoid is the **sum of the areas of each triangle**.

Area of trapezoid = area of Triangle 1 + area of Triangle 2

Area of trapezoid = $\frac{1}{2}b_1h + \frac{1}{2}b_2h$

Take out the **common factor** of $\frac{1}{2}h$ to give:

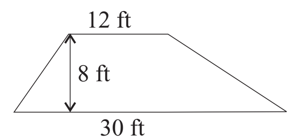
Area of trapezoid = $\frac{1}{2}h(b_1 + b_2)$



$$A = \frac{1}{2}h(b_1 + b_2)$$

Example 4

Find the area of the trapezoid shown.



Solution

Area of trapezoid = $\frac{1}{2}h(b_1 + b_2)$

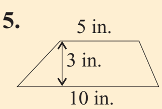
Substitute in the values given in the question and **evaluate**.

Area of trapezoid = $\frac{1}{2} \times 8\text{ ft} \times (12\text{ ft} + 30\text{ ft}) = \frac{1}{2} \times 8\text{ ft} \times 42\text{ ft} = \mathbf{168\ ft^2}$.

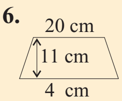
Don't forget:
It's always best to draw a diagram before attempting to solve an area question.
It's easy to make mistakes otherwise.

Guided Practice

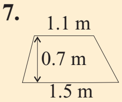
Find the areas of the trapezoids in Exercises 5–8, using the formula.

5.


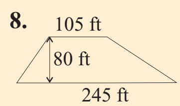
22.5 in²

6.


132 cm²

7.


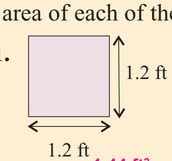
0.91 m²

8.


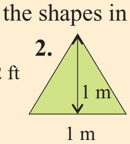
14000 ft²

Independent Practice

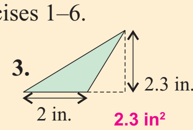
Find the area of each of the shapes in Exercises 1–6.

1.


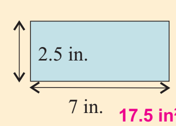
1.44 ft²

2.


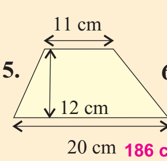
0.5 m²

3.


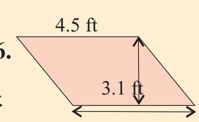
2.3 in²

4.


17.5 in²

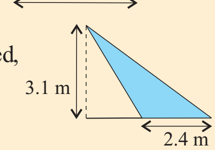
5.


186 cm²

6.


13.95 ft²

7. Miguel wants to know the area of his flower bed, shown opposite. Find the area using the correct formula. **3.72 m²**



Now try these:
Lesson 3.1.2 additional questions — p443

Round Up

Later you'll use these formulas to find the areas of irregular shapes. Make sure you practice all this stuff so that you're on track for the next few Lessons.

Solutions
For worked solutions see the Solution Guide

Lesson 3.1.3

Circles

In this Lesson, students review the terms "radius" and "diameter," and the formulas for calculating the circumference and area of a circle. They practice using these formulas, and also rearranging them to find an unknown radius or diameter.

Previous Study: In grade 6, students were introduced to the concept of the constant π, and learned the formulas for calculating the circumference and area of a circle.

Future Study: Later in this Section, students will use these formulas in finding the area and perimeter of more complex shapes. In Section 7.1, they will use the area formula to find surface areas of cylinders.

Lesson 3.1.3

Circles

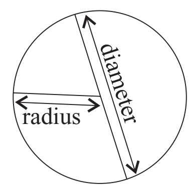
You've already met the special irrational number π or "pi". Now you're going to use it to find the circumference and area of circles.

Circles Have a Radius and a Diameter

The distance of any point on a circle from the center is called the **radius**.

The distance from one side of the circle to the other, through the center point, is called the **diameter**.

Notice the **diameter is always twice the radius**.



diameter = 2 • radius **$d = 2r$**

Example 1

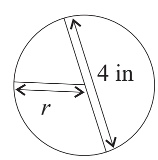
If a circle has a diameter of 4 in., what is its radius?

Solution

Use the formula: $d = 2r$.

Rearrange to give r in terms of d, so $r = \frac{d}{2}$

Substitute d from question: $r = 4 \div 2 =$ **2 in.**



Guided Practice

1. If a circle has a radius of 2 in., what is its diameter? **4 in.**
2. A circle has a diameter of 12 m. What is its radius? **6 m**

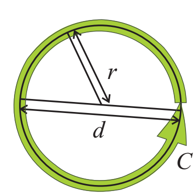
Circumference is the Perimeter of a Circle

The circumference is the **distance around the edge** of a circle. This is similar to the perimeter of a polygon.

There's a **formula** to find the circumference.

Circumference = π • diameter

$C = \pi d$



Because the diameter = 2 × radius,
Circumference = 2 × π × radius

$C = 2\pi r$

California Standards:

Measurement and Geometry 2.1
Use formulas routinely for finding the perimeter and area of basic two-dimensional figures, and the surface area and volume of basic three-dimensional figures, **including** rectangles, parallelograms, trapezoids, squares, triangles, **circles**, prisms, and cylinders.

What it means for you:

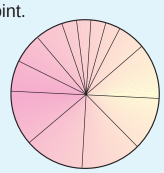
You'll find the circumference and area of circles using formulas.

Key words:

- pi (π)
- radius
- diameter
- irrational number
- circumference

Check it out:

Circles are not polygons — they don't have any straight sides.
A circle is formed from the set of all points that are an equal distance from a given center point.



Check it out:

There's a special π button on your calculator that will allow you to do very precise calculations. Otherwise, use the approximate values of 3.14 or $\frac{22}{7}$ in your calculations.

1 Get started

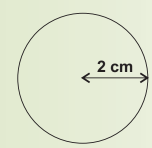
Resources:
- individual whiteboards
- string, scissors, rulers
- calculators
- cylindrical objects
- compasses

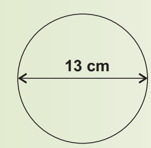
Warm-up questions:
- Lesson 3.1.3 sheet

2 Teach

Concept question

"Using a compass, draw a circle with a radius of 2 cm. What is the measure of its diameter? Draw a circle with a diameter of 13 cm. What is its radius?"





Guided practice

Level 1: q1–2
Level 2: q1–2
Level 3: q1–2

Universal access

The relationship between circumference and diameter can be explored using string, a ruler, and a calculator.

Set up some measuring stations. The stations should have objects that have circular bases — for example, compact discs or soda cans.

Students measure the distance around the outside of the circle with their string, then measure the length of the string with a ruler. They also measure the distance across the circle through the center (the diameter).

After all the measurements have been made, students divide each circumference measurement by the diameter measurement. This will lead to a discussion about π.

Solutions

For worked solutions see the Solution Guide

Strategic Learners and English Language Learners
After reviewing circle terms, use whiteboards to make quick checks of understanding. Ask students to draw a circle, label the center O, draw a radius AO, and a diameter AOB. Outline the circumference with a dark line, then shade the area.

Because both formulas for circumference and area involve π, students may need help distinguishing between the formulas. Use the "take notes, make notes" approach to review the formulas for the area and circumference of circles. Check the notes students add. For instance, they might write, "Area measures square units," or "The formula for the area of a circle has radius squared in it."

2 Teach (cont)

Additional examples

Find the circumference of the circle to the nearest hundredth.

1. Radius = 15 inches

94.25 inches

2. Diameter = 56.8 meters

178.44 meters

3. Radius = 43.62 inches

274.07 inches

Guided practice

Level 1: q3–5
Level 2: q3–7
Level 3: q3–8

Math background

π isn't equal to 3.14 or $\frac{22}{7}$. These numbers are only approximations of π that are used for the purposes of calculation. π is an irrational number, so it can't be written exactly — so it's not really correct to write $\pi = 3.14$. It's better to use the "approximately equal to" sign and write $\pi \approx 3.14$.

Common error

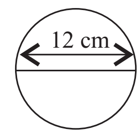
Students often use diameter measurements in formulas instead of radius measurements.

Encourage students to write out the correct formula each time they solve a problem, then substitute carefully for the radius or diameter.

Check it out:
π is the **ratio** between the circumference and the diameter of a circle.
For any size of circle:
circumference ÷ diameter = π.

Example 2

Find the circumference of the circle below. Use the approximation, $\pi \approx 3.14$.



Solution

Use the formula that has diameter in it.

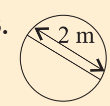
Substitute in the values and evaluate with your calculator.

$C = \pi d \approx 3.14 \times 12 \text{ cm} = 37.68 \text{ cm} \approx$ **37.7 cm**.

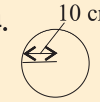
Check it out:
There are two formulas for the circumference. If you're given the radius in the question, use $C = 2\pi r$; if you're given the diameter, use $C = \pi d$.

Guided Practice

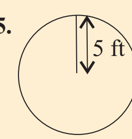
Find the circumference of the circles in Exercises 3–6.

3.


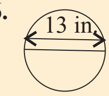
6.28 m

4.


62.8 cm

5.


31.4 ft

6.


40.8 in.

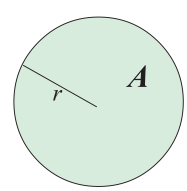
7. Find the radius of a circle that has a circumference of 56 ft. **8.91 ft**

8. Find the diameter of a circle that has a circumference of 7 m. **2.23 m**

The Area of a Circle Involves π Too

The **area** of a circle is the **amount of surface it covers**.

The area of a circle is **related to π** — just like the circumference. There's a **formula** for it:



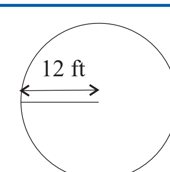
Area = π • (radius)²

$$A = \pi r^2$$

Check it out:
π is the **ratio** between the area and the square of the radius.
For any size of circle:
area ÷ (radius)^2 = π

Example 3

Find the area of the circle opposite, using $\pi \approx 3.14$.



Solution

Use the formula: $A = \pi r^2$

Substitute in the values and evaluate the area

$A \approx 3.14 \times (12 \text{ ft})^2 = 3.14 \times 144 \text{ ft}^2 = 452.16 \text{ ft}^2 \approx$ **452 ft²**

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to make a graph of the circumference of a circle as a function of its diameter. They will see that all the points lie roughly along a straight line — the slope of that line is π. This allows them to contemplate the relationship between the circumference, diameter, and π.

If you know the **area** of a circle you can calculate its **radius**:

Example 4

The area of a circle is 200 cm^2. What is the radius of this circle? Use $\pi \approx 3.14$.

Solution

The question gives the area, and you need to find the radius. This means rearranging the formula for the area of a circle to get r by itself.

$A = \pi r^2$

$\frac{A}{\pi} = r^2$ — **Divide both sides by π**

$r = \sqrt{\frac{A}{\pi}}$ — **Take the square root of both sides**

$r = \sqrt{\frac{200}{\pi}} \approx \sqrt{63.7} \approx$ **8 cm** — **Substitute in the values and find the radius**

Don't forget:
$r^2 = r \times r$, not $2r$.

Guided Practice

9. Find the area of a circle that has a diameter of 12 in. **113 in^2**

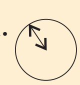
10. Find the area of a circle that has a radius of 5 m. **78.5 m^2**

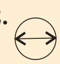
11. If a circle has an area of 45 in^2, what is its radius? **3.78 in.**

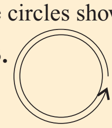
Don't forget:
You don't use the diameter in area problems, so it's useful to make sure you find the radius before attempting any other calculations. Remember, the radius is half the diameter.

Independent Practice

In Exercises 1–3, find the area of the circles shown.

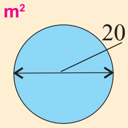
1.  $r = 8$ m **201 m^2**

2.  $d = 2$ in. **3.14 in^2**

3.  $C = 45$ cm **161 cm^2**

4. Find the circumference and area of a circle with a diameter of 6 m. **circumference ≈ 18.85 m, area ≈ 28.3 m^2**

5. Lakesha has measured the diameter of her new whirlpool bath as 20 ft. Find its surface area. **314 ft^2**



6. Find the circumference of the base of a glass with a 1.5 inch radius. **9.42 in.**

7. Find the area of the base of the glass in Exercise 6. **7.07 in^2**

8. A circle has an area of 36 cm^2. Find its radius and circumference. **r = 3.39 cm and C = 21.3 cm**

Now try these:
Lesson 3.1.3 additional questions — p443

Round Up

This Lesson is all about circles, and how to find their circumferences and areas. There are a few formulas that you need to master — make sure you practice rearranging them.

2 Teach (cont)

Concept question
"Which circle has a larger circumference? Explain your answer."

Circle A	*Circle B*
Radius = 14 in.	*Diameter = 20 in.*

Circle A has a larger circumference since its diameter is 28 inches. This is 8 inches longer than the diameter of circle B. As the circumference is obtained by multiplying each of these numbers by the same positive number greater than 1, the larger diameter results in the larger circumference.

Guided practice
Level 1: q9–10
Level 2: q9–10
Level 3: q9–11

Independent practice
Level 1: q1–5
Level 2: q1–7
Level 3: q1–8

Additional questions
Level 1: p443 q1–5, 8–10
Level 2: p443 q3–12
Level 3: p443 q3–12

3 Homework

Homework Book — Lesson 3.1.3
Level 1: q1, 2, 4, 6, 8
Level 2: q1–9
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 32 — Circles

Solutions
For worked solutions see the Solution Guide

Lesson 3.1.4

Areas of Complex Shapes

Students now use the area formulas for rectangles and triangles to calculate the areas of complex shapes. They learn to recognize situations where it's best to add areas together, and situations where it's best to subtract one area from another.

Previous Study: In grade 4, students used area formulas for rectangles and squares to find the areas of more complex figures.

Future Study: In Section 7.1 students will find the surface areas of solids by calculating the areas of their nets. They will also find the volumes and total edge lengths of complex 3-D figures.

1 Get started

Resources:
- geometric shapes cut from heavy construction paper (rectangles, squares, and triangles)
- graph paper

Teacher Resources CD-ROM
- Tangrams

Warm-up questions:
- Lesson 3.1.4 sheet

2 Teach

Universal access

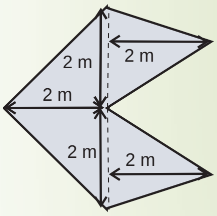
Give each student two rectangles, two squares, and two triangles. Make sure that they are all of different sizes. Encourage the students to make different irregular shapes with the shapes provided.

After this exploratory time, students should make irregular shapes with the manipulatives, trace around the outside, and hand the tracing to a partner. Their partner should then try to figure out the possible shapes used.

The activity can be extended by having students work out the areas for the irregular shapes.

Additional example

Find the area of the shape below.



8 m²

Lesson 3.1.4

Areas of Complex Shapes

California Standards:

Measurement and Geometry 2.2
Estimate and compute the area of more complex or irregular two- and three-**dimensional figures by breaking the figures down into more basic geometric objects**.

What it means for you:

You'll use the area formulas for regular shapes to find the areas of more complex shapes.

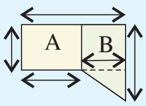
Key words:
- complex shape
- addition
- subtraction

Don't forget:

Areas should always have a squared unit. Check what it should be and make sure your answer includes it.

Check it out:

You could also have split the shape into a rectangle and a trapezoid.



You get the same answer however you split the shape.

You've practiced finding the areas of regular shapes. Now you're going to use what you've learned to find areas of more complex shapes.

Complex Shapes Can Be Broken into Parts

There are no easy formulas for finding the areas of **complex shapes**. However, complex shapes are often made up from **simpler shapes** that you know how to find the area of.

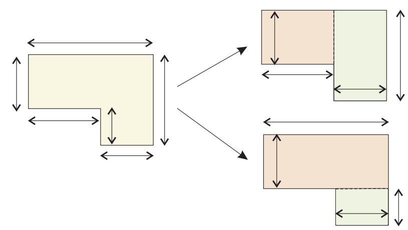
To find the area of a complex shape you:

1) **Break it up** into shapes that you know how to find the area of.

2) Find the area of each part **separately**.

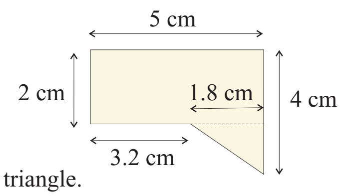
3) **Add the areas** of each part together to get the **total area**.

Shapes can often be broken up in **different ways**. Whichever way you choose, you'll get the **same total area**.



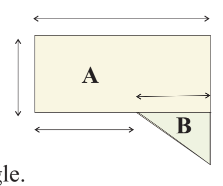
Example 1

Find the area of this shape.



Solution

Split the shape into a rectangle and a triangle.



Area A is a rectangle.
Area A = bh = 5 cm × 2 cm = 10 cm².

Area B is a triangle.

Area B = $\frac{1}{2}bh = \frac{1}{2}$ × 2 cm × 1.8 cm = 1.8 cm².

Total area = area A + area B = 10 cm² + 1.8 cm² = **11.8 cm²**

- **Strategic Learners**
  The Universal access activity described on the previous page is very suitable for strategic learners. It involves building up irregular shapes using manipulatives.

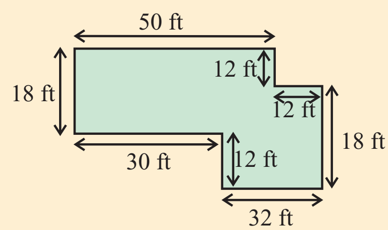
- **English Language Learners**
  Show an irregular figure on the overhead or board and have students copy the figure. Then ask the students to break it into at least two simple shapes. They should write the names and area formulas for the smaller shapes, and check their work with a partner.

Don't forget:

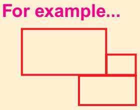
It's easy to count some pieces of the shape twice. Always draw the shape split into its parts first so you know exactly what you're dealing with.

Guided Practice

1. Find the area of the complex shape below.



For example...



1356 ft²

Check it out:

Most problems can be solved by either addition or subtraction of areas. Use whichever one looks simpler.

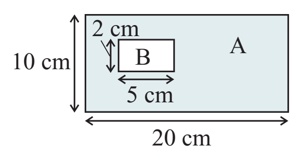
You Can Find Areas by Subtraction Too

So far we've looked at complex shapes where you **add together** the areas of the different parts.

For some shapes, it's easiest to find the area of a **larger shape** and **subtract** the area of a **smaller shape**.

Example 2

Find the shaded area of this shape.



Solution

First calculate the area of rectangle A, then subtract the area of rectangle B.

Area A = $lw = 20 \times 10 = 200$ cm²

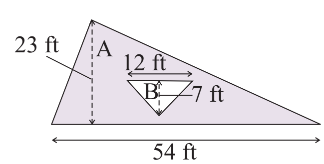
Area B = $lw = 5 \times 2 = 10$ cm²

Total area = area A – area B = 200 cm² – 10 cm² = **190 cm²**

Since there are **many stages** to these questions, always **explain** what you're doing and set your work out **clearly**.

Example 3

Find the shaded area of this shape.



Solution

First calculate the area of triangle A, then subtract the area of triangle B.

Area A = $\frac{1}{2}bh = \frac{1}{2} \times 54 \text{ ft} \times 23 \text{ ft} = 621 \text{ ft}^2$

Area B = $\frac{1}{2}bh = \frac{1}{2} \times 12 \text{ ft} \times 7 \text{ ft} = 42 \text{ ft}^2$

Total area = area A – area B = 621 ft² – 42 ft² = **579 ft²**

2 Teach (cont)

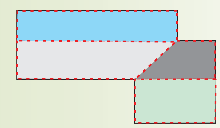
Guided practice

Level 1: q1
Level 2: q1
Level 3: q1

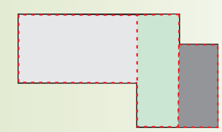
Concept question

"Suggest another way that you could split up the shape in Guided Practice Exercise 1."

Some examples are:
two trapezoids and two rectangles:



or, three rectangles:



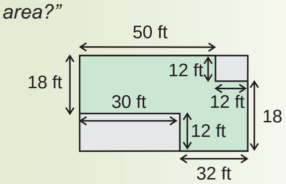
Common error

Students often have difficulty finding lengths of line segments that aren't directly given in the diagram.

Encourage students to mark horizontal and vertical lengths onto the diagram as they work them out, and then check that they look correct.

Concept question

"Look again at Guided Practice Exercise 1. How could you use the subtraction method to calculate the area?"



Area = area of whole rectangle – area of grey rectangles

Area of whole rectangle = (50 + 12) × (18 + 12) = 1860 ft²

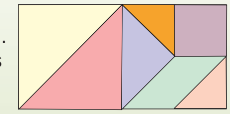
Area of grey rectangles = (12 × 12) + (12 × 30) = 144 + 360 = 504 ft²

Area = 1860 – 504 = 1356 ft²

Solutions

For worked solutions see the Solution Guide

● **Advanced Learners**
Tangrams are a good additional activity. Provide students with the Tangrams sheet from the **Teacher Resources CD-ROM**. Then ask them to work out the area of each piece. They can use this information to work out the areas of complex shapes made from tangram pieces.



2 Teach (cont)

Guided practice
Level 1: q2–3
Level 2: q2–4
Level 3: q2–4

Independent practice
Level 1: q1–2, 6
Level 2: q1–3, 6–7
Level 3: q1–7

Additional questions
Level 1: p444 q1–2, 4
Level 2: p444 q1–6
Level 3: p444 q1–7

3 Homework

Homework Book — Lesson 3.1.4
Level 1: q1a, 2a, 5a, 6
Level 2: q1, 2, 4–6
Level 3: q1–7

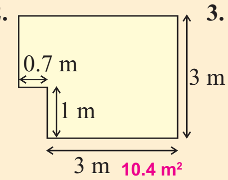
4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 33 — Area

Guided Practice

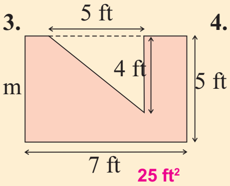
Use subtraction to find the areas of the shapes in Exercises 2–4.

2.



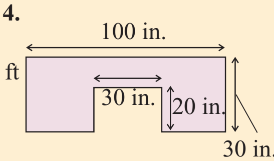
10.4 m²

3.



25 ft²

4.

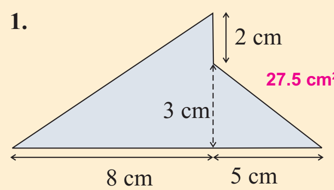


2400 in²

Independent Practice

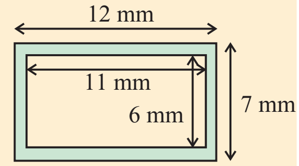
Use either addition or subtraction to find the areas of the following shapes.

1.



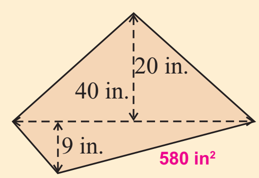
27.5 cm²

2.



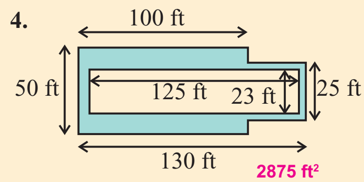
18 mm²

3.



580 in²

4.

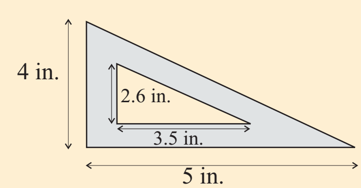


2875 ft²

5. Damion needs his window frame replacing. If the outside edge of the frame is a rectangle measuring 3 ft × 5 ft and the pane of glass inside is a rectangle measuring 2.6 ft by 4.5 ft, what is the total area of the frame that Damion needs? 3.3 ft²

6. Aisha has a decking area in her backyard. Find its area, if the deck is made from six isosceles triangles of base 4 m and height 5 m.
60 m²

7. Find the area of the metal bracket opposite.
5.45 in²



Now try these:
Lesson 3.1.4 additional questions — p444

Round Up

You can find the areas of complex shapes by splitting them up into shapes you know formulas for — squares, rectangles, triangles, trapezoids, parallelograms... Take care to include every piece though.

Solutions
For worked solutions see the Solution Guide

Lesson 3.1.5 More Complex Shapes

In this Lesson, students find the areas of complex shapes that include sectors of circles. They also find the areas of complex shapes that can most conveniently be broken down into trapezoids. Perimeter is also reviewed, using the context of complex shapes.

Previous Study: In the previous Lesson, students found the areas of complex shapes by breaking them up. Perimeter, and the area and circumference of a circle were also covered earlier in this Section.

Future Study: In Section 7.1 students will find the surface areas of solids by calculating the areas of their nets. They also find the volumes and total edge lengths of complex 3-D figures.

Lesson 3.1.5

More Complex Shapes

California Standards:

Measurement and Geometry 2.2
Estimate and compute the area of more complex or irregular two- and three-**dimensional figures by breaking the figures down into more basic geometric objects**.

What it means for you:

You'll find the areas of complex shapes by using the area formulas for simple shapes, including those for circles and trapezoids, and you'll also find perimeters of complex shapes.

Key words:

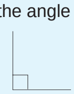
- sector
- semicircle
- circle
- complex shapes
- trapezoid

Don't forget:

If you're given the diameter of a circle in an area question, you need to halve it to find the radius before using the area formula.

Don't forget:

A little square in a corner means that the angle is a right angle.



In the last Lesson you found the areas of complex shapes by breaking them down into rectangles and triangles. Complex shapes can sometimes be broken down into other shapes — such as parts of circles or trapezoids. That's what you'll practice in this Lesson. You'll also look at finding the perimeters of complex shapes.

Complex Shapes Can Contain Circles

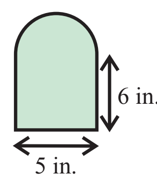
Some complex shapes involve **circles**, or **fractions of circles**.

To calculate the area, you first have to decide what **fraction** of the full circle is in the shape — for example, a half or a quarter.

Once you know what fraction of the circle you want, find the area of the **whole circle**, and then **multiply** that area by the fraction of the circle in the shape. For example, **a semicircle has half the area of a full circle**.

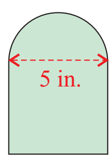
Example 1

Find the area of the complex shape opposite.



Solution

Split the shape into a **semicircle** and a **rectangle**.



The semicircle has half the area of a full circle, and has a diameter of 5 in. This means its radius is 2.5 in.

Area of full circle = $\pi r^2 = \pi \times (2.5 \text{ in})^2 = \pi \times 6.25 = 19.6 \text{ in}^2$

Area of semicircle = $0.5 \times$ area of full circle

$= 0.5 \times 19.6 \text{ in}^2 = 9.8 \text{ in}^2$

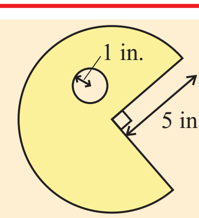
Area of rectangle = $lw = 5 \text{ in.} \times 6 \text{ in.} = 30 \text{ in}^2$

Total area = area of semicircle + area of rectangle = $9.8 \text{ in}^2 + 30 \text{ in}^2$

Total area = **39.8 in^2**

Guided Practice

1. Find the area of the complex shape opposite. 55.8 in²



1 Get started

Resources:
- construction paper shapes (including circles, semicircles, quarter-circles, trapezoids, parallelograms, triangles, squares, and rectangles)
- cards with shapes, and corresponding area and perimeter formulas on them

Teacher Resources CD-ROM
- Tangrams

Warm-up questions:
- Lesson 3.1.5 sheet

2 Teach

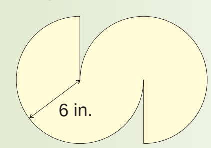
Concept question

"Write an expression for the area of one-third of a circle with a diameter of x."

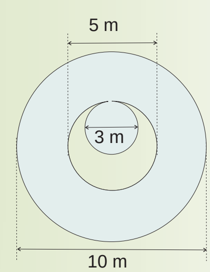
$\frac{1}{3}\pi\left(\frac{x}{2}\right)^2 = \frac{1}{3}\pi\left(\frac{x^2}{4}\right) = \frac{\pi x^2}{12}$

Additional examples

Find the areas of the following shapes:



169.6 in²



66.0 m²

Guided practice

Level 1: q1
Level 2: q1
Level 3: q1

Solutions
For worked solutions see the Solution Guide

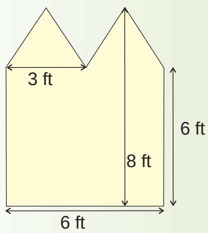
- **Strategic Learners**
  Ask students to use construction paper shapes to form given complex shapes that are made up from, say, circles, semicircles, quarter-circles, or trapezoids, in addition to squares and rectangles.

- **English Language Learners**
  To review formulas for area and perimeter, allow students to play **matching card games.** Include area formulas for trapezoids, parallelograms, and circles, as well as rectangles and triangles.

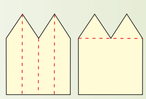
2 Teach (cont)

Concept question

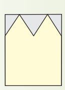
"Explain how you could find the area of this shape using:
1) the addition method
2) the subtraction method"



1) Split the shape into either four identical trapezoids, or a square and two triangles.

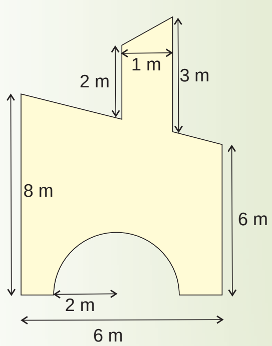


2) Find the area of the 6 ft by 8 ft rectangle and subtract the area of the triangles.



Additional example

Find the area of this shape:



38.2 m²

Guided practice

Level 1: q2
Level 2: q2
Level 3: q2

Look Out for Trapezoids and Parallelograms Too

Some complex shapes need to be broken into **more parts** than others. Often it's **not obvious** what the **best** way to break them up is.

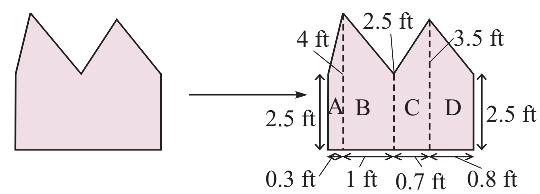
If you only look for triangles and rectangles you could miss the easiest way to solve the problem — look out for **trapezoids** and **parallelograms** too.

Don't forget:

If you see a problem that looks too tricky, don't panic! Break it down into small manageable chunks, instead of attempting the whole thing right away.

Example 2

Find the area of the complex shape below by breaking it down into trapezoids.



Solution

The equation for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$.

Area of trapezoid A = $\frac{1}{2} \times 0.3 \text{ ft} \times (2.5 \text{ ft} + 4 \text{ ft}) = 0.975 \text{ ft}^2$

Area of trapezoid B = $\frac{1}{2} \times 1 \text{ ft} \times (4 \text{ ft} + 2.5 \text{ ft}) = 3.25 \text{ ft}^2$

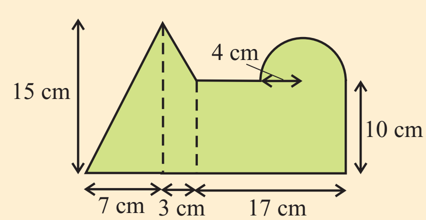
Area of trapezoid C = $\frac{1}{2} \times 0.7 \text{ ft} \times (2.5 \text{ ft} + 3.5 \text{ ft}) = 2.1 \text{ ft}^2$

Area of trapezoid D = $\frac{1}{2} \times 0.8 \text{ ft} \times (3.5 \text{ ft} + 2.5 \text{ ft}) = 2.4 \text{ ft}^2$

Total area = $0.975 \text{ ft}^2 + 3.25 \text{ ft}^2 + 2.1 \text{ ft}^2 + 2.4 \text{ ft}^2 =$ **8.725 ft^2**

Guided Practice

2. Find the area of the complex shape below.



285 cm²

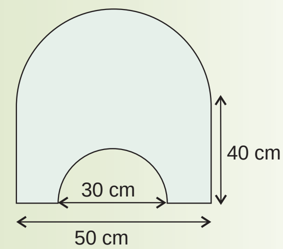
Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to create figures such as a cat, a ship, or a person using the Tangrams from the **Teacher Resources CD-ROM**. They should give the outline of their figure to their partner, who should try to work out which pieces were used. The perimeter and area of the figures can then be calculated.

2 Teach (cont)

Additional example

The rug below fits around a washbasin pedestal. What length of fringe is needed to go around its perimeter? Give your answer to the nearest centimeter.



226 cm

Common error

Students will often use an incorrect formula for the area or circumference of a circle. They will also often use the radius instead of the diameter, or the diameter instead of the radius. Students should write the correct formula down before starting their calculation.

Guided practice

Level 1: q3
Level 2: q3–4
Level 3: q3–4

Finding the Perimeter of Complex Shapes

The perimeter is the **distance** around the edge of a shape.

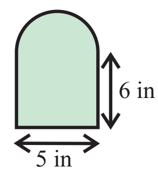
To find the perimeter of a complex shape, you need to **add the lengths of each side**.

It's likely that you won't be given the lengths of **all** the sides, so you may need to **find** some lengths yourself — for example, the circumference of a semicircle, which is half the circumference of a full circle.

Don't forget:
Remember — don't include the lengths of sides that don't form the outline of the final shape. For example, the fourth side of the rectangle isn't included here.

Example 3

Find the perimeter of the complex shape opposite.



Solution

The circumference of a semicircle is half the circumference of a full circle of the same radius.

Circumference of semicircle = $\frac{1}{2}$ • circumference of full circle

$= \frac{1}{2}\pi d = \frac{1}{2} \times \pi \times 5$ in. = 7.85 in.

Perimeter of 3 sides of rectangle = 6 in. + 5 in. + 6 in. = 17 in.

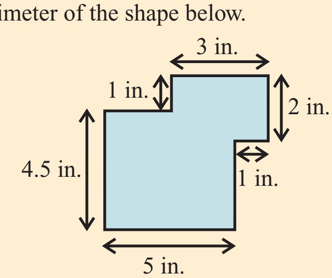
Total perimeter = circumference of semicircle + perimeter of rectangle.

Total perimeter = 7.85 in. + 17 in. = **24.85 in.**

Guided Practice

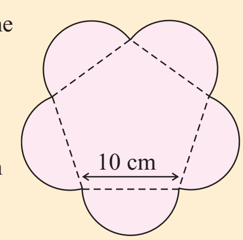
Don't forget:
Perimeter has units of length (for example, feet). Area has units of length squared (for example, ft^2).

3. Find the perimeter of the shape below.



23 in.

4. Davina has made a flower shape out of some wood by cutting out a regular pentagon and sticking a semicircle to each side of the pentagon, as shown. She wants to make a border for her shape by sticking some ribbon all around the edge. Find the length of ribbon that Davina will need.



78.5 cm

Solutions

For worked solutions see the Solution Guide

2 Teach (cont)

Independent practice
Level 1: q1–3
Level 2: q1–4
Level 3: q1–5

Common error
Students often miss an edge when calculating a perimeter.
Encourage them to mark each edge as they write down its length.

Additional questions
Level 1: p444 q1–4
Level 2: p444 q1–7
Level 3: p444 q1–7

3 Homework

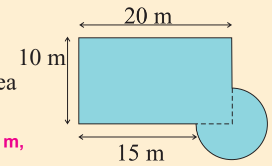
Homework Book — Lesson 3.1.5
Level 1: q1–3, 6, 8
Level 2: q1–5, 8, 9
Level 3: q1–9

4 Skills Review

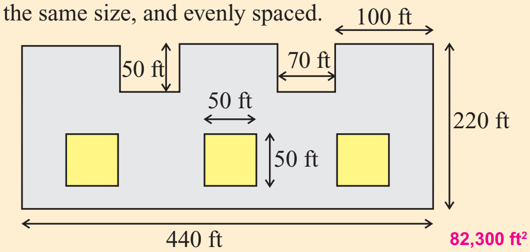
Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 31 — Perimeter
- Worksheet 32 — Circles
- Worksheet 33 — Area

Independent Practice

1. Kia's swimming pool is rectangular in shape with a circular wading pool at one corner, as shown. Find the total surface area of Kia's pool and the distance around the edge. **Surface area = 259 m², Perimeter = 73.6 m,**

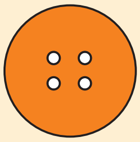


2. Find the area of the face of the castle below (don't include the windows). Assume that all the windows are the same size and that all the turrets are the same size, and evenly spaced.



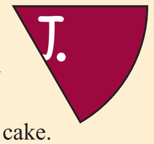
82,300 ft²

3. Find the area of the button shown below if each hole has a diameter of 0.1 in. and the button has a diameter of 1.2 in.



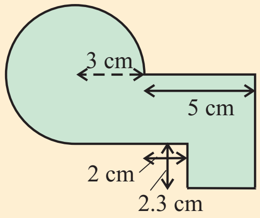
1.10 in²

4. T.J. has five friends coming to his 13th birthday party. He bakes a cake that is 12 inches in diameter. At the party, T.J. and his friends divide the cake equally between them, into identically shaped slices, as shown. Find the perimeter and area of the base of each slice of cake.



P = 18.3 in, A = 18.8 in²

5. Find the perimeter and area of the shape below.



P = 34.7 cm, A = 52.1 cm²

Check it out:
If there is a rectangle overlapping a circle, always try to find the area of the rectangle, then the area of the rest of the circle. You'll get in a mess if you try to solve it the other way around!

Now try these:
Lesson 3.1.5 additional questions — p444

Round Up

Now you know everything you need to know about finding the area and perimeter of complex shapes. The first step is to look at the shape and decide on the easiest way to break it up into simple shapes.

Solutions
For worked solutions see the Solution Guide

Exploration — Coordinate 4-in-a-row

Purpose of the Exploration

The purpose of the game is to introduce the coordinate plane to students, and to review how to plot points using (x, y) coordinates. A secondary concept is to have students see that a series of points can form a straight line. The secondary concept will be useful when students are asked to graph linear equations.

Section 3.2 introduction — an exploration into:

Coordinate 4-in-a-row

Ordered pairs are used to represent points on the coordinate plane. The goal of this game is to get as many points in a line as possible — the lines can be vertical, horizontal, or diagonal. You score for each row of four or more points that you make — the scoring system is below.

Scoring System	
4-in-a-row	1 point
5-in-a-row	2 points
6-in-a-row (or more)	3 points

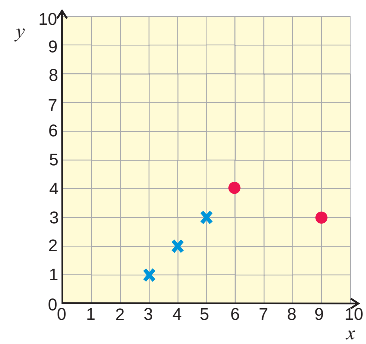
Example

You need part of a coordinate plane, like shown.
Players **take turns** calling out coordinates.

For example:

Player 1 (×): (4, 2) — right 4, up 2.
Player 2 (●): (9, 3) — right 9, up 3.
Player 1 (×): (5, 3) — right 5, up 3.
Player 2 (●): (6, 4) — right 6, up 4.
Player 1 (×): (3, 1) — right 3, up 1.

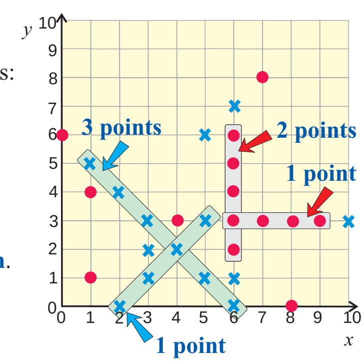
Now, Player 1 needs to get **(2, 0)** to get **4-in-a-row** and score **1 point**.
But it's Player 2's turn next, and if they choose (2, 0), they'll **block** the point.



After a while, your coordinate plane will look a bit like this:

Player 1 (×) has 4 points in total so far and
Player 2 (●) has 3 points so far.
So **Player 1** is winning at the moment.

If a player calls out a point that's already taken, or plots a different one to that which they called, they **lose their turn**.



Exercises

1. Play the game with another person. The person whose birthday is next goes first.
2. Play the game again. This time, players are allowed to pick two points in each turn.

Round Up

You can pinpoint a certain place on the coordinate plane using a pair of coordinates, and by plotting several points you can form a straight line.

Resources

- grid paper
- two different-colored markers for each pair of students
- rulers

Strategic & EL Learners

Strategic learners may benefit from playing the game on a smaller grid. A playing area that has maximum x and y values of 5 will make the game more manageable.

EL learners may get confused with the terms "horizontal", "vertical" and "diagonal." To avoid this confusion, circle sets of points to show the meaning of each term.

Universal access

Start the lesson by demonstrating how a set of coordinates is plotted in the first quadrant. Demonstrate how the game is played to the students.

After students have completed their second game, the activity can be extended by asking students to devise another variation on the game. For instance, they may choose to change the playing area to all four quadrants.

Common error

The main source of problems will be students switching the x and y coordinates. For example, a point with coordinates (2, 3) may be incorrectly graphed up two and right three. Remind students that it is across, then up or down.

Math background

Students should be familiar with the coordinate plane and the x and y-axis. They should also be able to graph a point in the first quadrant with little difficulty.

Lesson 3.2.1

Plotting Points

Plotting points and reading coordinates are reviewed in this Lesson. Students also learn to recognize which quadrant a point lies in from the signs of its coordinates.

Previous Study: In grade 5, students identified and graphed ordered pairs in the four quadrants of the coordinate plane.

Future Study: Later in this Section, students plot figures on the coordinate plane. In Geometry, students will prove theorems using coordinate geometry.

1 Get started

Resources:
- atlas, globe, or internet-based maps

Teacher Resources CD-ROM
- Coordinate Grid (or grid paper)

Warm-up questions:
- Lesson 3.2.1 sheet

2 Teach

Universal access

This is an alternative to the well-known game "Battleships" (described on the next page).

Break the class into two teams, and display a coordinate plane on the board or overhead projector.

The teams take it in turn to give the coordinates of the point they want. If the point is free then a letter representing their team is written on the point. If the point is taken, the team loses their turn. The goal is to get four points in a row, vertically, horizontally, or diagonally. Teams can try to block each other.

Concept question

"What are the coordinates of the point that is 10 units to the right of the y-axis and 6 units below the x-axis?"

(10, –6)

Lesson 3.2.1

California Standards:

Measurement and Geometry 3.2

Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

What it means for you:

You'll see how to use a grid system to plot numbered points.

Key words:
- coordinate
- x-axis
- y-axis
- quadrant

Check it out:

Coordinates are sometimes called ordered pairs.
This means that the order of the numbers matters — (1, 2) is different from (2, 1).

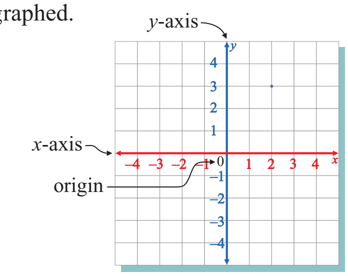
Section 3.2

Plotting Points

When you draw a graph you draw it on a coordinate plane. This is a flat grid that has a horizontal axis and a vertical axis. You can describe where any point on the plane is using a pair of numbers called coordinates.

You Plot Coordinates on a Coordinate Plane

The **coordinate plane** is a two-dimensional (flat) area where points and lines can be graphed.



The plane is formed by the intersection of a **vertical number line**, or **y-axis**, and a **horizontal number line**, or **x-axis**. They cross where they are **both equal to 0** — a point called the **origin**.

Coordinates Describe Points on the Plane

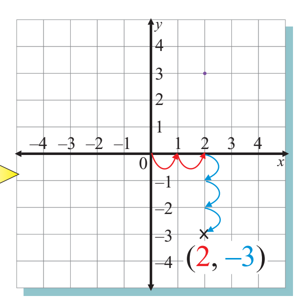
The **x and y coordinates** of a point describe where on the plane it lies. The coordinates are written as **(x, y)**.

x-coordinate → (2, –3) ← y-coordinate

When you plot points on the coordinate plane you plot them in relation to the **origin**, which has coordinates of **(0, 0)**.

- The **x-coordinate** tells you how many spaces along the **x-axis** to go.
  Negative values mean you go **left**.
  Positive values mean you go **right**.
- The **y-coordinate** tells you how many spaces up or down the **y-axis** to go.
  Positive values mean you go **up**.
  Negative values mean you go **down**.

So a point with the coordinates (2, –3) will be **two** units to the **right** of the origin, and **three** units **below**.



- **Strategic Learners**
  Get students to play the “Battleships” game in pairs. Each player draws on a coordinate plane a set of “battleships” (a “battleship” is a series of points forming a vertical or horizontal line of an agreed length — for example, (1, 2), (1, 3), (1, 4), (1, 5)). Students try to “sink” their opponent’s ships by guessing coordinate pairs, in turn. A correct guess is a “hit” — which both students record by, for example, circling the point on their grid. A ship is “sunk” when all its points have been hit.
- **English Language Learners**
  The term “plane” may be confusing. Distinguish the geometric term “plane” from the words “airplane” and “plain.”

Check it out:
A positive *x*-coordinate tells you to move right along the *x*-axis, while a negative *x*-coordinate tells you to move left along the *x*-axis.
A positive *y*-coordinate tells you to move up the *y*-axis, while a negative *y*-coordinate tells you to move down the *y*-axis.

Example 1

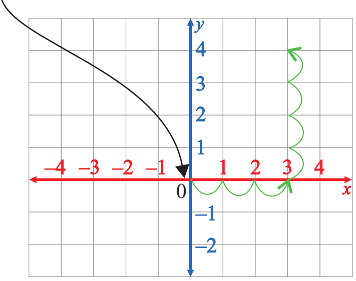
Plot the point with the coordinates (3, 4).

Solution

Step 1: start at the origin, (0, 0).

Step 2: move right along the *x*-axis 3 units.

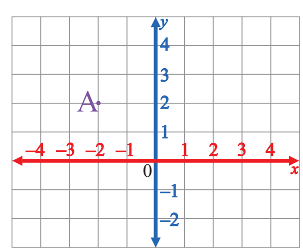
Step 3: now move straight up 4 units and plot the point.



When you are reading the coordinates of a point on a graph you can use the same idea.

Example 2

What are the coordinates of point A?



Solution

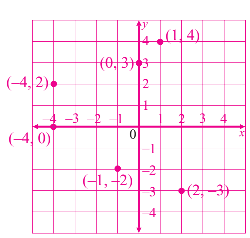
Start at (0, 0). To get to point A on the graph you need to move 2 units to the left. So the *x*-value of your coordinate is –2.
Then you need to go 2 units straight up. So the *y*-value is 2.

The coordinates of the point A are (–2, 2).

Guided Practice

Plot and label each of the coordinate pairs in Exercises 1–6 on a coordinate plane.

1. (1, 4) **2.** (2, –3)
3. (–1, –2) see left **4.** (–4, 2) see left
5. (0, 3) **6.** (–4, 0)

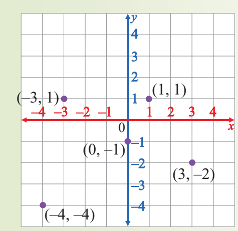


2 Teach (cont)

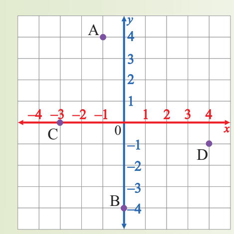
Universal access
Use the memory aid “go down the hall, then up the stairs” to help students remember to go horizontally along the *x*-axis, before going vertically.

Additional examples
Identify the points with the following coordinates.

1) (1, 1)
2) (–3, 1)
3) (3, –2)
4) (–4, –4)
5) (0, –1)



Additional examples
Give the coordinates of the points A–D.



A = (–1, 4)
B = (0, –4)
C = (–3, 0)
D = (4, –1)

Guided practice
Level 1: q1–6
Level 2: q1–6
Level 3: q1–6

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Using a globe, show how to describe a position on Earth using latitude and longitude. Have students locate points of interest using an atlas, globe, or internet-based map. Ask them to make a list of the places and their latitude/longitude coordinates.

2 Teach (cont)

Math background

In Roman numerals the letter I represents 1 and V represents 5. Two or three I's can be written in a row to represent 2 or 3.
Putting an I in front of a V indicates one less than five — so IV is 4.

(VI means one more than five, so VI is 6.)

Universal access

Divide the classroom into four equal sections with masking tape. Identify the center as the origin. Label the sections with the Roman numerals used for quadrants. Provide a sheet with a list of items in the classroom, such as the teacher's desk, pencil sharpener, door, windows, etc.

Ask students to identify which quadrant each item is in. Also ask them to provide directions to that point from the origin. The restrictions for the directions are that they must be left/right, then up/down.

A variation of this activity would be for students to move according to the directions given.

It's important to have all students facing the same direction for this activity — the grid should be oriented so that the front of the room is "up" relative to the coordinate system.

Common error

The quadrants are numbered in a counterclockwise direction on the plane. A common error is for students to identify the quadrants in a clockwise direction.

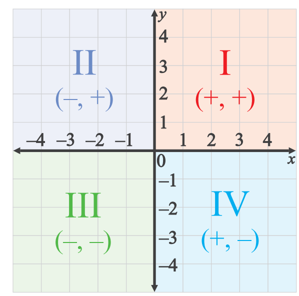
Concept question

"Write the coordinates of a point that is in the third quadrant."

Any pair of coordinates where the x and y values are both negative.

The Coordinate Plane is Divided into Four Quadrants

The ***x*-axis** and ***y*-axis** divide the coordinate plane into four sections. Each of these sections is called a **quadrant**. The quadrants are represented by Roman numerals, and are labeled **counterclockwise**.



The **signs** of the x and y values are **different** in each quadrant.
For instance, in **quadrant I** both the x and y values are **positive**.
But in **quadrant II** the x value is **negative** and the y value is **positive**.

You can tell which quadrant a point will fall in by looking at the signs of the x and y coordinates.

Check it out:
A point that is on either the x-axis or the y-axis is not in any of the quadrants.

Example 3

Which quadrant is the point (1, –4) in?

Solution

The x-value is 1. This is positive, so the point must be in quadrant I or IV.
The y-value is –4. This is negative, so the point must be in quadrant IV.

The point (1, –4) is in quadrant IV.

Example 4

Which quadrant is the point (–3, –6) in?

Solution

Both coordinates are **negative**, so **the point (–3, –6) is in quadrant III**.

Guided Practice

In Exercises 7–14 say which quadrant the point lies in.

7. (1, 3)	Quadrant I	**8.** (–2, –4)	Quadrant III
9. (7, –2)	Quadrant IV	**10.** (–3, 6)	Quadrant II
11. (–2, –2)	Quadrant III	**12.** (2, 2)	Quadrant I
13. (–1.5, 2.5)	Quadrant II	**14.** (1, –1)	Quadrant IV

Independent Practice

In Exercises 1–6 say which quadrant the point is in.

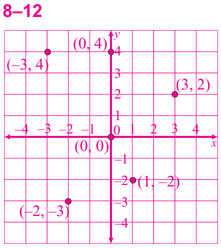
1. (1, 1)	Quadrant I	**2.** (–1, –1)	Quadrant III
3. (–1, 2)	Quadrant II	**4.** (–2, 1)	Quadrant II
5. (3, –2)	Quadrant IV	**6.** (–61, 141)	Quadrant II

7. Do the coordinate pairs (–3, 4) and (4, –3) correspond to the same point on the plane? No: the first coordinate is 3 units to the left of the origin and 4 up, the second is 4 units to the right of the origin and 3 down.

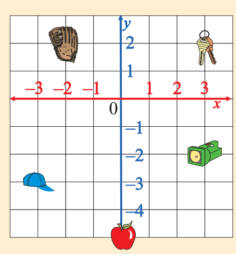
Plot each of the points in Exercises 8–13 on a coordinate plane.

8. (0, 0) **9.** (3, 2)
10. (–2, –3) see left **11.** (–3, 4) see left
12. (1, –2) **13.** (0, 4)

8–12



14. Sophie and Jorge are playing a game. Sophie marks out a coordinate plane on the beach, and buries some objects at different points. Jorge has to use the map below to find the objects.



a) What are the coordinates of the keys? (3, 2)
b) What are the coordinates of the baseball cap? (–3, –3)
c) What are the coordinates of the apple? (0, –5)
d) How many units are there between the keys and the flashlight? 4
e) How many units are there between the baseball mitt and the keys? 5

Now try these:
Lesson 3.2.1 additional questions — p445

Round Up

Coordinates allow you to describe where points are plotted — they're written as pairs of numbers, such as (1, –5). The first number tells you the horizontal or x-coordinate. The second tells you the vertical or y-coordinate. Plotting points on coordinate planes is a big part of drawing graphs, and will be used a lot in the rest of this Chapter and in the next Chapter.

2 Teach (cont)

Guided practice
Level 1: q7–12
Level 2: q7–14
Level 3: q7–14

Independent practice
Level 1: q1–4, 8–11, 14
Level 2: q1–14
Level 3: q1–14

Additional questions
Level 1: p445 q1–8
Level 2: p445 q1–11
Level 3: p445 q4–13

3 Homework

Homework Book — Lesson 3.2.1
Level 1: q1, 2, 4, 6, 8
Level 2: q1–8
Level 3: q1–8

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 28 — Graphing Linear Equations

Solutions
For worked solutions see the Solution Guide

Drawing Shapes on the Coordinate Plane

In this Lesson, students plot figures on the coordinate plane and use their knowledge of the properties of 2-D shapes to predict missing points. They then calculate lengths of line segments using absolute value, and use these lengths to find perimeters and areas.

Previous Study: In grade 5, students identified and graphed ordered pairs in the four quadrants of the coordinate plane. In Section 3.1, students calculated the area and perimeter of shapes.

Future Study: In Geometry, students will prove theorems using coordinate geometry.

1 Get started

Resources:
- internet computers

Teacher Resources CD-ROM
- Coordinate Grid (or grid paper)

Warm-up questions:
- Lesson 3.2.2 sheet

2 Teach

Universal access

Once again, the classroom can be transformed into a giant coordinate plane. Clear a section of the room of desks and other objects.
Use masking tape to create x- and y-axes.

Give students index cards that each have the coordinates of a different point. Then ask the students to move to stand at their correct positions in the coordinate plane.

Following this activity, place students in groups of four, and give each a different pair of coordinates. These coordinates should form squares, rectangles, parallelograms, and trapezoids.

As an extension activity leave one student without the coordinates of a point. Challenge students to form the shape and find the missing pair of coordinates.

Guided practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1–4

Lesson 3.2.2

Drawing Shapes on the Coordinate Plane

California Standards:
Measurement and Geometry 3.2
Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, and determine their image under translations and reflections.

What it means for you:
You'll see how to draw shapes on a grid by plotting points and joining them.

Key words:
- coordinate
- area
- perimeter

Check it out:
You have to join the points in order. So pentagon DEFGH must have D joined to E, E joined to F, F joined to G, etc... If you joined the points in a different order you'd get a completely different shape.

Check it out:
You can tell something about the shape you're drawing just from the number of coordinates you're given. If you are only given three points to plot, the shape must be a triangle. If you are given four points, then it's probably a quadrilateral (though it could be a triangle — if three of the points are in a line).

In the last Lesson you saw how to plot points on the coordinate plane. If you plot several points and then join them up, you get a shape.

You Can Make Shapes by Joining Points

You can **draw a shape** on the coordinate plane by **plotting points** and **joining them** together. The coordinates are the **corners** of the shape.

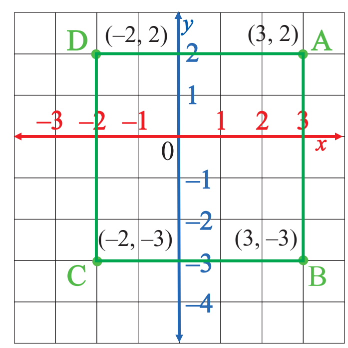
Example 1

Plot the shape ABCD on the coordinate plane, where A is (3, 2), B is (3, –3), C is (–2, –3), and D is (–2, 2).
Name the shape you have drawn.

Solution

Step 1: **Plot** and **label** the points A, B, C, and D.

Step 2: **Join** the points **in order**. So A joins to B, B joins to C, C joins to D, and D joins to A.



The shape has four sides of equal length and four right angles — so it's a **square**.

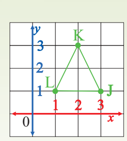
Guided Practice

Plot the shapes in Exercises 1–4 on the coordinate plane and name the shapes.

See below for coordinate planes

1. JKL J(3, 1) K(2, 3) L(1, 1) **Triangle (isosceles)**
2. EFG E(1, 2) F(2, –3) G(–3, –2) **Triangle (isosceles)**
3. RSTU R(–2, 1) S(2, 1) T(3, –1) U(–3, –1) **Trapezoid**
4. PQRS P(–1, 3) Q(3, 3) R(1, 0) S(–3, 0) **Parallelogram**

Solutions
For worked solutions see the Solution Guide

1.


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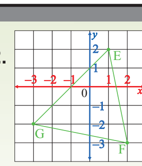

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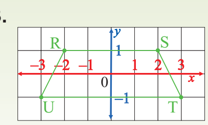

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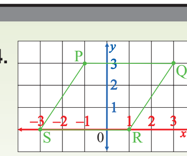


- **Strategic Learners**
  Provide students with the opportunity to play an extended version of the game “Battleships” (introduced in the previous Lesson), involving distances on the coordinate plane. One player draws a ship on the coordinate plane and their partner has five guesses to “hit” it. With each “miss” the player must give a numeric clue about how much the guess missed the ship by.

- **English Language Learners**
  Have students create designs on the coordinate plane and exchange lists of the coordinates plotted. A partner should then recreate the design from the list of points. The students should check each other’s work for accuracy.

Use the Shape’s Properties to Find Missing Points

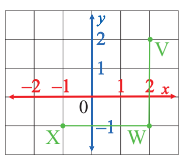
Sometimes you might be given a shape to graph with the coordinates of a corner **missing**. You can use the **properties** of the shape to work out the missing pair of coordinates.

Example 2

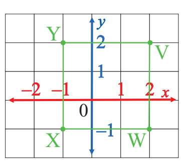
VWXY is a square on the coordinate plane, where V is (2, 2), W is (2, –1), and X is (–1, –1). What are the coordinates of point Y?

Solution

First **plot** points V, W, and X and **join** them in order.



You know that VWXY is a **square**. So it must have **four equal-length sides** that meet at **right angles**. The lines VW and WX are both 3 units long. So point Y must be 3 units left of V, and 3 units above X. Add it to the graph, and form the square.



Now read the coordinates of Y from the graph: **point Y is at (–1, 2).**

Guided Practice

In Exercises 5–8 find the missing point.

5. Square KLMN	K(–2, 2) L(2, 2) M(2, –2) N(?, ?)	**(–2, –2)**
6. Rectangle CDEF	C(1, 3) D(1, –1) E(–2, –1) F(?, ?)	**(–2, 3)**
7. Parallelogram ABCD	A(–3, –2) B(1, –2) C(3, 1) D (?, ?)	**(–1, 1)**
8. Isosceles triangle RST	R(1, –2) S(0, 1) T(?, ?)	**for example, (2, 1)**

Don’t forget:
An isosceles triangle is one that has at least two sides the same length and at least two angles of equal measure.

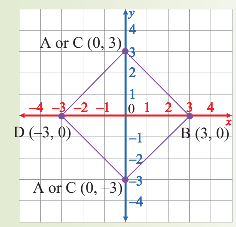
2 Teach (cont)

Common error

Sometimes students plot the vertices of a shape correctly, but then join them in the wrong order. Remind students that the vertices of a shape such as ABCD should be joined A to B, B to C, C to D, then D to A.

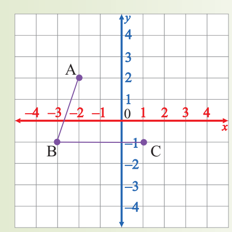
Concept question

“Point B (3, 0) is one of the vertices of square ABCD, which has its center at the origin. Plot and label the three other vertices so that the square is part of each quadrant.”



Concept question

“ABCD is a trapezoid, with side AD parallel to BC. Points A, B, and C are plotted below. What can you say about the coordinates of point D?”



The y-coordinate must be 2.
The x-coordinate must be greater than –2.

Note that using the definition that a trapezoid has exactly one pair of parallel sides, the x-coordinate can’t be 2.

Guided practice

Level 1: q5–6
Level 2: q5–7
Level 3: q5–8

Solutions

For worked solutions see the Solution Guide

● **Advanced Learners**
Challenge students to extend the coordinate system to three dimensions, using the z-axis, which is at right angles to the x- and y-axes. Ask students to give the coordinates of each vertex of a solid, such as a rectangular prism using three dimensions. As a follow-up activity, ask students to use the internet to research the legend explaining how the mathematician Descartes came up with the coordinate system. (He worked out a way to describe the position of a fly in his bedroom.)

2 Teach (cont)

Concept question

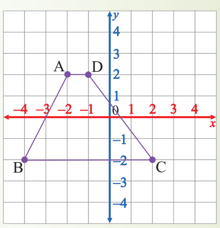
"Point F has the coordinates (4, 6). Write the coordinate of two points that are exactly 8 units away, and not in the same quadrant as F."

(–4, 6) and (4, –2)
(Other, noninteger answers are possible.)

Additional examples

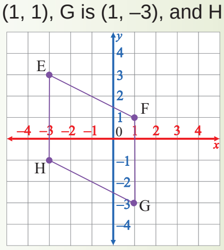
Plot the shapes below and calculate their areas.

1. ABCD, where A is (–2, 2), B is (–4, –2), C is (2, –2), and D is (–1, 2).



Area = 14 square units

2. EFGH, where E is (–3, 3), F is (1, 1), G is (1, –3), and H is (–3, –1).



Area = 16 square units

Find Lengths Using Absolute Value

Once you've **plotted** a shape on the coordinate plane you can find out its **area** or **perimeter** using the **formulas** that you saw in Section 3.1.

But first you'll need to find some **lengths** on the coordinate plane — such as the side lengths of the shapes.

You could do this by counting squares on the diagram. Another way of doing this is to use ***x*- and *y*-coordinate values**.

This is shown in the example below.

Don't forget:
If you need a reminder, then all the perimeter formulas that you might need are in Lesson 3.1.1. All the area formulas that you might need are in Lesson 3.1.2.

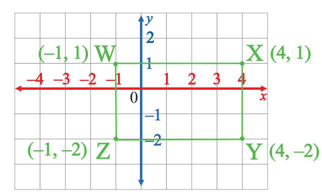
Example 3

Plot the rectangle WXYZ on the coordinate plane, where W is (–1, 1), X is (4, 1), Y is (4, –2), and Z is (–1, –2).
What are the perimeter and area of WXYZ?

Solution

First plot WXYZ on the coordinate plane.

To find the perimeter and area, you need to know the width and the length.



The sides WX and YZ give the length. They're the same, so find either. To find the **length** (l) using side WX, **subtract** the x-coordinate of W from the x-coordinate of X:

$l = |4 - (-1)| = 5$.

Sides WZ and XY give the width. They're the same, so find either. To find the **width** (w) using side XY, subtract the y-coordinate of Y from the y-coordinate of X:

$w = |1 - (-2)| = 3$.

Now just plug the length and width values into the formulas for perimeter and area.

Perimeter of WXYZ = $2(l + w) = 2(5 + 3) = 16$ units

Area of WXYZ = $l \cdot w = 5 \cdot 3 = 15$ units^2

Don't forget:
The bars around the calculation here show that the absolute value is being calculated.
This means it doesn't matter which coordinate you subtract from which — you'll always get the correct, positive answer.

Guided Practice

9. What is the perimeter of square ABCD, where A is (1, 1), B is (3, 1), C is (3, 3), and D is (1, 3)? perimeter = 8 units

10. What are the perimeter and area of rectangle EFGH, where E is (–2, 1), F is (3, 1), G is (3, –2), and H is (–2, –2)? perimeter = 16 units, area = 15 units^2

11. What is the area of triangle JKL, when J is (–1, –3), K is (3, –3), and L is (1, 0)? area = 6 units^2

Independent Practice

Plot and name the shapes in Exercises 1–3 on the coordinate plane.

1. ABC A(2, 3) B(3, –3) C(–2, –1)
2. TUVW T(4, –1) U(0, –1) V(0, 2) W(4, 2)
3. EFGH E(–2, –2) F(–1, 0) G(1, 0) H(2, –2)

See below for answers

4. Anthony is marking out a pond in his yard. It is going to be perfectly square. He is marking it out on a grid system, and has put the first three marker stakes in at (–1, –3), (–1, 1), and (3, 1). At what coordinates should he put in the last stake? (3, –3)

In Exercises 5–7, find the missing pair of coordinates.

5. Square CDEF C(1, 2) D(4, 2) E(4, –1) F(?, ?) (1, –1)
6. Rectangle TUVW T(–3, 3) U(–2, 3) V(–2, –2) W(?, ?) (–3, –2)
7. Parallelogram KLMN K(1, 0) L(–2, 0) M(–1, 2) N(?, ?) (2, 2)

8. What is the perimeter of rectangle BCDE, where B is (–2, 4), C is (3, 4), D is (3, 2), and E is (–2, 2)? Perimeter = 14 units

9. What is the area of triangle JKL, where J is (1, 2), K is (4, –1), and L is (7, –1)? Area = 4.5 units^2

10. What are the area and perimeter of rectangle PQRS, where P is (0, 0), Q is (3, 0), R is (3, –2), and S is (0, –2)? Perimeter = 10 units, Area = 6 units^2

11. A school has decided to set aside an area of their playing field as a nature reserve. A plan is made using a grid with 10-feet units. The coordinates of the corners of the area set aside are (0, 0), (4, 0), (2, –2), and (–2, –2). What area will the nature reserve cover? 800 feet^2

Now try these:
Lesson 3.2.2 additional questions — p445

Round Up

Drawing shapes on the coordinate plane just means plotting their corners from coordinates and joining them together. You can even use the known properties of some shapes to figure out the coordinates of any missing corners. Once you've got the shapes plotted, you can use the standard formulas to work out their perimeters and areas.

2 Teach (cont)

Guided practice
Level 1: q9–10
Level 2: q9–10
Level 3: q9–11

Independent practice
Level 1: q1–7
Level 2: q1–9
Level 3: q1–11

Additional questions
Level 1: p445 q1–2, 5–6
Level 2: p445 q1–6, 9
Level 3: p445 q1–9

3 Homework

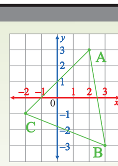
Homework Book — Lesson 3.2.2
Level 1: q1, 2a, 3, 5
Level 2: q1–6
Level 3: q1–8

4 Skills Review

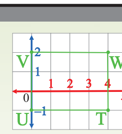
Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 28 — Graphing Linear Equations
- Worksheet 31 — Perimeter
- Worksheet 33 — Area

Solutions
For worked solutions see the Solution Guide

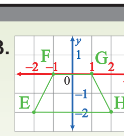
Independent Practice

1.


Triangle (scalene)

2.


Rectangle

3.


Trapezoid

Section 3.3 Exploration — Measuring Right Triangles

Purpose of the Exploration

The purpose of the Exploration is to have students discover the relationship between the legs and hypotenuse of a right triangle (the Pythagorean Theorem). Students will use a metric ruler to measure the sides. They will also learn that the hypotenuse is the longest side of the triangle.

Resources

- cm grid paper
- rulers
- calculators

Strategic & EL Learners

Strategic learners may benefit from having some triangles cut out into manipulatives for them to measure.

Like other students, EL learners may get confused with the terms “legs” and “hypotenuse.”

Universal access

Start the lesson by introducing the basic features of a right triangle. Stress to students that there is only one hypotenuse and two legs.

It may be useful to give students additional practice at identifying the hypotenuse in a right triangle.

Common errors

Students will confuse which side of the triangle is the hypotenuse. Remind students that the hypotenuse is the side opposite the right angle — the right-angle is formed by the legs.

Another problem occurs in measuring the lengths. Students will not always measure with sufficient accuracy — it may be a good idea to review measuring with a ruler.

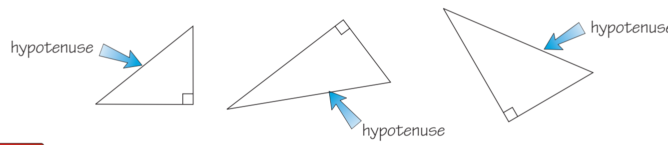
Math background

Students should be familiar with the different classifications of triangles. They need to be comfortable with the difference between a right triangle and other triangles. Students will also need to know how to measure with a metric ruler.

Section 3.3 introduction — an exploration into: Measuring Right Triangles

There’s a special relationship between the leg-lengths and the hypotenuse-length in a right triangle. The purpose of the Exploration is to discover this relationship.

The **hypotenuse** of a right triangle is the side **directly across from the right-angle**. The other sides are called **legs**. Some right triangles are shown below with their hypotenuses labeled.

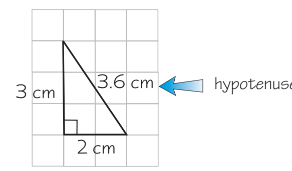


Example

On grid paper, draw a right triangle. Measure the length of each leg and the length of the hypotenuse.

Solution

You can draw any triangle, as long as it has a **right-angle**.
Legs = 2 cm and 3 cm
Hypotenuse = 3.6 cm



Exercises

1. Draw 5 right triangles on grid paper. Label them A-E. Then label the hypotenuse on each.
Any 5 different right triangles should be drawn.

2. For each right triangle, measure the length of each leg and the length of the hypotenuse. Measure in centimeters and record your measurements in a copy of this table.
Depends on students’ triangles.

Triangle	Leg 1 (cm)	Leg 2 (cm)	Hypotenuse (cm)
Example	2	3	3.6
A			
B			
C			
D			
E			

3. Explain how the length of a right triangle’s hypotenuse compares to the lengths of its legs. See below

4. Explain how the sum of the legs of each right triangle compares to the hypotenuse length. See below

5. Add three new columns to your table, like this:

Triangle	Leg 1 (cm)	Leg 2 (cm)	Hypotenuse (cm)	Leg 1 squared	Leg 2 squared	Hypotenuse squared
Example	2	3	3.6	4	9	13

Complete these columns, and then compare the squared side lengths for each triangle. What patterns do you notice? See below

Round Up

You should now have discovered how the leg-lengths of a right triangle are related to the hypotenuse-length. This is known as the Pythagorean Theorem — you’ll be using it in this Section.

Solutions

3. The hypotenuse is always longer than either of the legs.
4. The sum of the legs is always greater than the length of the hypotenuse.
5. When you square the legs and add them together they equal the hypotenuse squared.

Lesson 3.3.1 The Pythagorean Theorem

In this Lesson, students learn to recognize right triangles and identify their hypotenuse. They are then introduced to the Pythagorean theorem and explore it by looking at the areas of squares built onto the sides of right triangles.

Previous Study: In Chapter 2, students found powers of numbers, and square roots.

Future Study: In the next Lesson, students will calculate lengths using the Pythagorean theorem. In Trigonometry, they will prove that the identity $\cos^2(x) + \sin^2(x) = 1$ is equivalent to the Pythagorean theorem.

Lesson 3.3.1

California Standards:

Measurement and Geometry 3.3

Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments **and, in some situations, empirically verify the Pythagorean theorem by direct measurement.**

What it means for you:

You'll learn about an equation that you can use to find a missing side length of a right triangle.

Key words:

- Pythagorean theorem
- right triangle
- hypotenuse
- legs
- right angle

Check it out:

A right angle is an angle of exactly 90°. In diagrams, a right angle is shown as a small square in the corner like this:

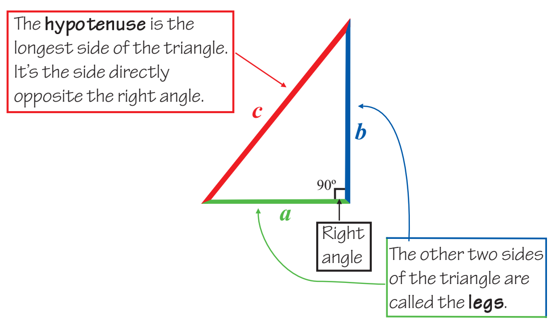
Any other angle is shown as a piece of a circle like this:

Section 3.3
The Pythagorean Theorem

You will have come across right triangles before — they're just triangles that have one corner that's a 90° angle. Well, there's a special formula that links the side lengths of a right triangle — it comes from the Pythagorean theorem.

The Pythagorean Theorem is About Right Triangles

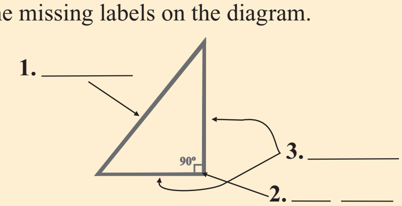
A **right triangle** is any triangle that has a **90° angle** (or right angle) as one of it corners. You need to know the names of the parts of a right triangle:



In diagrams of right triangles, the **hypotenuse** is usually labeled as ***c***, and the two **legs** as ***a*** and ***b***. It doesn't matter which leg you label *a*, and which you label *b*.

Guided Practice

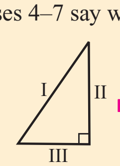
Complete the missing labels on the diagram.



1. Hypotenuse
2. Right angle
3. Legs

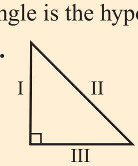
In Exercises 4–7 say which side of the right triangle is the hypotenuse.

4.



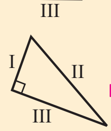
I

5.



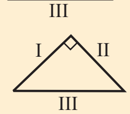
II

6.



II

7.



III

1 Get started

Resources:
- individual whiteboards
- graph paper
- internet computers

Warm-up questions:
- Lesson 3.3.1 sheet

2 Teach

Math background

Pythagoras of Samos lived from about 569 to about 475 B.C. He was a Greek philosopher, who made important developments in mathematics, astronomy, and music theory.

He's best known for the theorem that in right triangles, the square of the hypotenuse is equal to the sum of the squares of the two legs.

Common error

Students sometimes misunderstand the meaning of "right triangle."

Draw students' attention to right triangles in different orientations so that students recognize that the property that makes a triangle a right triangle is independent of the triangle's orientation.

Guided practice

Level 1: q1–7
Level 2: q1–7
Level 3: q1–7

Solutions

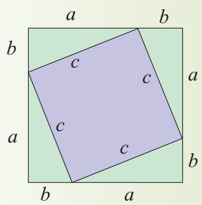
For worked solutions see the Solution Guide

- **Strategic Learners**
  Have students draw a right triangle in the center of a piece of grid paper, then construct squares off all three sides, as in the diagram below. Ask them to color, then cut out, the squares built off the legs of the right triangle. They should then use the smaller squares (cutting them up when necessary) to fill the square built off the hypotenuse. Explain how this illustrates the Pythagorean theorem.
- **English Language Learners**
  Review the vocabulary terms related to the theorem using individual whiteboards to check understanding. For instance, ask students to draw a right triangle, and label the hypotenuse, legs, and right angle. Ask them to find the squares and square roots of simple numbers.

2 Teach (cont)

Math background

There are various different proofs for the Pythagorean theorem. Here's a fairly simple one:



The area of the large square equals the area of the four congruent triangles, plus the area of the smaller square. So:

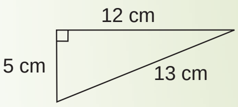
$(a + b)^2 = 4(\frac{1}{2}ab) + c^2$

Simplifying this gives the Pythagorean theorem:

$(a + b)(a + b) = 2ab + c^2$
$a^2 + 2ab + b^2 = 2ab + c^2$
$a^2 + b^2 = c^2$

Concept question

"A student is testing the Pythagorean theorem using the triangle below. Which lengths should be substituted for a, b, and c in the equation $a^2 + b^2 = c^2$?"



The longest length, 13 cm, should be substituted for c. The other lengths can be substituted for either a or b. It doesn't matter which is substituted for which.

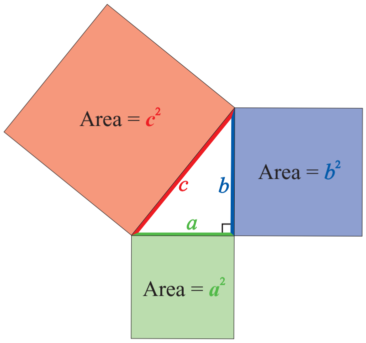
The Theorem Links Side Lengths of Right Triangles

Pythagoras was a Greek mathematician who lived around 500 B.C. A famous theorem about right triangles is named after him. It's called the **Pythagorean theorem**:

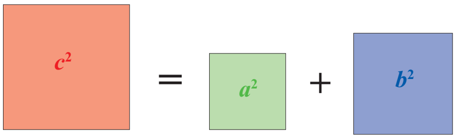
For any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs.

This all sounds very complicated, but it's not so bad once you know what it actually means.

Look again at the right triangle. Now add three squares whose side lengths are the same as the side lengths of the triangle:



What the Pythagorean theorem is saying is that the area of the red square is the same as the area of the blue square plus the area of the green square.



So this is what the Pythagorean theorem looks like written algebraically:

For any right triangle:
$$c^2 = a^2 + b^2$$

It means that if you know the **lengths** of **two sides** of a **right triangle**, you can always **find the length** of the **other side** using the equation.

- **Advanced Learners**
  Ask students to consider proofs for the Pythagorean theorem. One proof is given in the Math background section on the previous page. Students can research other similar proofs using the internet. Ask them to present a proof to the rest of the class in pairs.

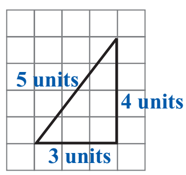
You Can Check the Theorem Using a Right Triangle

Don't forget:
It's really important to remember that the Pythagorean theorem **only** works on right triangles.
It won't work on any other type.

You can check for yourself that the theorem works by **measuring** the **side lengths** of **right triangles**, and putting the values into the **equation**.

Example 1

Use the right triangle below to verify the Pythagorean theorem.



Solution

$a = 3$ units $\quad b = 4$ units $\quad c = 5$ units

$c^2 = a^2 + b^2$

$5^2 = 3^2 + 4^2$

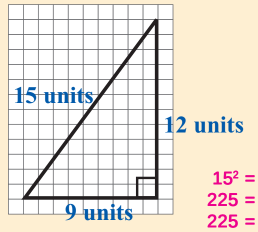
$25 = 9 + 16$

$25 = 25$

Guided Practice

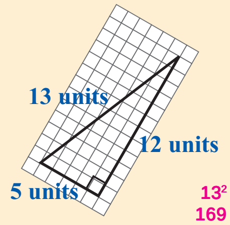
Use the right triangles in Exercises 8–11 to verify the Pythagorean theorem.

8.



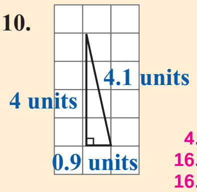
$15^2 = 12^2 + 9^2$
$225 = 144 + 81$
$225 = 225$

9.



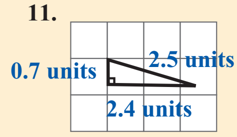
$13^2 = 12^2 + 5^2$
$169 = 144 + 25$
$169 = 169$

10.



$4.1^2 = 4^2 + 0.9^2$
$16.81 = 16 + 0.81$
$16.81 = 16.81$

11.

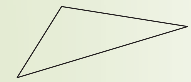


$2.5^2 = 2.4^2 + 0.7^2$
$6.25 = 5.76 + 0.49$
$6.25 = 6.25$

2 Teach (cont)

Concept question

"Can the Pythagorean theorem be used on the triangle below? Explain your answer."



No. It's not a right triangle. The Pythagorean theorem only applies to right triangles.

Common errors

Students will often multiply a number by two instead of squaring it. Alternatively, they will sometimes add the lengths of the legs and then square the sum.

Universal access

On grid paper, students can draw their own right triangles. They can then verify the theorem using the side lengths of their triangles.

Alternatively, use rectangular sheets of paper, books, or desks. Measure the side lengths and the diagonal and use the measurements to verify the theorem.

Guided practice

Level 1: q8–9
Level 2: q8–10
Level 3: q8–11

Solutions

For worked solutions see the Solution Guide

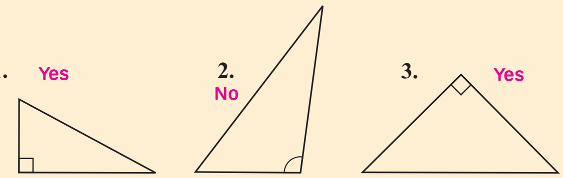
2 Teach (cont)

Independent practice
Level 1: q1–8
Level 2: q1–8, 11
Level 3: q1–11

Independent Practice

In Exercises 1–3 say whether the triangle is a right triangle or not.

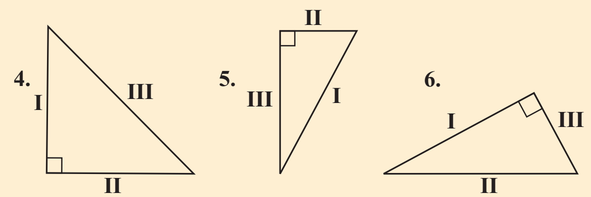
1. Yes
2. No
3. Yes



Don't forget:
The hypotenuse is always the longest side of a right triangle.

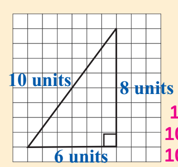
In Exercises 4–6 say which side of the right triangle is the hypotenuse.

4. III
5. I
6. II



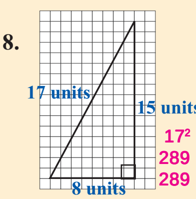
Use the triangles in Exercises 7–10 to verify the Pythagorean theorem.

7.



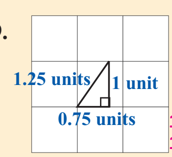
$10^2 = 8^2 + 6^2$
$100 = 64 + 36$
$100 = 100$

8.



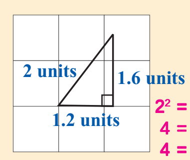
$17^2 = 15^2 + 8^2$
$289 = 225 + 64$
$289 = 289$

9.



$1.25^2 = 1^2 + 0.75^2$
$1.5625 = 1 + 0.5625$
$1.5625 = 1.5625$

10.



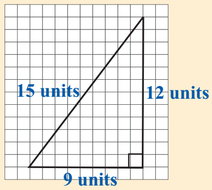
$2^2 = 1.6^2 + 1.2^2$
$4 = 2.56 + 1.44$
$4 = 4$

Now try these:
Lesson 3.3.1 additional questions — p445

11. Victor used the triangle shown on the right to try to verify the Pythagorean theorem. Explain why his work is wrong.

Victor's work:
$9^2 = 12^2 + 15^2$
$81 = 144 + 225$
$81 = 369$

Victor has not correctly identified the hypotenuse. The theorem says $c^2 = a^2 + b^2$ where c is the longest side, or hypotenuse. The first line of his work should be $15^2 = 9^2 + 12^2$.



Additional questions
Level 1: p445 q1–3
Level 2: p445 q1–7
Level 3: p445 q1–7

Round Up

The Pythagorean theorem describes the relationship between the lengths of the hypotenuse and the legs of a right triangle. It means that when you know the lengths of two of the sides of a right triangle, you can always find the length of the third side. You'll get a lot of practice at using it in the next few Lessons.

3 Homework

Homework Book — Lesson 3.3.1
Level 1: q1–4
Level 2: q1–4, 6, 7
Level 3: q1–7

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 14 — Squares and Square Roots
- Worksheet 37 — Types of Triangles

Solutions
For worked solutions see the Solution Guide

Lesson 3.3.2

Using the Pythagorean Theorem

In this Lesson, students use the Pythagorean theorem to find a missing side length of a right triangle. They also use the theorem to calculate the distance between two points on a coordinate plane.

Previous Study: In the last Lesson, students were introduced to the Pythagorean theorem and verified it using measurements.

Future Study: In the next Lesson, students will apply the Pythagorean theorem to real-life situations. In Trigonometry, they will prove that the identity $\cos^2(x) + \sin^2(x) \equiv 1$ is equivalent to the Pythagorean theorem.

Lesson 3.3.2

Using the Pythagorean Theorem

California Standards:

Measurement and Geometry 3.2

Understand and use coordinate graphs to plot simple figures, **determine lengths** and areas **related to them**, and determine their image under translations and reflections.

Measurement and Geometry 3.3

Know and understand the Pythagorean theorem and its converse **and use it to find the length of the missing side of a right triangle and the lengths of other line segments** and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

What it means for you:

You'll see how to use the Pythagorean theorem to find missing side lengths of right triangles.

Key words:

- Pythagorean theorem
- hypotenuse
- legs
- square root

Don't forget:

Finding the square root of a number is the opposite of squaring it. The symbol: $\sqrt{\ }$ represents the positive square root of the number inside it. For more about square roots, see Section 2.6.

In the last Lesson, you met the Pythagorean theorem and saw how it linked the lengths of the sides of a right triangle.

In this Lesson, you'll practice using the theorem to work out missing side lengths in right triangles.

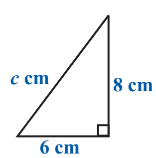
Use the Pythagorean Theorem to Find the Hypotenuse

If you know the **lengths** of the **two legs** of a **right triangle** you can use them to find the **length** of the **hypotenuse**.

The theorem says that $c^2 = a^2 + b^2$, where c is the length of the **hypotenuse**, and a and b are the lengths of the **two legs**. So if you know the lengths of the legs you can put them into the equation, and **solve** it to find the length of the hypotenuse.

Example 1

Use the Pythagorean theorem to find the length of the hypotenuse of the right triangle shown below.



Solution

$c^2 = a^2 + b^2$ — **First write out the equation**

$c^2 = 6^2 + 8^2$ — **Substitute in the side lengths that you know**

$c^2 = 36 + 64$

$c^2 = 100$ — **Simplify the expression**

$c = \sqrt{100}$ — **Take the square root of both sides**

$c = 10$ **cm**

A lot of the time your solution won't be a **whole number**. That's because the last step of the work is taking a **square root**, which often leaves a **decimal** or an **irrational number** answer.

1 Get started

Resources:

- grid paper
- maps of mountainous areas

Warm-up questions:

- Lesson 3.3.2 sheet

2 Teach

Universal access

Students often find it difficult to know whether to add or subtract once they have substituted the numbers into the equation.

Some simple warm-up problems can be used, such as:

Find c if: $12 + 13 = c$
Find b if: $14 + b = 26$

Another approach is to make a table for each student to have at their desk.

What are you looking for?	Leg (a or b)	Hypotenuse (c)
Operation	Subtract	Add
Example	$a^2 + 324 = 900$	$36 + 64 = c^2$

Additional examples

Find the hypotenuse for the given legs.

1. $a = 16$ cm, $b = 12$ cm

hypotenuse = 20 cm

2. $a = 7$ in., $b = 15$ in.

hypotenuse = $\sqrt{274}$ **in.**

3. $a = 13$ m, $b = 5$ m

hypotenuse = $\sqrt{194}$ **m**

- **Strategic Learners**
  Prepare students for finding leg lengths using the Pythagorean theorem by reviewing how to solve two-step equations. Start with simple examples, such as $2x + 3 = 11$. Discuss the importance of doing the same thing to both sides of an equation, and how operations can be "undone" using their inverse.

- **English Language Learners**
  Ask students to use grid paper to draw at least three different sized right triangles. They should use whole numbers for the lengths of the legs. Then ask them to estimate the length of the hypotenuse of each triangle before calculating it using the Pythagorean theorem. Discuss with them why it's hard to find the length of the hypotenuse by counting squares.

2 Teach (cont)

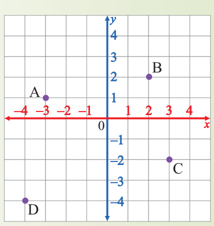
Math background

The $\sqrt{\ }$ sign means the positive square root only. As lengths are always positive, the sign indicates the correct value only.

Additional examples

Plot the following points:
A = (–3, 1), B = (2, 2),
C = (3, –2), D = (–4, –4)

Find the distance between:



1. A and B
$\sqrt{26}$ **units**

2. A and C
$\sqrt{45}$ **units**

3. A and D
$\sqrt{26}$ **units**

4. B and D
$\sqrt{72}$ **units**

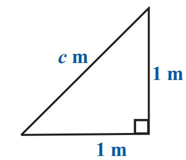
Guided practice

Level 1: q1–2
Level 2: q1–2, 4
Level 3: q1–4

Check it out:
Before you start, it's very important to work out which side of the triangle is the longest side — the hypotenuse. Otherwise you'll get an incorrect answer.

Example 2

Use the Pythagorean theorem to find the length of the hypotenuse of the right triangle shown.



Solution

$c^2 = a^2 + b^2$ First write out the equation

$c^2 = 1^2 + 1^2$ Substitute in the side lengths that you know

$c^2 = 1 + 1$

$c^2 = 2$ Simplify the expression

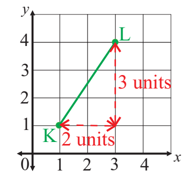
$c = \sqrt{2}$ **m** Cancel out the squaring by taking the square root

If you do this calculation on a calculator, you'll see that $\sqrt{2}$ **m** is approximately equal to **1.4 m**.

The Pythagorean theorem is also useful for finding **lengths on graphs** that aren't horizontal or vertical.

Example 3

Find the length of the line segment KL.



Solution

Draw a horizontal and vertical line on the plane to make a right triangle — then use the method above.

$c^2 = a^2 + b^2$ First write out the equation

$c^2 = 3^2 + 2^2$ Substitute in the side lengths that you know

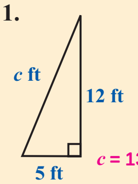
$c^2 = 9 + 4 = 13$ Simplify the expression

$c = \sqrt{13}$ units $\approx$ **3.6 units** Cancel out the squaring by taking the square root

Guided Practice

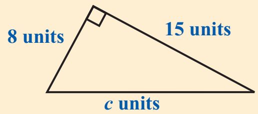
Use the Pythagorean theorem to find the length of the hypotenuse in Exercises 1–3.

1.



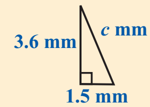
$c = 13$

2.



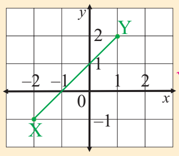
$c = 17$

3.



$c = 3.9$

4. Use the Pythagorean theorem to find the length of the line segment XY.



$\sqrt{18} \approx 4.2$ units

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
Give students maps with simple scales, and ask them to find (by measuring with a ruler and using the map scale) the horizontal distance between two points, for example between the top and bottom of a mountain. Then ask them to find the difference in height between these two points (using contour lines). Point out that these two distances could form the legs of a right triangle, and that its hypotenuse represents the minimum distance you'd cover if you walked from the bottom to the top of the mountain in a straight line. Have them calculate this distance, and consider what assumptions they have made and how they could adjust their answer to make it more realistic.
(For example, they've assumed that the slope is uniform. The "real-life" distance would almost certainly be greater than their answer.)

You Can Use the Theorem to Find a Leg Length

If you know the length of the **hypotenuse** and one of the **legs**, you can use the **theorem** to find the length of the **"missing" leg**. You just need to **rearrange the formula**:

$$a^2 + b^2 = c^2$$
$$a^2 = c^2 - b^2$$

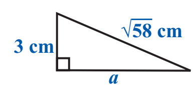
Subtract b^2 from both sides to get the a^2 term by itself.

Don't forget:
To keep an equation balanced, you have to do the same thing to both sides. Here b^2 is subtracted from each side. For a reminder on equations see Section 1.2.

Remember that it doesn't matter which of the **legs** you call ***a*** and which you call ***b***. But the **hypotenuse** is always ***c***.
Now you can **substitute** in values to find the missing leg length as you did with the hypotenuse.

Example 4

Find the missing leg length in this right triangle.

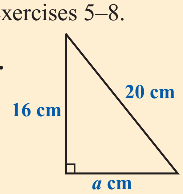


Solution

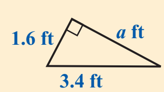
$c^2 = a^2 + b^2$	**First write out the equation**
$a^2 = c^2 - b^2$	**Rearrange it**
$a^2 = (\sqrt{58})^2 - 3^2$	**Substitute in the side lengths that you know**
$a^2 = 58 - 9$	
$a^2 = 49$	**Simplify the expression**
$a = \sqrt{49}$	**Take the square root of both sides**
$a = 7$ **cm**	

Guided Practice

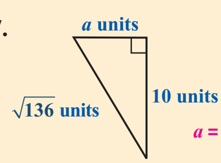
Use the Pythagorean theorem to calculate the missing leg lengths in Exercises 5–8.

5.


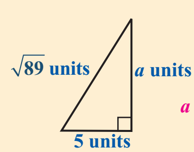
$a = 12$

6.


$a = 3$

7.


$a = 6$

8.


$a = 8$

2 Teach (cont)

Math background
The concept behind solving the equation to find a leg length is the same as for solving any two-step equation. Undo the operations in the opposite order to that in which they were done.

For example:

$121 + b^2 = 225$ can be rewritten as $121 + (b \times b) = 225$

The addition of 121 was the last operation to be done, so you undo this first by subtracting 121.

Additional examples
Find the missing leg lengths:
1. $a = 11$ cm, $c = 15$ cm

missing leg length = $\sqrt{104}$ cm

2. $b = 21$ in., $c = 35$ in.

missing leg length = 28 in.

3. $a = 20$ m, $c = 25$ m

missing leg length = 15 m

Guided practice
Level 1: q5–6
Level 2: q5–7
Level 3: q5–8

Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Independent practice

Level 1: q1–3, 6–8, 11
Level 2: q1–4, 6–9, 11
Level 3: q1–12

Concept question

"A student used the Pythagorean theorem to find the length of a hypotenuse. The lengths of the legs were 8 inches and 15 inches. She calculated the length of the hypotenuse to be 14 inches. Explain whether her answer is reasonable."

No, her answer is not reasonable because the hypotenuse should be the longest side of the triangle.

Additional questions

Level 1: p446 q1–4, 7
Level 2: p446 q1–8
Level 3: p446 q1–9

3 Homework

Homework Book — Lesson 3.3.2

Level 1: q1–5
Level 2: q1–8
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM

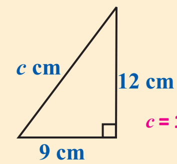
These worksheets may help struggling students:

- Worksheet 14 — Squares and Square Roots
- Worksheet 37 — Types of Triangles

Independent Practice

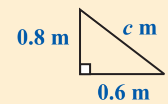
Use the Pythagorean theorem to find the value of c in Exercises 1–5.

1.



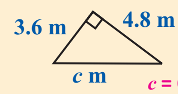
$c = 15$

2.



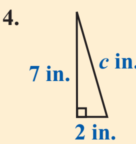
$c = 1$

3.



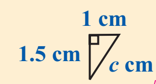
$c = 6$

4.



$c = \sqrt{53}$

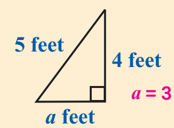
5.



$c = \sqrt{3.25}$

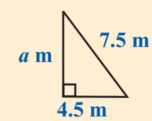
Calculate the value of a in Exercises 6–10.

6.



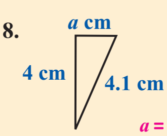
$a = 3$

7.



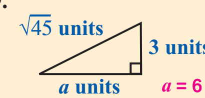
$a = 6$

8.



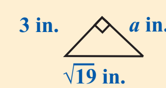
$a = 0.9$

9.



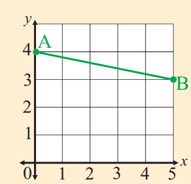
$a = 6$

10.



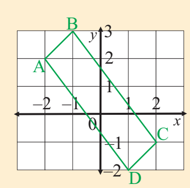
$a = \sqrt{10}$

11. Find the length of line AB.



$\sqrt{26} \approx 5.1$ units

12. Find the perimeter of quadrilateral ABCD.



$2\sqrt{2} + 10 \approx 12.8$ units

Now try these:
Lesson 3.3.2 additional questions — p446

Round Up

The Pythagorean theorem is really useful for finding missing side lengths of right triangles. If you know the lengths of both legs of a triangle, you can use the formula to work out the length of the hypotenuse. And if you know the lengths of the hypotenuse and one of the legs, you can rearrange the formula and use it to work out the length of the other leg.

Solutions

For worked solutions see the Solution Guide

Lesson 3.3.3

Applications of the Pythagorean Theorem

Students practice identifying right triangles in shapes and in real-life situations. They then apply the Pythagorean theorem to solve problems. Some of the problems are multistep — for instance, they require students to find a length and then use it to calculate an area.

Previous Study: In the last Lesson, students used the Pythagorean theorem to find the hypotenuse or missing leg length in a right triangle.

Future Study: In the next Lesson, students will use the converse of the Pythagorean theorem. In Trigonometry, they will prove that the identity $\cos^2(x) + \sin^2(x) = 1$ is equivalent to the Pythagorean theorem.

Lesson 3.3.3

Applications of the Pythagorean Theorem

California Standards:

Measurement and Geometry 3.3

Know and understand the Pythagorean theorem and its converse **and use it to find the length of the missing side of a right triangle and the lengths of other line segments** and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

What it means for you:

You'll see how the Pythagorean theorem can be used to find lengths in more complicated shapes and in real-life situations.

Key words:

- Pythagorean theorem
- right triangle
- hypotenuse
- legs
- right angle

Check it out:

When you use the Pythagorean theorem, it's important to remember that the longest side of the right triangle is the hypotenuse. Your first step in any problem involving the Pythagorean theorem should be to decide which side is the hypotenuse.

In the last two Lessons you've seen what the Pythagorean theorem is, and how you can use it to find missing side lengths in right triangles. Now you'll see how it can be used to help find missing lengths in other shapes too — by breaking them up into right triangles. It can help solve real-life measurement problems too.

Use the Pythagorean Theorem in Other Shapes Too

You can use the **Pythagorean theorem** to find lengths in lots of **shapes** — you just have to **split** them up into **right triangles**.

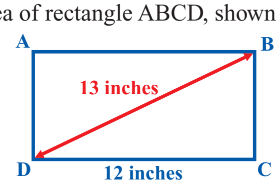
Here's a reminder of the formula.

$c^2 = a^2 + b^2$ Which rearranges to: $a^2 = c^2 - b^2$

(c is the hypotenuse length, and a and b are the leg lengths.)

Example 1

Find the area of rectangle ABCD, shown below.

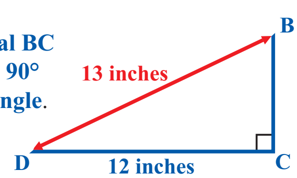


Solution

The formula for the area of a rectangle is **Area = length × width**.

You know that the **length** of the rectangle is **12 inches**, but you don't know the rectangle's **width**, **BD**.

But you do know the length of the **diagonal BC** and since all the corners of a rectangle are **90°** angles, you know that **BCD** is a **right triangle**.



You can use the **Pythagorean theorem** to find the length of side **BC**.

$BC^2 = BD^2 - CD^2$
$BC^2 = 13^2 - 12^2$
$BC^2 = 169 - 144$
$BC^2 = 25$
$BC = \sqrt{25} = 5$ inches

BC is the width of the rectangle. Now you can find its area.
Area = length × width
= 12 inches × 5 inches = **60 inches²**

1 Get started

Resources:

- empty cardboard box
- tape measures
- calculators
- flashlights

Warm-up questions:

- Lesson 3.3.3 sheet

2 Teach

Universal access

On a sunny day, take the students outside with a tape measure, pencil, and calculator.

In pairs, have the students stand on level ground, and measure their height and the length of their shadow. Point out that the distance from the top of their head to the top of the shadow's head is the hypotenuse of a right triangle.

Have students calculate the length of the hypotenuse and then measure it using the tape measure. They should compare the measurement with their calculation.

Note that the length of the shadow depends on the relative position of the sun.

(If the weather isn't sunny enough for this activity, students can use a strong flashlight as the light source.)

Math background

All rectangles and squares can be divided into right triangles — so a diagonal in these shapes can be a hypotenuse.

In any geometric shape where there is a right angle, there is a possibility that the Pythagorean theorem can be used to find a missing length.

- **Strategic Learners**
  Ask students to identify situations in the environment to which the Pythagorean theorem can be applied — for example, finding the length of a footpath across a lawn. They should make a sketch and mark the right angle and hypotenuse. Ask pairs of students to make up two word problems; one solving for the hypotenuse, and one for a missing leg length. They can then exchange problems with another pair and solve.

- **English Language Learners**
  Introduce real-life applications of the Pythagorean theorem with the "ladder problem:" A man wants to use a ladder to reach a window that is 18 feet from the ground. If he puts the ladder 5 feet from the wall, how long must the ladder be? Ask students to draw a picture of the problem and mark the right angle. They should decide what represents the hypotenuse, then solve and check with a partner.

2 Teach (cont)

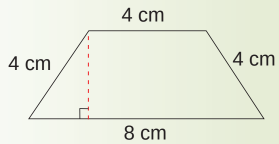
Common error

Students often apply the Pythagorean theorem to a problem and then stop, without properly answering the question. For example, if asked to find the area of a rectangle, a student might find the length of the rectangle and not carry out the rest of the problem.

Having a problem-solving checklist to work through, similar to the one described on p40, will reduce this by encouraging students to examine their answer in the context of the problem.

Additional example

An isosceles trapezoid is shown below. What is its vertical height?



$\sqrt{12}$ cm

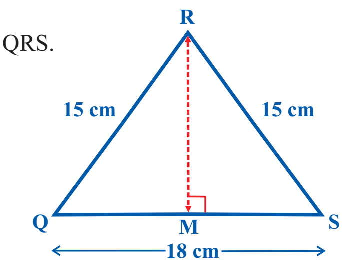
Guided practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1–4

Don't forget:
Isosceles triangles have two sides of equal length, and two angles of equal size.

Example 2

Find the area of isosceles triangle QRS.

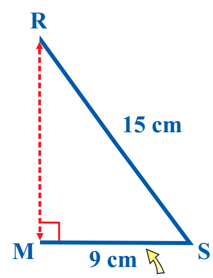


Solution

The formula for the area of a triangle is **Area** = $\frac{1}{2}$ **base** × **height**.

The **base** of the triangle is **18 cm**, but you don't know its **height**, **MR.**

Isosceles triangles can be split up into two right triangles.
Here's one:



This is half the base of the original triangle.

You can use the **Pythagorean theorem** to find the length of side **MR**.

$MR^2 = RS^2 - MS^2$
$MR^2 = 15^2 - 9^2$
$MR^2 = 225 - 81$
$MR^2 = 144$
$\mathbf{MR = \sqrt{144} = 12\text{ cm}}$

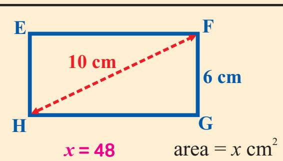
Now put the value of MR into the area formula:

$$\text{Area} = \frac{1}{2}\text{base} \times \text{height}$$
$$= \frac{1}{2}(18\text{ cm}) \times 12\text{ cm} = 9\text{ cm} \times 12\text{ cm} = \mathbf{108\text{ cm}^2}$$

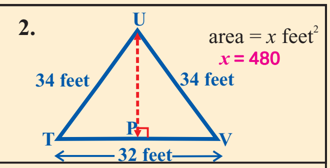
Guided Practice

In Exercises 1–4 use the Pythagorean theorem to find the missing value, x.

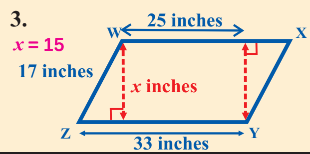
1.



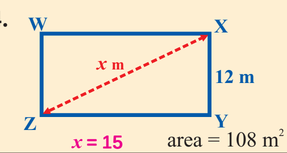
2.



3.



4.



Don't forget:
If you need a reminder of the area formulas of any of the shapes in this Lesson, see Section 3.1.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to consider how the Pythagorean theorem could be applied to 3-D shapes. For instance, ask them to figure out how to use it to find the diagonal inside an empty cardboard box, if the edge lengths are known.

The Pythagorean Theorem Has Real-Life Applications

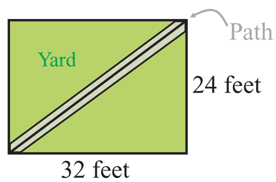
Because you can use the Pythagorean theorem to find lengths in many different shapes, it can be useful in lots of real-life situations too.

Example 3

Monique's yard is a rectangle 24 feet long by 32 feet wide. She is laying a diagonal gravel path from one corner to the other. One sack of gravel will cover a 10-foot stretch of path. How many sacks will she need?

Solution

The first thing you need to work out is the length of the path.
It's a good idea to draw a diagram to help sort out the information.



You can see from the diagram that the path is the hypotenuse of a right triangle. So you can use the Pythagorean theorem to work out its length.

$c^2 = a^2 + b^2$

$c^2 = 32^2 + 24^2$

$c^2 = 1024 + 576$

$c^2 = 1600$

$c = \sqrt{1600} = 40$ **feet**

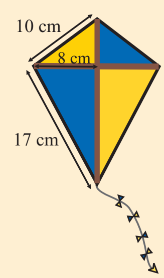
The question tells you that one sack of gravel will cover a 10-foot length of path. To work out how many are needed, divide the path length by 10.

Sacks needed = 40 ÷ 10 = **4 sacks**

Guided Practice

5. Rob is washing his upstairs windows. He puts a straight ladder up against the wall. The top of the ladder is 8 m up the wall. The bottom of the ladder is 6 m out from the wall. How long is the ladder? 10 m

6. To get to Gabriela's house, Sam walks 0.5 miles south and 1.2 miles east around the edge of a park. How much shorter would his walk be if he walked in a straight line across the park? 0.4 miles

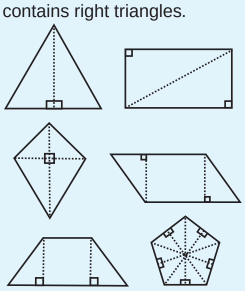


7. The diagonal of Akil's square tablecloth is 4 feet long. What is the area of the tablecloth? 8 feet²

8. Megan is making the kite shown in the diagram on the right. The crosspieces are made of thin cane. What length of cane will she need in total? 37 cm

Check it out:

Right triangles are found in many different shapes.
You can use the Pythagorean theorem on any shape that contains right triangles.



2 Teach (cont)

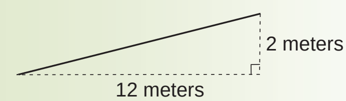
Concept question

"Explain the process for finding the area of a rectangle, given the diagonal and width."

First find the length using the Pythagorean theorem. The length is one of the legs and the diagonal is the hypotenuse. Then find the area by multiplying the length by the width.

Additional example

Plans for a ramp require that the highest point is 2 meters above the ground and that it has a horizontal length of 12 meters. What is the length of the surface of the ramp, to the nearest tenth of a meter?



12.2 meters

Guided practice

Level 1: q5–6
Level 2: q5–7
Level 3: q5–8

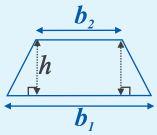
Solutions

For worked solutions see the Solution Guide

2 Teach (cont)

Independent practice
Level 1: q1–5
Level 2: q1–6
Level 3: q1–8

Don't forget:

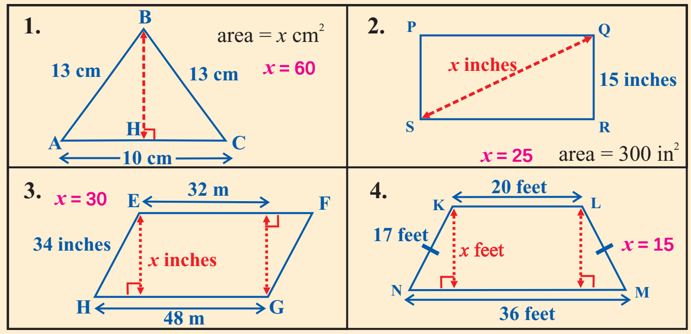


The formula for the area of a trapezoid is:
$A = \frac{1}{2}h(b_1 + b_2)$

Now try these:
Lesson 3.3.3 additional questions — p446

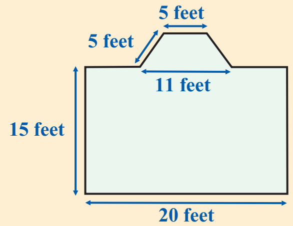
Independent Practice

In Exercises 1–4 use the Pythagorean theorem to find the missing value, x.



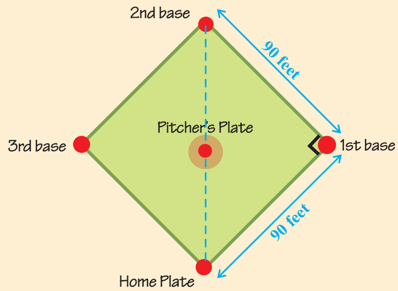
5. A local radio station is getting a new radio mast that is 360 m tall. It has guy wires attached to the top to hold it steady. Each wire is 450 m long. Given that the mast is to be put on flat ground, how far out from the base of the mast will the wires need to be anchored? 270 m

6. Luis is going to paint the end wall of his attic room, which is an isosceles triangle. The attic is 7 m tall, and the length of each sloping part of the roof is 15 m. One can of paint covers a wall area of 20 m². How many cans should he buy? 5 cans



7. Maria is carpeting her living room, shown in the diagram on the left. It is rectangular, but has a bay window. She has taken the measurements shown on the diagram. What area of carpet will she need? 332 feet²

8. The diagram on the right shows a baseball diamond. The catcher throws a ball from home plate to second base. What distance does the ball travel?
127 feet



Round Up

You can break up a lot of shapes into right triangles. This means you can use the Pythagorean theorem to find the missing lengths of sides in many different shapes — it just takes practice to be able to spot the right triangles.

Additional questions
Level 1: p446 q1–3
Level 2: p446 q1–5
Level 3: p446 q1–6

3 Homework

Homework Book — Lesson 3.3.3
Level 1: q1, 2, 3b, 4
Level 2: q1–5
Level 3: q2–7

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 14 — Squares and Square Roots
- Worksheet 33 — Area
- Worksheet 37 — Types of Triangles

Solutions
For worked solutions see the Solution Guide

Lesson 3.3.4 Pythagorean Triples & the Converse of the Theorem

Students learn that Pythagorean triples are sets of whole numbers that fit the equation $a^2 + b^2 = c^2$. They test whether sets of numbers are Pythagorean triples or not. They then use the converse of the Pythagorean theorem to check if triangles are actually right triangles.

Previous Study: Earlier in this Section, students practiced using the Pythagorean theorem to calculate missing lengths in triangles.

Future Study: In Trigonometry, students will prove that the identity $\cos^2(x) + \sin^2(x) = 1$ is equivalent to the Pythagorean theorem.

Lesson 3.3.4

Pythagorean Triples & the Converse of the Theorem

California Standards:

Measurement and Geometry 3.3

Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

What it means for you:

You'll learn about the groups of whole numbers that make the Pythagorean theorem true, and how to use the converse of the theorem to find out if a triangle is a right triangle.

Key words:

- Pythagorean theorem
- Pythagorean triple
- converse
- right triangle
- acute
- obtuse

Don't forget:

The longest side of a right triangle is always the hypotenuse. The other two sides are the legs.

Don't forget:

Lengths are always positive, so you can't have a negative integer in a Pythagorean triple.
Positive integers can also be called **whole numbers**.

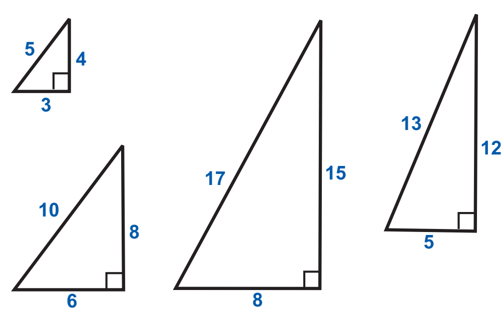
Up to now you've been using the Pythagorean theorem on triangles that you've been told are right triangles. But if you don't know for sure whether a triangle is a right triangle, you can use the theorem to decide. It's kind of like using the theorem backwards — and it's called using the converse of the theorem.

Pythagorean Triples are All Whole Numbers

You can draw a **right triangle** with any length **legs** you like, so the list of **side lengths** that can make the equation $c^2 = a^2 + b^2$ true never ends. Most sets of side lengths that fit the equation include at least one **decimal** — that's because finding the length of the hypotenuse using the equation involves taking a **square root**.

There are some sets of side lengths that are **all integers** — these are called **Pythagorean triples**. You've seen a lot of these already. For example:

(3, 4, 5)
(6, 8, 10)
(8, 15, 17)
(5, 12, 13)



You can find more Pythagorean triples by **multiplying** each of the numbers in a triple by the same number. For example:

$(3, 4, 5) \xrightarrow{\times 2} (6, 8, 10)$
$(3, 4, 5) \xrightarrow{\times 3} (9, 12, 15)$
$(3, 4, 5) \xrightarrow{\times 4} (12, 16, 20)$

These are all Pythagorean triples.

To test if three integers are a **Pythagorean triple**, put them into the equation $c^2 = a^2 + b^2$, where c is the biggest of the numbers.
If they make the equation **true**, they're a **Pythagorean triple**.
If they **don't**, they're **not**.

1 Get started

Resources:
- strips of paper (see Strategic Learners activity on the next page)

Warm-up questions:
- Lesson 3.3.4 sheet

2 Teach

Universal access

Have students investigate Pythagorean triples by starting with the basic (3, 4, 5) right triangle. Have students multiply the three side lengths by the numbers 1 through 10.

Then ask pairs of students to test the numbers with the Pythagorean theorem. This activity can lead to a discussion about what a Pythagorean triple is and what the requirements are.

Math background

A Pythagorean triple is "primitive" if the greatest common factor of all the numbers is 1.
For example, a (3, 4, 5) triangle is primitive, while a (6, 8, 10) triangle is not primitive.

Application

Carpenters can use the Pythagorean theorem to check whether two lines meet at a right angle.

The angle is checked by measuring out 3 feet and 4 feet from the intersection of the lines.
The diagonal between these two points should measure 5 feet.
If the diagonal doesn't measure 5 feet then the angle isn't 90°.

- **Strategic Learners**
  Create different stations that each have three strips of paper. For example, the first station may have strips of 3 inches, 4 inches, and 5 inches. Instruct students to try to make a right triangle using the three strips. After the students meet with success or failure, they should substitute the numbers into the Pythagorean theorem. After several stations, students should see that the Pythagorean theorem made a true statement where they were able to make a right triangle.
- **English Language Learners**
  Students may have difficulty understanding the term "converse." Help them to make up "if... then..." statements using familiar contexts, and then write the converse of the statements. For example: statement — "If Maria is Cecelia's aunt, then Cecelia is Maria's niece." Converse: "If Cecelia is Maria's niece, then Maria is Cecelia's aunt." Remind students that the converse may not always be true.

2 Teach (cont)

Common error
Students sometimes broaden the concept of a Pythagorean triple to any three numbers that satisfy the Pythagorean theorem. For example, they often classify $(1, 1, \sqrt{2})$ as a Pythagorean triple.

Guided practice
Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Concept question
"Write a Pythagorean triple where all three numbers are greater than 100."

Multiply a Pythagorean triple (such as 3, 4, 5) by any number so that the products are all larger than 100.

For example, multiply (3, 4, 5) by 34 to get (102, 136, 170).

Math background
The converse is formed when the hypothesis and conclusion are switched in a conditional statement.

For example:

Statement — "If it is Saturday, then there is no school."

Converse — "If there is no school, then it is Saturday."

The converse of this true statement is not necessarily true.

Don't forget:
The greatest number is substituted for *c*.

Example 1

Are the numbers (72, 96, 120) a Pythagorean triple?

Solution

To see if the numbers are a Pythagorean triple, put them into the equation.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 120^2 &= 72^2 + 96^2 \\ 14{,}400 &= 5184 + 9216 \\ 14{,}400 &= 14{,}400 \quad \text{— so which is true} \end{aligned}$$

These numbers are a Pythagorean triple.

Guided Practice

Are the sets of numbers in Exercises 1–6 Pythagorean triples or not? If they are not, give a reason why not.

1. 5, 12, 13 Pythagorean triple
2. 0.7, 0.9, 1.4 No. Not integers / don't fit equation
3. 8, 8, $\sqrt{128}$ No. Not all integers.
4. 25, 60, 65 Pythagorean triple
5. 6, 9, 12 No. Don't fit equation.
6. 18, 80, 82 Pythagorean triple

The Converse of the Pythagorean Theorem

The **Pythagorean theorem** says that the **side lengths** of any **right triangle** will satisfy the equation $c^2 = a^2 + b^2$, where c is the hypotenuse and a and b are the leg lengths.

You can also say the **opposite** — if a triangle's **side lengths** satisfy the equation, it is a **right triangle**. This is called the **converse** of the theorem:

Check it out:
The converse of a theorem or statement is formed by changing it around.
For example:
If the statement is,
"if a shape is a square, then it has four equal-length sides,"
then its converse is,
"if a shape has four equal length sides, then it is a square."
The converse of a true statement isn't always true — this one isn't. A rhombus is a shape with four equal-length sides — but it's not a square.

> **The converse of the Pythagorean theorem:**
> If the side lengths of a triangle a, b, and c, where c is the largest, satisfy the equation $c^2 = a^2 + b^2$, then the triangle is a right triangle.

Example 2

A triangle has side lengths 2.5 cm, 6 cm, and 6.5 cm. Is it a right triangle?

Solution

Put the side lengths into the equation $c^2 = a^2 + b^2$, and evaluate both sides.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 6.5^2 &= 2.5^2 + 6^2 \\ 42.25 &= 6.25 + 36 \\ 42.25 &= 42.25 \quad \text{— which is true} \end{aligned}$$

The longest side of a right triangle is the hypotenuse, *c*.

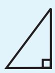
It is a right triangle.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask advanced learners to write "if... then..." statements about mathematical situations, such as properties of shapes. Then ask them to write the converses and consider if they are true. An example of this is given in the Student Textbook margin on the previous page.

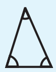
Don't forget:

A right triangle has one 90° angle.



An obtuse triangle has one angle between 90° and 180°.

An acute triangle has all three angles under 90°.



Don't forget:

When you evaluate c^2 and $a^2 + b^2$, remember that c is the longest side length — the hypotenuse is always the longest side.

Test Whether a Triangle is Acute or Obtuse

If a triangle isn't a right triangle, it must either be an **acute triangle** or an **obtuse triangle**. By seeing if c^2 is greater than or less than $a^2 + b^2$, you can tell what type of triangle it is.

If $c^2 > a^2 + b^2$ then the triangle is obtuse.
If $c^2 < a^2 + b^2$ then the triangle is acute.

Example 3

A triangle has side lengths of 2 ft, 2.5 ft, and 3 ft.
Is it right, acute, or obtuse?

Solution

Check whether $c^2 = a^2 + b^2$, with $c = 3$, $a = 2$ and $b = 2.5$.

$c^2 = 3^2 = \mathbf{9}$ and

$a^2 + b^2 = 2^2 + 2.5^2 = 4 + 6.25 = \mathbf{10.25}$

$9 < 10.25$ **$c^2 < a^2 + b^2$, so this is an acute triangle.**

Guided Practice

Are the side lengths in Exercises 7–12 of right, acute, or obtuse triangles?

7. 50, 120, 130 right
8. 8, 9, 10 acute
9. 3, 4, 6 obtuse
10. 12, 6, $\sqrt{180}$ right
11. 0.25, 0.3, 0.5 obtuse
12. 0.5, 0.52, 0.55 acute

Independent Practice

1. "Every set of numbers that satisfies the equation $c^2 = a^2 + b^2$ is a Pythagorean triple." Explain if this statement is true or not.
Not true: a Pythagorean triple must be made up of positive integers.

Say if the side lengths in Exercises 2–7 are Pythagorean triples or not.

2. 8, 15, 17 Pythagorean triple
3. 1, 1, $\sqrt{2}$ Not
4. 0.3, 0.4, 0.5 Not
5. 300, 400, 500 Pythagorean triple
6. 12, 29, 40 Not
7. 15, 36, 39 Pythagorean triple

8. In triangle ABC, side AB is longest.
If $AB^2 > AC^2 + BC^2$ then what kind of triangle is ABC? Triangle ABC is obtuse.

Are the following side lengths those of right, acute, or obtuse triangles?

9. 5, 10, 14 obtuse
10. 10, 11, 12 acute
11. 12, 16, 20 right
12. 5.1, 5.3, 9.3 obtuse
13. 2.4, 4.5, 5.1 right
14. 3.7, 4.7, 5.7 acute

15. Justin is going to fit a new door. He measures the width of the door frame as 105 cm, the height as 200 cm, and the diagonal of the frame as 232 cm. Is the door frame perfectly rectangular? No: its sides and diagonal form an obtuse triangle, not a right triangle.

Now try these:

Lesson 3.3.4 additional questions — p447

Round Up

The converse of the Pythagorean theorem is a kind of "backward" version. You can use it to prove whether a triangle is a right triangle or not — and if it's not, you can say if it's acute or obtuse.

2 Teach (cont)

Additional examples

Determine whether these triangles are right, obtuse, or acute.

1) A triangle with sides that measure 9 inches, 16 inches, and 20 inches.

$c = 20$, $a = 9$, $b = 16$
$20^2 = \underline{400}$
$9^2 + 16^2 = 81 + 256 = \underline{337}$
$400 > 337$
So the triangle is obtuse

2) A triangle with sides measuring 13.5 inches, 18 inches, and 22.5 inches.

$c = 22.5$, $a = 13.5$, $b = 18$
$22.5^2 = \underline{506.25}$
$13.5^2 + 18^2 = 182.25 + 324 = \underline{506.25}$
$506.25 = 506.25$
So the triangle is a right triangle

3) A triangle with sides that measure 12 inches, 15 inches, and 18 inches.

$c = 18$, $a = 12$, $b = 15$
$18^2 = \underline{324}$
$12^2 + 15^2 = 144 + 225 = \underline{369}$
$324 < 369$
So the triangle is acute

Guided practice

Level 1: q7–9
Level 2: q7–12
Level 3: q7–12

Independent practice

Level 1: q1–7, 9
Level 2: q1–12, 15
Level 3: q1–15

Additional questions

Level 1: p447 q1–5
Level 2: p447 q1–7
Level 3: p447 q1–9

3 Homework

Homework Book — Lesson 3.3.4

Level 1: q1, 3, 4a–b, 6
Level 2: q1–9
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 14 — Squares and Square Roots
- Worksheet 37 — Types of Triangles

Solutions

For worked solutions see the Solution Guide

Section 3.4

Exploration — Transforming Shapes

Purpose of the Exploration

The initial goal of the Exploration is to have students plot figures on the coordinate plane. Then they will predict the effects of manipulating the coordinates of the figures. Students will then apply these changes to test their predictions.

Resources

- cm grid paper
- rulers
- calculators

Strategic & EL Learners

Strategic learners may benefit from having solid shapes that are cut out. These students can then place the cutouts on the plane and experiment with different coordinates.

EL learners may have difficulty with the words "quadrants" and "coordinates." Remind them that the quadrants are the four sections of the graph, and the coordinates are the pairs of numbers used to plot a point.

Common errors

Students may have difficulty predicting the changes that will be made to the figures. They may be able to make the changes to the coordinates but will not be able to visualize the outcome without actually graphing. Throughout the Exploration, remind them that a prediction doesn't necessarily have to be correct. A prediction is only an educated guess.

Math background

Students should feel comfortable with graphing points on the coordinate plane. They should know the signs of x and y values in each quadrant.

Section 3.4 introduction — an exploration into: Transforming Shapes

Geometric figures, like triangles, rectangles and so on, can be plotted on the coordinate plane. The purpose of this Exploration is to predict and discover what changes will occur when the coordinates of a figure are changed in a particular way.

If you change the coordinates of figure, ABC, you normally label the changed figure **A'B'C'**.

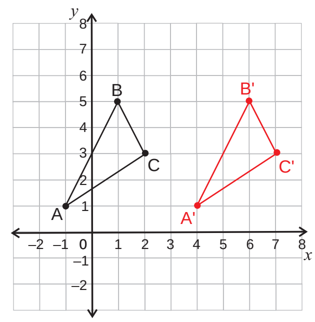
Example

Plot triangle ABC on a coordinate plane. A(–1, 1), B(1, 5), C(2, 3).
Add 5 to each x-value of the coordinates, and plot the new triangle A'B'C'.
Describe the change.

Solution

The new coordinates are
A'(**–1 + 5**, 1) = **A'(4, 1)**
B'(**1 + 5**, 5) = **B'(6, 5)**
C'(**2 + 5**, 3) = **C'(7, 3)**

The shape has moved 5 units to the right.



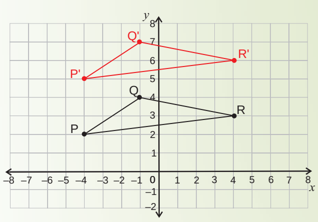
Exercises

1. Draw a coordinate plane. Your x and y axes should both go from –8 to +8.
   Plot triangle PQR — P(–4, 2), Q(–1, 4), R(4, 3). see left
2. Predict how triangle PQR will change if 3 is added to the y-values of each coordinate pair. Then test your prediction by performing the change.
   Triangle PQR will be moved up three units above the original figure. See left for diagram.
3. Draw a coordinate plane. Your x and y axes should both go from –6 to +6.
   Plot trapezoid EFGH on the coordinate plane. E(–3, –2), F(5, –2), G(2, –5), H(–1, –5).
   see below
4. Predict how trapezoid EFGH will change if the y-values are changed from negative to positive. Then test your prediction by performing the change.
   Trapezoid EFGH will be flipped/reflected over the x-axis. See below for diagram.
5. Draw a coordinate plane. Your x and y axes should both go from –8 to +8.
   Plot rectangle RSTU — R(1, –2), S(7, –2), T(7, –4), U(1, –4). see below
6. Predict how rectangle RSTU will change if the signs of the x-values are changed to the opposite sign. Then test your prediction by performing the change.
   Rectangle RSTU will be flipped/reflected over the y-axis. See below for diagram.
7. What was the effect on the size and shape of all of the figures after the changes were made to the coordinates? None.

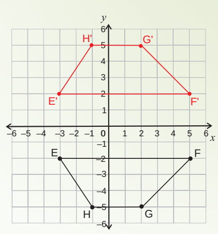
Round Up

You've looked at two types of transformation in this Exploration — translations and reflections. In translations, the shape is slid across the grid. In reflections, it's "flipped" over.

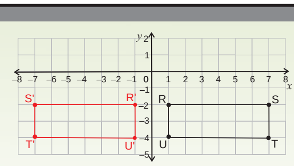
Solutions
1 and 2.



3 and 4.



5 and 6.



Lesson 3.4.1 Reflections

Students plot shapes on the coordinate plane and reflect them in the x- and y-axes. The effects of such reflections on the signs of the coordinates are examined to find general rules.

Previous Study: In grade 4 students identified figures with bilateral symmetry. In Section 2, students practiced plotting points in all four quadrants of the coordinate plane.

Future Study: Later in this Section, students will investigate rotations and translations.

Lesson 3.4.1

California Standards:

Measurement and Geometry 3.2

Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, **and determine their image under** translations and **reflections**.

What it means for you:

You'll learn what it means to reflect a shape. You'll also see how to draw and describe reflections.

Key words:

- reflection
- image
- flip
- prime
- coordinates
- x-axis/y-axis

Check it out:

A' is read as "A prime."

Section 3.4
Reflections

The next few Lessons are about transformations. A transformation is a way of changing a shape. For example, it could be flipping, stretching, moving, enlarging, or shrinking the shape.

The first type of transformation you're going to meet is reflection.

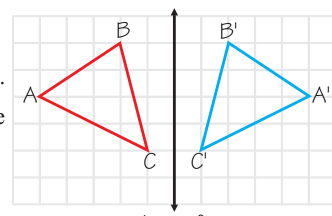
A Reflection Flips a Figure Across a Line

A **reflection** takes a shape and makes a **mirror image** of it on the other side of a given line.

Here triangle **ABC** has been **reflected** or "flipped" across the line of reflection.

The reflections of points A, B, and C are labeled **A'**, **B'**, and **C'**.

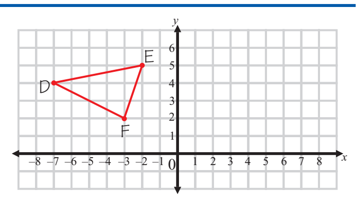
The whole reflected triangle A'B'C' is called the **image** of ABC.



Line of reflection

Example 1

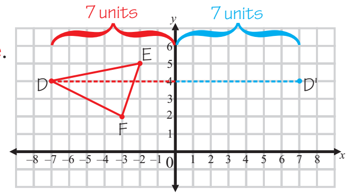
Reflect triangle DEF across the y-axis.



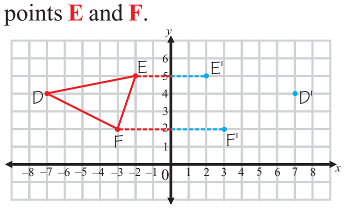
Solution

Step 1: Pick a point to reflect. Point **D** is **7 units** away from the y-axis.

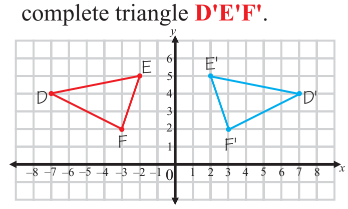
Move **across** the y-axis and find the point **7 units** away on the **other side**. This is where you plot the point **D'**.



Step 2: Repeat step 1 for points **E** and **F**.



Step 3: **Join** the points to complete triangle **D'E'F'**.



1 Get started

Resources:

- construction paper
- small mirrors

Teacher Resources CD-ROM

- Coordinate Grid (or grid paper)

Warm-up questions:

- Lesson 3.4.1 sheet

2 Teach

Universal access

Ask students to draw a coordinate plane on graph paper with a triangle in quadrant II.

Using a mirror, they should reflect the triangle in the x-axis, the y-axis, and the line $y = x$. They should draw reflected images where the reflections appear.

On a new set of axes, ask the students to draw a rectangle in quadrant I and predict where the reflections will appear using the same lines of symmetry.

Universal access

Cut out two copies of a variety of figures. Some should have line symmetry, and others should not.

Using a mirror, students should place each pair of figures either side of a mirror line, so they are reflections of each other. They should be led to discover that if the figures don't have line symmetry, one must be turned over, and that a nonsymmetrical figure and its image cannot be superimposed.

- **Strategic Learners and English Language Learners**
  Give each student a thin piece of paper. Instruct them to place their hand on one half of the paper and draw around it, then fold the paper in half and trace to get the reflected image. They should then unfold the paper and draw a line along the crease and label it as a line of reflection. Students will have drawn around their hands in different positions and these can be compared across the class.

2 Teach (cont)

Guided practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1–4

Common error

Students often incorrectly label the reflected image. Each corresponding point should be the same distance away from the mirror line.

Concept question

"A triangle is plotted in the second quadrant. In what quadrant will its reflection in the x-axis be?"

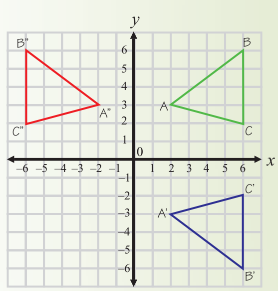
The third quadrant.

Additional example

Plot triangle ABC on the coordinate plane, where A is (2, 3), B is (6, 6), and C is (6, 2).

1. Reflect ABC across the x-axis. Label the image A'B'C'.

2. Reflect ABC across the y-axis. Label the image A''B''C''.

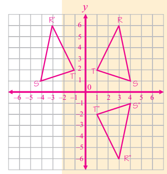


Check it out:

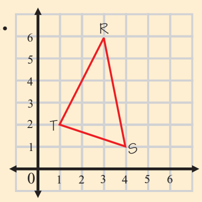
Suppose you're drawing more than one image of a shape called ABC. The first image should be called A'B'C', the second is A''B''C'', the third is A'''B'''C''', and so on.

Guided Practice

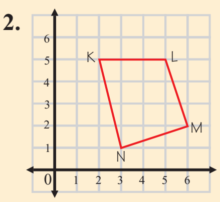
In Exercises 1–4, copy each shape onto a set of axes, then draw its reflections across the y-axis and the x-axis. Draw a new pair of axes for each Exercise, ranging from –6 to 6 in both directions.

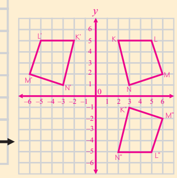


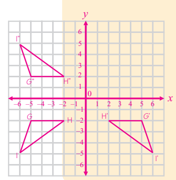
1.



2.







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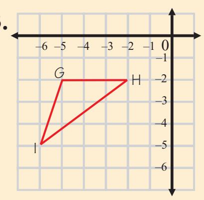

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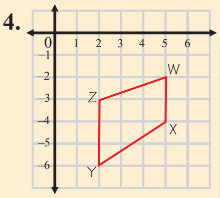
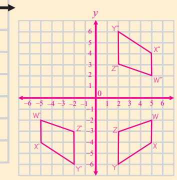




Don't forget:

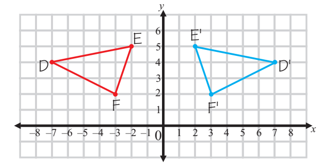
For a reminder about the coordinate plane see Lesson 3.2.1.

Reflections Change Coordinate Signs

A reflection across the ***x*-axis** changes **(x, y)** to **(x, $-y$)**.
A reflection across the ***y*-axis** changes **(x, y)** to **($-x$, y)**.

To see this, look again at the reflection from Example 1.
The **coordinates** of the corners of the triangles are shown below.

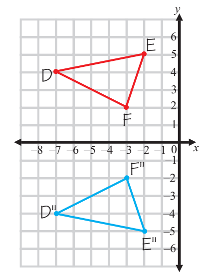
D (–7, 4)
E (–2, 5)
F (–3, 2)



D' (7, 4)
E' (2, 5)
F' (3, 2)

When DEF is reflected across the ***y*-axis**, the y-coordinate **stays the same** and the x-coordinate **changes** from negative to positive.

If you reflect DEF across the ***x*-axis**, the x-coordinate **stays the same** and the y-coordinate **changes** from positive to negative.



D (–7, 4)
E (–2, 5)
F (–3, 2)

D'' (–7, –4)
E'' (–2, –5)
F'' (–3, –2)

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask advanced learners to create patterns that have multiple lines of symmetry. For instance, they might create a pattern centered on the origin that is symmetric about the x-axis, the y-axis, the line $y = x$, and the line $y = -x$.

Guided Practice

In Exercises 5–8, give the coordinates of the image produced.

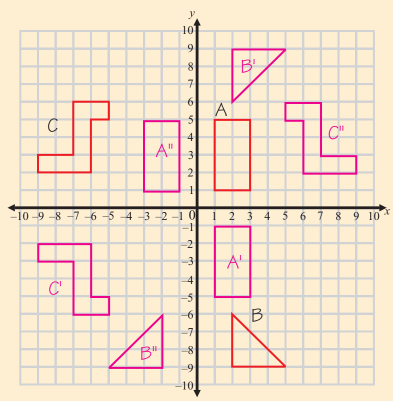
5. A: (5, 2), (4, 7), (6, 1). Triangle A is reflected over the x-axis. (5, –2), (4, –7), (6, –1)
6. B: (9, 9), (–4, 8), (–2, 6). Triangle B is reflected over the y-axis. (–9, 9), (4, 8), (2, 6)
7. C: (–2, 10), (2, 10), (5, 5), (0, –3), (–5, 5).
Pentagon C is reflected over the x-axis. (–2, –10), (2, –10), (5, –5), (0, 3), (–5, –5)
8. Pentagon C from Exercise 7 is reflected over the y-axis. (2, 10), (–2, 10), (–5, 5), (0, –3), (5, 5)

Exercises 9–11 give the coordinates of the corners of a figure and its reflected image. Describe each reflection in words.
9. D: (5, 2), (6, 3), (8, 1), (4, 1); D' (5, –2), (6, –3), (8, –1), (4, –1) Reflection over x-axis
10. E: (–6, –1), (–3, –6), (–9, –4); E' (6, –1), (3, –6), (9, –4) Reflection over y-axis
11. F: (0, 0), (0, 5), (3, 3); F' (0, 0), (0, 5), (–3, 3) Reflection over y-axis

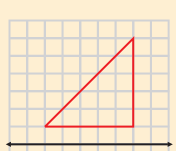
Independent Practice

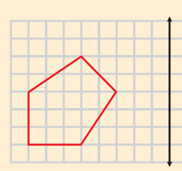
Copy the grid and figures shown below, then draw the reflections described in Exercises 1–6.

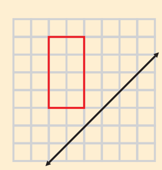
1. Reflect A across the x-axis. Label the image A'.
2. Reflect A across the y-axis. Label the image A''.
3. Reflect B across the x-axis. Label the image B'.
4. Reflect B across the y-axis. Label the image B''.
5. Reflect C across the x-axis. Label the image C'.
6. Reflect C across the y-axis. Label the image C''.



In Exercises 7–9, copy the figures onto graph paper and reflect each one over the line of reflection shown.

7.

see below

8.

see below

9.

see below

Now try these:
Lesson 3.4.1 additional questions — p447

Round Up

Don't forget that a reflection makes a back-to-front image — like the image you see when you look in a mirror. Unless the original is symmetrical, the image shouldn't be the same way around as the original. If it is the same way around, that's a translation, not a reflection. You'll learn about translations in the next Lesson.

2 Teach (cont)

Guided practice
Level 1: q5–8
Level 2: q5–10
Level 3: q5–11

Independent practice
Level 1: q1–6
Level 2: q1–9
Level 3: q1–9

Additional questions
Level 1: p447 q1–4
Level 2: p447 q1–8
Level 3: p447 q1–8

3 Homework

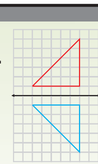
Homework Book — Lesson 3.4.1
Level 1: q1–3, 4a–b, 5
Level 2: q2–6
Level 3: q2–6

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 38 — Symmetry

Solutions
For worked solutions see the Solution Guide

Independent practice

7.


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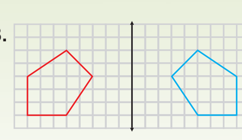

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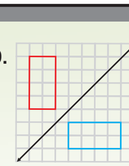


Lesson 3.4.2

Translations

In this Lesson, students develop an understanding of what it means to translate a point or a figure. They also learn to describe translations by writing down the effect they have on the coordinates of points.

Previous Study: In Section 3.2, students practiced plotting points in all four quadrants of the coordinate plane. In the previous Lesson students reflected figures across the x- and y-axes.

Future Study: Later in this Section, students will investigate rotations.

1 Get started

Resources:
- overhead projector and transparencies

Teacher Resources CD-ROM
- Coordinate Grid (or grid paper)

Warm-up questions:
- Lesson 3.4.2

2 Teach

Universal access

Translation is a topic that can be taught well using an overhead projector.

Take two pieces of transparency paper. On one draw a grid and graph a figure on it, then trace the figure onto the second sheet. Use different colors for each figure.

Tape one transparency down on the projector, and place the other on top of it. Translations can then be demonstrated by moving the top transparency the required number of spaces. This gives the visual effect of sliding the figure.

Students can also be given small squares of transparency paper and experiment with translations themselves.

Guided practice

Level 1: q1–4
Level 2: q1–4
Level 3: q1–4

Lesson 3.4.2

Translations

California Standards:
Measurement and Geometry 3.2
Understand and use coordinate graphs to plot simple figures, determine lengths and areas related to them, **and determine their image under translations** and reflections.

What it means for you:
You'll see how to draw and describe translations of shapes.

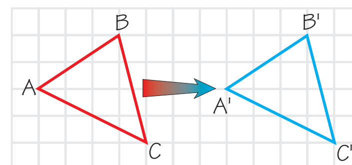
Key words:
- translation
- slide
- image
- coordinates

Don't forget:
The image of point A is the point A' (A prime).

A translation is another type of transformation. When you translate a shape, you slide it around. You can translate a shape up, down, left, or right, or any combination of these.

A Translation Slides a Figure

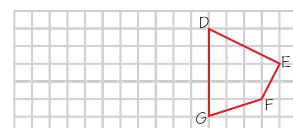
A **translation** takes a shape and **slides** every point of that shape a **fixed distance** in the same **direction**.



The image is the **same size** and **shape**, and the **same way around** as the original figure.

Example 1

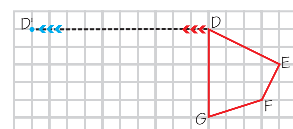
Translate DEFG 10 units to the left.



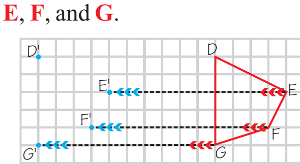
Solution

Step 1: Pick a point to translate — we'll start with point **D**.

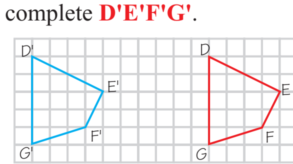
Move **across** the grid and find the point **10 units** to the **left** of point D. This is where you plot the point **D'**.



Step 2: Repeat step 1 for points **E**, **F**, and **G**.



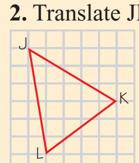
Step 3: **Join** the points to complete **D'E'F'G'**.

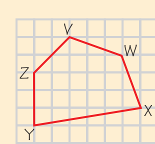


Guided Practice

In Exercises 1–4, copy each shape onto graph paper. Remember to leave enough space to draw the translations.

1. Translate JKL up 7 units. see left
2. Translate JKL left 8 units and down 1 unit. see below

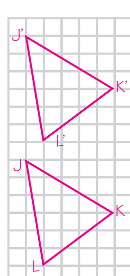




see below

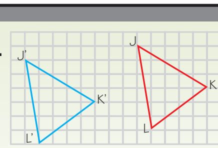
3. Translate VWXYZ left 9 units.
4. Translate VWXYZ down 3 units and right 7 units. see below

1.

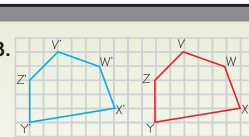


Solutions
For worked solutions see the Solution Guide

2.



3.



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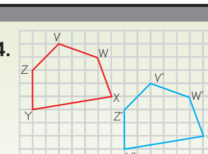


- **Strategic Learners**
  Ask students to cut out a small rectangle from graph paper and label the corners A–D. Have them place their rectangle on another piece of graph paper, draw around it, and label the corners A–D. Next, the students should slide the rectangle to a new location, 5 units to the left, then draw around it and label the corners of the image A'–D'. Repeat with vertical translations, then combine horizontal and vertical translations.

- **English Language Learners**
  Make two sets of matching cards — one set should have descriptions of translations on, and the other set should show the original shape and the translated image. Instruct the students to match the cards, or play games such as pairs with them.

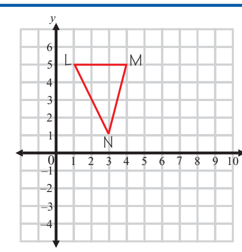
You Can Describe Translations with Coordinates

When you **translate** a shape, the **coordinates** of every point change by the **same amount**. So you can use coordinates to **describe** the translation.

Example 2

Translate the triangle LMN using the translation:

$$(x, y) \to (x + 4, y - 3)$$



Solution

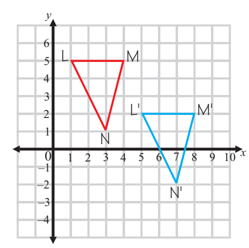
The question tells you how to change the coordinates of the points of LMN.

Start by finding the coordinates of L, M, and N:

L = (1, 5) **M = (4, 5)** **N = (3, 1)**

Now you can apply the transformation:

(x, y)	$\to$	$(x + 4, y - 3)$	
L (1, 5)	$\to$	L' (1 + 4, 5 – 3)	= **(5, 2)**
M (4, 5)	$\to$	M' (4 + 4, 5 – 3)	= **(8, 2)**
N (3, 1)	$\to$	N' (3 + 4, 1 – 3)	= **(7, –2)**



Once you've figured out the coordinates of the image L'M'N', you can draw it on the coordinate grid.

Guided Practice

In Exercises 5–8, copy the shapes and axes shown onto graph paper.

Apply the following translations to triangle PQR:

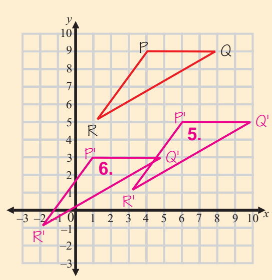
5. $(x, y) \to (x + 2, y - 4)$ see below

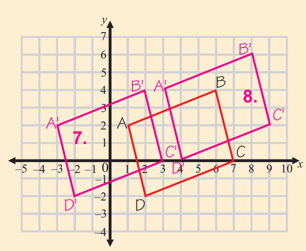
6. $(x, y) \to (x - 3, y - 6)$ see below

Apply the following translations to the quadrilateral ABCD:

7. $(x, y) \to (x - 4, y)$ see below

8. $(x, y) \to (x + 2, y + 2)$ see below





2 Teach (cont)

Math background

In a translation, the size and orientation of a shape stay the same.

Also note that the points of the original image and the translated image are labeled in the same direction. For example, if the points A, B, and C are graphed clockwise, the points A', B', and C' in the translated image will also be graphed clockwise.

Note that the points of a **reflected** image are graphed in the reverse direction to the points in the original image.

Additional example

The triangle LMN has the coordinates L(–9, –5), M(–6, –2), and N(–6, –7).

It has this translation applied to it: $(x, y) \to (x - 2, y - 5)$

What are the coordinates of the points L', M', and N'?

L' = (–11, –10), M' = (–8, –7), N' = (–8, –12)

Guided practice

Level 1: q5–8
Level 2: q5–8
Level 3: q5–8

Check it out:

A lot of people prefer to use coordinates to do translations. If you use the method of counting how many squares to move, it's easy to miscount and put a point in the wrong place.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to combine a reflection across the x- or y-axis with a translation. They should do this, then show the original shape and the image to a partner. The partner should identify the two separate transformations. There will be two different answers, as the translation could have been done before the rotation, or vice versa.

2 Teach (cont)

Additional examples

1. Point S has the coordinates (8, 4). The image of S has the coordinates (11, 4). Describe the translation that has been performed on S.

S has been translated 3 units to the right (the x-coordinate has changed from 8 to 11), and 0 units up or down (the y-coordinate hasn't changed).
So the translation is $(x, y) \rightarrow (x + 3, y)$

2. Triangle TUV has a translation performed on it. Point T has the coordinates (0, –2). After being translated it is at (1, 1).

Point U was originally at (–3, 2). Where will it be after the translation?

T has been translated 1 unit to the right (the x-coordinate has changed from 0 to 1), and 3 units up (the y-coordinate has changed from –2 to 1).
So the translation is $(x, y) \rightarrow (x + 1, y + 3)$

The same translation is performed on U:
U $(-3, 2) \rightarrow (-3 + 1, 2 + 3) = (-2, 5)$

So U will be translated to (–2, 5).

Guided practice

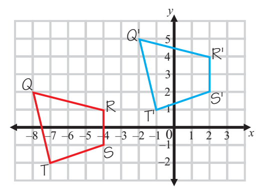
Level 1: q9–12
Level 2: q9–14
Level 3: q9–14

Find the Translation by Looking at the Coordinates

When you look at a shape and its translated **image**, you can figure out what **translation** was used to make the image.

Example 3

Describe the translation from QRST to Q'R'S'T' in coordinate notation.



Solution

Find the coordinates of Q and Q'.

Q = **(–8, 2)** Q' = **(–2, 5)**

The ***x*-coordinate** has changed from –8 to –2.
This is an **increase** of **6**.

The ***y*-coordinate** has changed from 2 to 5, so it has **increased** by **3**.

So the translation is **$(x, y) \rightarrow (x + 6, y + 3)$**

You can check that this is the right answer by seeing if this translation changes R, S, and T to R', S', and T'.

R (–4, 1) → (–4 + 6, 1 + 3) = (2, 4) = R'
S (–4, –1) → (–4 + 6, –1 + 3) = (2, 2) = S'
T (–7, –2) → (–7 + 6, –2 + 3) = (–1, 1) = T'

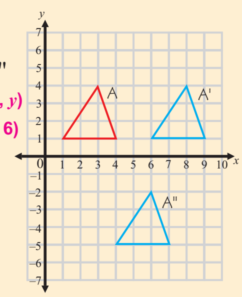
It does — so you have the right answer.

Guided Practice

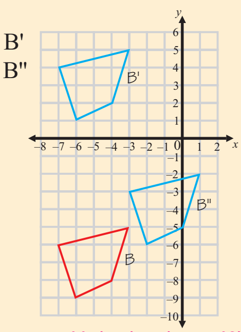
In Exercises 9–14, describe the following translations in coordinates.

9. A to A'
10. A to A"

9. $(x, y) \rightarrow (x + 5, y)$
10. $(x, y) \rightarrow (x + 3, y - 6)$

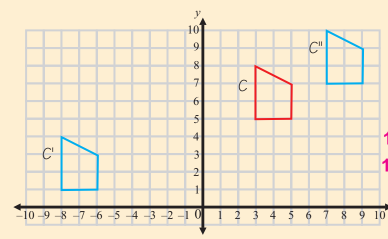


11. B to B'
12. B to B"



11. $(x, y) \rightarrow (x, y + 10)$
12. $(x, y) \rightarrow (x + 4, y + 3)$

13. C to C'
14. C to C"



13. $(x, y) \rightarrow (x - 11, y - 4)$
14. $(x, y) \rightarrow (x + 4, y + 2)$

Check it out:
You don't just use the prime symbol to indicate the image of a single point.
Sometimes it's used when the whole shape is named by a single letter, like in these Exercises.

Solutions
For worked solutions see the Solution Guide

Independent Practice

Copy the shapes and axes shown onto graph paper for Exercises 1–6.

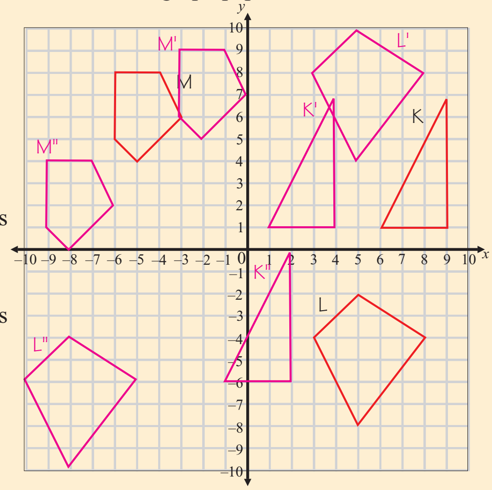
1. Translate K 5 units to the left. Label the image K'.

2. Translate K 7 units left and 7 units down. Label the image K".

3. Translate L 12 units up. Label the image L'.

4. Translate L 13 units left and 2 units down. Label the image L".

5. Translate M 1 unit up and 3 units right. Label the image M'.



6. Translate M 3 units left and 4 units down. Label the image M".

Use coordinate notation to describe the following translations:

7. K to K' **7.** $(x, y) \to (x - 5, y)$ **8.** K to K" **8.** $(x, y) \to (x - 7, y - 7)$

9. L to L' **9.** $(x, y) \to (x, y + 12)$ **10.** L to L" **10.** $(x, y) \to (x - 13, y - 2)$

11. M to M' **11.** $(x, y) \to (x + 3, y + 1)$ **12.** M to M" **12.** $(x, y) \to (x - 3, y - 4)$

Copy the axes and triangle shown onto graph paper for Exercises 13–16.

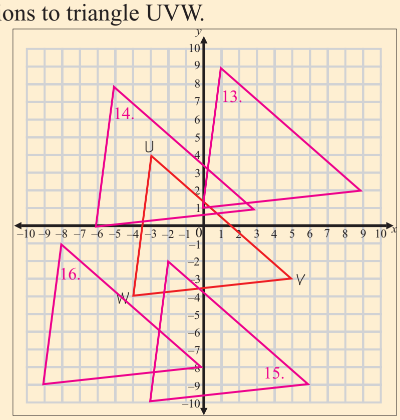
Apply the following translations to triangle UVW.

13. $(x, y) \to (x + 4, y + 5)$

14. $(x, y) \to (x - 2, y + 4)$

15. $(x, y) \to (x + 1, y - 6)$

16. $(x, y) \to (x - 5, y - 5)$



Round Up

Coordinates are really useful for drawing translations, and can help you check your answers. But don't forget that one of the most important checks is to look at the image you've drawn and see if it looks the same as the original.

Now try these:
Lesson 3.4.2 additional questions — p447

Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Independent practice
Level 1: q1–6
Level 2: q1–12
Level 3: q1–16

Concept questions
"Point A is at (1, 1). What can you say about a translation that moves point A to quadrant III?"

It must move left by more than 1 unit, and down by more than 1 unit.

So both the x and y values must be decreased by more than 1.

"What can you say about a translation that moves point A to quadrant IV?"

If it moves left, this must be by less than 1 unit. It can move right by any amount. It must move down by more than 1 unit.

Additional questions
Level 1: p447 q1–5
Level 2: p447 q1–7
Level 3: p447 q1–9

3 Homework

Homework Book — Lesson 3.4.2
Level 1: q1a, 2a, 3, 4
Level 2: q1–5
Level 3: q1–6

Lesson 3.4.3

Scale Factors

In this Lesson, students apply scale factors to simple figures to change their size. They also compare pairs of figures and identify the scale factor that was used to change one figure into the other.

Previous Study: In grade 3 students measured lengths using appropriate tools and units. In grade 6 students expressed in symbolic form simple relationships arising from geometry.

Future Study: In the next Lesson students will read and create scale drawings. In Chapter 7, they will compare solid figures that have had scale factors applied.

1 Get started

Resources:
- school photos of various sizes
- magazine pictures (see Advanced Learners activity)
- construction paper rectangles (see below)
- rulers

Warm-up questions:
- Lesson 3.4.3 sheet

2 Teach

Universal access

The concept of enlargements can be introduced by working backward. Provide three construction paper rectangles that are all in proportion to one another. For instance, 1 cm by 3 cm, 3 cm by 9 cm, and 4 cm by 12 cm.

Provide each student with a ruler and get them to measure the length and width of each rectangle. Discuss how the lengths and widths relate to one another.

Concept question

"What is the result of applying a scale factor of 1 to a figure?"

The figure doesn't change size.

Additional examples

1. A square has sides of 3 cm. An enlargement of the square has sides of 30 cm. What scale factor was used?

A scale factor of 10.

2. Point H(5, 6) and point J(10, 6) form line segment HJ. What is the length of H'J' if HJ is enlarged by a scale factor of 4?

Length of HJ is 5 units.
The length of H'J' would be 20 units.

Lesson 3.4.3

Scale Factors

California Standards:
Measurement and Geometry 1.2
Construct and read drawings and models **made to scale**.

What it means for you:
You'll learn how to use scale factors to produce images that have the same shape as another figure, but a different size.

Key words:
- scale factor
- image
- multiply

Check it out:
A scale factor of 1 leaves the shape exactly the same size.

In this Section so far you've seen two types of transformation — reflections and translations. These both give an image that's the same size as the original.

Another type of transformation changes the size of shapes. The scale factor tells you by how much the size changes.

A Scale Factor of More Than 1 Makes a Shape Bigger

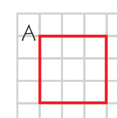
Sometimes an image is **identical** to the original apart from its **size**. The **scale factor** tells you how much larger or smaller the image is.

The scale factor is a **number**. You **multiply** all the lengths in the **original** by the scale factor to get the lengths in the **image**.

So: original length × scale factor = image length

Example 1

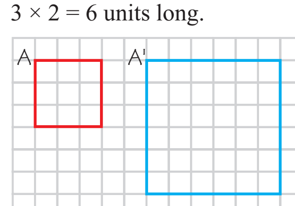
Draw an image of square A using a scale factor of 2.



Solution

The sides of A are 3 units long.

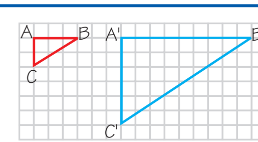
So if you apply a scale factor of 2, the sides of the image will be $3 \times 2 = 6$ units long.



So the image **A'** is a square with side length **6 units**.

Example 2

What scale factor has been used to enlarge ABC to A'B'C'?



Solution

The scale factor is given by dividing any length in the **image** by the corresponding length in the **original**.

Length of A'B' = 9
Length of AB = 3

So the scale factor is:

Length of A'B' ÷ Length of AB = 9 ÷ 3 = **3**

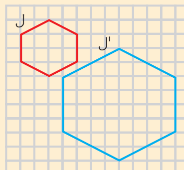
- **Strategic Learners**
  Ask students to compare and contrast translations (sliding), enlargements (resizing), and reflections. Show examples of each using a silhouette of a nonsymmetrical object, such as the side view of a coffeepot. Ask students to identify the transformation each time.

- **English Language Learners**
  Show different sizes of a school photo (for example, wallet size, 4" × 6", and 8" × 10") as examples of resizing. Have students work in partners to see how their pupils change in size when they stare at a light, or cover their eyes with their hands. Create a word bank of related words such as dilation, enlargement, expansion, contraction, and compression.

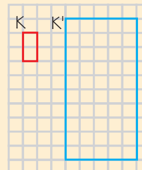
Guided Practice

In Exercises 1–5, find the scale factor that has produced each image.

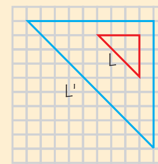
1. 2



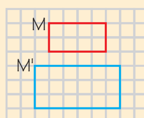
2. 5



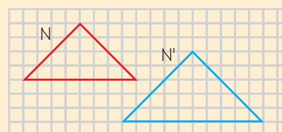
3. 3



4. 1.5

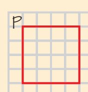


5. 1.25

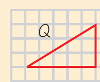


Copy the shapes shown in Exercises 6–9 onto graph paper.
Draw the image produced by applying the given scale factor.

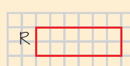
6. Scale factor 3
see left



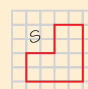
7. Scale factor 2
see below



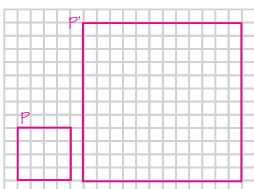
8. Scale factor 2.5
see left



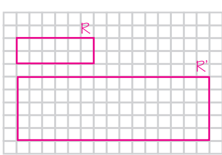
9. Scale factor 1.5
see below



6.



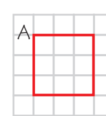
8.



A Scale Factor of Less Than 1 Makes a Shape Smaller

Example 3

Draw an image of square A using a scale factor of $\frac{1}{3}$.



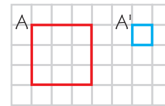
Solution

The sides of A are 3 units long.

So if you apply a scale factor of $\frac{1}{3}$, the sides of the image will be

$$3 \times \frac{1}{3} = 1 \text{ unit long.}$$

So the image A' is a square with side length **1 unit**.

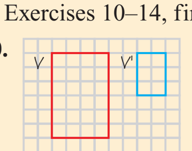


Check it out:
Decimals could also be used for scale factors. The same method applies — multiply the original dimension by the decimal scale factor to find the dimension of the image.

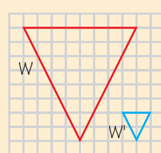
Guided Practice

In Exercises 10–14, find what scale factor has produced each image.

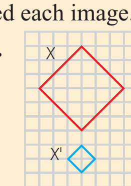
10. $\frac{1}{2}$



11. $\frac{1}{4}$



12. $\frac{1}{3}$



2 Teach (cont)

Guided practice

Level 1: q1–3, 6–7
Level 2: q1–5, 6–8
Level 3: q1–9

Application

Maps and plans are drawn to certain scales. For instance, a map might have a scale of 1 : 50,000. This means that every inch on the map represents 50,000 inches in reality. A map of 1 : 25,000 would show a greater amount of detail — things on it would appear twice the size.

There's more on using maps and plans drawn to particular scales in the next Lesson.

Concept questions

1. "A picture is enlarged by a scale factor of 4 to produce a poster for a wall display. What scale factor must be applied to get back to the original size from the poster?"
$\frac{1}{4}$

2. "A drawing has a scale factor of $\frac{1}{10}$ applied so that it fits on a postage stamp. What scale factor must be applied to the postage stamp image to get back to the original size drawing?"
10

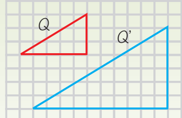
Guided practice

Level 1: q10–12, 15–16
Level 2: q10–12, 15–16
Level 3: q10–18

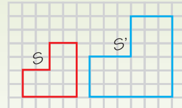
Solutions

For worked solutions see the Solution Guide

7.



9.



- **Advanced Learners**
  Provide students with pictures cut from magazines of things of known approximate sizes (such as people, or vehicles) next to things of unknown sizes (such as buildings). Then ask the students to use the height of the known object to find the approximate scale factor. Then they can use the scale factor to estimate the height of the object of unknown size.

2 Teach (cont)

Common error
Students will often automatically divide a longer side length by a shorter side length, and always get a scale factor greater than 1.
Remind them that the scale factor is the image length divided by the original length, and it can be less than 1.

Independent practice
Level 1: q1–2, 7–9
Level 2: q1–4, 7–9
Level 3: q1–10

Additional questions
Level 1: p448 q1–7
Level 2: p448 q1–7
Level 3: p448 q1–7

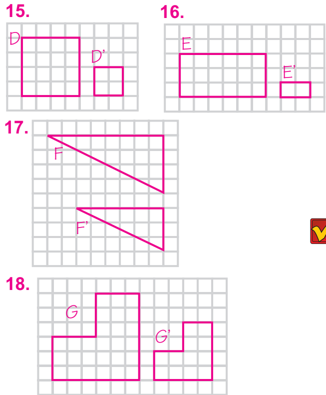
3 Homework

Homework Book — Lesson 3.4.3
Level 1: q1a–b, 3, 4a–b
Level 2: q1–5
Level 3: q1–6

4 Skills Review

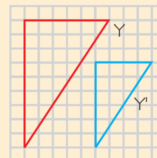
Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 39 — Scale and Proportion

Don't forget:
Go back to Lesson 2.3.3 if you need a reminder on multiplying fractions by integers.

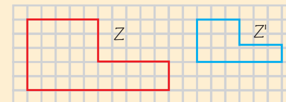


Now try these:
Lesson 3.4.3 additional questions — p448

13. $\frac{2}{3}$

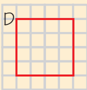


14. $\frac{3}{5}$

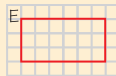


Copy the shapes shown in Exercises 15–18 onto graph paper. Draw the image produced by applying the given scale factor.

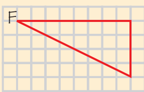
15. Scale factor $\frac{1}{2}$ see left



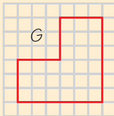
16. Scale factor $\frac{1}{3}$ see left



17. Scale factor $\frac{3}{4}$ see left



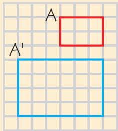
18. Scale factor $\frac{2}{3}$ see left



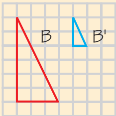
Independent Practice

In Exercises 1–6, find what scale factor has produced each image.

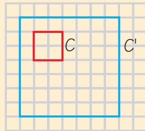
1. 2



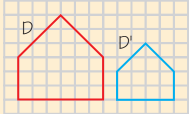
2. $\frac{1}{3}$



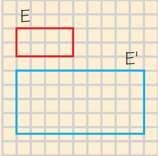
3. 3.5



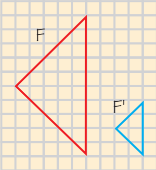
4. $\frac{2}{3}$



5. 2.25

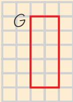


6. $\frac{2}{5}$

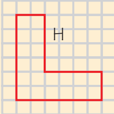


Copy the shapes shown in Exercises 7–10 onto graph paper. Draw the image produced by applying the given scale factor.

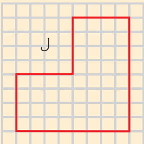
7. Scale factor 2 see below



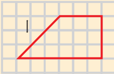
8. Scale factor $\frac{1}{2}$ see below



9. Scale factor $\frac{1}{4}$ see below



10. Scale factor $1\frac{1}{3}$ see below

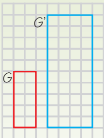


Round Up

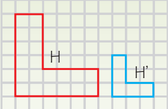
The scale factor tells you how much bigger or smaller than the original object an image is. You'll use scale factors to make and understand scale drawings, which you'll learn about in the next Lesson.

Solutions
For worked solutions see the Solution Guide

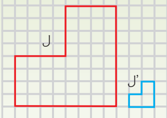
7.



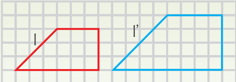
8.



9.



10.



Lesson 3.4.4 Scale Drawings

In this Lesson, students use their knowledge of scale factors to calculate how big real-life objects should be in scale drawings, such as maps and plans. They also calculate real-life measurements using lengths from scale drawings.

Previous Study: In the previous Lesson, students found scale factors and applied these to figures. In grade 6 students used proportions to solve problems.

Future Study: In Chapter 7 students will apply scale factors to 3-D objects and calculate their new dimensions.

Lesson 3.4.4

Scale Drawings

California Standards:
Measurement and Geometry 1.2
Construct and read drawings and models made to scale.

What it means for you:
You'll learn how to draw pictures that accurately represent real places or objects. You'll also use drawings to find information about the places or objects they represent.

Key words:
- scale drawing
- scale factor
- measurement
- distance
- scale

Don't forget:
Scales are often written as ratios. Ratios are a way of comparing two numbers — you should have learned about them in grade 6.
You can also write a ratio as a fraction, so the scale can be written as $\frac{1 \text{ inch}}{4 \text{ feet}}$.
The ratio of the length on the drawing to the real-life length must be equivalent to the scale ratio. That's why you can write the proportion as $\frac{1 \text{ inch}}{4 \text{ feet}} = \frac{x}{24 \text{ feet}}$.

Scale drawings often show real objects or places — maps are good examples of scale drawings. All the measurements on the drawing are related to the real-life measurements by the same scale factor. So if you know the scale factor, you can figure out what the real-life measurements are.

To Make Scale Drawings You Need Real Measurements

To make a **scale drawing** of an object or place, you need two things.

First, you need the **real-life measurements** of what you're going to draw.

Second, you need a **scale**. This will tell you what the distances on the drawing **represent**. The scale is usually written as a **ratio**.
If **1 inch** on the drawing represents **10 feet** in real life, the scale is **1 inch : 10 feet**.

Example 1

A rectangular yard has a length of 24 feet and a width of 20 feet. Make a scale drawing using a scale of 1 inch : 4 feet.

Solution

You need to find the **length and width of the yard in the drawing**.

To convert the real-life **length** into a length for the drawing, set up a proportion using the scale given.

Let x be the length the yard in the drawing should be.

$$\frac{\text{Drawing length}}{\text{Real-life length}} = \frac{1 \text{ inch}}{4 \text{ feet}} = \frac{x}{24 \text{ feet}}$$

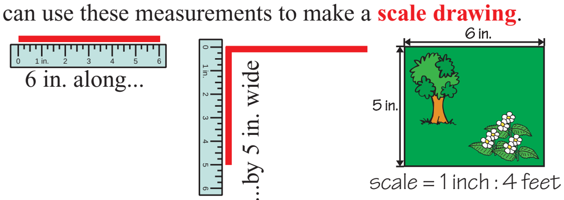
$$\frac{1 \text{ inch} \times 24 \text{ feet}}{4 \text{ feet}} = x \times \frac{\cancel{24 \text{ feet}}}{\cancel{24 \text{ feet}}} = x$$ **Multiply both sides by 24 ft**

So, $x = 1 \text{ in.} \times \frac{24 \text{ feet}}{4 \text{ feet}} = 1 \text{ in.} \times 6 = $ **6 in.**

Repeat the process using y for the **width** of the drawing and you find:

$y = 1 \text{ in.} \times \frac{20 \text{ feet}}{4 \text{ feet}} = 1 \text{ in.} \times 5 = $ **5 in.**

You can use these measurements to make a **scale drawing**.



1 Get started

Resources:
- large sheet of paper with grid lines
- meter rulers/tape measures
- furniture/kitchen/bathroom catalogs

Warm-up questions:
- Lesson 3.4.4 sheet

2 Teach

Universal access
Making a scale drawing of the classroom is a good way to introduce this Lesson.
Explain that the room will be drawn from a bird's-eye view — as if it were looked at from above.

Use a large piece of paper with inch or two-inch grid lines on it and decide on a suitable scale appropriate to the size of the classroom and the piece of paper. Draw the outline of the classroom on it.

Assign each pair of students to measure a certain item, such as the teacher's desk or the door. They also need to measure how far from the walls or corner their item is. Next they add what they've measured to the class picture.

The class should then discuss the drawing and see if anything looks off-scale or in the wrong place.

Math background
A proportion is an equation that states that two ratios are equal. When setting up a proportion, the units for one ratio's numerator and denominator must agree with the units for the other.

Correct proportion: $\frac{\text{in.}}{\text{ft}} = \frac{\text{in.}}{\text{ft}}$

Incorrect proportion: $\frac{\text{in.}}{\text{ft}} = \frac{\text{ft}}{\text{in.}}$

Additional example
Use the scale of 2 inches : 3 feet to make a scale drawing of a rectangle with a length of 6 feet and width of 3 feet.

The scale drawing should be a rectangle measuring 4 inches by 2 inches.

- **Strategic Learners**
  Ask students to make scale drawings (by hand or computer) that match their interests (for example, a football pitch, basketball court, etc.). Suggest suitable scales to use.

- **English Language Learners**
  Making a plan of the classroom by measuring objects in pairs is a useful activity for English language learners, especially if they are paired up with language buddies. This is described in detail in the margin of the previous page.

2 Teach (cont)

Guided practice
Level 1: q1–2
Level 2: q1–3
Level 3: q1–4

Additional examples
1. The distance on a map between points A and F is 4.5 inches.
The scale of the map is 2 inches : 3 feet.
What is the actual distance between points A and F?

$\frac{2\text{ in.}}{3\text{ ft}} = \frac{4.5\text{ in.}}{y}$

y = 6.75 feet

2. A 20-inch model of a 150-foot ship is being designed.
What is 1 inch of the model equal to on the ship?

$\frac{20\text{ in.}}{150\text{ ft}} = \frac{1\text{ in.}}{y}$

y = 7.5 feet

Guided practice
Level 1: q5–8
Level 2: q5–8
Level 3: q5–8

Guided Practice

In Exercises 1–4, make the following scale drawings:

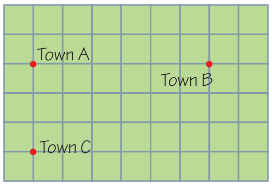
1. A square of side length 4 m, using the scale 1 cm : 1 m. **see below**
2. A rectangle measuring 40 in. by 60 in., using the scale 1 in. : 20 in.
3. A rectangular room measuring 6 ft by 12 ft, using the scale 1 in. : 3 ft.
4. A circular pond with diameter 3 m, using the scale 1 cm : 2 m.

You Can Use Scale Drawings to Find Actual Lengths

The **size** of real objects can be found by **measuring** scale drawings.

Example 2

This map shows three towns.
Find the real-life distances between:
- Town A and Town B
- Town A and Town C



Scale — 1 grid square : 2.5 miles

Solution

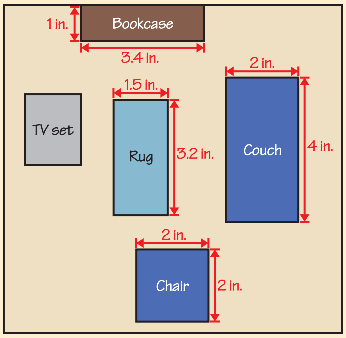
The distance between Town A and Town B on the map is **6 grid squares**.

The scale tells us that 1 grid square represents 2.5 miles, so the distance between Town A and Town B is 6 × 2.5 miles = **15 miles**.

Town A and Town C are **3 grid squares** apart on the map.
In real life this is equal to 3 × 2.5 miles = **7.5 miles**.

Guided Practice

This picture shows a scale drawing of the living room in Lashona's house.
The scale used is 2 in. : 3 feet.



In Exercises 5–8, find the real-life measurements of:
5. The chair **3 ft × 3 ft**
6. The couch **3 ft × 6 ft**
7. The bookcase **1.5 ft × 5.1 ft**
8. The rug **2.25 ft × 4.8 ft**

Check it out:
When you're buying new furniture, such as kitchen units, you might use a scale drawing of the room to decide where you want the furniture (or which furniture would fit).

Solutions
For worked solutions see the Solution Guide

1. A square with sides of 4 cm.
2. A rectangle of width 2 in. and length 3 in.
3. A rectangle of width 2 in. and length 4 in.
4. A circle of diameter 1.5 cm.

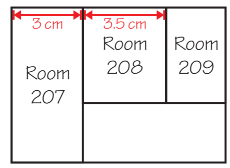
- **Advanced Learners**
  Ask students to create scale diagrams of the furniture they'd like in different rooms of a house, using catalogs that state furniture dimensions. Also, ask them to add up the cost of their selections.

You Can Sometimes Find Real Lengths Without a Scale

If you know **one** of the real-life lengths shown on a scale drawing, then you can **figure out** the others without a scale.

Example 3

This scale drawing shows three classrooms at Gabriel's school. Gabriel measures the drawing. His measurements are shown in red.



Gabriel knows Room 207 is 4.2 m wide in real life. What is the real-life width of Room 208?

Solution

You can find the answer by **setting up a proportion**, similar to the one in Example 1. Use x for the real-life width of Room 208.

$$\frac{\text{Real-life width}}{\text{Drawing width}} = \frac{4.2\text{ m}}{3\text{ cm}} = \frac{x}{3.5\text{ cm}}$$

$$x = 4.2\text{ m} \times \frac{3.5\text{ cm}}{3\text{ cm}} = 4.9\text{ m}$$

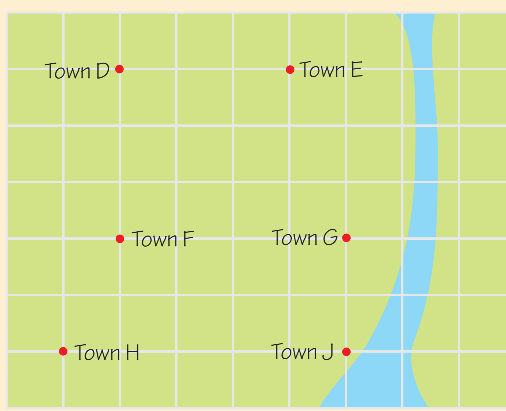
So the width of Room 208 is 4.9 m in real life.

Check it out:
Another way of doing this is to find the scale factor that's been used and use it to find the other real-life lengths, as you did before.
The scale factor is the ratio between the real-life length and the drawing length.

So in Example 3, the scale factor would be 4.2 m ÷ 3 cm, which is 1.4 m/cm.
To find the real-life width of room 208, you'd just multiply the drawing width by the scale factor — 3.5 cm × 1.4 m/cm = **4.9 m**. This gives the same answer as the other method.

Guided Practice

Use the map below to answer Exercises 9–14.



It is 18 miles from Town D to Town E. Calculate the distance from:

9. Town D to Town F **18 miles**
10. Town F to Town G **24 miles**
11. Town G to Town J **12 miles**
12. Town H to Town J **30 miles**
13. Town D to Town G **30 miles**

14. Find the number that completes the following sentence:
The scale on this map is 1 grid square : __6__ miles.

2 Teach (cont)

Additional example

A distance of 15 miles is represented by 2 inches on a map.
How many inches on the map would represent a distance of 75 miles?

$$\frac{2\text{ inches}}{15\text{ miles}} = \frac{x}{75\text{ miles}}$$

x = 10 inches

Common errors

Students often set up proportions incorrectly, with numerators and denominators that don't have the same units. Errors also occur when writing the units for an answer.

Encourage students to write the units in their calculations, so they can check that their proportions are correct.

Unit analysis (covered in Lesson 1.2.7) allows students to check what units their answers should have.

Guided practice

Level 1: q9–14
Level 2: q9–14
Level 3: q9–14

Solutions

For worked solutions see the Solution Guide

2 Teach (cont)

Independent practice
Level 1: q1–4
Level 2: q1–7
Level 3: q1–11

Additional questions
Level 1: p448 q1–10
Level 2: p448 q1–12
Level 3: p448 q1–13

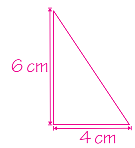
3 Homework

Homework Book — Lesson 3.4.4
Level 1: q1–3, 5
Level 2: q1–7
Level 3: q1–7

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 39 — Scale and Proportion

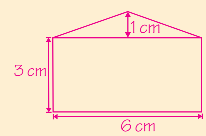
Independent Practice

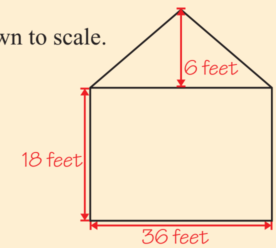


1. A sail for a boat is in the shape of a right triangle. The actual height of the sail is 18 feet, and it has a base of 12 feet. Make a scale drawing of the sail using a scale of 1 cm : 3 ft. **see left**

2. This sketch of a house has not been drawn to scale.

Make a scale drawing of the house using a scale of 1 cm : 6 ft.



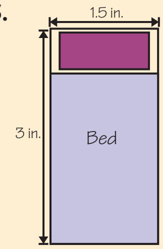


3. A scale model of a town uses a scale of 1 inch : 30 feet. Find the actual height of a building that is 2.5 in. tall in the model.
75 feet

4. On a map, 2 inches represents 45 miles. What does one inch represent on this map? **22.5 miles**

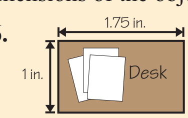
Amanda is drawing a plan of her bedroom using a scale of 1 in. : 2 ft. Exercises 5–7 show objects from the plan. Calculate the real-life dimensions of the objects.

5.



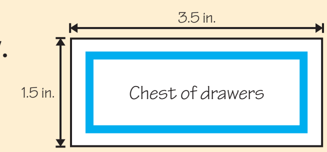
3 ft × 6 ft

6.



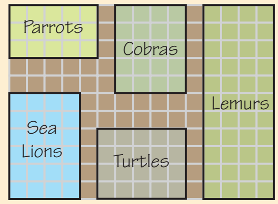
2 ft × 3.5 ft

7.



3 ft × 7 ft

The scale drawing below shows part of a zoo. The parrot enclosure measures 40 m by 24 m. Find the following real-life measurements:



8. The length and width of the sea lion enclosure. **48 m × 32 m**

9. The length and width of the lemur enclosure. **88 m × 32 m**

10. The perimeter of the turtle enclosure. **144 m**

11. The area of the cobra enclosure. **1280 m²**

Now try these:
Lesson 3.4.4 additional questions — p448

Round Up

Pictures that are drawn to scale can be very useful. If maps weren't made to scale, they would be much harder to use. And if plans and blueprints for buildings or machines weren't done as scale drawings, it would be difficult to build them the right size and shape.

Solutions
For worked solutions see the Solution Guide

Lesson 3.4.5

Perimeter, Area, and Scale

When a scale factor is applied, the perimeter and area of a shape changes. In this Lesson, students practice calculating the new areas and perimeters of resized shapes. They also calculate scale factors by comparing areas and perimeters.

Previous Study: In previous grades, students learned to calculate perimeters and areas of rectangles, squares, triangles, and circles. This was reviewed and extended earlier in this Chapter.

Future Study: In Chapter 7, students will investigate the effect on surface areas and volumes of applying a scale factor to a 3-D figure.

Lesson 3.4.5

Perimeter, Area, and Scale

California Standards:

Measurement and Geometry 1.2
Construct and read drawings and models **made to scale.**

Measurement and Geometry 2.0
Students compute the perimeter, area, and volume **of common geometric objects** and use the results to find measures of less common objects. **They know how perimeter, area,** and volume **are affected by changes of scale.**

What it means for you:

You'll see the effect that multiplying by a scale factor has on perimeter and area.

Key words:

- perimeter
- area
- scale factor
- image

Don't forget:

The formula for the perimeter of a rectangle is $P = 2(l + w)$. For more about perimeter see Section 3.1.

Check it out:

You'll need to use the Pythagorean theorem to help you find the perimeter of these triangles. See Section 3.3 for more.

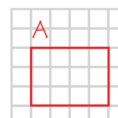
So far you've been looking at how length and width are altered by applying a scale factor. In this Lesson, you're going to see how applying scale factors affects perimeter and area.

Applying a Scale Factor Changes the Perimeter

When you change the **size** of a shape, the **perimeter** changes too.

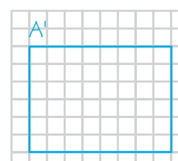
Example 1

Gilberto draws an image of rectangle A using a scale factor of 2. Find the perimeter of rectangle A. What is the perimeter of the image A'?



Solution

Rectangle A is **3** units wide and **4** units long.
So A' will be $2 \times 3 =$ **6 units wide** and $2 \times 4 =$ **8 units long**.



The perimeter of A is $2(3 + 4) = 2 \times 7 =$ **14 units**.
The perimeter of A' is $2(6 + 8) = 2 \times 14 =$ **28 units**.

In the example above, the perimeter of the **image** is **double** the perimeter of the **original**. This is because all the **lengths** that you add together to find the perimeter have been **multiplied by 2**.

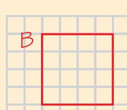
The perimeter of the image is the perimeter of the original **multiplied by the scale factor**. This is true for any shape and any scale factor, so:

original perimeter × scale factor = image perimeter

Guided Practice

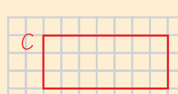
In Exercises 1–6, find the perimeter of the image you would get if you applied the given scale factor to the figure shown.
Give your answers in units. You do not need to draw the images.

1. Scale factor $\frac{1}{2}$



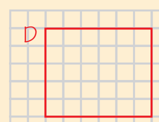
8 units

2. Scale factor 3



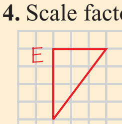
60 units

3. Scale factor 5



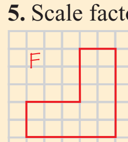
110 units

4. Scale factor 2



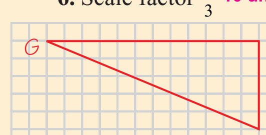
24 units

5. Scale factor 2.5



50 units

6. Scale factor $\frac{1}{3}$ 10 units



1 Get started

Resources:
- grid paper

Warm-up questions:
- Lesson 3.4.5 sheet

2 Teach

Universal access

In pairs, have students draw a 2 × 3 rectangle on squared paper — this rectangle will be the original. Then ask them to make a chart with the headings: scale factor, length, width, perimeter, and area, and complete it for the original rectangle. They should continue by increasing the size of the rectangle by scale factors of 2, 3, 4, and 5, and completing the table for each. Ask students to report any patterns they find.

Concept question

"A square tablecloth requires 4.5 meters of lace to go around the edge. A smaller tablecloth is made by applying a scale factor of two-thirds. How much lace is needed to go around the edge of the smaller cloth?"

3 meters

Guided practice

Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  The Universal access activity described on the previous page is particularly suitable for strategic learners. It may be best to focus students' attention first on how the perimeter changes. Then ask them to add an extra column for area calculations so that they can look for patterns in how the area increases.

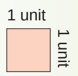
- **English Language Learners**
  Review the terms area and perimeter, and provide examples of area and perimeter calculations for simple shapes. The Strategic Learners activity above is also valuable for English language learners — they should be paired with a language buddy for support.

2 Teach (cont)

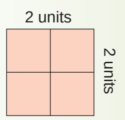
Common error

Students often assume that when a scale factor is applied to a shape, everything is multiplied by the scale factor, including angle measures and area.

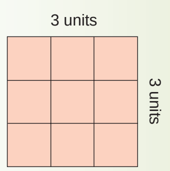
Illustrate that this is not the case using squared units:



Area = 1 unit^2



Scale factor = 2
Area = 4 unit^2



Scale factor = 3
Area = 9 unit^2

The angle measures stay at 90°.

Universal access

Students can investigate the effect of applying a scale factor on the area of a circle. They should create a table for circles of different sizes, like this:

Scale factor	Radius	Area (πr^2)
1	1	π
2	2	4π
3	3	9π

They should see that when a scale factor is applied, the area of a circle increases in the same way as the area of other shapes.

Guided practice

Level 1: q7–10
Level 2: q7–11
Level 3: q7–12

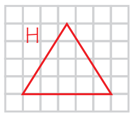
Areas Also Change When You Apply Scale Factors

Area also gets larger or smaller as a figure changes **size**.

Don't forget:
The formula for the area of a triangle is $A = \frac{1}{2}(b \times h)$.
For more about area see Section 3.1.

Example 2

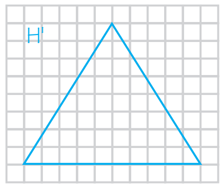
Chelsea uses a scale factor of 2 to draw an image of triangle H. Find the area of H and of the image H'.



Solution

Triangle H has a base of **5 units** and height of **4 units**.

So the base of H' will be 2 × 5 = **10 units** and its height will be 2 × 4 = **8 units**.



The area of H is $\frac{1}{2}(5 \times 4) = \frac{1}{2} \times 20 = $ **10 units^2**.

The area of H' is $\frac{1}{2}(10 \times 8) = \frac{1}{2} \times 80 = $ **40 units^2**.

In the example above, the area of the **image** is **4 times** the area of the **original**. The **lengths** that you multiply together to find the area have both been **multiplied by 2**, so the area is multiplied by 2 × 2 = 4.

The area of the image is the area of the original **multiplied by the scale factor squared**. This is true for **any shape** and **any scale factor**.

original area × (scale factor)^2 = image area

Example 3

Alejandra draws an image of shape J. She uses a scale factor of 3. If the area of shape J is 5 cm^2, what is the area of the image J'?

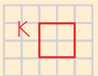
Solution

$$\begin{aligned}\text{The area of the image} &= (\text{area of the original}) \times (\text{scale factor})^2 \\ &= 5\text{ cm}^2 \times 3^2 \\ &= 5\text{ cm}^2 \times 9 = \mathbf{45\text{ cm}^2}\end{aligned}$$

Guided Practice

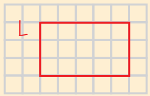
In Exercises 7–12, find the area of the image you would get if you applied the given scale factor to the figure shown.
Give your answers in units^2. You do not need to draw the images.

7. Scale factor 4



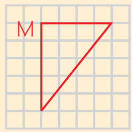
64 units^2

8. Scale factor 3



135 units^2

9. Scale factor 10

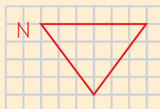


1000 units^2

Solutions
For worked solutions see the Solution Guide

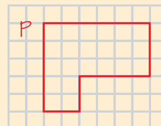
- **Advanced Learners**
  Ask advanced learners to consider how the amount of gift wrap required to cover a gift in the shape of a rectangular prism increases as the dimensions of the prism increase (they should keep all the dimensions constant except for one).
  The concept question below is also particularly suitable for advanced learners.

10. Scale factor 2



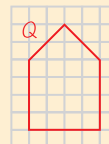
48 units^2

11. Scale factor 5



550 units^2

12. Scale factor 20



8000 units^2

Independent Practice

Roger draws a figure with a perimeter of 8 units.
Find the perimeter of the image if Roger multiplies his figure by:

1. Scale factor 2 16 units
2. Scale factor 11 88 units
3. Scale factor 4.5 36 units
4. Scale factor 0.25 2 units

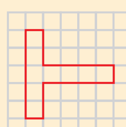
Daesha draws a figure with an area of 10 cm^2.
Find the area of the image if Daesha multiplies her figure by:

5. Scale factor 2 40 cm^2
6. Scale factor 3 90 cm^2
7. Scale factor 7.5 562.5 cm^2
8. Scale factor 0.5 2.5 cm^2

In Exercises 9–10, find the perimeter of the image you would get if you applied the given scale factor to the figure shown. Give your answers in units. You do not need to draw the images.

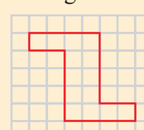
9. Scale factor 9

180 units



10. Scale factor $\frac{1}{4}$

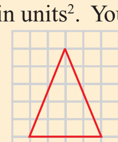
5.5 units



In Exercises 11–12, find the area of the image you would get if you applied the given scale factor to the figure shown. Give your answers in units^2. You do not need to draw the images.

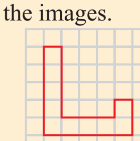
11. Scale factor 2

40 units^2



12. Scale factor 7

490 units^2



What scale factor has been used in the following transformations?

13. Perimeter of original = 13 cm, perimeter of image = 26 cm 2
14. Perimeter of original = 22 in., perimeter of image = 77 in. 3.5
15. Perimeter of original = 50 in., perimeter of image = 5 in. $\frac{1}{10}$
16. Perimeter of original = 15 cm, perimeter of image = 3.75 cm $\frac{1}{4}$
17. Area of original = 10 in^2, area of image = 90 in^2 3
18. Area of original = 1 cm^2, area of image = 36 cm^2 6
19. Area of original = 8 in^2, area of image = 128 in^2 4
20. Area of original = 5 cm^2, area of image = 125 cm^2 5

Now try these:
Lesson 3.4.5 additional questions — p449

Don't forget:
You'll need to find square roots in some of these Exercises. You learned about them in Section 2.5.

Round Up

The effects of scale factor on perimeter and area can be confusing, but they do make sense.
Try to remember them, because understanding them is an important part of geometry in general.

Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Independent practice
Level 1: q1–12
Level 2: q1–16
Level 3: q1–20

Concept question
"A horse owner has a square field, with an area of one acre. She remembers that she needed about 280 yards of fencing to enclose the field. She buys a new, square, two-acre field and buys 560 yards of fencing to enclose it with. Is this the right amount of fencing?"

No. If the area has doubled, the perimeter won't have. The perimeter will increase by a factor of $\sqrt{2}$. This is about 1.41, so she'll need approximately 395 yards of fencing.

Additional questions
Level 1: p449 q1–2, 6
Level 2: p449 q1–8
Level 3: p449 q1–8

3 Homework

Homework Book — Lesson 3.4.5
Level 1: q1–6
Level 2: q1–7
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 31 — Perimeter
- Worksheet 33 — Area
- Worksheet 39 — Scale and Proportion

Congruence and Similarity

In this Lesson, students learn the definitions of "congruent" and "similar" and practice identifying congruent and similar shapes.

Previous Study: In grade 4, students identified congruent figures, and also those with bilateral and rotational symmetry.

Future Study: In geometry, students will prove basic theorems involving congruence and similarity. They will also 'prove' whether or not two triangles are congruent or similar.

1 Get started

Resources:
- shapes made from heavy construction paper (see Universal access activity below and Strategic Learners activity on next page)
- additional construction paper
- strips of paper of varying lengths (see Universal access activity on the next page, and the Advanced Learners activity)
- paper fasteners (split pin type)

Warm-up questions:
- Lesson 3.4.6 sheet

2 Teach

Universal access

Distribute 12 triangles of varying sizes to each student. Be sure to include some congruent triangles, some similar triangles, and triangles that have no match. Students should organize the triangles by a classification system of their own choosing. Ask students to justify their method of classification.

Use this as a starting point for discussing the concepts of congruent figures, figures that are similar to one another, and figures that have no similarities.

Ask students to make congruent matches for the triangles that lacked a partner. They can do this by drawing around the triangles onto construction paper.

Math background

The concepts of similar and congruent figures are connected to the concept of resizing.

A figure that is similar to another can be made congruent, by shrinking or enlarging it by the appropriate scale factor. Enlarging a figure by a scale factor of 1 will make a figure which is congruent to the original.

Guided practice

Level 1: q1–8
Level 2: q1–8
Level 3: q1–8

Lesson 3.4.6

Congruence and Similarity

California Standards:

Measurement and Geometry 3.4

Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

What it means for you:

You'll learn the meaning of the terms congruent and similar. You'll find out how to tell if two shapes are congruent, similar, or neither.

Key words:
- congruent
- similar
- size
- shape
- scale factor

Don't forget:

If a shape is flipped over, it's called a reflection. A shape and its reflection are congruent.

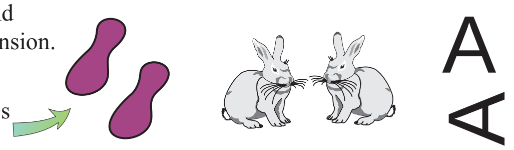
Congruent figures are shapes that are exactly the same size and shape as each other. That means that if you could lift them off the page, there would always be a way to make them fit exactly on top of each other, just by flipping them over or turning them around.

Congruent Figures Have the Same Size and Shape

Two figures are **congruent** if they match perfectly when you place them on top of each other.

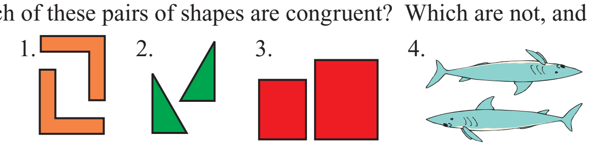
They can be **turned around** or **flipped over**, but they always have the same **size**, **shape**, and **length** of each dimension.

These pairs of shapes are all congruent.



Example 1

Which of these pairs of shapes are congruent? Which are not, and why?



Solution

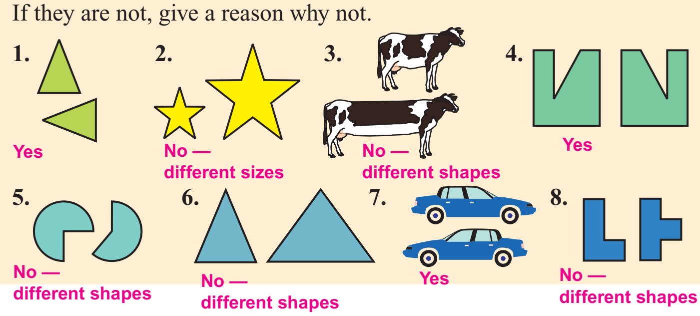
In pairs 1 and 4, each shape is identical to the other, but upside down. So pairs **1** and **4** are **congruent**.

Pair **2** is also **congruent**, as each shape is a mirror image of the other.

The rectangles in pair **3** are the same shape but they're **not** the same **size**, so they're **not congruent**.

Guided Practice

In Exercises 1–8, say whether or not each pair of shapes is congruent. If they are not, give a reason why not.



Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
Cut out congruent figures and noncongruent figures from heavy construction paper. Ask students to try to fit them one on top of another. A similar activity can be done with similar and nonsimilar figures — angles can be measured, marked on the shapes, and compared.

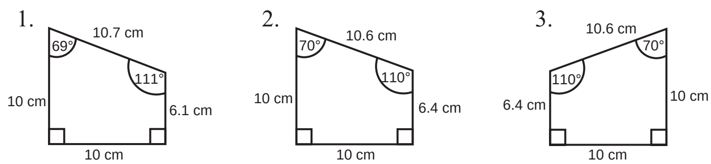
- **English Language Learners**
Use the “take notes, make notes” format to distinguish between “congruent,” “similar,” and “neither congruent nor similar shapes.” Include both the lengths of sides and the sizes of angles as deciding factors. Students should share “make notes” with a partner.

Congruent Polygons Have Matching Sides and Angles

Sometimes two **polygons** might look quite **alike**. You can tell for sure if they’re **congruent** if you know the **measures** of their sides and angles.

Example 2

Which two of these quadrilaterals are congruent?



Solution

Quadrilaterals 1 and 2 look alike, but you can see from the angle measures and side lengths that they’re **not identical**.

The angle measures tell you that Quadrilateral 3 is a **mirror image** of Quadrilateral 2.

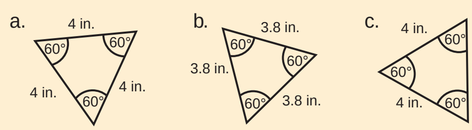
So Quadrilaterals **2 and 3** are **congruent**.

Check it out:
The angles of congruent polygons have the same measures, in the same order. Suppose that, going clockwise around its vertices, a quadrilateral has angles of 70°, 80°, 100°, and 110°. Then a congruent polygon’s vertices will have the same measures in the same order, either clockwise or counterclockwise.

Guided Practice

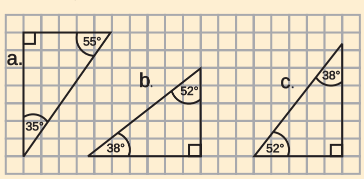
In Exercises 9–12, say which two out of each group of shapes are congruent. Give a reason why the other one is not.

9.



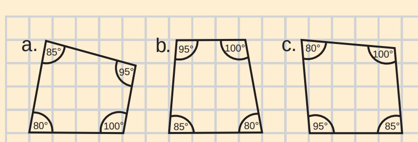
a & c — b has different side lengths

10.



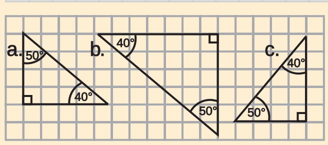
b & c — a has different angles (and different side lengths)

11.



a & b — all have the same angles, but in c they are in a different order, giving different side lengths.

12.



a & c — b is a different size

2 Teach (cont)

Common error
Students often classify figures as being congruent or similar based solely on their appearance. Students need to examine the measurements of the angles and lengths of the sides.

Universal access
This activity demonstrates to students that congruent shapes have congruent sides, but noncongruent polygons (except triangles) can be made from congruent sides.

Have students work in pairs. Distribute eight paper fasteners and eight strips of paper to each pair of students. There should be four identical small, two identical medium, and two identical large strips. Each strip should be hole-punched at both ends so that paper fasteners can be used to attach them.

Ask students to each make a triangle using one small, medium, and one large strip of paper (with paper fasteners to join the strips), such that the two triangles made within each pair are different — not congruent. Students should discover that this is impossible (you may need to remind them that rotated or reflected figures are still congruent).

Next, ask students, “So, if three congruent parts always make congruent triangles... do four congruent parts always make congruent quadrilaterals?”

Have students should repeat the exercise, using all four strips of paper to make a quadrilateral each. Pairs of students should discover that noncongruent quadrilaterals can be made from congruent parts.

Guided practice
Level 1: q9–12
Level 2: q9–12
Level 3: q9–12

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask advanced learners to investigate the minimum amount of information you need to know about two triangles to determine if they are congruent. The conditions for congruence are given in the margin notes on the next page. Students may find it useful to have some strips of paper and angle templates. Encourage students to ask themselves questions like: "If I know two side lengths, and the enclosed angle between them, how many different triangles can be formed?"

2 Teach (cont)

Concept question

"Two triangles have the same angle measures. What can you say about their relative shapes?"

They are either congruent or similar, depending on whether any pair of corresponding sides have the same or different lengths.

Concept question

"Say whether each of the following transformations produces a shape congruent to the original, or a shape similar to the original:

reflection, translation, or applying a scale factor."

Reflection and translation produce congruent shapes. Applying a scale factor produces a similar shape.

Guided practice

Level 1: q13–18
Level 2: q13–18
Level 3: q13–18

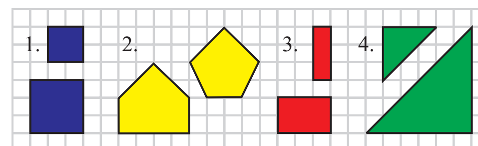
Similar Figures Can Be Different Sizes

Similar figures have angles of the same measure and have the same shape as each other, but they can be different sizes.

So two figures are **similar** if you can **apply** a **scale factor** and get a congruent pair.

Example 3

Which of these pairs of shapes are similar?



Solution

Pair 1 is a **similar pair**. They are both squares, and the only difference is the **size**.

Pair 2 is **not** a similar pair. The shapes are different — they have different angles.

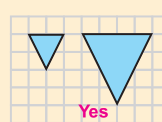
Pair 3 is **not** a similar pair. You can't multiply either of them by any **scale factor** to get a rectangle **congruent** to the other one.

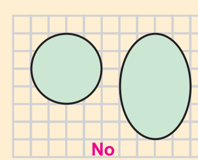
Pair 4 is a **similar pair**. If you multiply the smaller triangle by a scale factor of **2**, you will get a triangle **congruent** to the larger one.

Check it out:
You could also turn pair 4 from Example 3 into a congruent pair by applying a scale factor of $\frac{1}{2}$.

Guided Practice

In Exercises 13–18, say whether or not each pair of shapes is similar.

13.


14.


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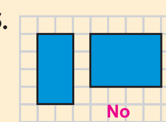

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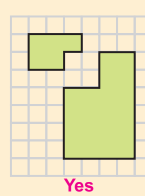
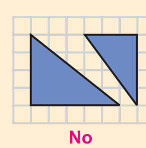


17.


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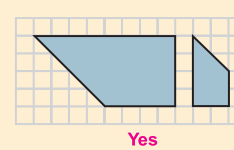
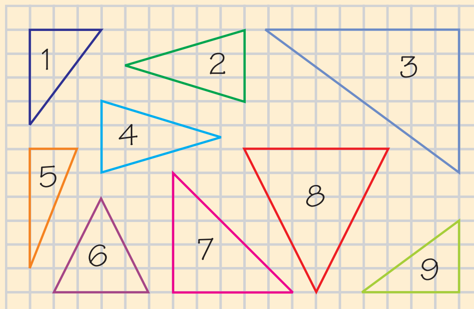


Solutions
For worked solutions see the Solution Guide

Independent Practice

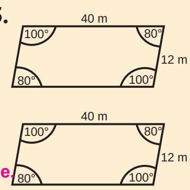
Use the triangles shown below to answer Exercises 1–4.



1. Which triangle is congruent to triangle 1? **9**
2. Which triangle is similar to triangle 6? **8**
3. Which triangle is congruent to triangle 4? **2**
4. Which two triangles are similar to triangle 3? **1 and 9**

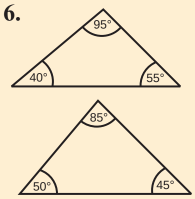
In Exercises 5–8, identify each pair of shapes as congruent, similar, or neither. Explain your answers.

5.



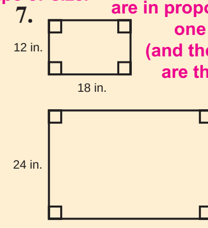
5. Congruent. They are the same size and shape.

6.



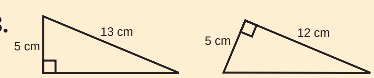
6. Neither. The triangles are not the same shape or size.

7.



7. Similar. The sides are in proportion to one another (and the angles are the same)

8.



8. Congruent. The triangles are the same size and shape.

9. Explain the difference between congruency and similarity when examining two figures. **Congruent shapes have the same shape and size. Similar shapes have the same shape, but not the same size.**

10. Triangle ABC has sides measuring 5 in., 6 in., and 8 in. Write the side lengths of a triangle that would be similar to ABC.
Any lengths that are multiples of (5, 6, 8), like (10, 12, 16) or (0.5, 0.6, 0.8).

11. "You can tell whether two shapes are congruent just by looking at the lengths of the sides. It is not necessary to look at the measures of the angles."
Is this statement true or false? Give a reason why.
False (except for triangles). For example, a rectangle and a parallelogram could have the same lengths, but these two figures are not congruent.

Round Up

You'll learn more about congruence and similarity — particularly with triangles — in later grades. For now, make sure you know what each term means, and don't forget which is which.

Now try these:
Lesson 3.4.6 additional questions — p449

Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Concept question
"Two isosceles triangles both have two equal-length sides of 6 cm. Does this mean that they are congruent?"

No — the third side / the angle between the two equal-length sides is not necessarily the same length in both triangles.

Math background
You don't need to know the lengths of all three sides and all three angle measures to tell if two triangles are congruent. Any of the following is enough information to tell that two triangles are congruent:

SSS — three sides of one triangle are congruent to three sides of another triangle.

SAS — two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle.

ASA — two angles and the included side of one triangle are congruent to the corresponding parts of another triangle.

AAS — two angles and a nonincluded side of one triangle are congruent to the corresponding parts of another triangle.

HL — the hypotenuse and a leg of one right triangle are congruent to the corresponding parts of another right triangle.

Independent practice
Level 1: q1–8
Level 2: q1–10
Level 3: q1–11

Additional questions
Level 1: p449 q1–3
Level 2: p449 q1–5
Level 3: p449 q1–6

3 Homework

Homework Book — Lesson 3.4.6
Level 1: q1–4
Level 2: q1–7
Level 3: q1–8

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 40 — Congruence

Lesson 3.5.1

Constructing Circles

In this Lesson, students practice drawing circles of given sizes and adding chords of given lengths to them using a compass and a ruler. They also practice drawing central angles using a protractor.

Previous Study: In grade 5, students measured, identified, and drew angles, perpendicular and parallel lines, rectangles, and triangles using appropriate tools, such as rulers, compasses, and protractors.

Future Study: Later in this Section, students will draw other constructions, including angle bisectors and perpendicular bisectors. These are revisited and extended in Geometry.

1 Get started

Resources:
- compasses
- protractors
- rulers
- circular/cylindrical objects

Warm-up questions:
- Lesson 3.5.1 sheet

2 Teach

Universal access

Start the Lesson with various cylindrical objects, such as cans, film canisters, plastic plates, etc., on each table. The items should labeled with letters, and should be small enough that the compass can open up to the radius of the object.

Ask students to estimate the radius of an object and write down their estimate, for example, "object A, radius 3 cm." Then have students use a compass to draw a circle with their estimate as its radius:
On their paper, have students make a dot that will be the center. Walk through the procedure of opening up the compass to the correct radius (using a ruler to measure), placing its point on the marked dot, and sweeping a circle.

Now have students place the object in the circle they have drawn, to see how good their estimate of the radius was. They should then adjust their estimate of the object's radius.

This process can be repeated with the remaining objects. Provide students with additional layers of paper or cardboard between the desk and the paper so that the compass does not leave holes in the desk.

Common error

Students often confuse the radius with the diameter, resulting in a circle that is the incorrect size.

Guided practice

Level 1: q1–4
Level 2: q1–4
Level 3: q1–4

Lesson 3.5.1

Section 3.5

Constructing Circles

California Standards:

Measurement and Geometry 3.1
Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, angle bisectors, and perpendicular bisectors; **central angles, radii, diameters, and chords of circles) by using a compass and straightedge.**

What it means for you:
You'll learn what chords and central angles of circles are, and how to draw them.

Key words:
- circle
- compass
- radius
- chord
- central angle
- arc

Don't forget:
360° is the measure of a full circle.

Check it out:
The part-circle you make if you don't join the ends of your curve in a full circle is called an arc.

Don't forget:
You open your compass to the length of the radius — not the diameter.
If you're asked to draw a circle with a certain diameter, halve it to find the radius.

You already know a lot about circles. Earlier in this Chapter, you learned about the radius, diameter, circumference, and area of circles. In this Lesson, you'll learn some more words relating to circles. You'll also draw circles and mark features on them using a compass.

A Compass Can Help You to Draw Shapes

You might have used a **compass** in math lessons in earlier grades. A compass is a tool that can help you **draw** many types of shape.

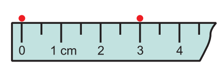
The easiest shape to draw with a compass is a **circle**.

Example 1

Use a compass and ruler to construct a circle with radius 3 cm.

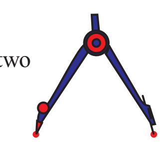
Solution

Step 1: Draw a **point** that will be the **center**.

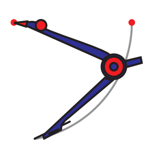


Step 2: Draw another point **3 cm** away from the center.

Step 3: Open the compass to the **length** between the two points. This length is the **radius** of the circle.

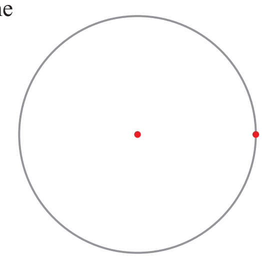


Step 4: Slowly sweep the **pencil end** of the compass 360°.



Keep the **pointed end** of the compass on top of the **center point**.

Make sure the ends of the curve **join** to make a complete circle.



Guided Practice

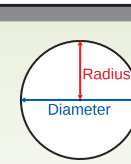
Use a compass and ruler to construct the following circles:

1. Radius 4 cm see below
2. Radius 2.5 cm see below
3. Diameter 10 cm see below
4. Diameter 7 cm see below

Solutions
For worked solutions see the Solution Guide

Guided Practice: Have the students check each other's answers by measuring the radius or diameter of the circles they have drawn.

If they need it, provide them with a labeled diagram like the one on the right showing the radius and diameter of a circle.



- **Strategic Learners**
  Show students how to construct circles using two pencils and a piece of string. Place one pencil on a center point, tie string to it, and tie the second pencil to the other end of the string. Help students to draw and label the center, radius, diameter, circumference, arc, and a chord. Discuss how the string length is the distance all the points on the circle are from the center.

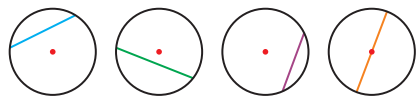
- **English Language Learners**
  Give students the opportunity to construct circles using a compass. They should draw and label the center, radius, diameter, circumference, arc, and a chord. They should leave these displayed throughout the Lesson for reference.

A Chord Joins Two Points of a Circle

Check it out:
A line segment is a straight line between two points. You'll learn more about line segments next Lesson.

A **chord** is a **line segment** that joins two points on the **circumference** of a **circle**. The **length** of a chord can be less than or equal to the length of the **diameter**.

A **compass** can help you to draw chords.

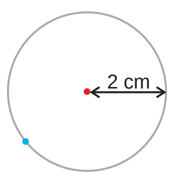


Example 2

Use a compass and ruler to construct a circle, then draw a chord of length 3 cm.

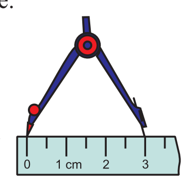
Solution

Step 1: Start by drawing a **circle**.

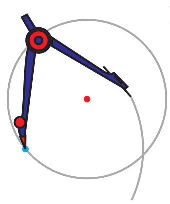


Remember that the length of the chord is less than or equal to the diameter. So the **diameter** must be at least **3 cm** — which means the **radius** must be at least 3 ÷ 2 = **1.5 cm**.
A circle of radius **2 cm** will do nicely.

Step 2: Mark a **point** on the circle. This will be one **endpoint** of the chord.



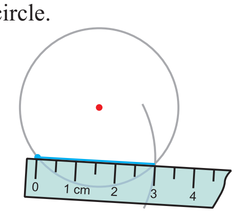
Step 3: Open the compass to a length of **3 cm** — the length of the chord you want to draw.



Step 4: Put the **pointed end** of the compass on the point you marked on the circle.
Draw an **arc** that crosses the circle.

Check it out:
In step 4 of this method, there will be two possible places where your arc could cross the circle. You only need it to cross at one of these points.

Step 5: Draw a straight line from the point you drew in Step 2 to the point where the arc **crosses** the circle. Measure the chord you've drawn to **check** that it is 3 cm long.



Guided Practice

In Exercises 5–10, use a ruler and compass to construct the following circles and chords

5. Circle of radius 3 cm, chord of length 5 cm
6. Circle of radius 1.5 cm, chord of length 2 cm
7. Circle of radius 2.2 cm, chord of length 3.5 cm
8. Circle of diameter 11 cm, chord of length 10 cm
9. Circle of diameter 6.8 cm, chord of length 4 cm
10. Circle of diameter 5.2 cm, chord of length 3.2 cm

Students can check the accuracy of their partner's work by measuring the radius/diameter and the chord with a ruler.

2 Teach (cont)

Math background

The diameter is a chord — it's a line segment that joins two points on the circumference of a circle.

The difference between a line and a line segment is that lines go on forever, whereas line segments have a start and end point.

Guided practice

Level 1: q5–8
Level 2: q5–9
Level 3: q5–10

Solutions

For worked solutions see the Solution Guide

Advanced Learners

Challenge students to construct regular polygons, such as pentagons, inside circles. They'll need to divide the circle into sectors of the correct size. For instance, to construct a regular pentagon, the circle needs to be divided into five equal sectors — these should each have a central angle of 360° ÷ 5 = 72°.

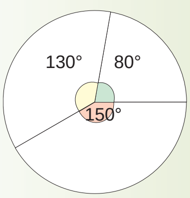
Interesting patterns can also be created by constructing equal-sized chords end to end. Ask students to try to predict when the chords will meet up again.

2 Teach (cont)

Math background

A full turn is 360°. So if you drew several radii in a circle, and added up all the central angles formed, you'd always get 360°.

For example,



130° + 80° + 150° = 360°

Guided practice

Level 1: q11–13
Level 2: q11–15
Level 3: q11–16

Independent practice

Level 1: q1–6
Level 2: q1–6
Level 3: q1–6

Additional questions

Level 1: p450 q1–6
Level 2: p450 q1–8
Level 3: p450 q1–8

3 Homework

Homework Book — Lesson 3.5.1

Level 1: q1a–b, 2, 3, 5
Level 2: q1–6
Level 3: q1–7

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 32 — Circles
- Worksheet 41 — Using a straightedge / compass / protractor

Don't forget:

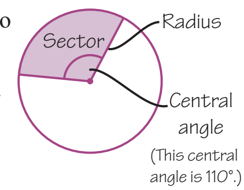
Radii is the plural of radius.

A Central Angle is Formed by Two Radii

A **central angle** of a circle is an angle made by two **radii** of the circle.

The size of a central angle is between **0°** and **360°**.

When a circle is divided by radii, the parts that it splits into are called **sectors**.

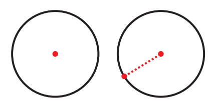


Example 3

Construct a central angle of a circle with a measure of 130°.

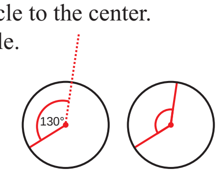
Solution

Step 1: Start by using a compass to draw a **circle**.



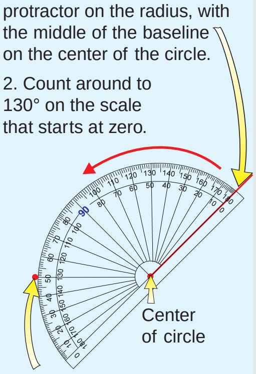
Step 2: You can now use a ruler or straightedge to join any point on the circle to the center. This is a **radius** of the circle.

Step 3: Use a **protractor** to make another radius at an angle of **130°** to the first one.



Don't forget:

1. Place the baseline of the protractor on the radius, with the middle of the baseline on the center of the circle.
2. Count around to 130° on the scale that starts at zero.



3. Make a dot next to the correct number of degrees.
4. Join this dot to the center of the circle with a straight line.

Guided Practice

For each of Exercises 11–16, construct a circle and draw central angles with the following measures: see below

11. 90° **12.** 75° **13.** 45°
14. 125° **15.** 150° **16.** 10°

Independent Practice

In Exercises 1–2, draw the following onto a circle of radius 5 cm:

1. A chord of length 7 cm
2. A central angle measuring 60°

Have students check each other's answers by measuring the radius, chord, and central angle of the circle they have drawn.

In Exercises 3–4, draw the following onto a circle of diameter 5.8 cm:

3. A chord of length 3.5 cm
4. A central angle measuring 85°

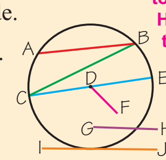
Have students check each other's answers by measuring the diameter, chord, and central angle of the circle they have drawn.

5. Terrell is constructing a circle with a diameter of 6 inches. He opens his compass so that it is 6 inches wide. Explain what error Terrell has made.

Terrell opened his compass to the length of the diameter. He should have opened it to the length of the radius (3 in.).

6. Identify the chords in this circle.

AB, BC, and CE



Now try these:

Lesson 3.5.1 additional questions — p450

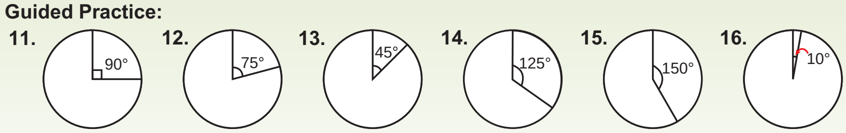
Round Up

If you need to draw a circle, always use a compass. It's pretty much impossible to draw a perfect circle without one. Practice drawing circles, chords, and central angles until you're really confident.

Solutions

For worked solutions see the Solution Guide

Guided Practice:



Lesson 3.5.2

Constructing Perpendicular Bisectors

In this Lesson, students use their compasses to copy line segments accurately, and also to construct perpendicular bisectors. Constructing a perpendicular bisector allows students to pinpoint the midpoint of a line segment.

Previous Study: In grade 5, students measured, identified, and drew angles, perpendicular and parallel lines, rectangles, and triangles using appropriate tools, such as rulers, compasses, and protractors.

Future Study: Later in this Section, students will draw other constructions, including angle bisectors and altitudes. These are revisited and extended in Geometry.

Lesson 3.5.2

California Standards:

Measurement and Geometry 3.1

Identify and construct basic elements of geometric figures (e.g., altitudes, **midpoints**, diagonals, angle bisectors, **and perpendicular bisectors**; central angles, radii, diameters, and chords of circles) **by using a compass and straightedge.**

What it means for you:

You'll learn what midpoints and perpendicular bisectors are, and how to draw them.

Key words:

- line segment
- midpoint
- bisect
- perpendicular bisector
- right angle

Check it out:

This method of constructing line segments will be used for other more complex drawings over the next few Lessons.

Constructing Perpendicular Bisectors

You should have gotten used to using a compass to draw circles and chords in the last Lesson. Now you can start using it for more complex drawings that don't have anything to do with circles. For starters, this Lesson shows you a neat way to use a compass to divide a line segment exactly in half.

A Line Segment is Part of a Line

If you **join** two **points** on a page using a straightedge, you make what you'd normally call a **line**. But to mathematicians, a **line** carries on **forever** in both directions.

When you join two points, you draw **part of a line**. In math that's called a **line segment**.

This line segment joins points **A** and **B**. A and B are called the **endpoints**. The line segment is called **AB**.

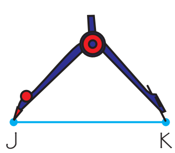
You can use a compass to make an accurate copy of a line segment **without** measuring its length.

Example 1

Use a compass and straightedge to copy the line segment JK. Label the copy LM.

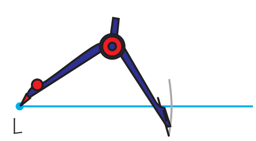
Solution

Step 1: Draw a line segment longer than JK. Label one of its endpoints L.



Step 2: Open up your compass to the length of JK.

Step 3: With the compass point on L, draw an arc that crosses your new segment.



Step 4: The point where the arc crosses the line segment is point M, the second endpoint of your new line segment.

Guided Practice

In Exercises 1–4, use a ruler to draw a line segment with the length given, then copy it using a compass.

1. 5 cm **2.** 6.5 cm

3. 3.9 cm **4.** 2.6 cm

You'll use these line segments again for Guided Practice Exercise 11.

Have students check each other's answers by measuring the lengths of the lines drawn.

1 Get started

Resources:

- compasses
- rulers
- index cards for matching game (see English Language Learner activity)
- thin paper

Warm-up questions:

- Lesson 3.5.2 sheet

2 Teach

Common error

When drawing line segments students will sometimes place arrows on the ends. This shows that they don't understand the distinction between a line and a line segment.

Guided practice

Level 1: q1–4
Level 2: q1–4
Level 3: q1–4

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  The paper-folding activity described in the Universal access section below is particularly suitable for strategic learners.
  Also, to reinforce the concept of "perpendicular," ask students to identify perpendicular line segments in the classroom environment.

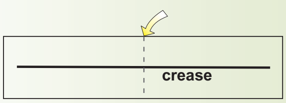
- **English Language Learners**
  Create a set of cards for matching games. Use a pair of cards for each vocabulary term, such as midpoint and perpendicular bisector.
  One card should have the vocabulary word and its matching card should have the definition and an example or illustration. Ask students to play games such as pairs with them.

2 Teach (cont)

Universal access

Ask students to draw a line segment on a thin piece of paper and fold the paper so that the two endpoints are directly on top of each other.

Carefully creasing the paper will produce the perpendicular bisector of the line segment — the crease crosses the line at the midpoint, and at a right angle.



Guided practice

Level 1: q5–7
Level 2: q5–9
Level 3: q5–10

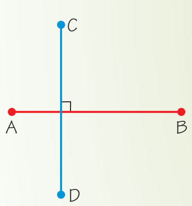
Concept question

"A line segment is drawn starting at 5 cm on a ruler and ending at 17 cm. Where is the midpoint of the segment on the ruler?"

The midpoint is at 11 cm.

Concept question

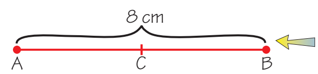
"Is the line segment CD a perpendicular bisector of line segment AB? Explain your answer."



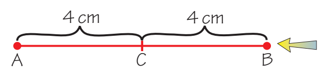
No. CD is perpendicular to AB, but it's not a bisector — it does not split AB into two equal sections.

The Midpoint Splits a Line Segment in Half

The midpoint of a line segment is the point that divides it into **two** line segments of **equal measure**. Dividing a line segment into two equal parts like this is called **bisecting** the line segment.



In this diagram, C is the **midpoint** of the line segment **AB**.

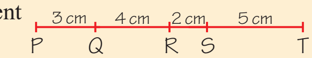


AB is **bisected** into two line segments of equal length, **AC** and **CB**.

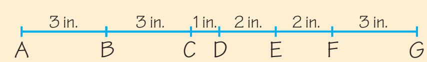
Guided Practice

5. The line segment DE is 8 in. long. F is the midpoint of DE. How long is the line segment DF? **4 in.**

6. Which is the midpoint of line segment PT — point Q, point R, or point S? **R**



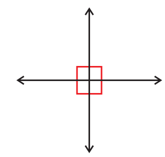
Use the diagram below to answer Exercises 7–10.



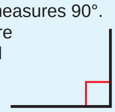
7. Which point is the midpoint of line segment AG? **D**
8. Which point is the midpoint of line segment BE? **C**
9. Which point the midpoint of line segment DF? **E**
10. Which line segment is point B the midpoint of? **AC**

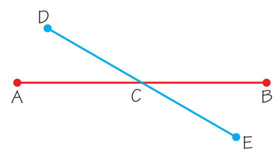
A Perpendicular Bisector Crosses the Midpoint at 90°

Two lines are **perpendicular** if the angles that are made where they cross each other are **right angles**.



Don't forget:
A right angle measures 90°. Right angles are usually marked with a little square.



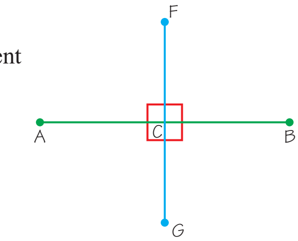


A **bisector** is a line or line segment that crosses the **midpoint** of another line segment.

In this diagram, C is the midpoint of AB, so DE is a bisector of AB.

A **perpendicular bisector** of a line segment is a bisector that passes through the **midpoint** at a **right angle**.

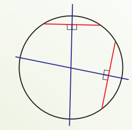
In this diagram, FG is a perpendicular bisector of AB.



Solutions

For worked solutions see the Solution Guide

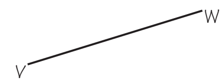
- **Advanced Learners**
  Challenge more able students to find the exact center of a circle by constructing only chords and perpendicular bisectors. The perpendicular bisector of a chord always goes through the center of a circle. Therefore, the intersection of the perpendicular bisectors of two different chords must be at the center. The emphasis should be on students explaining the methods used.



You need a **compass** to draw a **perpendicular bisector**.

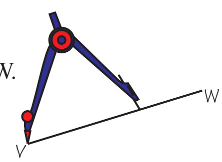
Example 2

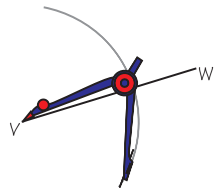
Use a compass and straightedge to draw the perpendicular bisector of VW. Label the bisector YZ.



Solution

Step 1: Place compass point on V. Open the compass more than half the length of VW.

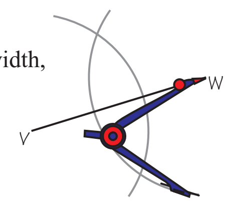


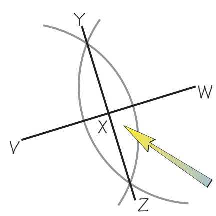


Step 2: Sweep a large arc that goes above and below line segment VW.

Step 3: Keeping the compass open to the same width, place the compass point on W and repeat step 2.

The two arcs should cross in two places. If they don't you might need to extend them.





Step 4: Draw a line segment that passes through the points where the arcs cross.

This is the perpendicular bisector of VW, so label its endpoints Y and Z.

The bisector crosses VW at the midpoint, X.

Check it out:
This method has two uses. It is the way to draw a perpendicular bisector using a compass and straightedge, but it also shows you where the midpoint is.

Guided Practice

11. Use a compass and straightedge to draw the perpendicular bisectors of each of the line segments you copied in Exercises 1–4. Label the midpoint of each line segment. **see below**

Independent Practice

S is the midpoint of line segment RT. The length of RS is 12.6 in.
1. What is the length of RT? **25.2 in.** **2.** What is the length of ST? **12.6 in.**

In Exercises 3–8, draw a line segment of the given length, then construct its perpendicular bisector. Mark the midpoint X.

3. 8 cm	**4.** 5 in.	**5.** 4.5 cm
6. 9.3 cm **see below**	**7.** 3.5 in. **see below**	**8.** 6.5 in. **see below**

Now try these:
Lesson 3.5.2 additional questions — p450

Round Up

A compass isn't just useful for drawing circles and arcs. You also use it to find midpoints and draw perpendicular bisectors. This is something else that you need to practice until you are happy with it.

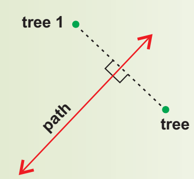
2 Teach (cont)

Concept question

"A footpath is to be built between two trees. Any point on the footpath is to be an equal distance from each tree.

Describe how this footpath would look from a "bird's-eye view."

The footpath will be a straight line that perpendicularly bisects a straight line drawn between the trees.



Guided practice
Level 1: q11
Level 2: q11
Level 3: q11

Independent practice
Level 1: q1–6
Level 2: q1–7
Level 3: q1–8

Additional questions
Level 1: p450 q1–9
Level 2: p450 q1–9
Level 3: p450 q1–9

3 Homework

Homework Book — Lesson 3.5.2
Level 1: q1–3, 6, 7
Level 2: q1–8
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 41 — Using a straightedge / compass / protractor

Solutions
For worked solutions see the Solution Guide

Guided Practice Exercise 11 and Independent Practice Exercises 3–8:
Students should check each other's diagrams, using a ruler to check that the line drawn goes through the midpoint of the original line, and a protractor to check that the lines are at right angles.

Lesson 3.5.3

Perpendiculars, Altitudes, and Angle Bisectors

In this final Lesson on constructions, students learn to construct accurate perpendicular line segments, altitudes in triangles, and angle bisectors.

Previous Study: In grade 5, students measured, identified, and drew angles, perpendicular and parallel lines, rectangles, and triangles using appropriate tools, such as rulers, compasses, and protractors.

Future Study: Constructions are revisited in Geometry. They are extended to include the construction of lines parallel to a given line, through a point off the line.

1 Get started

Resources:
- compasses
- straightedges
- thin paper

Warm-up questions:
- Lesson 3.5.3 sheet

2 Teach

Universal access

The perpendicular and altitude constructions can also be shown using full circles instead of arcs. This helps students see how all the pieces fit together.

Although this approach is useful for some students, the diagram often gets messy.

Common error

If students are using blunt pencils, their constructions will be inaccurate. Make sure they are using sharpened pencils.

Lesson 3.5.3

Perpendiculars, Altitudes, and Angle Bisectors

California Standards:

Measurement and Geometry 3.1
Identify and construct basic elements of geometric figures (e.g., altitudes, midpoints, diagonals, **angle bisectors**, and perpendicular bisectors; central angles, radii, diameters, and chords of circles) **by using a compass and straightedge.**

What it means for you:
You'll learn what perpendiculars, altitudes, and angle bisectors are, and how to draw them.

Key words:
- perpendicular
- altitude
- triangle
- angle bisector

Don't forget:
If you need a reminder of how to construct a perpendicular bisector, look at Lesson 3.5.2.

Last Lesson you learned to draw perpendicular bisectors, which cross other line segments exactly in their center, at 90°. Now you're going to use the skills you learned to draw other perpendiculars too.

You're also going to learn about angle bisectors. These do just what it sounds like they do — divide an angle exactly in half.

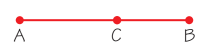
Perpendicular Line Segments Meet at Right Angles

You've seen how to construct a **perpendicular bisector**. That's a line segment that crosses the **midpoint** of another line segment at a **90°** angle.

A line or line segment that makes a 90° angle with another line segment, but isn't a **bisector**, is sometimes called a **perpendicular**.

You can **construct** a perpendicular that passes through a specific point using a **compass** and **straightedge**.

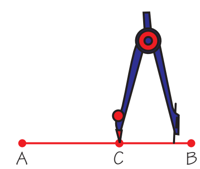
Example 1

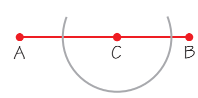


Use a compass and straightedge to construct a line segment perpendicular to AB that passes through point C.

Solution

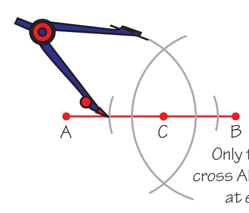
Step 1: Put the compass point on C.
Open the compass to a length between C and the nearest end of the line segment — B in this case.



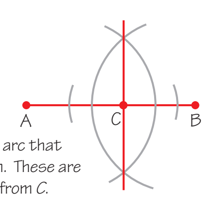


Step 2: Draw an arc that crosses AB twice. (In fact, you only need to draw the parts of this arc where it cross AB — see the next step.)

Step 3: Follow the steps for constructing a perpendicular bisector. Use the points where the arc crosses the line segment as endpoints.



Only the parts of the arc that cross AB are now shown. These are at equal distances from C.



- **Strategic Learners**
  Give students a selection of diagrams of triangles. Some should have altitudes drawn on them, while others should have lines drawn on them that aren't altitudes. Ask students to identify which diagrams show an altitude. For the other diagrams, ask them to explain why the lines aren't altitudes.

- **English Language Learners**
  The term "altitude" can be a source of confusion for English language learners (and for other students too). The word altitude is often used in the real world for height — as in the altitude of an airplane. In math, it's used both for the height of a triangle, and for the line segments drawn in this Lesson. Warn students that they need to decide which meaning is intended from the context.

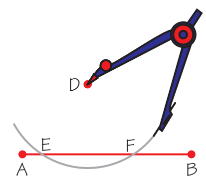
The point you need to pass through isn't always on the line segment you want to cross.

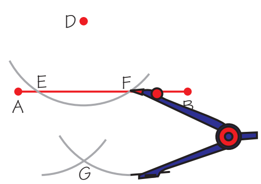
Example 2

Use a compass and straightedge to construct a line segment perpendicular to AB that passes through point D.

Solution

Step 1: Put the compass point on D and draw an arc that crosses AB twice. Call the points where AB crosses the arc E and F.

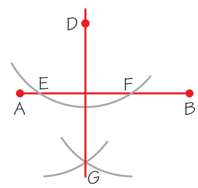




Step 2: With the compass point on E, draw an arc on the opposite side of AB to point D.

Step 3: Move the compass point to F. Keep the compass setting the same and draw an arc that crosses the one you drew in step 2. Call the point where the two arcs cross G.

Step 4: Draw a line segment passing through D and G.



Guided Practice

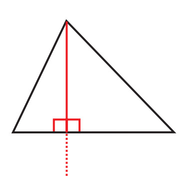
Draw a line segment, JK, that is 12 cm long. Mark the following points on the line. Draw a perpendicular through each of those points.

1. Point L, 3 cm away from J
2. Point M, 2 cm away from K
3. Point N, 5 cm away from J
4. Point O, 5.5 cm away from K *see below*

5. Draw a line segment, PQ, that is 8 cm long. Draw one point, S, above PQ and one point, T, below PQ. Construct a perpendicular to PQ that passes through S and another that passes through T. *see below*

An Altitude is a Line Showing the Height of a Triangle

An **altitude** of a **triangle** is a line segment that starts from one corner of the triangle and crosses the opposite side at a **90°** angle.



The way to **construct** an altitude is very similar to the method for constructing a **perpendicular** through a point not on the line — just use the **corner** of the triangle as the point.

2 Teach (cont)

Common error

Students sometimes close the compass after sweeping an arc, when they need the same width for future sweeps. Encourage students to keep the compass open and unchanged after they've used it, unless they need to alter it right away to draw a new arc.

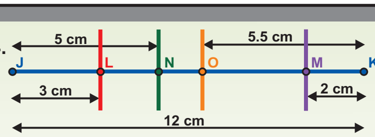
Guided practice

Level 1: q1–4
Level 2: q1–4
Level 3: q1–5

Solutions

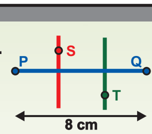
For worked solutions see the Solution Guide

1–4.



(Not to scale.)

5.

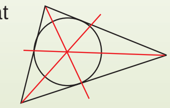


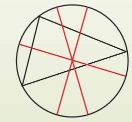
(Not to scale.)

This is one possible answer: any diagram with S above PQ and T below PQ and correct perpendiculars is acceptable.

Advanced Learners

Ask students to investigate the construction of circles inside and around triangles. They should find that the point where all three angle bisectors intersect is the center of a circle drawn inside a triangle touching each side. Also, the point of intersection of the perpendicular bisectors of the sides of a triangle is the same as the center of the circle drawn around the triangle, touching each vertex.





2 Teach (cont)

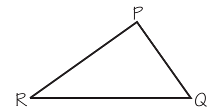
Concept question

"One triangle can have three different altitudes. Explain why."

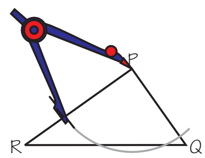
Three altitudes can be drawn for any triangle, each one starting from a different vertex. (In an equilateral triangle, all the altitudes will be of equal length. In an isosceles triangle, two will be equal in length. In a scalene triangle, all the altitudes will be different lengths.)

Example 3

Use a compass and straightedge to construct an altitude from P through RQ.



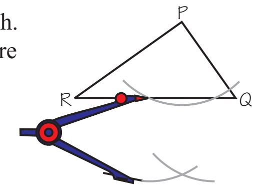
Solution

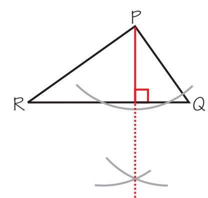


Step 1: Put the compass point on P. Draw an arc that crosses RQ in two places.

Step 2: Keep the compass open the same width. Put the compass point at one of the points where the arc crosses RQ. Draw an arc below RQ.

Step 3: Repeat step 2 from the other point where the arc and RQ cross. Make sure the two new arcs cross.

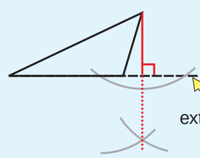




Step 4: Draw a line segment from P to the opposite side of the triangle, toward the point where the two arcs cross.

Check it out:

Drawing an altitude from one of the acute corners of an obtuse triangle is a little more tricky. You need to extend the opposite side of the triangle for the method to work.



Guided Practice

In Exercises 6–8, you need to draw triangles. Start each one by drawing a line, AB, that is 6 cm long. Choose the lengths of the other sides to suit the question. see below

6. Draw an acute triangle ABC. Construct an altitude from C.

7. Draw a right triangle ABC. Construct an altitude from C.

8. Draw an obtuse triangle ABC. Construct an altitude from C.

Guided practice

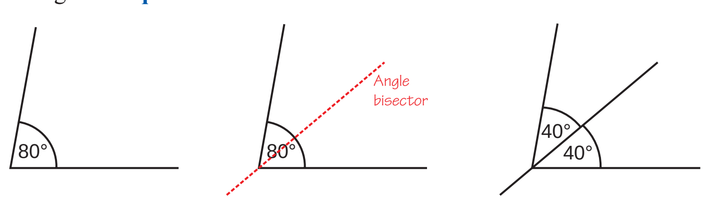
Level 1: q6–8
Level 2: q6–8
Level 3: q6–8

Don't forget:

An acute triangle has three angles of less than 90°.
A right triangle has one angle of exactly 90°.
An obtuse triangle has one angle of more than 90°.

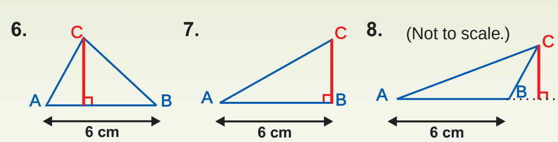
An Angle Bisector Divides an Angle Exactly in Half

An **angle bisector** is a line or line segment that **divides** an angle into two new angles of **equal measure**.



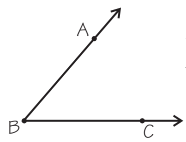
Solutions

For worked solutions see the Solution Guide



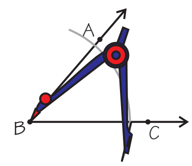
These are sample answers: any diagram with the correct type of triangle and accurate altitude is acceptable.

Example 4



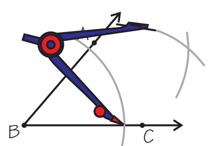
Use a compass and straightedge to bisect the angle ABC.

Solution



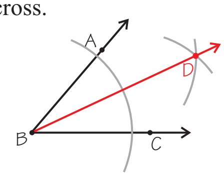
Step 1: Put the compass point on B and draw an arc that crosses the line segments AB and BC.

Step 2: Put the compass point where the arc crosses AB and draw a new arc in the middle of the angle. Keep the compass open at the width you've just used.



Step 3: Using the same compass width, repeat step 2 with the compass point at the spot where the first arc crosses BC. Make sure the two new arcs cross.

Step 4: Join point B to the point where the two arcs cross. The angle ABC has been bisected into two equal angles, ABD and DBC.



Check it out:

The method for constructing an angle bisector works for any angle — acute, obtuse, or right.

Guided Practice

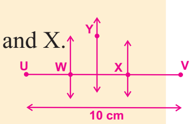
Use a protractor to draw the following angles. Bisect them using a compass and straightedge. see left

9. 90° **10.** 65° **11.** 20° **12.** 129°

Students should check the work of their partner. They should use a protractor to check the size of the angle, then to check that each half is the same size.

Independent Practice

1. Draw a 10 cm long line segment UV.
Mark two points on the line segment, and label them W and X.
Mark a point Y above the line.
Draw perpendiculars to UV through W, X, and Y.



2. Draw an acute triangle and an obtuse triangle. Construct altitudes from all three corners of each triangle. What is different about the points where the three altitudes meet (or will meet if extended)? see below

3. Draw one acute, one right, and one obtuse triangle. Start each one by drawing a line that is 5 cm long. Construct an angle bisector for the largest angle in each triangle. see below

Now try these:

Lesson 3.5.3 additional questions — p450

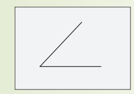
Round Up

This Lesson gives you two methods for the price of one. The method for drawing an altitude of a triangle is the same as for drawing a perpendicular through a point that's not on the line. Remember, watch out for those tricky obtuse triangles, where you might need to extend one side.

2 Teach (cont)

Universal access

Paper folding is useful for examining angle bisectors. Distribute a thin piece of paper to each student with an angle marked on it. For example:



Have students fold the paper so that the rays line up perfectly with each other. They should make a crease and then open the paper out again.

Students can use a protractor to check that they've created two equal angles.

Next follow the procedure for constructing an angle bisector using a compass. This activity helps establish a connection between constructing an angle bisector and what it actually is.

Guided practice

Level 1: q9–12
Level 2: q9–12
Level 3: q9–12

Independent practice

Level 1: q1–3
Level 2: q1–3
Level 3: q1–3

Additional questions

Level 1: p450 q1–13
Level 2: p450 q1–13
Level 3: p450 q1–13

3 Homework

Homework Book — Lesson 3.5.3

Level 1: q1–4, 7
Level 2: q1–9
Level 3: q1–9

4 Skills Review

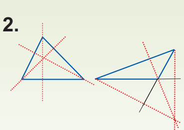
Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 41 — Using a straightedge / compass / protractor

Solutions

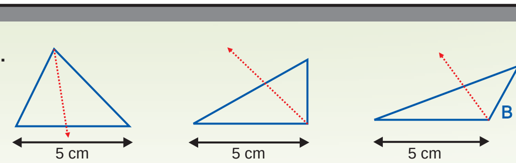
For worked solutions see the Solution Guide

2.



In acute triangles, the altitudes meet inside the triangle. In obtuse triangles they meet outside.

3.



Lesson 3.6.1 Geometrical Patterns and Conjectures

In math, conjectures are mathematical statements that are thought likely to be true — but haven't been proved to be true. In this Lesson, students practice making conjectures, and either disproving them with a counterexample, or justifying them with careful reasoning.

Previous Study: In previous grades, students have identified relationships and patterns. They have also started to develop generalizations and to formulate and justify conjectures.

Future Study: In Geometry, students will write proofs, including proofs by contradiction. Students will construct and judge the validity of a logical argument and give counterexamples to disprove a statement.

1 Get started

Resources:
- fish tank of water and an assortment of objects

Warm-up questions:
- Lesson 3.6.1 sheet

2 Teach

Universal access

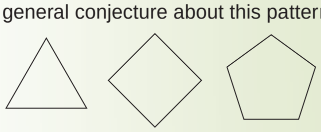
Lay out the dot pattern for Example 1 on the board or the overhead, then ask students to write down three or more observations about it.

As students share their observations with the class, organize them into three groups: "simple observations," "specific conjectures," and "general conjectures."

When there are several specific and several general conjectures collated, ask students to suggest new specific and new general conjectures.

Additional example

Write a specific conjecture and a general conjecture about this pattern:



Instance 1 Instance 2 Instance 3

For example:
Specific conjecture: Instance 4 will have six sides.
General conjecture: The number of sides is always two more than the instance number.

Lesson 3.6.1

Geometrical Patterns and Conjectures

California Standards:

Mathematical Reasoning 1.2
Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

Mathematical Reasoning 2.4
Make and test conjectures by using both inductive and deductive reasoning.

Measurement and Geometry 3.3
Know and understand the Pythagorean theorem and its converse **and use it to find the length of the missing side of a right triangle and the lengths of other line segments** and, in some situations, empirically verify the Pythagorean theorem by direct measurement.

What it means for you:
You'll make conjectures, or "educated guesses," about problems and use counterexamples and reasoning to decide whether your conjectures are true or false.

Key words:
- conjecture
- limiting case
- justifying
- reasoning
- instance

Check it out:
There's no real right or wrong answer with conjectures. The only rule is that you should be able to explain why you've made that conjecture, and why you think it's likely to be true.

In this Lesson, you're going to learn about testing and justifying conjectures. You make conjectures all the time in math, and also in everyday life. A conjecture is just an educated guess that is based on some good reason.

A Conjecture is an Educated Guess

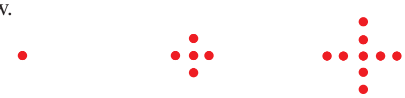
A **conjecture** is what's called an **educated guess** or an **unproved opinion**, such as, "it'll rain soon because there are gray clouds." This is unproved because we don't actually know whether it will rain soon or not.

You can make conjectures about mathematical situations such as **patterns** or **data**. For example, given the pattern 2, 4, 6... you could make a conjecture that the pattern increases by 2 each time.

There are two main types of conjecture in mathematical patterns:
- **Specific** conjectures about a **new instance** of a pattern.
- **General** conjectures about a pattern.

Example 1

Make three specific conjectures and three general conjectures about the pattern below.



Instance 1 Instance 2 Instance 3

Solution

Specific conjectures:
1. Instance 4 will have 13 dots.
2. Instance 4 will be in the shape of a cross.
3. Instance 5 will have 17 dots.

These are specific conjectures because they describe instances 4 and 5 only.

General conjectures:
1. Each instance is the shape of a plus sign.
2. Each instance has rotational symmetry.
3. Each instance has four more dots than the instance before it.

These are general conjectures because they describe the entire pattern.

There are usually **lots** of conjectures you could make about a pattern, and you have to select which you think are the **most important** to mention.

Not every conjecture has to be true, but if you make a conjecture you should either **think it is true**, or think it has the **possibility** of being true. So, although we don't know that instance 4 will definitely have 13 dots, we make the conjecture because it seems the most sensible guess based on what we know so far.

- **Strategic Learners**
  Fill a fish tank three-quarters full of water. Bring in several objects, some of which which will sink and some of which will float (for example, a brick and a plastic cup). Have students write down conjectures about which items will sink and which will float. Test and justify or disprove the conjectures.

- **English Language Learners**
  The Universal access activity on the previous page is particularly effective with English language learners if a “think–pair–share” approach is used. The “think” step involves students writing down their observations, which they then talk over with a partner in the “pair” step. This helps students work out the language needed for the “share” step, which is sharing the observations with the class.

2 Teach (cont)

Guided practice
Level 1: q1
Level 2: q1
Level 3: q1

Guided Practice

1. Below is the first three instances of a dot pattern.
Make at least one specific and one general conjecture.

Instance 1 Instance 2 Instance 3 See left

For example:
Specific conjecture: “Instance 4 will have 9 dots,” “Instance 4 will be n-shaped,” or “Instance 5 will have 11 dots.”
General conjecture: “Each instance is n-shaped,” “Each instance has one axis of reflection symmetry,” or “Each instance has two more dots than the instance before it.”

A Counterexample Shows that a Conjecture is False

It only takes **one instance** where the conjecture **doesn’t** apply to show that the conjecture is **not true**. For example, if you made the conjecture that it never rains on Mars, only one drop of rain would have to fall on Mars to prove you wrong.

You can show that some math conjectures aren’t true by finding a counterexample. A counterexample is a single case that makes a conjecture false.

To **find a counterexample**, consider some instances of the situation. Try to think about any **extreme** or **limiting cases**. These may be cases such as negative numbers, zero, or the most regular or irregular shapes.

Don’t forget:
Limiting cases are generally the ones that are most out of the ordinary.
For example, one limiting case for the parallelogram is the rhombus, which has all sides the same length.

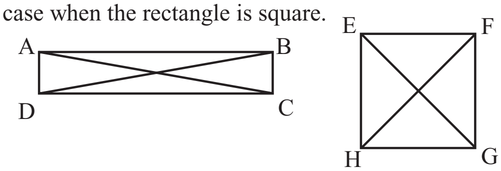
Example 2

Test the following conjecture about rectangles:

“The diagonals of a rectangle are never perpendicular.”

Solution

Perpendicular means that the lines are at 90° to each other.
First find the **limiting cases**.
For rectangles, try the case where the rectangle is very long and thin, and the case when the rectangle is square.



For the conjecture to be true, it must be true for **every possible case**.
So if this conjecture is true, it must hold for every possible rectangle.

The special case of the long and thin rectangle clearly **does not** have perpendicular diagonals. But the special case of the square **does** have perpendicular diagonals.

So the conjecture is **false** since it isn’t true for every case.
The square is a **counterexample**.

Deciding whether a conjecture is true or not by looking at specific limiting examples is called **justifying through cases**.

Math background
There are lots of conjectures that have been made in mathematics that are not yet either proven or disproven.

If a conjecture is proven, it becomes a theorem. Sometimes it takes a long time for a conjecture to be proven or disproven.

An example is the four color map theorem. This states that any plane separated into regions, such as a map of the counties of a state, can be colored using no more than four colors so that no two adjacent regions have the same color.

The conjecture was first proposed in 1852 when Francis Guthrie was coloring a map showing the counties of England. For more than a century, various people tried to prove the conjecture, but they failed. It wasn’t proven until 1976 — it was the first major theorem to be proven using a computer.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to make up their own mathematical conjectures, such as, “adding two odd numbers together always gives an even number.” Ask them to try to justify by reasoning why their conjecture is likely to be true.

2 Teach (cont)

Guided practice

Level 1: q2
Level 2: q2
Level 3: q2

Math background

In more rigorous mathematics, all conjectures are written as if-then statements. So the conjectures in Examples 2 and 3 would be written as:

If a figure is a rectangle, then its diagonals are never perpendicular.

If a figure is a rectangle, then its diagonals are congruent.

A counterexample is then defined as a case where the “if” part is true and the “then” part is false.

So to find a counterexample to disprove the conjecture in Example 2, you need to find a rectangle (making the “if” part true) whose diagonals are perpendicular (making the “then” part false).

Don't forget:
Quadrilaterals are four-sided shapes.

Guided Practice

2. Consider the following conjecture:

“All quadrilaterals have four right angles.”

Decide whether it is false or could possibly be true by examining limiting cases. See below

You Can Justify Conjectures Through Reasoning

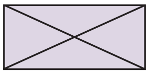
It’s hard to show that a conjecture is **definitely** true — there could always be an example that you haven’t found that would disprove the conjecture.

Justifying through reasoning means that you use **algebra** or **principles** to show that a conjecture is true for **all possible cases**.

Example 3

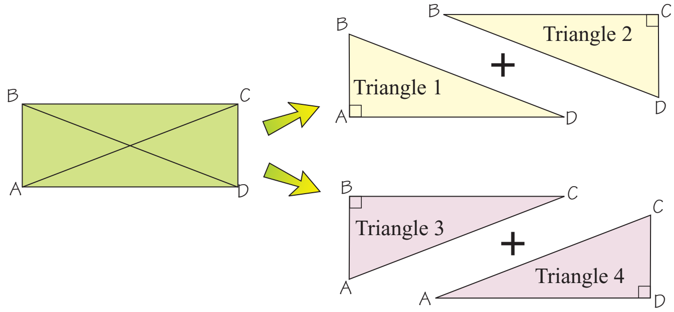
Test the following conjecture about rectangles:

“The diagonals of a rectangle are congruent.”



Solution

Congruent means the same in size and shape.
Split the rectangle along both the diagonals to make **4 triangles**, where each diagonal becomes the **hypotenuse** of a right triangle.



Don't forget:
Go back to Lesson 3.3.1 if you need a reminder of the Pythagorean theorem.

The **Pythagorean theorem** says that the square of the length of the hypotenuse is equal to the sum of the squares of the two legs.

Triangle 1: $(BD)^2 = (AB)^2 + (AD)^2$
Triangle 3: $(AC)^2 = (AB)^2 + (BC)^2 = (AB)^2 + (AD)^2$

Rectangles have two pairs of equal sides, so BC = AD.

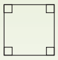
$(BD)^2 = (AC)^2$, so **BD and AC must be the same length**.

This means the diagonals of the rectangle are the same length, and so they’re congruent.

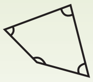
Solutions

For worked solutions see the Solution Guide

2. Examine the limiting cases:



A square is a regular quadrilateral. All its corners are right angles.



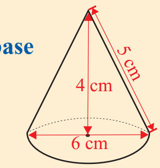
This is an irregular quadrilateral. Its corners are not all right angles.

The conjecture is false since it isn’t true for every case — the irregular quadrilateral is a counterexample.

Guided Practice

3. Consider the following conjecture:

"The vertical height of a cone will be 4 cm, if the base has diameter 6 cm and the slant height is 5 cm."



Decide whether the conjecture is true or false by justification through reasoning.

The cone's height, slant height, and base radius form a right triangle. Use the Pythagorean theorem to show that the conjecture is true. Let h = cone height.
$5^2 = (0.5 \times 6)^2 + h^2 \Rightarrow 25 = 9 + h^2 \Rightarrow h^2 = 16 \Rightarrow h = 4$. So the conjecture is true.

Independent Practice

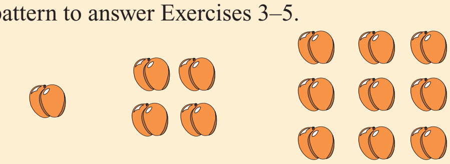
1. Make two specific conjectures and two general conjectures about the following number sequence.

"1, 9, 25, 49..."

Use some of your conjectures to find the next two numbers in the series. **See left**

2. If n is an odd number, make a conjecture about $n + 1$.
Accept any valid conjecture, such as "n + 1 is even."

The pattern below shows the first three instances of a pattern. Use the pattern to answer Exercises 3–5.



Instance 1 Instance 2 Instance 3

3. Make three specific conjectures about the pattern. **See left**
4. Make three general conjectures about the pattern. **See left**
5. Draw the next two instances of the pattern.

Instance 4 Instance 5

6. Consider the following conjecture:

"Parallelograms never have a line of symmetry."

Decide if this conjecture is true or false by testing limiting cases. **See below**

7. Consider the following conjecture:

"There are <u>exactly</u> three ways that you can split a rectangle into four smaller rectangles, so that all four smaller rectangles are congruent to each other."

Show that the conjecture is false by finding a counterexample. **See left**

Round Up

So that's conjectures. You'll make conjectures about all kinds of things in math — often without even thinking about it — and you might have to show whether they're true or not using counterexamples or careful reasoning.

Check it out:

You can use the Pythagorean theorem to justify whether this conjecture is true or false too.

1. Any two specific conjectures, such as "Instance 5 will be 81," "Instance 5 will be a square number," or "Instance 5 will be larger than instance 4."
Any two general conjectures, such as, "Each instance is a square number," or "Each instance is an odd number."
The next two numbers in the sequence are 81 and 121.

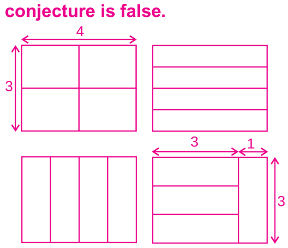
3. Any three specific conjectures, such as, "Instance 4 will be square," "Instance 4 will have 16 fruits," or "Instance 4 will have 4 rows."

4. Any three general conjectures, such as, "Each instance has a square number of fruit," "Each instance has the same number of rows as columns," or "Each instance has one more row than the instance before."

Now try these:

Lesson 3.6.1 additional questions — p451

7. This rectangle can be split into four smaller congruent rectangles in four ways. The conjecture is false.



2 Teach (cont)

Guided practice
Level 1: q3
Level 2: q3
Level 3: q3

Additional example
Find a counterexample to the following conjecture:

"All whole numbers greater than 1 have an even number of factors."

There are many possible counterexamples. The numbers that are counterexamples are perfect squares. 4 is the smallest of these. It has three factors: 1, 2, and 4.

Math background
Parallelograms are quadrilaterals whose opposite sides are parallel.

Independent practice
Level 1: q1–5
Level 2: q1–6
Level 3: q1–7

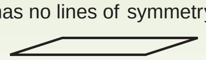
Additional questions
Level 1: p451 q1–12
Level 2: p451 q1–13
Level 3: p451 q1–14

3 Homework

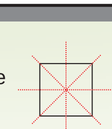
Homework Book — Lesson 3.6.1
Level 1: q1–3, 6–8
Level 2: q1–10
Level 3: q1–10

Solutions
For worked solutions see the Solution Guide

6. A long thin parallelogram has no lines of symmetry.



But a square is a type of parallelogram too, and a square has multiple lines of symmetry. So the conjecture is false.



Lesson 3.6.2

Expressions and Generalizations

It is important for students to be able to move beyond one particular problem and apply their results to other situations. In this Lesson, students make generalizations about patterns and write expressions for the nth term.

Previous Study: In previous grades, students have been developing their ability to generalize results and to apply them in other situations.

Future Study: In Algebra I, students will determine whether algebraic statements are true sometimes, always, or never.

1 Get started

Warm-up questions:

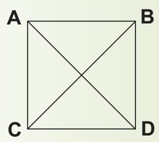
- Lesson 3.6.2 sheet

2 Teach

Universal access

One way to approach to the topic of generalizations is through making clear the distinction between specific and general.

For example, consider this 1–inch square and its diagonals:



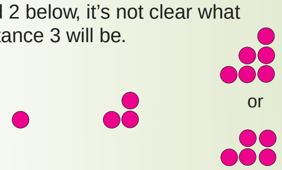
What is specific to this square is that AB = 1 inch, and that its area is 1 inch squared.
What is general is that AB = BC and AC and BD are perpendicular.

Specific items will be different for different size squares, but the generalizations will hold for all squares.

Math background

It's important to make students aware that you shouldn't generalize about a pattern unless you have sufficient information. You need to be fairly confident that you can predict what's coming next.

For example, given just instances 1 and 2 below, it's not clear what instance 3 will be.



Instance 1 Instance 2 Instance 3

Guided practice

Level 1: q1
Level 2: q1
Level 3: q1

Lesson 3.6.2

Expressions and Generalizations

California Standards:

Algebra and Functions 1.1
Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

Mathematical Reasoning 2.2
Apply strategies and results from simpler problems to more complex problems.

Mathematical Reasoning 3.3
Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

What it means for you:
You'll learn how to generalize a pattern from simple examples so that you can find any given instance of that pattern.

Key words:

- generalization
- number pattern
- number sequence
- conjecture

Don't forget:
It's always a good idea to test your generalizations. With patterns, do this by checking that a generalization or formula works on a new instance.

You met specific and general conjectures in the previous Lesson.

A generalization is a special kind of general conjecture. It allows you to work out quickly what any instance of a pattern will be.

A Generalization Comes from a General Conjecture

A **generalization** is a way of **extending** a general conjecture.

Making a generalization means finding some kind of **expression** or **formula** that you could use for **any instance**. Generalizations can be **mathematical expressions or word descriptions**.

You might use the first three instances of a pattern to make a the generalization that would tell you how to find the **nth instance** — the nth instance could be any instance whatsoever.

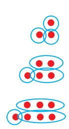
Example 1

Make a generalization for the number of dots in instance n of this pattern:

Instance 1 Instance 2 Instance 3

Solution

Look at what happens in each instance.



Instance 1 has **$1 + 1 + 1 = 3$ dots**.

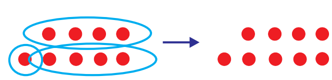
Instance 2 has **$1 + 2 + 2 = 5$ dots**.

Instance 3 has **$1 + 3 + 3 = 7$ dots**.

We can therefore say that Instance n will have $\mathbf{1 + n + n = 2n + 1}$ dots.

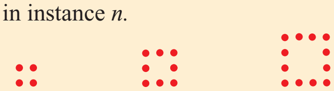
Check this is correct by testing on instance 2:
$2n + 1 = (2 \times 2) + 1 = 5$ dots, which is correct.

Test on instance 4:
$2n + 1 = (2 \times 4) + 1 = 9$ dots.
This is what you would have expected.



Guided Practice

1. Make a generalization of the pattern below by writing an expression for the number of dots in instance n.



Instance 1 Instance 2 Instance 3 Instance 4

$4n - 4$ or $4(n - 1)$

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners and English Language Learners**
  Ask students to make a generalization about all the students in the room (for example, they are all in the seventh grade). Then ask them to make a generalization about all the students in the school. Discuss why it's harder to do this.

  In pairs, ask students to make at least one generalization about a circle (for example, the diameter of a circle is equal to twice its radius). Discuss why it's helpful to know this generalization — it allows us to make calculations for all circles, whatever their size. Ask students to share their generalizations with another pair of students, then repeat with a generalization about a square, about parallel lines, etc.

Use Generalizations to Solve Problems

Example 2

In the pattern below, find the number of dots in instance 10.

Instance 1 Instance 2 Instance 3

Solution

In Example 1 we found a generalization for this pattern.

This was that the number of dots in instance n is $2n + 1$.

Instance 10 therefore has $2n + 1 = (2 \times 10) + 1 =$ **21 dots**.

Another way to do this is to notice that the top line has the same number of dots as the instance number, and the bottom line has one more dot than the top line. The top line of instance 10 will have 10 dots, and the bottom line will have $10 + 1 = 11$ dots. The total number of dots is $10 + 11 =$ **21 dots**.

When you look at a pattern, there can be **more than one** generalization to make.

Example 3

By making a generalization about the pattern below, find the sum of the 7th line of the pattern.

$1 = 1$

$1 + 3 = 4$

$1 + 3 + 5 = 9$

Solution

The pattern is the **sum of consecutive odd numbers**, adding one more odd number each line. You could say that the sum of the 7th line will be the sum of the first 7 odd numbers. A generalization would be that **the sum of the nth instance is the sum of the first n odd numbers**.

You could also generalize that the sum of each of the lines is the square of the instance number. So the **nth line will sum to n^2**.

So for line 7, the sum will be $n^2 = 7^2 =$ **49**.

Guided Practice

2. Use a generalization to find the 50th odd number. Generalization: $2n - 1$ 50th odd number: 99

3. Use a generalization to find the 9th term in the following pattern:

   4, 6, 8, 10, 12... Generalization: $2n + 2$ 9th term: 20

Don't forget:

The examples opposite ask you to find expressions for the numbers in patterns. One way to do this is to compare each instance number with the corresponding value in the pattern. A table can help you do this. For example:

Instance	Sum
1	1
2	4
3	9
4	16

Check it out:

It's often useful to make a table of simple cases. For example, a table for Exercise 2 would look like this:

Instance	Number
1	1
2	3
3	5
4	7

2 Teach (cont)

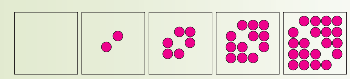
Common error

Some students believe that generalizations are different from conjectures in that generalizations must be true.

The fact is that most generalizations in math and in life are generalized conjectures because the process of proof is too laborious (or impossible). For instance, to prove many generalizations requires proof by induction, a topic usually not addressed until later in high school or college.

Additional example

Make a generalization about the dot pattern below.



1 2 3 4 5

For example, the number of dots in each instance is the instance number subtracted from the square of the instance number.

So the nth instance will have $n^2 - n$ dots.

Guided practice

Level 1: q2
Level 2: q2
Level 3: q3

Solutions

For worked solutions see the Solution Guide

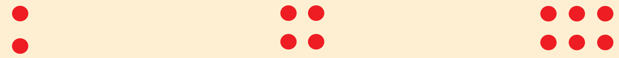
- **Advanced Learners**
  Provide advanced learners with the opportunity to apply their skills to a more complicated problem. For example, ask them to calculate the number of different handshakes there would be between 50 people if everyone shook hands with everyone else. Encourage students to start off with simple cases (for example, 2 people, 1 handshake; 3 people, 3 handshakes, 4 people, 6 handshakes) and then generalize their results. (With 50 people, there would be 1225 handshakes. With n people, there could be $\frac{n(n-1)}{2}$ handshakes.)

2 Teach (cont)

Independent practice
Level 1: q1–4
Level 2: q1–9
Level 3: q1–12

Independent Practice

Use the dot pattern below to answer Exercises 1–3.



Instance 1 Instance 2 Instance 3

1. The number of dots is always twice the instance number.
Or: Each instance has two rows, each containing a number of dots equal to the instance number.

1. Generalize the pattern using words. see left

2. Generalize the pattern by finding an expression for the number of dots in the nth instance. $2n$

3. Draw Instance 10.

4. Maggie created a pattern in which the nth instance had $5n - 1$ dots in it. Draw the first three instances of Maggie's pattern.
For example,

Use this pattern to answer Exercises 5–6.

$$2 = 2$$
$$2 \times 2 = 4$$
$$2 \times 2 \times 2 = 8$$

5. Extend the pattern for two more lines. $2 \times 2 \times 2 \times 2 = 16$, $2 \times 2 \times 2 \times 2 \times 2 = 32$

6. Find a generalization and use it to find the product of the 9th line. 2^n, product of 9th line $= 2^9 = 512$

For Exercises 7–9 use the pattern of numbers:

7, 10, 13, 16, 19...

7. Describe the pattern in words. The pattern starts at 7 and increases by 3 each time.

8. Write an expression for the nth number in the pattern. $3n + 4$

9. Find the 30th number in the sequence. 94

Now try these:
Lesson 3.6.2 additional questions — p451

Use this pattern for Exercises 10–12:

$$1 = 1$$
$$3 + 5 = 8$$
$$7 + 9 + 11 = 27$$

10. Find the next line of the sequence. $13 + 15 + 17 + 19 = 64$

11. Find an expression for the sum of the nth line. n^3

12. Find the sum of the 10th line. 1000

Additional questions
Level 1: p451 q1–5
Level 2: p451 q1–8
Level 3: p451 q3–12

Round Up

Generalizing is really useful in problem solving. If you're asked to find the 100th instance in a pattern, it'll take you ages to write or draw all 100 out — better to start simple and then generalize.

3 Homework

Homework Book — Lesson 3.6.2
Level 1: q1–3, 5
Level 2: q1–8
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
- Worksheet 20 — Variables and Expressions

Solutions
For worked solutions see the Solution Guide

Chapter 3 Investigation — Designing a House

Purpose of the Investigation

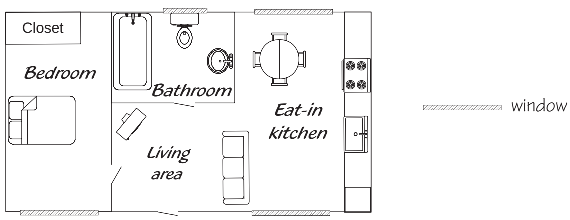
The Investigation provides students with the opportunity to apply their knowledge of finding the area of regular and complex shapes to a real-life situation. Students are asked to design a house that is livable and meets certain requirements. Students also practice using scale factors to produce scale drawings of their designs.

Chapter 3 Investigation
Designing a House

Before starting expensive construction, architects will make scale drawings, so that the size and shape of everything is clear. Often, constructions will be complex, rather than regular shapes.

You work for an architect company, which is designing **single-floor, one-bedroom houses**. The houses will have one bedroom, a living area/eat-in kitchen and one bathroom. They must have a floor area of **900 square foot**.

An example layout is shown here.



Part 1:
Design a house that satisfies these requirements. Be sure to include the **dimensions** of each room.

Part 2:
Calculate the area of each room. Add up the area of each room to check that it comes to 900 square feet.

Things to think about:
- You don't have to design a rectangular or square house — you could make it a **complex shape**.
- Try to make the room dimensions seem **reasonable** — a bathroom that is a 2-foot by 2-foot square wouldn't be usable.
- Be sure to include **doorways**, **windows** and other useful things, like a closet.

Extensions

1) Make a scale drawing of the house using a scale of 1 cm : 2 feet.

2) Make a scale drawing of the house using a scale of your own choosing.

Open-ended Extensions

1) One buyer wants the bathroom to be accessible from the bedroom and the living area. He also wants a separate kitchen. Design a house that would incorporate this concept.

2) A second buyer wants the rooms in the house to flow from one into the other, but to still offer privacy. Design a house that uses partial walls to separate rooms.

Round Up

When you're designing something, you'll often have certain limitations — like the area that the house should be. But you also have to think about how to make it usable — for example, you need enough space for a bed in the bedroom. But after that, you can take preferences into account — like where you'd prefer to put the door and the kitchen sink.

Resources
- large sheets of cm grid paper
- rulers
- outline "furniture" manipulatives (for strategic learners — see below)

Strategic & EL Learners
Some students may have difficulty dividing the house into reasonably proportioned rooms. Provide plan views of important pieces of furniture, such as a bed, sofa and dining table, drawn to scale. Students can then make sure each room is reasonably sized.

Sketch a range of different house shapes to reinforce to EL learners that they do not need to follow the example shown on the page — and to reinforce the term "complex" shapes.

Investigation Notes on p213 B-C

Investigation — Designing a House

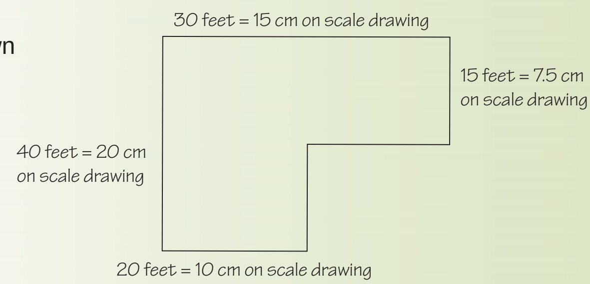
Mathematical Background

The Investigation is focused on creating shapes with a **given area**. The shape will form the outline of a house. Students can tackle this at their own level — they can either limit themselves to **square or rectangular** house designs, or challenge themselves to create a **more complex shape** with the given area.

The area of a complex shape is found by dividing the figure into smaller recognizable geometric figures. The area of these geometric shapes are calculated and then **added together.** The total area of the shape students design must equal 900 square feet.

Whichever shape students design, they then have to divide it into at least three different areas — one for each room. Again, these can be simple rectangles or more complex shapes. Part 2 requires students to calculate the area of each room.

In the Extension activities, students are required to produce scale drawings of their proposed design. The first Extension asks them to use the scale 1 cm : 2 feet. This means that each 2-foot length of the real house is shown by 1 cm on the scale drawing — or each foot is shown by 0.5 cm. This diagram shows real lengths and scale drawing lengths of an example.



You can work out lengths on the scale drawing by setting up a proportion using the given scale.

Let x be the length the 30-foot wall should be in the drawing.

$$\frac{\text{Drawing length}}{\text{Real life length}} = \frac{1\text{ cm}}{2\text{ feet}} = \frac{x}{30\text{ feet}}$$

The drawing length and the real-life length are always in the same ratio.

$$\frac{1\text{ cm}\times 30\text{ feet}}{2\text{ feet}} = \frac{x \times \cancel{30\text{ feet}}}{\cancel{30\text{ feet}}} = x$$

Isolate x by multiplying both sides of the proportion by 30 feet.

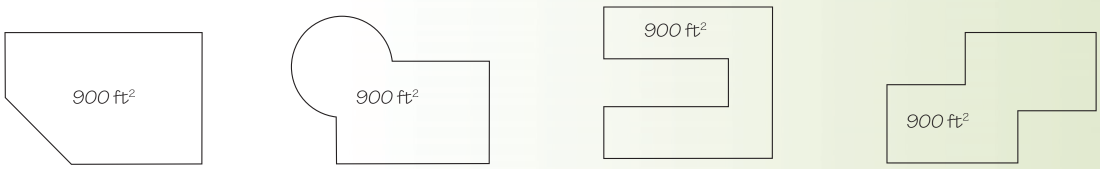
$$x = 1\text{ cm}\times\frac{30\,\cancel{\text{feet}}}{2\,\cancel{\text{feet}}} = 1\text{ cm}\times 15 = \mathbf{15\text{ cm}}$$

Simplify to find x.

Approaching the Investigation

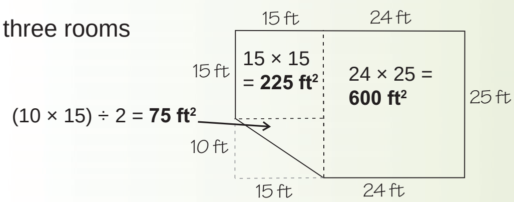
Part 1:
Introduce the Investigation by discussing what is a reasonable living space. Create a list of rooms and items that would usually be found in an apartment or house, and what realistic minimum dimensions of them would be.

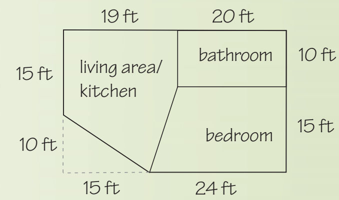
Explain the task to the students and encourage them to make some quick sketches before using a ruler and committing to one design. Encourage advanced learners to try a fairly complex design, like one of those shown below.



Students should split their house up into three rooms and mark the dimensions of each room. There's an example on the right:

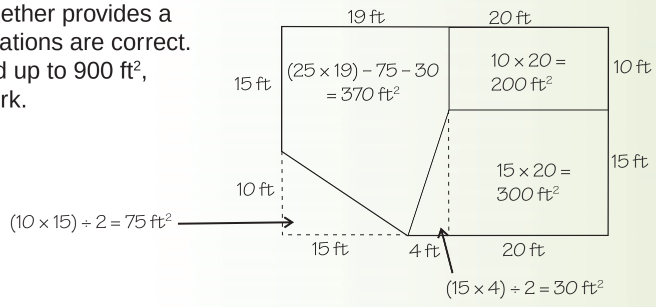
Students may want to add extra details at this stage.





Part 2:
In this part, students must calculate the area of each room. They may need to break the rooms into simple, geometric shapes.

Adding the room areas together provides a check that students' calculations are correct. If the room areas don't add up to 900 ft^2, they should check their work.



Living area/kitchen = 370 ft^2
Bathroom = 200 ft^2
Bedroom = 300 + 30 = 330 ft^2
Total area = 370 + 200 + 330 = **900 ft^2**

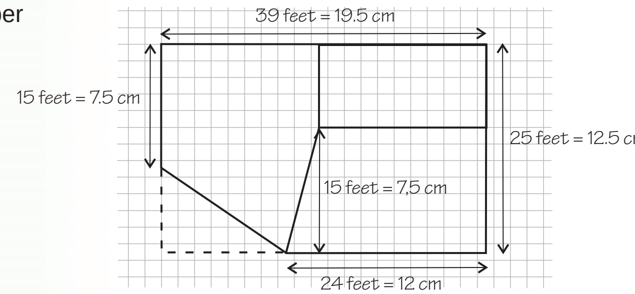
Investigation — Designing a House

Extensions

1) Students should now create a scale drawing of their design, using a scale of 1 cm = 2 feet. This is made much easier by using centimeter grid paper. They could find the length each vertical and horizontal side should be by setting up proportions. There's an example of this on the previous page.

   However, students are likely to realise that with this scale, the number of centimeters on the drawing is half the number of feet in real life.

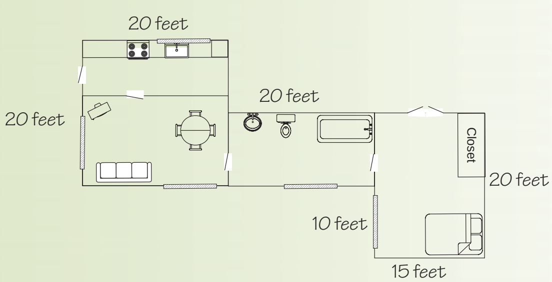
2) For this part, students are asked to redraw their design to a scale of their own choosing. They will need to look at the dimensions of their diagram, and the size of their grid paper.
   For instance, if students are using large grid paper that is 48 cm long and 36 cm wide, and their house design is 36 feet by 25 feet, they could use a scale of 1 cm : 1 foot.



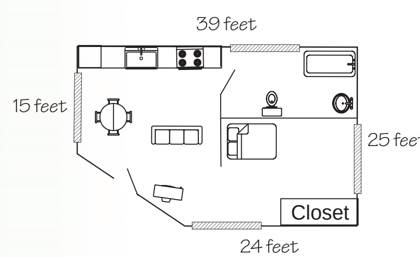
Open-Ended Extensions

These extensions are designed so that students still have to meet the area requirements, but also have to include a design style. There are lots of different designs that students may come up with for each set of design preferences. Some examples are shown below.

1) In this design, the bathroom is accessible from the bedroom and living area, and there is a separate kitchen.



2) In this design, the bathroom is the only room that can be fully enclosed. A partial wall separates the bedroom and living area.



Chapter 4

Linear Functions

How Chapter 4 fits into the K-12 curriculum

Section 4.1 — Graphing Linear Equations		
Section 4.1 covers Algebra and Functions 1.1, 1.5, 3.3 **Objective: To graph equations by plotting points, to understand systems of linear equations, and to calculate slope**		
Previous Study In grade 4 students learned how to plot points and join them up to make the line graph of an equation. At grades 5 and 6 students learned how to solve linear equations using this method.	**This Section** First students plot points and join them up in order to graph a linear equation. Then they write systems of equations from word problems, and graph both equations on one set of axes. Finally, they learn how to calculate slope of a graph.	**Future Study** In Algebra I students will learn how to find the equation of a line that passes through a point, and will learn how to graph quadratic equations and inequalities.

Section 4.2 — Rates and Variation		
Section 4.2 covers Measurement and Geometry 1.3, Algebra and Functions 3.4, 4.2 **Objective: To understand rates, ratios, and direct proportion**		
Previous Study In grade 6 students were introduced to ratios and rates for the first time. They used ratios and rates to compare quantities and solve problems about speed, distance and time.	**This Section** Students are reintroduced to rates and ratios, and then graph them on coordinate axes. They then apply the principles to distance, speed, and time problems. Finally, they learn about direct variation and learn to recognize a graph showing direct proportion.	**Future Study** In Algebra I students will use algebraic methods to help solve problems involving ratio and rate. They will also learn to graph linear equations and to find their slopes and intercepts.

Section 4.3 — Units and Measures		
Section 4.3 covers Measurement and Geometry 1.1, 1.3, Algebra and Functions 4.2 **Objective: To convert measures between unit systems**		
Previous Study In grade 6 students learned about the customary and metric measurement systems, and how to convert between different units.	**This Section** Students review the customary and metric measurement systems, and then learn to convert between systems. They then learn the concept of dimensional analysis, and finally convert between units of speed.	**Future Study** In Algebra I, Geometry, and all science subjects, students will be expected to be competent in manipulating units. In Algebra I students learn how to solve complex rate problems using algebraic techniques.

Section 4.4 — More on Inequalities		
Section 4.4 covers Algebra and Functions 1.1, 4.1 **Objective: To solve two-step linear inequalities**		
Previous Study In Section 1.3 students learned how to write one- and two-step linear inequalities, and how to graph one-step inequalities on a number line. They also met systems of equations in Section 4.1.	**This Section** Students review inequalities, then learn to solve one-step inequalities involving addition, subtraction, multiplication, and division. Finally, they go on to solve two-step inequalities.	**Future Study** In Algebra I students will learn how to solve multistep problems involving linear inequalities in one variable and provide justification for each step of their working.

Pacing Guide – Chapter 4

40- to 50-Minute Class Periods

If your class periods are 40-50 minutes, we recommend allowing **19 days** for teaching Chapter 4.
As well as the **14 days of basic teaching**, you have **5 days** remaining to allocate 5 of the 7 optional activities (to be delivered at any appropriate point during the Chapter).

The table shows the 14 teaching days as well as all of the **optional days** you may choose for Chapter 4, in the order we recommend.

Day	Lesson	Description
Section 4.1 — Graphing Linear Equations		
Optional		*Exploration — Block Patterns*
1	4.1.1	Graphing Equations
2	4.1.2	Systems of Linear Equations
3	4.1.3	Slope
Optional		*Assessment Test — Section 4.1*
Section 4.2 — Rates and Variation		
Optional		*Exploration — Pulse Rates*
4	4.2.1	Ratios and Rates
5	4.2.2	Graphing Ratios and Rates
6	4.2.3	Distance, Speed, and Time
7	4.2.4	Direct Variation
Optional		*Assessment Test — Section 4.2*
Section 4.3 — Units and Measures		
8	4.3.1	Converting Measures
9	4.3.2	Converting Between Unit Systems
10	4.3.3	Dimensional Analysis
11	4.3.4	Converting Between Units of Speed
Optional		*Assessment Test — Section 4.3*
Section 4.4 — More on Inequalities		
12	4.4.1	Linear Inequalities
13	4.4.2	More on Linear Inequalities
14	4.4.3	Solving Two-Step Inequalities
Optional		*Assessment Test — Section 4.4*
Chapter Investigation		
Optional		*Investigation — Choosing a Route*

Accelerating and Decelerating

- To **accelerate** Chapter 4, allocate fewer than 5 days to the optional material. This will give you extra days to allocate to other Chapters.
  Note that you may use the remaining optional days at the end of the 160-day course.
- To **decelerate** Chapter 4, consider allocating more than 5 days to the optional Assessment Tests, Section Explorations, or Chapter Investigation, or spend longer teaching some Lessons. Also consider preparing students for difficult Lessons by reviewing previous coverage of math topics on related Skills Review Worksheets.
  Note that decelerating Chapter 4 will result in fewer days being available for teaching other Chapters.

90-Minute Class Periods

If you are following a block schedule with 90-minute class periods, we recommend allowing **9.5 days** for teaching Chapter 4.
The basic teaching material will take up **7 days**, and you can allocate the remaining **2.5 days** to the **optional material**.
To accelerate or decelerate a block schedule, follow the same advice as given above.

Exploration — Block Patterns

Purpose of the Exploration

The purpose of the Exploration is for students to discover how linear patterns can be related to straight-line graphs. The students make patterns that can be represented by straight lines, and then use the lines to predict later instances of the pattern. Students will see that they can find later pattern instances without having to write the pattern out.

Resources

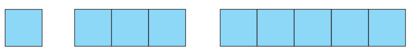
- pattern blocks
- graph paper
- ruler

Section 4.1 introduction — an exploration into: Block Patterns

In this Exploration, you'll use pattern blocks to make patterns that could carry on forever. These patterns all result in different straight lines when graphed on the coordinate plane.

Example

Pattern 1 is developed using pattern blocks. **Describe** the pattern and **record** it in a table.



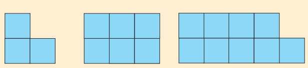
Solution

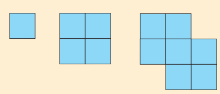
The pattern of 1, 3, 5, 7... continues on forever. Two blocks are added each time.

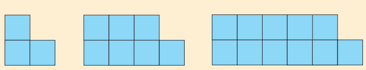
Figure in pattern	1	2	3
Number of blocks	1	3	5

Exercises

1. Describe each pattern and record it in a table.

a. 

b. 

c. 

see below

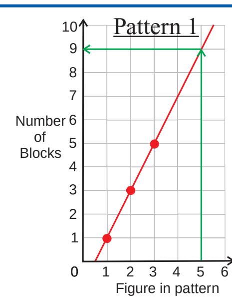
The numbers from each pattern can be **graphed on the coordinate plane**. This data can be used to make **predictions** about other figures in the pattern.

Example

Graph the data for Pattern 1 and predict the number of blocks in the fifth figure in the pattern.

Solution

Reading from the graph as shown, there will be **9 blocks** in the fifth figure.



Exercises

2. Graph the data for each pattern in Exercise 1. see right

3. Predict the number of blocks in the fifth and sixth figures of each pattern.
a. fifth = 15, sixth = 18, b. fifth = 13, sixth = 16, c. fifth = 19, sixth = 23

4. Find the slope of the graph of each pattern. What do you notice about each slope?
a. 3, b. 3, c. 4. It's the same as the number of blocks added each time.

Round Up

The figures in each pattern increase by the same number of blocks each time. So, when you plot the data, you get straight line graphs. You can use these to predict later figures in each pattern.

Strategic & EL Learners

Strategic learners would benefit from having the graph axes already drawn for them. This allows them to simply plot the data.

EL learners may not be familiar with the words "pattern" or "slope." Discuss where they see patterns in real-life with shapes or with numbers. Tell these students that a pattern is something where they can see what the next items will be by looking at the previous items. Explain slope by tilting a book to various degrees of steepness.

Math background

Students will need to be able to create a graph and plot points. Key parts of creating a graph are choosing an appropriate scale and labeling the axes.

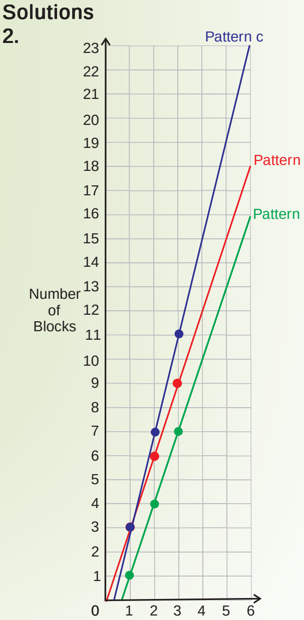
Math background

Explain to students about how slope is the steepness of the line. Slope is the relationship between the change in y as compared to the change in x. You find it dividing the change in y by the change in x.

The change in x is 1 for each new figure which is added to the pattern.

Solutions

2.



Solutions

1. a.

Figure in pattern	1	2	3
Number of blocks	3	6	9

b.

Figure in pattern	1	2	3
Number of blocks	1	4	7

c.

Figure in pattern	1	2	3
Number of blocks	3	7	11

Lesson 4.1.1

Graphing Equations

In this Lesson, students are reminded how to find ordered pairs that are solutions to linear equations in two variables. They see how to plot these points on the coordinate plane to produce a straight line that represents all the solutions of the equation.

Previous Study: In grade 4 students learned how to plot points and join them up to draw the line graph of an equation. At grades 5 and 6 students learned how to solve linear equations using this method.

Future Study: In Algebra I students will learn how to find the equation of a line that passes through a point. They will learn how to graph systems of equations, quadratic equations, and inequalities.

1 Get started

Resources:
- individual whiteboards and pens
- sticky tack
- large x- and y-axes
- about 4 m of ribbon
- grid paper

Teacher Resources CD-ROM
- Number and Operator Tiles
- Coordinate Grid

Warm-up questions:
- Lesson 4.1.1 sheet

2 Teach

Universal access

Give students some practice at rearranging equations into the form $y = mx + b$.

Provide everyone with three sets of cards saying y, x, 2, 3, –, and + (from the Number and Operator Tiles on the **Teacher Resources CD-ROM**). Give everyone an equals sign and some sticky tack. They should stick the = card to the table so it can't move.

Give them the equations $y + 2 = x$ and $3 = 2x - y$, and have them make the equations using the cards. Then ask them to rearrange the equations into the form $y = mx + b$. Remind them that to keep an equation true they must do the same thing to both sides at each step.

When they have finished, ask them to write out both equations, and to write beside them what m and b are in each.

Guided practice

Level 1: q1–3
Level 2: q1–4
Level 3: q1–6

Common error

Students will sometimes mix up the x- and y-values and so wrongly say that a point is, or is not, a solution to an equation. For example, they may say that the point (2, 1) is a solution of $y = x + 1$, rather than the point (1, 2).

Remind them that although an equation is usually written in the form $y = mx + b$, an ordered pair is always written in the form (x, y).

Lesson 4.1.1

California Standards:

Algebra and Functions 1.5
Represent quantitative relationships graphically, and interpret the meaning of a specific part of a graph in the situation represented by the graph.

Algebra and Functions 3.3
Graph linear functions, noting that the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same, and know that the ratio ("rise over run") is called the slope of a graph.

What it means for you:
You'll learn how to plot linear equations on a coordinate plane.

Key words:
- linear equation
- variables
- graph

Check it out:
The $y = mx + b$ equation represents a "function." A function is a rule that assigns each number to one other number. If you put a value for x into the function, you get one value for y out.

Don't forget:
Ordered pairs are often called coordinates, or coordinate pairs.

Check it out:
There are an infinite number of points on a line. So there are an infinite number of solutions to a linear equation.

Section 4.1

Graphing Equations

Equations like $y = 3x$, $y = x + 1$, and $y = 2x + 3$ are known as linear equations because if you plot them on a grid, you get straight lines. In this Lesson you'll learn how to plot linear equations.

Linear Equations Have the Form $y = mx + b$

A linear equation can have **one or two variables**. The variables must be **single powers**, and if there are two variables, they must be in **separate terms**.

Linear Equations	Nonlinear Equations
$y = x - 7$	$xy = 7$
$y = 8x + 1$	$y = 2x^2 - 3$
$4y - 2x = -7$	$y = x^3 + 1$
$y = 3x$	$y = 3x^2$

A linear equation can always be written in the form below (but you might have to rearrange it first):

y is a variable. — m and b are constants, so they're numbers like 1 or 3.

$$y = mx + b$$

x is a variable too.

The m and b values can be 0, so $y = 3x$ and $y = 4$ are linear equations too.

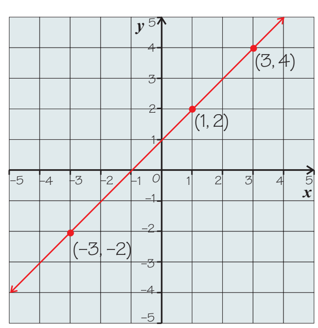
Guided Practice

In Exercises 1–6, state whether the equation is a linear equation or not.

1. $y = 2x - 5$ yes **2.** $7y - 9x = -1$ yes **3.** $y = x^2 + 4$ no
4. $2y = 4x + 3$ yes **5.** $y^3 = x^3 - 1$ no **6.** $y = x$ yes

Every Point on the Line is a Solution to the Equation

The graph of a linear equation is always a **straight line**. Every point on the graph is an **ordered pair (x, y)** that is a **solution to the equation**.



This is the graph of the equation $y = x + 1$. The point (1, 2) lies on the graph, so $x = 1$, $y = 2$ must be a solution to the equation.

You can test this by **substituting** the x- and y-values into the equation and checking that they make the equation **true**:

$y = x + 1 \rightarrow 2 = 1 + 1$

This makes the equation true, so $x = 1$, $y = 2$ is a solution to the equation.

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Use individual whiteboards to review the concept of plotting and joining points, and check that everyone understands. Ask students to draw the x-axis, checking that they draw a horizontal line. Then ask them to add the y-axis. Then have them number both axes from –4 to 4, mark the origin (0, 0), and plot the points (1, 4) and (–2, –3), checking they have the x- and y-coordinates the correct way around.
- **English Language Learners**
  Have students use the "take notes, make notes" strategy to review plotting points and drawing graphs. Ask them to make a table for an equation like $y = 2x + 1$ in their notebooks, finding five ordered pairs that fit the equation. Have them plot the points, draw a graph, and make notes around the graph.

Guided Practice

Show that the following are solutions to the equation $y = x + 1$.

7. $x = 3, y = 4$ see below **8.** $x = -3, y = -2$ see below

Using the graph on the previous page, explain whether the following are solutions to the equation $y = x + 1$.

9. $x = 1, y = 4$ see below **10.** $x = -4, y = -3$ see below

Find Some Solutions to Plot a Graph

To graph a linear equation, you need to **find some ordered pairs** to plot that are **solutions** to the linear equation.

You do this by putting some **x-values** into the equation and finding their **corresponding y-values**.

Check it out:
You only really need two ordered pairs to draw a straight-line graph.
But you should work out at least one more than this to make sure you haven't made any errors.

Example 1

Find the solutions to the equation $y = 2x + 1$ that have x-values of –2, –1, 0, 1, and 2.
Use these to write ordered pairs that lie on the graph of $y = 2x + 1$.

Solution

Step 1: Draw a **table** that allows you to fill in the y-values next to the corresponding x-values. Make a column to write the ordered pairs in.

x	y	Ordered Pair (x, y)
–2		
–1		
0		
1		
2		

Step 2: Substitute each x-value into the **equation**, to get the corresponding y-value. Here are a few examples:

For $x = -2$:
$y = 2x + 1 = 2(-2) + 1 = -3$

For $x = -1$:
$y = 2x + 1 = 2(-1) + 1 = -1$

Step 3: Write each set of x- and y-values as an **ordered pair (x, y)**.

x	y	Ordered Pair (x, y)
–2	–3	(–2, –3)
–1	–1	(–1, –1)
0	1	(0, 1)
1	3	(1, 3)
2	5	(2, 5)

2 Teach (cont)

Guided practice
Level 1: q7, 9
Level 2: q7–9
Level 3: q7–10

Math background
Students learned how to plot points on the coordinate plane in Section 3.2, which also covered ordered pairs and the quadrants of the plane.

Additional examples
Say if the following points lie on the graph of the equation $y = 4x - 3$.

1) (0, –3) **Yes**
2) (3, 7) **No**
3) (–4, –19) **Yes**
4) (15, 47) **No**
5) (20, 77) **Yes**

Concept question
"What is the y-value of the equation $y = 2x + 10$ when $x = 2$?"
14

"And what is the y-value when $x = -2$?"
6

Solutions
For worked solutions see the Solution Guide

7. $4 = 3 + 1$
$4 = 4$

8. $-2 = -3 + 1$
$-2 = -2$

9. The point (1, 4) isn't on the graph of $y = x + 1$, so $(x, y) = (1, 4)$ isn't a solution to the equation.

10. The point (–4, –3) is on the graph of $y = x + 1$, so $(x, y) = (-4, -3)$ is a solution to the equation.

- **Advanced Learners**
  Provide students with some linear graphs plotted on a set of axes. Have them look at the graphs and try to figure out the equation of each one. When they have done this, ask them to compare the equations they have written with the generalized formula for linear graphs, $y = mx + b$. Ask students if they can see any connection between the formula and their equations. They should realize that the constant b is the same as the graph's y-intercept. They may also figure out that the constant, m, is the slope of the graph.

2 Teach (cont)

Guided practice

Level 1: q11
Level 2: q11–12
Level 3: q11–12

Universal access

Draw out large x- and y-axes on strips of paper. Label both axes from –8 to 8. Lay them out on the floor of the classroom, or outside if you wish.

Write the equation $y = 2x - 2$ on the board. Remind the class that it is a linear equation, so it should give a straight-line graph. Draw a table with the x-values –2, –1, 0, 1, and 2. Have the class work together to fill in the y-values.

Select five students to be the points of the graph. Have each one in turn pick a point to be. They should come and stand on the origin of the graph, then walk out their coordinates to get to the right point on the plane.

When they are all in place, give the one who is at the bottom left of the grid the end of a long piece of ribbon. The students should then pass the ribbon along, until the fifth "point" has the other end.

The points should form a straight line. If they don't, someone is in the wrong place — the class should work out who, and help them correct the error.

Guided Practice

11. Find the solutions to the equation $y = 5x - 4$ that have x-values equal to –2, –1, 0, 1, and 2. Use your solutions to write a set of ordered pairs that lie on the graph of $y = 5x - 4$. see below

12. Find the solutions to the equation $y = 2x - 6$ that have x-values equal to –6, –3, 0, 3, and 6. Use your solutions to write a set of ordered pairs that lie on the graph of $y = 2x - 6$. see below

Check it out:
If your points aren't in a straight line, you must have made a mistake — go back and check.
That's why you need more than two points — you wouldn't know if you'd made a mistake if you only had two points to join up.

Plot the Points and Join Them Up

You draw the graph of an equation by first plotting **ordered pairs** that represent the solutions to the equation. They should lie in a **straight line**.

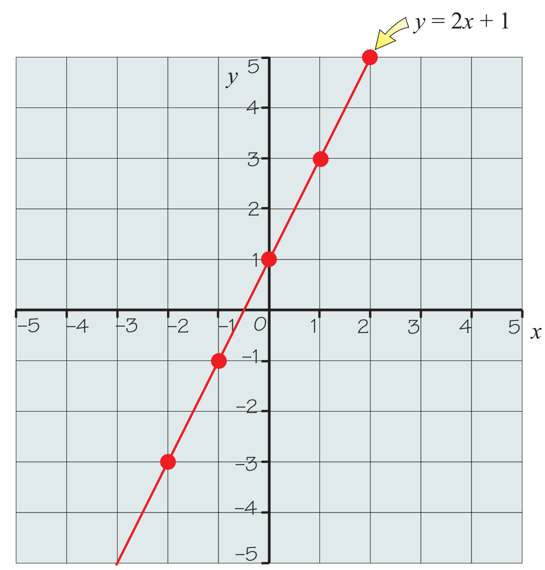
Example 2

Draw the graph of $y = 2x + 1$ by plotting the ordered pairs you found in Example 1.

Solution

The ordered pairs that fit the equation are (–2, –3), (–1, –1), (0, 1), (1, 3), and (2, 5).

Plot these points and draw a straight line through them.



Check it out:
Always label your graph with its equation.

Solutions
For worked solutions see the Solution Guide

Guided Practice:

11.

x	y	Ordered Pair (x, y)
–2	–14	(–2, –14)
–1	–9	(–1, –9)
0	–4	(0, –4)
1	1	(1, 1)
2	6	(2, 6)

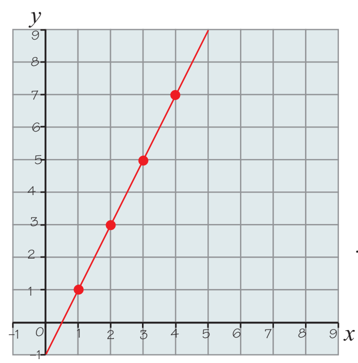
12.

x	y	Ordered Pair (x, y)
–6	–18	(–6, –18)
–3	–12	(–3, –12)
0	–6	(0, –6)
3	0	(3, 0)
6	6	(6, 6)

Example 3

Plot the ordered pairs (1, 1), (2, 3), (3, 5), and (4, 7).
Do these ordered pairs lie on a linear graph?

Solution



Plot the points on a coordinate plane.

When you join the points together, you get a straight line.

So, the coordinates do lie on a linear graph.

(In fact, they lie on the graph of $y = 2x - 1$.)

Guided Practice

13. Plot the graph of the function $y = 5x - 4$.
You found some x and y pairs in Guided Practice Exercise 11. **See right**

14. Plot the graph of the function $y = 2x - 6$.
You found some x and y pairs in Guided Practice Exercise 12. **See right**

Independent Practice

In Exercises 1–3, use the values of x to evaluate the following equation: $y = 2x - 9$

1. $x = 4$ $y = -1$ **2.** $x = -6$ $y = -21$ **3.** $x = -10$ $y = -29$

4. Fill in a table for the equation $y = -5x + 3$ ready for it to be graphed on a coordinate plane. Use the x-values –1, 0, 1, and 2. **See left**

x	y	Ordered Pair (x, y)
–1	8	(–1, 8)
0	3	(0, 3)
1	–2	(1, –2)
2	–7	(2, –7)

In Exercises 5–6, determine whether the set of ordered pairs lies on a linear graph.

5. (0, 1), (1, 3), (2, 3), (3, 5). **No** **6.** (1, 1), (2, 2), (3, 3), (4, 4). **Yes**

7. Explain whether the point D with coordinates (4, –6) is on the line $y = -5x + 14$. **Yes — the x- and y-values of the ordered pair satisfy the equation.**

8. Construct a table to find some points on the graph of $y = 3x - 5$.
Plot the values on a coordinate plane and draw the graph. **See below**

9. Construct a table to find some points on the graph of $y = \frac{2}{3}x - 6$.
Plot the values on a coordinate plane and draw the graph. **See below**

Now try these:

Lesson 4.1.1 additional questions — p452

Round Up

When you draw a graph of a linear equation you always get a straight line, and all the points on the graph represent solutions to the equation. It's important to understand this when you look at solving systems of equations in the next Lesson.

2 Teach (cont)

Concept question

"Do the ordered pairs (a, 2), (b, 2), (c, 2), and (d, 2) lie on a linear graph?"

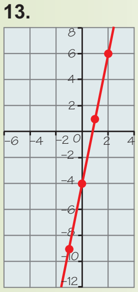
Yes — they lie on the graph of $y = 2$.

Guided practice

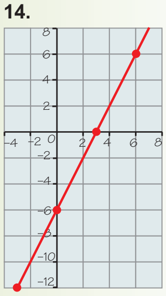
Level 1: q13
Level 2: q13
Level 3: q13–14

Solutions

13.



14.



Independent practice

Level 1: q1–2, 4–5, 8
Level 2: q1–2, 4–8
Level 3: q2–7, 9

Additional questions

Level 1: p452 q1–5, 7
Level 2: p452 q1–7, 9
Level 3: p452 q1–10

3 Homework

Homework Book — Lesson 4.1.1

Level 1: q1–5
Level 2: q1–7
Level 3: q3–8

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:

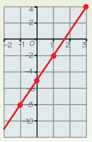
- Worksheet 28 — Graphing Linear Equations

Solutions

For worked solutions see the Solution Guide

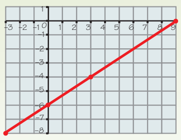
8.

x	y	Ordered Pair (x, y)
–1	–8	(–1, –8)
0	–5	(0, –5)
1	–2	(1, –2)
3	4	(3, 4)



9.

x	y	Ordered Pair (x, y)
–3	–8	(–3, –8)
0	–6	(0, –6)
3	–4	(3, –4)
9	0	(9, 0)



Lesson 4.1.2

Systems of Linear Equations

Students are introduced to systems of linear equations for the first time. They write a system of linear equations from a word problem, and see how the intersection of the graphs gives the solution to the system of equations.

Previous Study: At grade 6 students learned how to write and solve one-step linear equations containing one variable. They also learned how to graph a linear equation.

Future Study: Students will solve systems of linear equations using both graphic and algebraic methods in Algebra I.

1 Get started

Resources:

- red and blue card squares
- overhead projector and transparencies

Teacher Resources CD-ROM

- Coordinate Grid (or grid paper)

Warm-up questions:

- Lesson 4.1.2 sheet

2 Teach

Universal access

Set the following logic puzzle:

"Bill works out that in two years' time he will be exactly twice the age that his brother is now. The sum of their ages is 25. How old is Bill?"

Ask students to think about how you could use the information from the problem to write a system of equations. It might look like this:

Let Bill's age = x.
Let his brother's age = y.
$x + 2 = 2y$ and $x + y = 25$

To solve the problem, they need to find a pair of x- and y-values that fits both equations — they need to find a solution to the system.

This can be done using trial and error, but they could also do it using a graph — which is what the next part of the Lesson is about.

Have students come back to the problem after they have learned how to solve systems of equations graphically. Ask them to graph the system of equations that they wrote, and use their graph to find the solution of the system.

In Algebra I, students will solve problems like this using algebraic methods, such as substitution:

$x + 2 = 2y \Rightarrow x = 2y - 2.$
$(2y - 2) + y = 25$
$3y - 2 = 25$
$3y = 27$
$y = 9$
$x = 25 - y = 25 - 9 = 16$

Guided practice

Level 1: q1
Level 2: q1–2
Level 3: q1–2

Lesson 4.1.2

Systems of Linear Equations

California Standards:

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression, an equation, an inequality, or **a system of equations** or inequalities **that represents a verbal description** (e.g., three less than a number, half as large as area A).

Algebra and Functions 1.5

Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

What it means for you:

You'll learn what systems of linear equations are and understand how their solutions are shown by graphs.

Key words:

- system of equations
- linear equation
- solving
- intersection

In the last Lesson you graphed linear equations and saw how every point on a line is a solution to the equation of the line. In this Lesson you'll use this idea to solve a system of equations.

A System is a Set of Linear Equations

A system of linear equations is a **set** of two or more linear equations in the **same variables**. The equations $y = 2x + 2$ and $y = -3x - 8$ are a system of equations in the two variables x and y.

Example 1

Write a system of linear equations to represent the following statement:

"y is three times x and the sum of y and x is 8"

Solution

You need to write **two equations** that **both** need to be true for the statement to be true.

The first part says, "y is three times x," so $y = 3x$.
The second part says, "the sum of y and x is 8," so $y + x = 8$.

These two equations form a **system of linear equations**.

The **solutions** to a system of equations have to satisfy all the equations at the same time. So the solution to the system of equations $y = 3x$ and $y + x = 8$ is $x = 2$ **and** $y = 6$. These values make **both** equations true.

Guided Practice

Write systems of equations to represent the following statements.

1. x subtracted from y is 3, and y is twice x. $y - x = 3,\ y = 2x$

2. Bob buys two melons at \$$y$ each and three avocados at \$$x$ each. He is charged \$9 altogether. Melons cost \$2 more than avocados.
$2y + 3x = 9,\ y - x = 2$

You Can Solve Systems of Equations Graphically

All points on the graph of a linear equation have **x- and y-values that make that equation true**. Points on the graph of another linear equation in a system have x- and y-values that make that equation true.

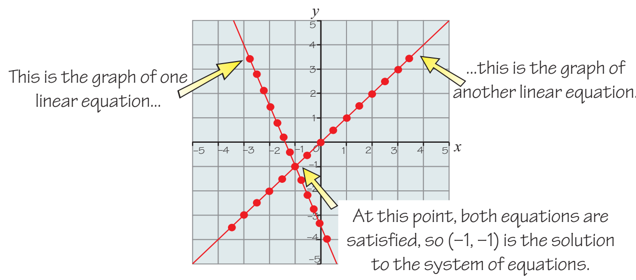
Where the graphs of two linear equations in a system **intersect**, the x- and y-values satisfy **both equations**. This intersection point is a **solution to both equations**, and so is the **solution to the system**.

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Give each pair of students five red and five blue card squares. Tell them they're going to lay the squares out in a pattern. But before starting, they must write two rules about how their squares can be arranged — for example, "the pattern must be a rectangle," or "each red must touch another red." Then they lay out the squares following both rules. The pattern they make is a solution that satisfies their system of rules.

- **English Language Learners**
  Have students write word problems that describe a system of two equations. If it helps them, they could write the two equations first, and then think of a situation to fit around it, or use a word problem from the previous page as a template. When they have finished they should give the problem to a partner, and see if they can write the correct two equations to represent it.

Check it out:
Two straight lines that aren't parallel, or exactly the same, can only intersect each other once. So there will only ever be one solution to a system of linear equations like this. Parallel lines never cross, so there would be no solution. And if the lines are the same, there would be infinitely many solutions.



The **solution** to a system of linear equations in two variables is the **point of intersection (x, y)** of their graphs.

So you can solve a system of linear equations by **plotting the graph** of each equation and finding out where they cross.

Example 2

Solve the following system of equations by graphing:

$y = 2x - 1$ $\qquad$ $y = x - 2$

Solution

Draw tables to find coordinates of some points on each graph.

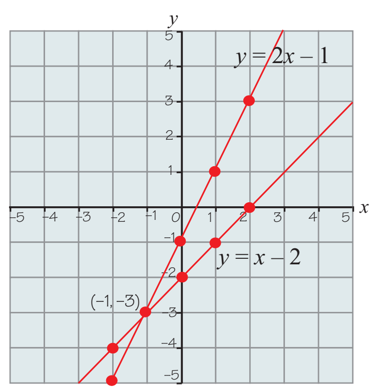
$y = 2x - 1$

x	y	Ordered Pair (x, y)
2	3	(2, 3)
1	1	(1, 1)
0	–1	(0, –1)
–1	–3	(–1, –3)
–2	–5	(–2, –5)

$y = x - 2$

x	y	Ordered Pair (x, y)
2	0	(2, 0)
1	–1	(1, –1)
0	–2	(0, –2)
–1	–3	(–1, –3)
–2	–4	(–2, –4)

Now **plot both graphs on the same coordinate plane**.



Read off the point of intersection — it is (–1, –3).
So (–1, –3), or $x = -1$, $y = -3$, is the solution to the system of equations.

Don't forget:
This is the same method that you learned last Lesson for plotting linear equations. If you can't remember the steps, go back and do a bit more practice on graphing equations first.

2 Teach (cont)

Concept question
"The line graphs of two equations cross at the origin. What solution do both equations share?"
(0, 0), or $x = 0$, $y = 0$

Universal access
Demonstrate how to graph a system of equations using an overhead and transparencies.

Draw a set of x- and y-axes on a transparency, and copy it exactly onto another. Put the first transparency on the overhead. Write one equation on it, for example, $y = 2x + 2$, and graph the equation on the axes. This is a good opportunity to review the process of graphing a line from a table.

Repeat the process with the second transparency and a different equation, for example, $y = x + 1$.

Now overlay the transparencies so that the axes are lined up. Pick a point on one of the lines, and put its coordinates into both equations. Repeat the same process for the second line. Students will see that in both cases the coordinates work for one of the lines but not the other. Finally, use the coordinates of the point of intersection. This is the only point whose coordinates work for both equations — it is the only solution to the system of linear equations.

Math background
You can also solve systems of equations by rearranging and substituting. An example of this technique is shown in the Universal access activity on page 220. Students will learn this method in Algebra I.

- **Advanced Learners**
  Give advanced learners some more complicated word problems to solve by graphing systems of equations. For example:
  1. 12 pencils cost the same as 10 erasers. Bob buys five erasers, and spends 50¢ more than Paz, who buys four pencils. How much does a pencil cost? How much does an eraser cost? **Pencil = 25 ¢, eraser = 30 ¢**
  2. Two men place orders at a bakery. The first is for 15 rolls and four loaves, and totals \$19.42. The second is for six rolls and two loaves, and totals \$8.66. How much does a roll cost? How much does a loaf cost? **Roll = 70 ¢, loaf = \$2.23**

2 Teach (cont)

Guided practice
Level 1: q3
Level 2: q3–4
Level 3: q3–4

Concept question
"Is (3, 9) a solution to the equations $y = 2x + 3$ and $3y = x + 24$?"
Yes

Guided practice
Level 1: q5
Level 2: q5–6
Level 3: q5–6

Independent practice
Level 1: q1, 3
Level 2: q1–3
Level 3: q2–4

Additional questions
Level 1: p452 q1–5
Level 2: p452 q1–7
Level 3: p452 q1–9

3 Homework

Homework Book — Lesson 4.1.2
Level 1: q1–3, 4a, 5a
Level 2: q1–5, 7, 8
Level 3: q1–8

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 28 — Graphing Linear Equations

Guided Practice

Solve the systems of equations in Exercises 3–4 by graphing.
3. $y = x + 2$ and $y = -2x + 5$ **$x = 1, y = 3$**
4. $y = x - 6$ and $y = -x + 2$ **$x = 4, y = -2$**

Always Check Your Solution

It's easy to make mistakes when graphing, so you should always **test the solution** you get by putting it into both equations and checking it makes them both **true**.

Don't forget:
The first number in an ordered pair is always the x-value — (x, y).

Example 3

Check that (–1, –3) is a solution to the system of equations $y = 2x - 1$ and $y = x - 2$.

Solution

Check the solution by **substituting** the x- and y-values into both equations:
Equation 1: $y = 2x - 1 \Rightarrow -3 = 2(-1) - 1 \Rightarrow -3 = -3$ — **true**
Equation 2: $y = x - 2 \Rightarrow -3 = -1 - 2 \Rightarrow -3 = -3$ — **true**
So (–1, –3) is a solution to the system of equations.

Guided Practice

5. Check your answer from Guided Practice Exercise 3 by substituting it back into the equations. **Equation 1: $y = x + 2 \Rightarrow 3 = 1 + 2 \Rightarrow 3 = 3$ — true**
Equation 2: $y = -2x + 5 \Rightarrow 3 = -2(1) + 5 \Rightarrow 3 = 3$ — true
6. Check your answer from Guided Practice Exercise 4 by substituting it back into the equations. **Equation 1: $y = x - 6 \Rightarrow -2 = 4 - 6 \Rightarrow -2 = -2$ — true**
Equation 2: $y = -x + 2 \Rightarrow -2 = -4 + 2 \Rightarrow -2 = -2$ — true

Independent Practice

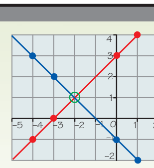
Now try these:
Lesson 4.1.2 additional questions — p452

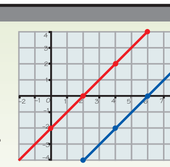
1. Explain whether it is possible to have two solutions to a system of two linear equations. **No — their graphs will only cross once. (If the lines were the same, there'd be an infinite number of solutions.)**
2. Describe the situation in which there is no solution to a system of two linear equations. **The lines are parallel**
3. Solve the following system of equations by graphing. Check your solution by substituting.
$y = x + 3$ $\qquad$ $y = -x - 1$ **see below**
4. Graph the following two equations. Explain why this system of equations has no solution.
$y = x - 2$ $\qquad$ $y = x - 6$ **see below**

Round Up

In this Lesson you learned how to write systems of linear equations, and how their single solution can be read from a graph. In grade 8 you'll also solve systems of linear equations algebraically.

Solutions
For worked solutions see the Solution Guide

3.

$x = -2, y = 1$
$y = x + 3 \Rightarrow 1 = -2 + 3 \Rightarrow 1 = 1$
$y = -x - 1 \Rightarrow 1 = -(-2) - 1 \Rightarrow 1 = 1$

4. The lines of the graphs are parallel, so they never cross.


Lesson 4.1.3

Slope

In this Lesson, students learn what the slope of a graph is and how to calculate it. They also learn to tell if the slope of a given line is positive, negative, or zero.

Previous Study: At grade 6 students used tables and graphs to help solve problems involving rates, ratios, and proportions. They solved problems involving average speed, distance, and time.

Future Study: In Algebra I students will learn how to derive the equation of a line from its slope, and x- and y-intercepts. They will also learn to apply the point-slope formula to find a line's equation.

Lesson 4.1.3

Slope

California Standards:

Algebra and Functions 3.3

Graph linear functions, noting that the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same, and know that the ratio ("rise over run") is called the slope of a graph.

What it means for you:

You'll learn what the slope of a graph is and how to calculate it.

Key words:
- slope
- steepness
- ratio

Check it out:

The distance you go across a graph is known as the "run," and the distance you go up is known as the "rise." So the ratio $\frac{\text{change in } y}{\text{change in } x}$ is often called the "rise over run."

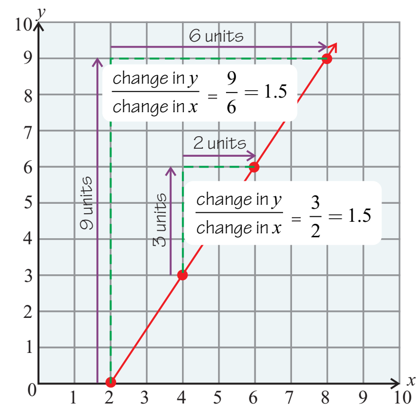
Check it out:

You can use any points on the line to calculate slope — it's normally easiest to choose points that have integer x- and y-values though.

Over the past few Lessons you've been graphing linear equations — which have straight-line graphs. Some straight-line graphs you've drawn have been steep, and others have been more shallow. There's a measure for how steep a line is — slope. In this Lesson you'll learn how to find the slope of a straight-line graph.

The Slope of a Line is a Ratio

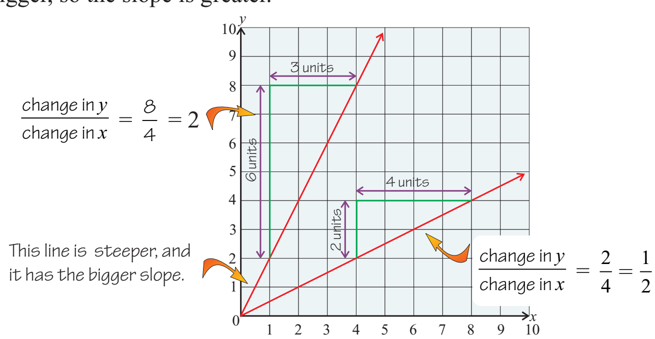
For any straight line, the ratio $\frac{\text{change in } y}{\text{change in } x}$ is always **the same** — it doesn't matter which two points you choose to measure the changes between.



This ratio, $\frac{\text{change in } y}{\text{change in } x}$, is the **slope** of the graph.

Slope is a Measure of Steepness of a Line

A larger change in y for the same change in x makes the ratio $\frac{\text{change in } y}{\text{change in } x}$ bigger, so the slope is greater.



So a slope is a measure of the **steepness** of a line — **steeper lines** have **bigger slopes**.

1 Get started

Resources:
- wooden board and toy car
- grid paper
- rulers

Warm-up questions:
- Lesson 4.1.3 sheet

2 Teach

Math background

Students studied ratios and how to evaluate them in grade 6.

To evaluate a ratio, you simply divide the first number (or the numerator) by the second (or denominator). A ratio has no units.

Common error

Students often have trouble remembering that to calculate slope you divide the change in y by the change in x, and not vice versa.

If it helps them, they could remember it in another form, for example, $\frac{\text{rise}}{\text{run}}$, $\frac{\text{rise}}{\text{tread}}$, or $\frac{\text{up}}{\text{along}}$.

Universal access

Introduce the topic of slope by talking about how we describe it in real life. You could bring in a picture of a "Steep Grade Ahead" road sign, or a map with contour lines showing a hilly area.

Ask students to think about what the numbers describing slope mean — if a hill has a slope of 5%, this means that for every 20 m you go forward, you go 1 m up or down. Have them think about the fact that the steeper the hill, the larger the percent slope will be.

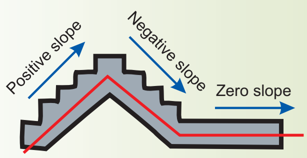
- **Strategic Learners**
  Put students into small groups. Give each group a table with the names of three made-up people, and how much they pay for their cell phones each month. Have them extend the table to show how much each person will have paid in total after 2 months and 6 months, then use it to draw a graph showing the rate each person pays. How can they tell by looking at the graph who pays the most?

- **English Language Learners**
  Reinforce the terms "slope," "steep," "uphill," and "downhill" using a board and a toy car. Prop the board up at various levels of steepness. Have students make conjectures about the rate at which the car will roll compared to the steepness of the board. Have volunteers share and test their conjectures (for example, "the car should roll faster the steeper the slope of the board is").

2 Teach (cont)

Universal access

Have students draw a picture to help them remember what a positive, negative, and zero slope look like. They can illustrate it any way they like — maybe with a mountain, a slide, or a staircase, as in the example below.



Whenever they have trouble remembering which is which, they can visualize their picture.

Guided practice

Level 1: q1
Level 2: q1–2
Level 3: q1–2

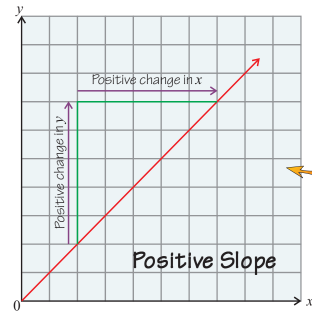
Common error

Students sometimes write a pair of x- and y-values in the wrong order when they substitute them into the formula:

$\dfrac{y_2 - y_1}{x_2 - x_1}$ ✔ $\qquad \dfrac{y_1 - y_2}{x_2 - x_1}$ ✘

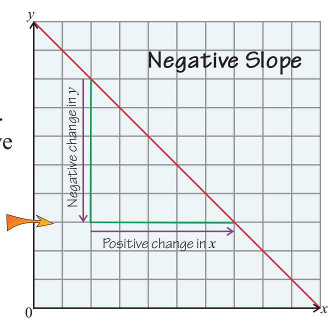
Remind them to subtract **both** the x- and y-coordinates of one point from the x- and y-coordinates of the other.

Slopes Can Be Positive, Negative, or Zero



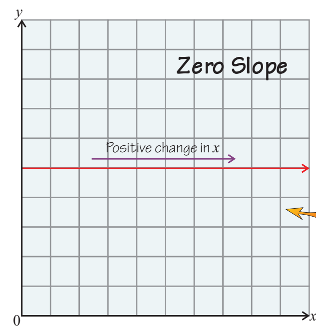
A **positive slope** is an "**uphill**" **slope**. The changes in x and y are both positive — as one increases, so does the other.

$$\frac{\text{positive change in } y}{\text{positive change in } x} = \text{positive slope}$$



A **negative slope** is a "**downhill**" **slope**. The change in y is negative for a positive change in x. y decreases as x increases.

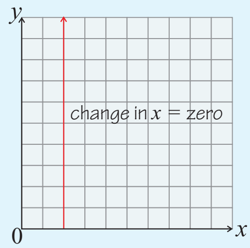
$$\frac{\text{negative change in } y}{\text{positive change in } x} = \text{negative slope}$$



A line with **zero slope** is **horizontal**. There is no change in y.

$$\frac{0 \text{ change in } y}{\text{positive change in } x} = 0 \text{ slope}$$

Check it out:
The slope of a vertical line is undefined. There's a change of zero on the x-axis, and you can't divide by zero.



Guided Practice

1. Plot the points (1, 3) and (2, 5) on a coordinate plane. Find the slope of the line connecting the two points. 2

2. Does the graph of $y = -x$ have a positive or negative slope? Explain your answer. **Negative — it's downhill from left to right.**

Compute Slopes from Coordinates of Two Points

Instead of counting unit squares to calculate slope, you can use the **coordinates** of any two points on a line. There's a formula for this:

For the line passing through coordinates (x_1, y_1) and (x_2, y_2):

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Check it out:
It doesn't matter which points you use for (x_1, y_1) and (x_2, y_2). But you have to make sure you subtract both the x-and y-values of one pair from the x- and y-values of the other pair.

Solutions

For worked solutions see the Solution Guide

Advanced Learners

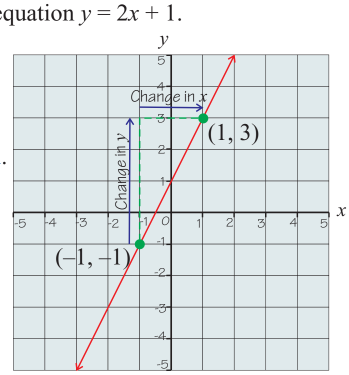
Challenge students to calculate the slope of two different staircases. They can do this by measuring the riser and tread of an individual step. Ask them to record their work, and to make a small-scale drawing of each staircase on grid paper to illustrate it. Ask them to write down under their drawings which staircase has the greater slope, and to compare this to their relative steepnesses.

Don't forget:

Subtracting a negative number is the same as adding a positive number. So 3 – (–1) is the same as 3 + 1. Be careful with this when you're calculating slopes.

Example 1

The graph below is the graph of the equation $y = 2x + 1$. Find the slope of the line.



Solution

Start by **drawing a triangle connecting two points on the graph**.

Choose two points that are easy to read from the graph, for example:
$(x_1, y_1) = (-1, -1)$
$(x_2, y_2) = (1, 3)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

This is the change in y.

$$= \frac{3-(-1)}{1-(-1)} = \frac{3+1}{1+1} = \frac{4}{2} = 2$$

This is the change in x.

So the slope of the graph is 2.

Don't forget:

You can always count the units on the graph to check — but be careful. The scale might not always be one square to one unit.

Example 2

Find the slope of the line connecting the points C (–2, 5) and D (1, –4).

Solution

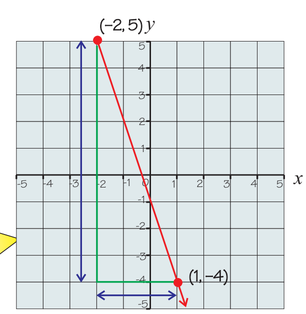
You don't need to draw the line to calculate the slope — you are given the coordinates of two points on the line.
$(x_1, y_1) = (-2, 5)$ and $(x_2, y_2) = (1, -4)$.

Substitute the coordinates into the formula for slope:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-5}{1-(-2)} = \frac{-9}{3}$$

Slope = –3

If you plot these points and draw a line through them, you can see that the slope is negative (it's a "downhill" line).



Check it out:

Always check your answer is reasonable.
Look at the graph you're finding the slope of, and check that if it's downhill, your slope is negative, and if it's uphill, your slope is positive.

Guided Practice

3. Plot the points (–2, 3) and (2, 5) on a coordinate plane. Find the slope of the line connecting the two points. $\frac{1}{2}$

4. Plot the graph of the equation $y = 4x - 2$ and find its slope. 4

2 Teach (cont)

Universal access

Ask everyone to draw the graph of the equation $y = x + 2$, and to mark the points (0, 2) and (2, 4).

Have them count units to find the rise and run between these two points, and then divide to find the slope of the line.

Then ask them to calculate the slope by putting the two coordinates into the formula Slope = $\frac{y_2 - y_1}{x_2 - x_1}$.

The students should get the same answer both times — emphasize that this is because both methods are really doing the same thing.
$(y_2 - y_1)$ = rise and $(x_2 - x_1)$ = run.

Concept question

"Without drawing a graph, say what the slope is of the line passing through the points (4, 2) and (–6, 2)."

0. The y values are the same, so the line must be horizontal.

Guided practice

Level 1: q3
Level 2: q3–4
Level 3: q3–4

Solutions

For worked solutions see the Solution Guide

2 Teach (cont)

Math background

The slope of a line is equal to its rate of change. Students have covered rates and ratios at grade 6, and the topic is covered again in the next Section.

Independent practice

Level 1: q1–3, 6–7, 10
Level 2: q1, 3–4, 7–8, 11–13
Level 3: q1, 4–5, 8–9, 12–14

Additional questions

Level 1: p452 q1–7
Level 2: p452 q1–9
Level 3: p452 q1–11

3 Homework

Homework Book — Lesson 4.1.3

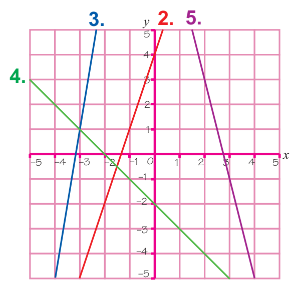
Level 1: q1a–b, 2, 3, 4a–b, 6
Level 2: q1–6
Level 3: q1–7

4 Skills Review

Skills Review CD-ROM

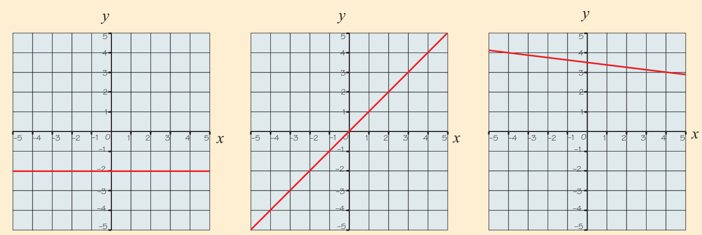
This worksheet may help struggling students:
• Worksheet 28 — Graphing Linear Equations

For example:



Independent Practice

1. Identify whether the slope of each of the lines below is positive, negative, or zero.



zero positive negative

On a coordinate plane, draw lines with the slopes given in Exercises 2–5.

2. 3 **3.** 6 **4.** –1 **5.** –4 see left

In Exercises 6–9, find the slope of the line passing through the two points.

6. W (3, 6) and R (–2, 9) $\frac{-3}{5}$
7. Q (–5, –7) and E (–11, 0) $\frac{-7}{6}$
8. A (–12, 18) and J (–10, 6) –6
9. F (2, 3) and H (–4, 6) $\frac{-1}{2}$

10. The move required to get from point C to D is up six and left eight units. What is the slope of the line connecting C and D? $\frac{-3}{4}$

11. Point G with coordinates (7, 12) lies on a line with a slope of $\frac{3}{4}$.
Write the coordinates of another point that lies on the same line.
Answers will vary. Example: (11, 15)

12. On the coordinate plane, draw a line through the points E (–2, 5) and S (4, 1). Find the slope of this line. $-\frac{2}{3}$
On the same plane, draw a line through the points P (–2, –2) and N (4, –6). Find the slope of this line. $-\frac{2}{3}$
What can you say about the two lines you have drawn and their slopes?
They're parallel and their slopes are the same.

13. Consider the statement: "The slope of a line becomes less steep if the distance you have to move along the line for a given change in *y* increases." Determine whether this statement is true or not. True

14. Is it possible to calculate the slope of a vertical line?
Explain your answer. No — it's undefined, as you cannot divide by zero.

Now try these:
Lesson 4.1.3 additional questions — p452

Round Up

The slope of a line is the ratio of the change in the y-direction to the change in the x-direction when you move between two points on the line — it's basically a measure of how steep the line is. Positive slopes go "uphill" as you go from left to right across the page, and negative slopes go "downhill." Slope is actually a rate — and you'll be looking at rates over the next few Lessons.

Solutions
For worked solutions see the Solution Guide

Section 4.2

Exploration — Pulse Rates

Purpose of the Exploration

The purpose of the Exploration is to have students see the concept of a unit rate. They will count their pulse over 15 seconds, and then calculate several different unit rates.

Resources

- stop watch
- calculators

Section 4.2 introduction — an exploration into: Pulse Rates

In this Exploration, you'll measure your pulse rate and convert it to several different unit rates.

A **rate** is a comparison of two amounts that have **different units of measure**.

For example: 100 miles in 2 hours or $\frac{100 \text{ miles}}{2 \text{ hours}}$.

A **unit rate** is a rate where the second amount is **1**.
You find a unit rate by **dividing the first amount by the second**.

Example

A car travels 100 miles in 2 hours. What is its unit rate?

Solution

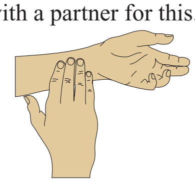
$\frac{100 \text{ miles}}{2 \text{ hours}}$ ⇒ **Unit rate = 50 miles/hour, or 50 miles per hour**

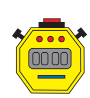
Exercises

1. Write the unit rate for each.
   a. 90 words in 3 minutes — 30 words per minute
   b. 10 feet for 2 inches of height — 5 feet per inch of height
   c. 100 miles on 4 gallons — 25 miles per gallon

You'll now make some measurements involving **heart rate**. Work with a partner for this.

Find your pulse on your left wrist, using two fingers of your right hand, as shown. When you've found your pulse, your partner should start the stopclock and say "go." Count how many **pulse beats** you feel, until your partner calls "stop," after **15 seconds**. Write this number down.





Now swap, so that your partner counts his or her pulse, and you time 15 seconds for them.

Exercises

2. Write your results in this form: _____ **beats in 15 seconds**. Now change it into a **unit rate**.
   Depends on students' results. For example, 20 beats in 15 seconds = 1.33 beats per second (or beats/second).
3. Pulse rate is usually given in beats per minute (bpm). Calculate your unit pulse rate in beats per minute.
   Depends on students' results. For example, 20 beats in 15 seconds = 80 bpm.
4. Approximately how many times will your heart beat in:
   a. 1 hour **b.** 1 day **c.** 1 week **d.** 1 year (365 days in a nonleap-year)
   Depends on students' results. For example, 80 bpm: a) 4800 beats, b) 115,200, c) 806,400, d) 42,048,000.

Round Up

The unit rate is generally more useful than other rates — it makes it easier to compare things. For example, it's difficult to compare 18 beats in 15 seconds with 23 beats in 20 seconds — it's much easier to compare 72 beats per minute with 69 beats per minute.

Strategic & EL Learners

Strategic learners may need to be reminded of time conversion factors. For example, 60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day.

EL learners may not be familiar with the word "per." Explain that this means "for each."

Math background

Students may confuse ratio and rate. Remind them that a ratio is a comparison like red marbles to blue marbles — the units are the same. Rates have different units, like beats to minutes.

Universal access

Remind students that, in this experiment, they will all have different results from their neighbor. The pulse rates of students are likely to vary greatly.

Lesson 4.2.1

Ratios and Rates

This Lesson introduces rates. Students learn how to find ratios, rates, and unit rates, which they use to determine the "better buy."

Previous Study: In grade 6 students were introduced to ratios and rates for the first time. They used ratios and rates to compare quantities and solve problems about speed, distance, and time.

Future Study: In Algebra I students will use algebraic methods to help solve problems involving rate and ratio.

1 Get started

Resources:
- jump rope
- stopwatches
- internet-enabled computers

Warm-up questions:
- Lesson 4.2.1 sheet

2 Teach

Math background

Ratios and rates both employ the same concept of comparing numbers using division.

Students should be reasonably familiar with the basics of ratios and rates, having studied them in grade 6.

Concept question

"If the ratio of boys to girls in a class is 5:6, then what is the ratio of girls to boys?" **6:5**

Universal access

Introduce the students to the concept of rates by having them find the rate at which they can do various activities.

Set up a series of activity stations. Some possible examples are:
1) Students see how many times they can jump rope in 30 seconds.
2) Students count how many times their hearts beat in 15 seconds.
3) Students solve as many addition problems as they can in 2 minutes.

Give everyone a copy of a table with the numbers of the stations, and space to record their results at each one.

After visiting each station, ask them to work out the rate at which they did each activity, complete with units. For example, the units for station 1 (above) would be "jumps per 30 seconds," for station 2, they'd be "beats per 15 seconds," and for station 3 they'd be "problems per 2 minutes."

Lesson 4.2.1

California Standards:

Measurement and Geometry 1.3
Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) **to solve problems; check the units of the solutions**; and use dimensional analysis to check the reasonableness of the answer.

What it means for you:
You'll learn what rates are and how you can use them to compare things — such as which size product is better value.

Key words:
- rate
- ratio
- fraction
- denominator
- unit rate

Don't forget:
You can simplify ratios by dividing both numbers by a common factor.
Rates can be simplified in the same way. For example — you can divide top and bottom of the rate $\frac{3 \text{ inches}}{9 \text{ years}}$ by 3 to get $\frac{1 \text{ inches}}{3 \text{ years}}$.

Check it out:
You must always write the units after a rate.
The unit miles per hour could also be written as miles/hour.

Section 4.2
Ratios and Rates

Rates are used a lot in daily life. You often hear people talk about speed in miles per hour, or the cost of groceries in dollars per pound. A rate tells you how much one thing changes when something else changes by a certain amount. Imagine buying apples for \$2 per pound — the cost will increase by \$2 for every pound you buy.

Ratios are Used to Compare Two Numbers

You might remember ratios from grade 6. Ratios **compare two numbers**, and **don't have any units**. For example, the ratio of boys to girls in a class might be 5 : 6. There are three ways of expressing a ratio.
The ratio 5 : 6 could also be expressed as "5 to 6" or as the fraction $\frac{5}{6}$.

Example 1

There are four nuts between three squirrels.
What is the ratio of nuts to squirrels?

Solution

There are 4 nuts to 3 squirrels so the ratio of nuts to squirrels is **4 : 3**.
This could also be written "4 to 3" or $\frac{4}{3}$.

Rates Compare Quantities with Different Units

A rate is a **special kind of ratio**, because it compares two quantities that **have different units**. For example, if you travel at 60 miles in 3 hours you would be traveling at a rate of $\frac{60 \text{ miles}}{3 \text{ hours}}$.

You'd normally write this as a **unit rate**. That's one with a denominator of 1. So $\frac{60 \text{ miles}}{3 \text{ hours}} = \frac{20 \text{ miles}}{1 \text{ hour}}$, or 20 miles per hour.

Example 2

John takes 110 steps in 2 minutes. What is his unit rate in steps per minute?

Solution

110 steps in 2 minutes means a rate of:

$$\frac{110 \text{ steps}}{2 \text{ minutes}} = \frac{55 \text{ steps}}{1 \text{ minute}} = \textbf{55 steps per minute}.$$

- **Strategic Learners**
  Develop students' understanding of rates by exploring a work-rate scenario, such as: "Marc paints a room in 5 hours. Tia paints the same size room in 4 hours." Put students in groups. Give each group a question, like, "How much of the room can each one paint in 1 hour?," "What are their unit painting rates?," or "How long would each one take to paint a room half the size?" Have the groups share their findings.

- **English Language Learners**
  Write three examples of rates on the board — for example, heartbeats per minute, miles per hour, and words typed per minute. Put everyone in pairs, and have each pair come up with as many different examples of rates as they can think of in 2 minutes. Have each pair share their results with another pair, or with the whole class.

Check it out:
Rates don't have to be expressed as fractions or whole numbers. Rates can also be decimals.

Check it out:
The unit rate that's calculated here is average speed — there's more on this in Lesson 4.2.3.

Numerator ÷ Denominator Gives a Unit Rate

Dividing the numerator by the denominator of a rate gives the **unit rate**.

So, if it costs 2 dollars for 3 apples, the unit rate is the price per apple,

which is $\frac{\text{2 dollars}}{\text{3 apples}}$ = 2 dollars ÷ 3 apples = 0.66 dollars per apple.

Example 3

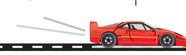
A car goes 54 miles in 3 hours. Write this as a unit rate in miles per hour.

Solution

Divide the top by the bottom of the rate.

$\frac{\text{54 miles}}{\text{3 hours}}$ = (54 ÷ 3) miles per hour = **18 miles per hour**.

This is a unit rate because the denominator is now 1 (it's equivalent to $\frac{18}{1}$ mi/h).

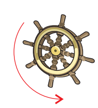


Example 4

If a wheel spins 420 times in 7 minutes, what is its unit rate in revolutions per minute?

Solution

The rate is $\frac{420}{7}$ revolutions per minute.



Divide the top by the bottom of the rate.

(420 ÷ 7) revolutions per minute = **60 revolutions per minute**.

This is a unit rate because 60 revolutions per minute has a denominator of 1 ($60 = \frac{60}{1}$).

Guided Practice

In Exercises 1–3, find the unit rates.

1. $3.60 for 3 pounds of tomatoes. **$1.20 per pound of tomatoes**

2. $25 for 500 cell phone minutes. **$0.05 per minute**

3. 648 words typed in 8 minutes. **81 words per minute**

4. Joaquin buys 2 meters of fabric, which costs him $9.50. What was the price per meter? **$4.75 per meter**

5. Mischa buys a $19.98 ticket for unlimited rides at a fairground. She goes on six rides. How much did she pay per ride? **$3.33 per ride**

2 Teach (cont)

Math background

To solve problems about rates, students must be able to work out what units their answer should have.

Dimensional analysis, a technique for finding the right units to go with an answer, is covered in Lesson 4.3.3.

Universal access

Return to the Universal access activity on the previous page. Have students work out their unit rate for each activity.

For example, the rates for the examples given would become "jumps per second (or minute)," "beats per minute," and "problems per minute."

Additional examples

Find the unit rates of the following:

1) $7 for 2 lb of pears **$3.50 per lb**

2) 96 pupils on 3 buses **32 pupils per bus**

3) 10 cm in 4 months **2.5 cm per month**

Common error

Students sometimes think that to find a rate you must always divide the first quantity given by the second. In fact, either way is a valid rate, as long as the correct units are given.

Consider this question: "A car went 60 miles in 3 hours. What was its unit rate?" You would probably give your answer as a speed — 20 miles per hour. But it would also be valid to say that its unit rate was 0.05 hours per mile.

Guided practice

Level 1: q1–3
Level 2: q1–4
Level 3: q1–5

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
Exchange rates are an important part of a country's economy. Over the course of a week, have students use the internet to track the rate of exchange between the U.S. dollar and the currency of another country of their choosing. Ask them to imagine they have a \$100 bill. Each day have them calculate what amount they would have if they exchanged their bill for the foreign currency, and record the information in a table.

2 Teach (cont)

Universal access

Have a class discussion about unit rates of cost and better buys.

Give students the cereal prices from Example 5. Have them work out how much it would cost to buy 48 ounces of cereal — \$9.60 for three 16 ounce boxes or \$8.64 for two 24 ounce boxes. Ask if they can tell from this which has the lower unit cost, and if so, how.

Ask them to think about whether the lowest unit cost is always the best choice in real life. As suggested in the student margin, it may not pay to buy in bulk if you only want a small amount.

A good real-life example to discuss is cell-phone tariffs. Give students two cell-phone price plans to look at, one charging \$0.03 a minute for all calls, and one charging \$15 a month, for which you get 1000 minutes of calls. Have them think about which plan they would pick if they made 375 minutes of calls a month, and which they would pick if they made 750 minutes of calls a month.

Guided practice

Level 1: q6
Level 2: q6–7
Level 3: q6–7

Independent practice

Level 1: q1–3, 7
Level 2: q1–5, 7–8
Level 3: q1–9

Additional questions

Level 1: p453 q1–8, 10–11
Level 2: p453 q1–12
Level 3: p453 q1–14

3 Homework

Homework Book — Lesson 4.2.1

Level 1: q1–7
Level 2: q2–10
Level 3: q2–11

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 29 — Rates

Use Unit Rates to Find the "Better Buy"

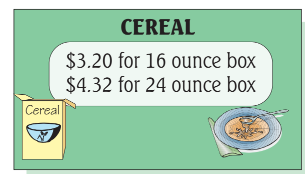
Stores often sell **different sizes of the same thing**, such as clothes detergent or fruit juice. A bigger size is often a **better buy** — meaning that you get more product for the same amount of money. But this isn't always the case, so it's useful to be able to work out which is the **better buy**.

You can do this by finding the **price for a single unit** of each product. The units can be ounces, liters, meters, or whatever is most sensible.

Check it out:
There are situations where the "better buy" is not actually the most sensible option. For example, a store might sell a product at a much cheaper unit price when you buy 20 — but if you only want 1 of the product, it's silly paying the extra if you're not going to use it all.

Example 5

A store sells two sizes of cereal. Which is the better buy?



Solution

16 ounce box: Rate is $\frac{3.20 \text{ dollars}}{16 \text{ ounces}}$.
Unit rate = (3.20 ÷ 16) dollars per ounce = **\$0.20 per ounce**

24 ounce box: Rate is $\frac{4.32 \text{ dollars}}{24 \text{ ounces}}$.
Unit rate = (4.32 ÷ 24) dollars per ounce = **\$0.18 per ounce**

The 24 ounce box is the better buy — the price per ounce is lower.

Guided Practice

6. Determine which phone company offers the better deal:
Phone Company A: \$40 for 800 minutes.
Phone Company B: \$26 for 650 minutes. **Phone Company B**

7. Determine which is the better deal on carrots: \$1.20 for 2 lb or \$2.30 for 5 lb. **\$2.30 for 5 lb**

Independent Practice

In Exercises 1–6, write each as a unit rate. **See below**

1. \$4.50 for 6 pens **2.** 100 miles in 8 h **3.** 200 pages in 5 days
4. 120 miles in 2 h **5.** \$400 for 10 items **6.** \$36 in 6 hours

7. Peanuts are either \$1.70 per pound or \$8 for 5 pounds. Which is the better buy? **\$8 for 5 pounds**

8. Lemons sell for \$4.50 for 6, or \$10.50 for 15. Which is the better buy? **\$10.50 for 15**

9. "\$40 for 500 pins or \$60 for 800 pins." Which is the better buy? **\$60 for 800 pins**

Now try these:
Lesson 4.2.1 additional questions — p453

Round Up

Rates compare one thing to another and always have units. A unit rate is a rate that has a denominator of one. In the next Lesson you'll see how rate is related to the slope of a graph.

Solutions
For worked solutions see the Solution Guide

1. \$0.75 per pen **2.** 12.5 miles per hour
3. 40 pages per day **4.** 60 miles per hour
5. \$40 per item **6.** \$6 per hour

Lesson 4.2.2

Graphing Ratios and Rates

In this Lesson, students learn how to graph the two quantities of a ratio or a rate. They discover that the slope of such a graph is equivalent to the rate.

Previous Study: At grade 4 students learned how to use a table of data to plot one quantity against another and produce a linear graph. In Lesson 4.1.3 students learned about slope.

Future Study: In Algebra I students will learn how to solve problems involving ratios, rates, and linear relationships using algebraic techniques.

Lesson 4.2.2

Graphing Ratios and Rates

California Standards:

Algebra and Functions 3.4

Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the ratio of the quantities.

What it means for you:

You'll learn how to graph two quantities in a ratio or rate and understand what the slope means in this context.

Key words:

- straight-line graph
- slope
- rate

When you're buying apples, the price you pay increases steadily the more apples you buy. If you plot a graph of the weight of apples against the cost, you get a straight line. The slope of this line is the same as the unit rate — the cost per pound.

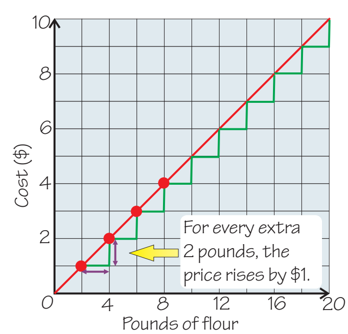
Quantities in Ratios Make Straight-Line Graphs

When you **increase** one quantity in a ratio or rate, the other quantity **increases in proportion** with it.

For example, if flour cost **$0.50 per pound**, you know that two pounds of flour would cost $1. This is because if you double the amount of flour, you also double the cost. In the same way, if you buy ten times as much flour, it costs ten times as much.

You can represent the cost of different amounts of flour on a **graph**:

Pounds of flour	Cost
2	2 × $0.50 = $1
4	4 × $0.50 = $2
6	6 × $0.50 = $3
8	8 × $0.50 = $4



By joining these points you get a **straight-line graph**. You get a **straight-line graph** whenever you plot quantities in a ratio or rate.

Example 1

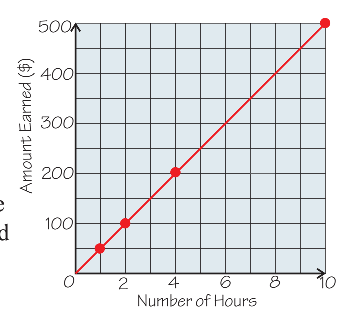
Suzi the decorator earns $50 per hour. Plot a graph to show how the amount Suzi earns increases with the amount of time she works.

Solution

Step 1: You know she earns $50 per hour. Draw a table of her earnings for different numbers of hours.

Number of Hours	Amount She Earns
1	$50
2	2 × $50 = $100
4	4 × $50 = $200
10	10 × $50 = $500

Step 2: Plot a graph from your table to show the number of hours worked against the amount she earns.



Check it out:

When one quantity in a ratio is zero, the other quantity must be zero also. So you know a graph of quantities in a ratio must go through the point (0, 0).

1 Get started

Resources:

- grid paper
- marbles
- card
- scissors
- wooden/plastic board (at least 30 cm long)
- stopwatches
- water, corn oil, and honey

Teacher Resources CD-ROM

- Money Tiles

Warm-up questions:

- Lesson 4.2.2 sheet

2 Teach

Math background

When two quantities change in proportion to each other they are said to show direct variation — they have a linear relationship.

This means that a graph of one quantity against the other will always be linear.

The concepts of direct variation and proportionality are covered in more depth in Lesson 4.2.4.

Concept question

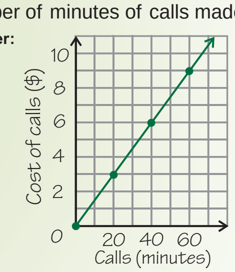
"If granola costs $0.20 an ounce, how much would it cost me to buy 10 ounces? If I bought $1.20 worth of granola, what weight would I buy?"

$2, 6 ounces

Additional example

A cell-phone plan charges $0.15 a minute for all calls. Plot a graph to show how the bill increases with the number of minutes of calls made.

Answer:



- **Strategic Learners**
  Put students into pairs. Give each pair a set of Money Tiles (printed from the **Teacher Resources CD-ROM**) and 10 marbles. Tell them that 3 marbles cost $1.20, and have them work out how much 6 marbles and 12 marbles would cost (**$2.40 and $4.80**). Ask them to draw a graph of the number of marbles against the cost. Have them use it to work out how much 5 marbles would cost ($2), and ask them to check their answer using the manipulatives.

- **English Language Learners**
  Have students think about two cars going at different speeds, one at 55 mph and one at 80 mph. Have them draw a table showing how far each car will have gone after 2, 4, and 6 hours, and then graph it, with the time on the horizontal axis. Get them to find the slope of each line. Ask them which line is steeper, and discuss what the slope value signifies.

2 Teach (cont)

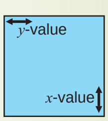
Guided practice

Level 1: q1
Level 2: q1–2
Level 3: q1–2

Universal access

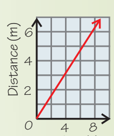
Have students make a card right angle to help them read graphs.

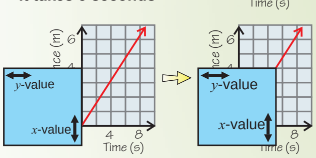
Give everyone a card. Ask them to draw and cut out a 15 cm square. At the top-left corner have them add a left arrow, and write "y-value" next to it. At the bottom-right corner have them draw a down arrow and write "x-value" next to it. This should give them a square like the one below.

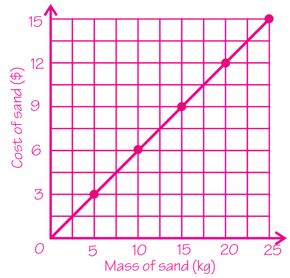


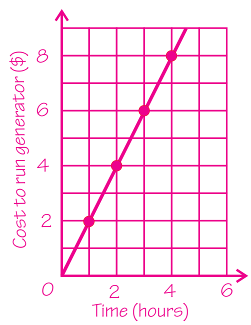
When they need to use a graph to find an unknown value, they line up the arrow with the given x- or y-value, and, keeping it lined up, slide the square across until the corner touches the line. Then they can read off the unknown value from the other axis. An example of this method is shown below.

Use the graph on the right to find out how long it takes to go 4 m.
It takes 5 seconds









Guided Practice

1. You can buy 5 kg of sand from a toy store for $3.00. If the sand always costs the same amount per kilogram, draw a graph to represent the relationship between the cost and the mass of sand. **See left.**

2. It costs $1 to run an electricity generator for half an hour. Draw a graph to represent the relationship between cost and time. **See below left.**

Use the Graph to Find Unknown Values

Once you've drawn a graph, you can use it to find **unknown values**.

Example 2

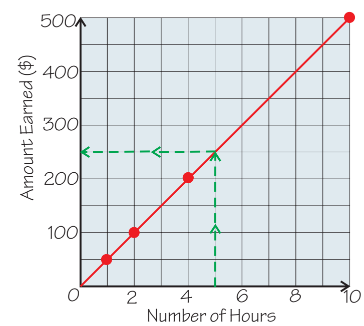
Suzi the decorator worked for 5 hours on Monday.
Use the graph in Example 1 to work out how much she earned.

Solution

She worked for 5 hours, so find 5 hours on the horizontal axis.

Go up to the line, and then across to find the amount earned for 5 hours' work.

On Monday Suzi earned $250 for 5 hours' work.



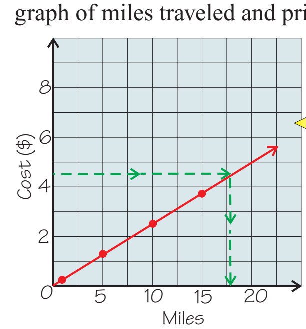
Example 3

A car rental company charges $0.25 per mile driven. Plot a graph to show this rate and use it to find how far you could drive for $4.50.

Solution

Draw a table that will allow you to plot a straight-line graph of miles traveled and price charged.

Miles	Cost
1	$0.25
5	$1.25
10	$2.50
15	$3.75



Plot the graph on a coordinate plane.
To find the number of miles you could drive for $4.50, find $4.50 on the vertical axis.
Go across to the line, and read off the corresponding **number of miles**.
You can drive 18 miles for $4.50.

Check it out:
You could use the graph to find the cost for any distance driven — for example, 5.3 miles. In reality, the company would probably charge to the nearest mile.

Solutions

For worked solutions see the Solution Guide

Advanced Learners

Do a classroom experiment looking at rates. Bring in a board that is at least 30 cm long, with a scale marked along the edge, and prop up one end to a height of about 15 cm. Bring in a small amount of three liquids of different viscosities, such as water, corn oil, and honey. Place a teaspoon of each substance in turn at the top of the board and have students record how long the liquid takes to flow 10 cm, 20 cm, and 30 cm. Then have them draw a graph showing the distance traveled by each liquid against time. Ask them to use their graph to find the rate of flow of each liquid, and to predict how long each one would have taken to travel 50 cm.

Guided Practice

3. Rita is filling a sand box at the day camp where she works. She needs 8 kg of sand. Use your graph from Guided Practice Exercise 1 to find the approximate cost of the sand. **$4.80**

4. Use your graph from Guided Practice Exercise 2 to find the approximate price of running the generator for 3 hours. **$6**

The Slope of the Graph Tells You the Rate

The **slope** of a graph of two quantities is the **unit rate**. It tells you **how much** the quantity on the vertical axis changes when the quantity on the horizontal axis changes by **one unit**.

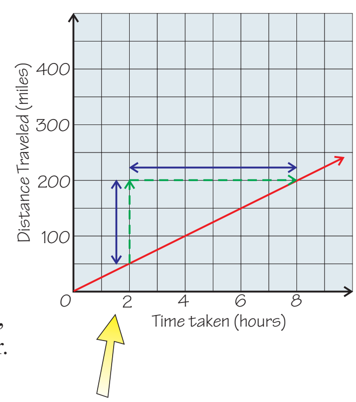
The **slope** of a straight-line graph is found using this formula:

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

Example 4

This graph shows the progress of Selina, who is traveling at a constant rate.

Use the graph to find how many miles per hour Selina is traveling at.



Solution

The slope is the **change in y divided by the change in x**.

On this graph, this is the **distance traveled divided by the time taken**, which is a unit rate in miles per hour.

Find **two points on the line**, and find the vertical change and the horizontal change between them by drawing a triangle onto the graph.

Change in y = **150 miles**

Change in x = **6 hours**

So, $\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{150 \text{ miles}}{6 \text{ hours}}$

Rate $= (150 \div 6)$ miles per hour

$=$ **25 miles per hour**

Selina is traveling at 25 miles per hour.

2 Teach (cont)

Guided practice

Level 1: q3
Level 2: q3–4
Level 3: q3–4

Math background

The slope of a graph at any point represents the **rate of change** of one quantity as compared to another at that point.

With a linear relationship, the slope of the graph, and therefore the rate of change, is the same at all points.

For more information about slope, see Lesson 4.1.3.

Concept question

"I drew a graph, with the distance I drove in my car in kilometers on the y-axis and the time it took me in hours on the x-axis. The slope of the resulting straight-line graph was 60. What average speed did I travel at?"

60 kilometers per hour

Common error

When drawing graphs, students may have trouble working out which axis to put each quantity on.

Remind them that, since the slope of the graph is change in y over change in x, the quantity they want to be in the numerator of the rate should go on the y-axis.

For instance, if they are looking for a speed in meters per second, distance in meters must go on the y-axis, and time in seconds on the x-axis.

Check it out:

The slope of a distance-time graph (with time on the horizontal axis) is always speed.

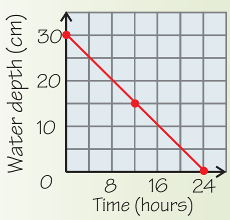
Solutions

For worked solutions see the Solution Guide

2 Teach (cont)

Guided practice
Level 1: q5–6
Level 2: q5–7
Level 3: q5–7

Math background
It is possible for a rate of change to be negative. For example, if you measured the depth of water in a tank that was leaking, and plotted a graph of the depth against time, the slope of your graph would be negative:



Rate of change = –1.25 cm per hour

A negative rate of change indicates a decrease. So you could say that the depth of water in the tank decreases at a rate of 1.25 cm per hour.

For more about negative slope, see Lesson 4.1.3.

Independent practice
Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Additional questions
Level 1: p453 q1, 3–5
Level 2: p453 q1–5
Level 3: p453 q1–6

3 Homework

Homework Book — Lesson 4.2.2
Level 1: q1–4, 6
Level 2: q1–7
Level 3: q2–8

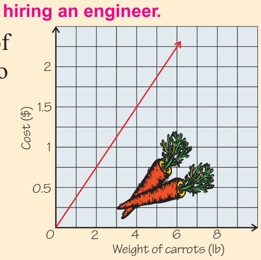
4 Skills Review

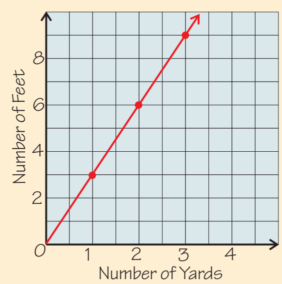
Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 28 — Graphing Linear Equations
- Worksheet 29 — Rates

Guided Practice

5. The *y*-axis of a graph shows the cost of hiring an engineer, and the *x*-axis shows the number of hours you get the engineer's services for. What does the slope of the graph tell you? **The cost per hour of hiring an engineer.**

6. The graph on the right shows the price of carrots in a grocery store. Use the graph to find the unit rate for the price of carrots. **$0.38 per lb**





7. The graph on the left shows the number of feet against the equivalent number of yards. Use the graph to find the number of feet in a yard. **3 ft per yd**

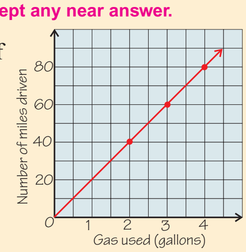
Independent Practice

1. Water is dripping into an empty tank from a pipe. The water depth is increasing by a depth of 4 inches every 24 hours. Draw a graph and use it to find the depth of water in the tank after 36 hours. **6 inches**
Use the graph to find the unit rate of water depth increase. **0.167 in./h**

2. A store earns about $100,000 over seven months. Draw a graph and use it to estimate the store's earnings for a year. **About $170,000. Accept any near answer.**

3. There are 12 inches in a foot. Draw a graph and use it to find the number of inches in 9 feet. Then use it to estimate the number of feet in 50 inches. **108 inches and about 4.2 feet. Accept any near answer.**

4. The graph on the right shows the number of miles John drives, and the number of gallons of gas he uses. Find the number of miles John's car does per gallon. **20 mpg**



5. Plot a graph of circle diameter against circumference. Put diameter on the horizontal axis. Find the slope of the graph. What does this slope represent? **Slope ≈ 3.1. The slope represents π.**

6. Plot a graph of circle radius against circumference. Put radius on the horizontal axis. Compare the slope of your graph to the slope of your graph from Exercise 5. Explain any differences.
The slope is twice as steep: it represents 2π rather than π.

Now try these:
Lesson 4.2.2 additional questions — p453

Don't forget:
Circumference = π × diameter
Diameter = 2 × radius

Round Up

When you graph quantities that are always in the same ratios you get straight-line graphs. You can use these graphs to convert from one quantity to another. They aren't the only way to convert quantities — the next Lesson is about conversion factors, which are another way.

Solutions
For worked solutions see the Solution Guide

Lesson 4.2.3

Distance, Speed, and Time

This Lesson focuses on solving word problems involving speed, distance, and time. Students learn the formula that links speed, distance, and time, and practice using it.

Previous Study: In grade 6 students solved problems involving average speed, distance, and time, using the formula that relates the three variables.

Future Study: In Algebra I students will learn how to solve more complex problems involving rates, using algebraic techniques.

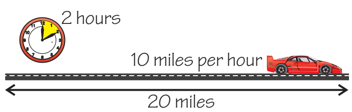
Lesson 4.2.3

Distance, Speed, and Time

Speed is a rate — it's the distance you travel per unit of time.
55 miles per hour is the speed limit on some roads. If you drive steadily at this speed, you'll travel 55 miles every hour.
There's a formula that links speed, distance, and time — and you're going to use it in this Lesson.

Speed is a Rate

Speed is a rate. It is the **distance** traveled in a certain amount of **time**.



Speed can be measured in lots of different units, such as miles per hour, meters per second, inches per minute...

The formula for speed is: $\textbf{speed} = \dfrac{\textbf{distance}}{\textbf{time}}$

Example 1

Gila walked 6 miles in 8 hours. What was Gila's average speed?

Solution

Use the formula, and **substitute** in the values from the question.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$
$$= \frac{6 \text{ miles}}{8 \text{ hours}}$$
$$= (6 \div 8) \text{ miles per hour}$$
$$= 0.75 \text{ miles per hour}$$

Gila's average speed was 0.75 miles per hour.

Rearrange the Equation to Find Other Unknowns

You can **rearrange** the speed formula, and use it to find **distance** or **time**.

To change the equation $\textbf{speed} = \dfrac{\textbf{distance}}{\textbf{time}}$ into an equation that gives **distance** in terms of speed and time, **multiply both sides of the equation by time**.

$$\text{speed} \times \text{time} = \frac{\text{distance} \times \cancel{\text{time}}}{\cancel{\text{time}}}$$

$$\textbf{distance} = \textbf{speed} \times \textbf{time}$$

California Standards:

Algebra and Functions 4.2

Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

What it means for you:

You'll learn the formula for speed, and how to use it to solve problems.

Key words:

- speed
- distance
- time
- formula

Check it out:

Gila might not have walked at a steady speed for the entire 8 hours. That's why we have to calculate **average speed**.

Check it out:

Miles per hour is often shortened to mi/h or mph. This means the same as miles ÷ hours, and $\frac{\text{miles}}{\text{hours}}$.

Don't forget:

Speed can be measured in **any unit of distance per unit of time**. If you divide a distance in kilometers by a time in hours, your answer will be in kilometers per hour.

1 Get started

Resources:

- stopwatches
- tape measure
- individual whiteboards and pens

Warm-up questions:

- Lesson 4.2.3 sheet

2 Teach

Universal access

Ask students to think about a car traveling at a steady speed of 40 miles per hour. Ask them to make a table showing how far the car will have gone after 0 hours, 2 hours, 4 hours, and 5 hours.

Ask them to graph the distance traveled by the car against the time taken. Then have them work out the slope of the graph. This will be 40 mph.

This exercise introduces the relationship between speed, distance, and time. It also acts as a clear illustration that speed is a rate, since it is the slope of the distance-time graph.

Concept question

"Damien walked 2 miles in 1 hour. Martha walked 4 miles in 2 hours. Whose average speed was faster?"

Damien and Martha had the same average speed.

Math background

Students learned how to rearrange equations in Section 1.2.

To keep the equation balanced you must do the same thing to both sides. For example, here both sides are being multiplied by "time."

- **Strategic Learners**
  On the playing field mark out a start and finish line 50 yards apart. Put students into small groups. Ask each student to cover the 50 yard distance at three different speeds: slow, walking pace, and running. Another person from the group should time them with a stopwatch. Then go back inside, and introduce the "speed = distance ÷ time" formula. Have students use the formula to find their own speed for each pace.

- **English Language Learners**
  Check students are familiar with all the units of speed, distance, and time that they will encounter. Call out questions, such as, "How many meters are in a kilometer?" and "How many hours are in a day?" Have students use individual whiteboards to write down their answer, and hold them up as a check. Then ask students to work in pairs to think of and list as many different speed, distance, and time related units as they can.

2 Teach (cont)

Universal access

It can be tricky for students to find the right unit to match their answer. Teach them to always write units out in full as they do their work, and apply the same operations to the units as they do to the numbers.

For instance, in Example 2:

$$\begin{aligned}\text{Distance} &= \text{Speed} \times \text{Time}\\ &= 11\,\frac{\text{kilometers}}{\text{hours}} \times 0.5\text{ hours}\\ &= (11 \times 0.5) \times \left(\frac{\text{kilometers}}{\cancel{\text{hours}}} \times \cancel{\text{hours}}\right)\\ &= 5.5\text{ kilometers}\end{aligned}$$

This technique is called dimensional analysis. There is more information about it in Lesson 4.3.3.

Concept question

"How long will it take me to walk x yards if I walk at a constant speed of 1 yard per second?"
x seconds

Additional examples

1) I walk 11 km in 2 hours. What is my average speed? **5.5 km per hour**

2) If I run at a constant speed of 6 meters per second for 1 minute, what distance will I cover? **360 meters**

3) I drove 105 miles at an average speed of 35 miles per hour. How long did my journey take? **3 hours**

Guided practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1–5

Example 2

Alyssa runs for 0.5 hours at a speed of 11 kilometers per hour.
How far does she run?

Solution

Use the formula for distance, and substitute the values for speed and time.

$$\begin{aligned}\text{Distance} &= \text{speed} \times \text{time}\\ &= 11\text{ kilometers per hour} \times 0.5\text{ hours}\\ &= \mathbf{5.5\text{ kilometers}}\end{aligned}$$

You can find the equation for time in terms of speed and distance in a similar way.

Example 3

Andy is planning a walk. He walks at an average speed of 3 miles per hour, and plans to cover 15 miles. How long should his walk take him?

Solution

You need to rearrange the speed formula first.
distance = speed × time

$$\frac{\text{distance}}{\text{speed}} = \frac{\cancel{\text{speed}} \times \text{time}}{\cancel{\text{speed}}}$$ **Divide both sides by speed**

$$\textbf{time} = \frac{\textbf{distance}}{\textbf{speed}}$$

Now you can use the formula to answer the question:

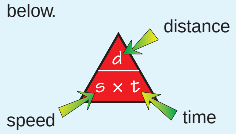
$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{15\text{ miles}}{3\text{ miles per hour}}$$

time = (15 ÷ 3) hours = 5 hours

Andy's walk should take him 5 hours.

Check it out:

Formula triangles help you find the formula you want. The formula triangle for speed-distance-time is shown below.



To use it, cover up the thing you want to find, and the equation is what's left.
So if you want to find distance, cover up "d" and you're left with "$s \times t$."
If you want to find time, cover up "t" and you're left with "$\frac{d}{s}$."

Guided Practice

1. Juan ran in a marathon that was 26 miles long.
If his time was 4 hours, what was his average speed? **6.5 miles per hour**

2. Moesha goes to school every day by bike. The journey is 6 miles long, and takes her 0.6 hours. What is her average speed? **10 miles per hour**

3. Monica travels 6 miles to work at a speed of 30 miles per hour.
How long does the journey take her each morning? **0.2 hours or 12 minutes**

Josh has been walking for 5 hours at a speed of 4 miles per hour.
4. His walk is 22 miles long. How far does he have left to walk? **2 miles**
5. How much longer will he take if he continues at the same speed? **0.5 h or 30 min**

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Give students distance and time data about four different sports. For example, you could give them the race lengths and winning times for a 100 m race, a motor race, a California-based marathon, and a speed-skating race. Ask the students to work out the average speed of each winner over the course of their race.

Some Problems Might Need More than One Step

Example 4

On a three-hour bike ride, a cyclist rode 58 miles. The first two hours were downhill, so the cyclist rode 5 miles per hour quicker than she did for the last hour.

a) What was her speed for the first two hours?

b) What was her speed for the last hour?

Solution

Let the cyclist's speed for the first two hours be **$(x + 5)$ miles per hour**.
So her speed for the last hour = **x miles per hour**.
You need to write an **equation** using the information you're given.

$$\text{Total distance} = \begin{array}{c}\text{distance traveled}\\ \text{in first two hours}\end{array} + \begin{array}{c}\text{distance traveled}\\ \text{in last hour}\end{array}$$

$$58 = (x + 5) \times 2 + (x) \times 1$$

distance = speed × time

$58 = 2x + 10 + x$

$58 = 3x + 10$

$48 = 3x \Rightarrow x = 16$

a) The speed for the first two hours was $(x + 5) = 16 + 5 =$ **21 mi/h**

b) So the speed for the last hour was $x =$ **16 mi/h**

Check it out:
This answer seems reasonable — the cyclist rode 5 mi/h faster for the first two hours. 21 mi/h and 16 mi/h fit this and sound reasonable speeds for a cyclist.

Guided Practice

6. Train A travels 20 mi/h faster than Train B. Train A takes 3 hours to go between two cities, and Train B takes 4 hours to travel the same distance. How fast does each train travel? **Train A = 80 mi/h, Train B = 60 mi/h**

Independent Practice

1. A mouse ran at a speed of 3 meters per second for 30 seconds. How far did it travel in this time? **90 meters**

2. A slug crawls at 70 inches per hour. How long will it take it to crawl 630 inches? **9 hours**

3. A shark swims at 7 miles per hour for 2 hours, and then at 9 miles per hour for 3 hours. How far does it travel altogether? **41 miles**

4. Bike J moves at a rate of x miles per hour for 2 hours. Bike K travels at $0.5x$ miles per hour for 4 hours. Which bike travels the furthest? **Both travel the same distance**

5. On a two-day journey, you travel 500 miles in total. On the first day you travel for 5 hours at an average speed of 60 mi/h. On the second day you travel for 4 hours. What's your average speed for these 4 hours? **50 mi/h**

Now try these:
Lesson 4.2.3 additional questions — p453

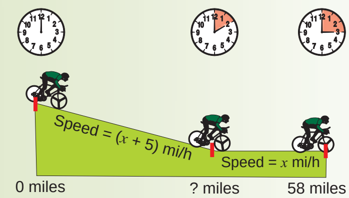
Round Up

You need to remember the formula for speed. If you know this, you can rearrange it to figure out the formulas for distance and time when you need them — so that's two less things to remember.

2 Teach (cont)

Universal access

It may help students to draw a diagram to illustrate a complex problem like Example 4. They should put all the information they are given on the diagram. For example:



This can help them to visualize the problem before answering.

Guided practice

Level 1: q6
Level 2: q6
Level 3: q6

Independent practice

Level 1: q1–3
Level 2: q1–4
Level 3: q1–5

Additional questions

Level 1: p453 q1–5
Level 2: p453 q1–6
Level 3: p453 q1–7

3 Homework

Homework Book — Lesson 4.2.3

Level 1: q1–4, 6, 7
Level 2: q2–9
Level 3: q2–9

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 29 — Rates

Solutions

For worked solutions see the Solution Guide

Lesson 4.2.4

Direct Variation

In this Lesson, students learn that direct variation is when two variables are in direct proportion to each other. They also learn to recognize a graph showing direct proportion, and understand that such a graph always passes through the origin.

Previous Study: At grade 6 students learned about ratios and proportions, and how to use them to solve math problems. Earlier in this Section they studied rates.

Future Study: In Algebra I students will learn to graph linear equations, and to find their slopes and x- and y-intercepts. They will also apply algebraic techniques to solve problems involving ratios.

1 Get started

Resources:
- measuring cylinder, water, balance
- grid paper

Warm-up questions:
- Lesson 4.2.4 sheet

2 Teach

Universal access

Have pairs of students come up with as many examples as they can of variables that show direct variation.

Some examples are:
- Distance walked at a constant speed against time taken.
- Number of any item bought against the total cost.
- Circumference of a circle against its diameter.
- Amount of sales tax paid against the total cost of an item.
- The distance between any two points on a map and the distance between the two points in real life.

Math background

Students learned how to set up and solve proportions at grade 6.

This problem could also be solved using cross-multiplication:

When $n = 8$,
$$\frac{4}{2} = \frac{m}{8}$$
$$2m = 4 \times 8$$
$$2m = 32$$
$$m = 16$$

Lesson 4.2.4

Direct Variation

California Standards:
Algebra and Functions 4.2
Solve multistep problems involving rate, average speed, distance, and time or **a direct variation.**

What it means for you:
You'll learn how to use the fact that two things are in proportion to solve problems.

Key words:
- direct variation
- proportion
- ratio
- variables
- constant of proportionality

Check it out:
You could also divide the quantities the other way around ($x \div y$) to get a different constant of proportionality.

Check it out:
Corresponding lengths in similar shapes show direct variation. If you divide a length on one shape by the corresponding length on another, you get a constant that's the same whichever length you pick — this is the scale factor.
See Lesson 3.4.3.

Check it out:
You could have divided n by m to get a constant of proportionality, k, of 0.5. This means your equation is $\frac{n}{m} = k$.
This rearranges to $m = \frac{n}{k}$, so when you substitute in $k = 0.5$ and $n = 8$, you still get 16.

Direct variation is when two things change in proportion to each other — this means that the ratio between the two quantities always stays the same. For example, if fencing is sold at \$15.99 per meter, then you can say that the length of fencing bought and the cost show direct variation.

Direct Variation Means Proportional Change

Two quantities show direct variation if the ratio between them is always the same.

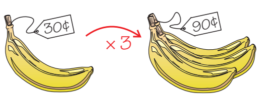
If you have two quantities, x and y, that show **direct variation**, the ratio between them, $\frac{y}{x}$, is **always the same** — it's a constant.

If you call this constant k, then $\frac{y}{x} = k$. This rearranges to $y = kx$.

$$y = kx$$

k is known as the "**constant of proportionality**."

You've seen things that show direct variation before when you learned about rates. For example, imagine a store selling bananas at a certain price per banana. The price per banana is **constant** and doesn't change no matter how many bananas you have:



What your bananas cost = price per banana × number of bananas.

What your bananas cost and the number of bananas are the **variables**, and the price per banana is the **constant of proportionality**.

Example 1

If m and n show direct variation, and $m = 4$ when $n = 2$, find m when $n = 8$.

Solution

First find the constant of proportionality: $k = \frac{m}{n} = \frac{4}{2} = 4 \div 2 = 2$

The constant of proportionality, $k = 2$.

The formula rearranges to $m = kn$.
So substitute in the value for k and the new value for n.

$m = kn$
$= 2 \times 8 = 16$

So $m = 16$ when $n = 8$.

- **Strategic Learners**
  Use a simple example to explain direct variation. Ask students to imagine they are walking at a constant speed of 3 mi/h. If they double the length of time they walk for, the distance they walk will also double. Have them generate a table of corresponding times and distances for the constant speed.
- **English Language Learners**
  The Strategic Learners activity above is also useful for English language learners. This can be extended by asking students to divide each time by the corresponding distance. They should notice that they get the same value each time — this is the constant of proportionality. (You could also divide the distances by the times to give a different constant of proportionality — the speed.)

2 Teach (cont)

Example 2

A person's earnings and the number of hours they work show direct variation. An employee earns \$600 for 40 hours' work.
Find their earnings for 60 hours' work.

Solution

First write a **direct variation equation**:
k **= earnings ÷ number of hours worked**

Now **substitute in** the pair of variables you know and find k:
k = \$600 ÷ 40 hours
k **= \$15 per hour.**

The direct variation equation rearranges to:
Earnings = k **× number of hours worked**

So, earnings for 60 hours = \$15 per hour × 60 hours
= **\$900**

Additional example

The maximum load that a horizontal beam can support is directly proportional to its width. If a beam 8 cm wide can support a maximum load of 220 kg, what width of beam would be needed to support 385 kg?
14 cm

Guided Practice

1. y and x show direct variation and $y = 4$ when $x = 6$.
Find x when $y = 9$. $x = 13.5$

2. s and t show direct variation and $s = 70$ when $t = 10$.
Find s when $t = 7$. $s = 49$

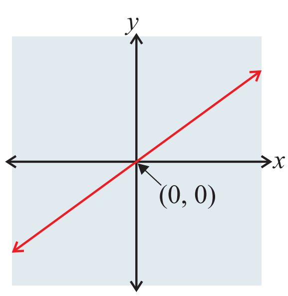
3. The cost of gas varies directly with the number of gallons you buy.
If 10 gallons of gas costs \$25, what is the cost per gallon of gas?
What does 18 gallons of gas cost? **\$2.50 per gallon; \$45**

Guided practice

Level 1: q1
Level 2: q1–2
Level 3: q1–3

Direct Variation Graphs are Straight Lines

Graphs of quantities that show **direct variation** are always **straight lines** through the **point (0, 0)**.



The **slope** of this direct variation graph is equal to $\frac{y}{x}$.

As $\frac{y}{x} = k$, the slope is the same as the **constant of proportionality**, k.

Check it out:

The line of a direct variation graph always crosses the origin (0, 0). If one quantity is 0, the other quantity will be 0 too.

Concept question

"A graph of the variable q against the variable p is a straight, sloping line passing through the origin. What can you say about p and q?"
p and q show direct variation. You'll always get the same value when you divide one by the other.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Put students into small groups and give each group a measuring cylinder, some water, and a balance. Ask them to predict whether the mass and the volume of water will show direct variation. Then have them measure out and weigh 10 cm³, 25 cm³, and 50 cm³ of water. They should record the data in a table, and plot a graph. This can be connected to the concept of density — if they divide the mass of the water by the volume, the constant of proportionality they get is the density of the water.

2 Teach (cont)

Common error

When students find the constant of proportionality between two real-life quantities that vary directly, they are often confused about which quantity they should divide by.

It doesn't matter if they use $\frac{a}{b} = k$ or $\frac{b}{a} = k$, as long as they use the same equation throughout, and substitute the variables correctly.

Guided practice

Level 1: q4
Level 2: q4
Level 3: q4–5

Independent practice

Level 1: q1, 3–5
Level 2: q1–5
Level 3: q1–6

Additional questions

Level 1: p454 q1–4, 8–10
Level 2: p454 q1–10
Level 3: p454 q1–12

3 Homework

Homework Book — Lesson 4.2.4

Level 1: q1–3, 5
Level 2: q1–8
Level 3: q1–8

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 28 — Graphing Linear Equations
- Worksheet 29 — Rates

Example 3

If y and x show direct variation, and $x = -1$ when $y = 2$:
a) Write an equation relating x and y.
b) Graph this equation.
c) Find the value of y when $x = 1$.

Solution

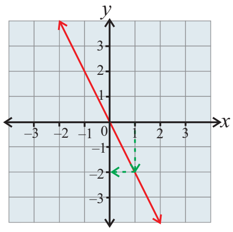
a) Because y and x show direct variation, $\frac{y}{x} = k$.

Now you need to find out the value of k: $k = \frac{y}{x} = \frac{2}{-1} = -2$

So $\frac{y}{x} = -2$, or $y = -2x$.

b) The graph of $y = -2x$ must go through **(0, 0)**. Because $x = -1$ when $y = 2$, it must go through **(–1, 2)**.
The **slope** of the line is equal to k, and $k = -2$.

c) Reading from the graph, when $x = 1$, $y = -2$.



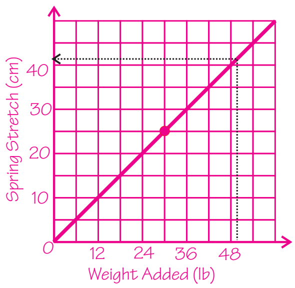
Check it out:
You could have found the value of y when $x = 1$ from the equation.
$y = -2x$, so if $x = 1$,
then $y = -2(1) = -2$.

Guided Practice

When you add a weight to the end of a spring, the spring stretches. The amount of weight you add (w) and the distance the spring stretches (d) show direct variation (until you have added 100 pounds).

4. If 30 pounds causes a stretch of 25 centimeters, write an equation relating w and d. $d = \frac{5}{6}w$ (when w < 100 pounds)

5. Graph the direct variation. Use the graph to estimate how far the spring stretches when 50 pounds is added.
For graph see left. When 50 lb is added the spring stretches about 42 cm.



Independent Practice

In Exercises 1–2, x and y show direct variation, with $y = kx$.

1. If $y = 36$ and $x = 6$, find k. $k = 6$ **2.** If $y = 12$ and $k = 2$, find x. $x = 6$

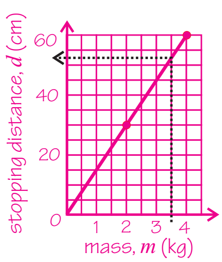
3. You can buy 6 pounds of apples for $4.50. Given that the cost and weight show direct variation, find the cost of 5 pounds of apples. $3.75

4. The graph of a line crosses the y-axis at 1. Explain whether the line could represent a direct variation. No. Graphs showing direct variation cross both axes at zero.

5. The stopping distance of a toy cart varies directly with its mass. A 2 kg cart stops after 30 cm. Write an equation linking the stopping distance and the mass of the cart. Graph this equation and find the stopping distance of a 3.5 kg cart. For graph, see left. For equation and calculations, see below.

6. A graph showing direct variation is drawn on the coordinate plane. The line goes from top left to bottom right. What can you say about the constant of proportionality? It's negative.

Now try these:
Lesson 4.2.4 additional questions — p454



Round Up

If two things vary directly, the ratio between them is constant. Most rates are examples of direct variation, like the price of something per kilogram — the price and weight stay in proportion.

Solutions
For worked solutions see the Solution Guide

Independent Practice
5. Stopping distance = d cm Cart mass = m kg, $d = 15 \times m$.
A 3.5 kg cart takes 52.5 cm to stop.

Lesson 4.3.1

Converting Measures

This Lesson reviews customary and metric measurement units. Students learn how to convert between units in the same measurement system by setting up proportions and solving them.

Previous Study: Students have done simple unit conversions (such as cm to m) since grade 3. In grade 6 students learned how to convert between different measurement systems (such as from cm to in.).

Future Study: Converting measures is an essential real-life math skill. It will be important for students to understand measurement systems, and be able to convert between units in further study of math and science.

Lesson 4.3.1

California Standards:

Measurement and Geometry 1.1

Compare weights, capacities, geometric measures, times, and temperatures **within** and between **measurement systems** (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

What it means for you:

You'll learn how to convert between different units of length, weight, and capacity in the customary and metric systems.

Key words:

- customary system
- metric system
- conversion table

Don't forget:

"Capacity" is often called "volume."

Don't forget:

You'll often see units abbreviated — for instance, mL for milliliters.
The strangest one is the pound — which is abbreviated to lb.

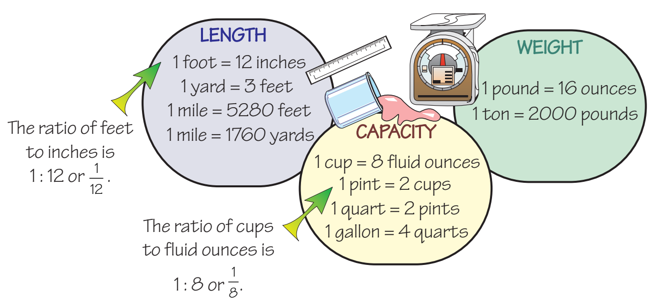
Section 4.3
Converting Measures

*There are lots of circumstances where you might want to **convert from one unit to another**. For instance — say you have 2 **pounds** of flour and you want to know how many cakes you can make that each need 6 **ounces** of flour. This Lesson is about how you **convert between different units of measurement**.*

The Customary System — Feet, Pounds, and Pints...

The **customary system** includes units such as **feet, pints, and pounds**.

To convert between the different units in the customary system you can use a **conversion table**. A conversion table tells you **how many of one unit is the same as another unit**.



Guided Practice

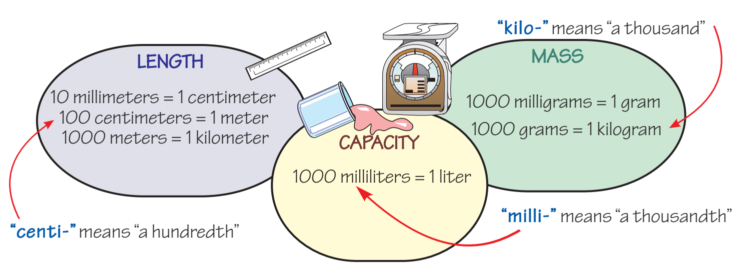
In Exercises 1–6, find the ratio between the units.

1. yards : feet **1 : 3** **2.** quarts : pints **1 : 2** **3.** tons : pounds **1 : 2000**

4. ounces : pounds **16 : 1** **5.** yards : miles **1760 : 1** **6.** quarts : gallons **4 : 1**

Metric Units Have the Same Prefixes

The **meter, the liter, and the gram** are metric units, and the prefixes "**kilo-**," "**centi-**," and "**milli-**" are used in this system:



1 Get started

Resources:

- cards showing the table of metric prefixes (see page 243)

Warm-up questions:

- Lesson 4.3.1 sheet

2 Teach

Universal access

Begin the lesson with a discussion about units of measurement, and how they are used in everyday life.

Ask students to think of places where they have come across the units — for example, they may have had their height measured in feet and inches, or cooked a recipe where the ingredients were measured in cups.

Ask them to try to come up with a common use of each of the units shown in the two diagrams. Write out their examples on a large sheet of paper, and stick them on the wall.

You may have to give them an example for some of the less usual units. For example, bridge weight limits are often given in tons, and liquid commodities like gasoline are sold by the gallon.

Guided practice

Level 1: q1–2
Level 2: q1–4
Level 3: q1–6

Math background

To understand and use the metric, or SI, prefixes students will need to be competent with using powers of 10.

kilo = 10^3 1 km = 1 × 10^3 m = 1000 m
centi = 10^{-2} 1 cm = 1 × 10^{-2} m = 0.01 m
milli = 10^{-3} 1 mm = 1 × 10^{-3} m = 0.001 m

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  Ask students to make two lists, one of customary units and one of metric units, using the two diagrams on page 241. Have them list the units along with their equivalents, for example "1 cm = 10 mm" and "1 mile = 1760 yards = 5280 feet." They can keep their list on hand, and use it as a reference whenever they tackle a unit conversion.

- **English Language Learners**
  The customary and metric measurement systems between them contain a lot of unit names to become familiar with. Have students make two tables, one for customary units and one for metric units. The tables should contain the names of the units in English, and in their own first language (if they know them), and also what type of measure each is (length, mass, or capacity).

2 Teach (cont)

Guided practice

Level 1: q7–8
Level 2: q7–10
Level 3: q7–12

Math background

Length units are often squared to give area units (for example, in^2 or cm^2). They are often cubed to give volume units (for example, cubic meters, or m^3).

1 liter = 1 decimeter^3 = 0.001 meters^3

Additional examples

1) How many feet are equivalent to half a mile? **2640 feet**
2) How many ounces are equivalent to two pounds? **32 ounces**
3) How many liters are equivalent to 500 milliliters? **0.5 liters**

Math background

Students learned about how to set up and solve proportions in grade 6.

A proportion is an equation made up of two equivalent ratios, for example:

$$\frac{2}{3} = \frac{8}{12}$$

Common error

Students may find it hard to remember which system which measures come from.

Remind them that for length, capacity, and mass, the only basic units that the metric system uses are the meter, the liter, and the gram. You can put prefixes onto them, but any measure not containing any of these words is not a metric unit, and so must be customary.

Guided Practice

In Exercises 7–12, find the ratio between the units.

7. millimeters : meters	1000 : 1	**8.** liters : milliliters	1 : 1000
9. grams : kilograms	1000 : 1	**10.** kilometers : meters	1 : 1000
11. milliliters : liters	1000 : 1	**12.** grams : milligrams	1 : 1000

Convert Between Units by Setting Up Proportions

You might remember proportions from grade 6. They're a good way of converting between different units.

Don't forget:

To cross-multiply the following proportion: $\frac{a}{b} = \frac{c}{d}$

1. Multiply both sides of the equation by b, and cancel.

$$\frac{a}{\cancel{b}} \times \cancel{b} = \frac{c}{d} \times b \quad \Rightarrow \quad a = \frac{c}{d} \times b$$

2. Multiply both sides of the equation by d, and cancel.

$$a \times d = \frac{c}{\cancel{d}} \times b \times \cancel{d}$$

$$\Rightarrow \quad a \times d = c \times b$$

So if $\frac{a}{b} = \frac{c}{d}$, then $a \times d = c \times b$

Example 1

How many yards are equivalent to 58 feet?

Solution

Step 1: The **ratio** of yards to feet is **1 : 3** or $\frac{1}{3}$.

Step 2: You want to find the number of yards in 58 feet.
So write another ratio — **the ratio of yards to feet is x : 58**, where x stands for the number of yards in 58 feet.

You know that the ratios 1 : 3 and x : 58 have to be **equivalent** — which means they **simplify to the same thing**, because there are always 3 feet in every yard.

So you can write these ratios as an equation — this is called a **proportion**.

$$\frac{1}{3} = \frac{x}{58}$$

Step 3: **Solve** the proportion for x using **cross-multiplication**.

$1 \times 58 = 3 \times x$
$58 = 3x$
$x = 58 \div 3 = 19.333... = $ **19.3**

So 58 feet is approximately equivalent to 19.3 yards.

Step 4: **Check the reasonableness of your answer.**
The conversion table tells you that there are 3 feet in every yard, so estimate:

20 yards × 3 ft per yard = 60 feet.

The estimation is close to the answer — so the answer is reasonable.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
When using very large and very small measurements, especially in science classes, students will come across the SI prefixes. They should be familiar with milli-, meaning 10^{-3}, or one-thousandth; centi-, meaning 10^{-2}, or one hundredth; and kilo-, meaning 10^{3}, or one thousand. Give them a card with the table from the math background section below on it. Give them some conversion questions using the different prefixes, such as, "How many micrometers are in 2 centimeters?" (20,000), and "How many hectoliters are equivalent to a deciliter?" (0.001). Challenge them to learn the prefixes, and give them a short quiz.

Example 2

How many kilometers are equivalent to 7890 meters?

Solution

There are 1000 meters in a kilometer, so the ratio of meters to kilometers is 1000 : 1 or $\frac{1000}{1}$.

Write a proportion where there are x kilometers in 7890 meters:

$$\frac{1000}{1} = \frac{7890}{x}$$

Cross-multiply and **solve** for x:

$1000 \times x = 7890 \times 1$
$1000x = 7890$
$x = 7890 \div 1000 = 7.89$

So 7.89 kilometers is equivalent to 7890 meters.

Check the reasonableness: there are 1000 meters in a kilometer, so estimate 8 km × 1000 m = 8000 m — **the answer is reasonable**.

Check it out:
Make sure you set up your proportion correctly. In this example, meters are on the top on both sides of the proportion, and kilometers are on the bottom.

$$\frac{1000 \text{ m}}{1 \text{ km}} = \frac{7890 \text{ m}}{x \text{ km}}$$

Guided Practice

In Exercises 13–18, find the missing value.

13. 6 miles = ? feet 31,680
14. 40 tons = ? pounds 80,000
15. 18 quarts = ? pints 36
16. 4560 ml = ? l 4.56
17. 45 g = ? kg 0.045
18. 670 km = ? m 670,000

Independent Practice

In Exercises 1–6, find the ratio between the units.

1. gallons : quarts 1 : 4
2. ounces : pounds 16 : 1
3. cups : pints 2 : 1
4. meters : centimeters 1 : 100
5. liters : milliliters 1 : 1000
6. milligrams : grams 1000 : 1

In Exercises 7–12, find the missing value.

7. 560 cm = ? m 5.6
8. 8.2 kg = ? g 8200
9. 9.67 l = ? ml 9670
10. 20 inches = ? feet 1.67
11. 5 cups = ? pints 2.5
12. 5 pounds = ? tons 0.0025

13. A recipe uses 8 ounces of butter and 12 ounces of flour. The supermarket sells butter and flour by the pound. How many pounds of butter and flour do you need for the recipe? 0.5 lb butter, 0.75 lb flour

14. Jackie wants to drink 2 liters of water a day. She sees a bottle of water that contains 250 milliliters. How many bottles would she need to drink in order to get the full 2 liters? 8

Now try these:
Lesson 4.3.1 additional questions — p454

Round Up

There are two main systems of measurement — the customary system and the metric system. You can convert between units in a system by setting up proportions and solving them. You'll often have to convert between the systems too, which you'll learn about in the next Lesson.

2 Teach (cont)

Math background

The table below shows some of the more commonly used SI prefixes. Students may come across these in math and science classes.

Prefix	Number	10^n	Decimal Equivalent
giga	Billion	10^9	1,000,000,000
mega	Million	10^6	1,000,000
kilo	Thousand	10^3	1000
hecto	Hundred	10^2	100
deca	Ten	10^1	10
—	One	10^0	1
deci	Tenth	10^{-1}	0.1
centi	Hundredth	10^{-2}	0.01
milli	Thousandth	10^{-3}	0.001
micro	Millionth	10^{-6}	0.000001
nano	Billionth	10^{-9}	0.000000001

The International System of Units (usually abbreviated to SI from the French "Système International d'unités") is the modern standardized form of the metric system.

Guided practice

Level 1: q13–14
Level 2: q13–16
Level 3: q13–18

Independent practice

Level 1: q1–4, 7–8, 13
Level 2: q1–5, 7–10, 13
Level 3: q1–14

Additional questions

Level 1: p454 q1–10
Level 2: p454 q1–11, 15
Level 3: p454 q1–16

3 Homework

Homework Book — Lesson 4.3.1

Level 1: q1–7
Level 2: q1–10
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
- Worksheet 30 — Measuring

Solutions
For worked solutions see the Solution Guide

Lesson
4.3.2

Converting Between Unit Systems

In this Lesson, students learn how to convert between units in different measurement systems. This leads to learning how to convert between the Celsius and Fahrenheit temperature scales using a formula.

Previous Study: Students have done simple unit conversions (such as cm to m) since grade 3. In grade 6 students learned how to convert between different measurement systems (such as from cm to in.).

Future Study: Converting measures is an essential real-life math skill. It will be important for students to understand measurement systems, and be able to convert between units in further study of math and science.

1 Get started

Resources:

- yard ruler and tape measures
- meter rulers
- cups
- measuring cylinders
- sets of unit snap cards (see Strategic Learners activity, p245)
- thermometer

Warm-up questions:

- Lesson 4.3.2 sheet

2 Teach

Math background

In Lesson 4.3.1 students saw how to set up and use a proportion to convert one unit to another within the same measurement system.

They can use exactly the same method to perform conversions between units that are in different measurement systems.

Universal access

Ask students to find conversion factors for themselves. For example:

1) How many meters are in a yard?
Give each group of students a yard ruler and have them measure it using a metric tape measure. They may need to convert the number of centimeters to meters.

2) How many liters are there in a cup?
Give each group a cup filled with water, and a metric measuring cylinder. Have them pour the water carefully into the cylinder, and read off the scale. Remind them that if the measuring cylinder scale is in cm^3, they will have to convert their reading to liters by dividing by 1000.

Additional examples

1) How many meters are equivalent to 9 yards? **About 8.19 meters**
2) How many liters are equivalent to 2 gallons? **About 7.57 liters**
3) How many pounds are equivalent to 4.5 kg? **About 9.9 pounds**

Lesson
4.3.2

Converting Between Unit Systems

California Standards:

Measurement and Geometry 1.1
Compare weights, capacities, geometric measures, times, **and temperatures within and between measurement systems** (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).

What it means for you:

You'll learn how to convert from a unit in one measurement system to a unit in another measurement system. You'll also convert between temperature scales using a formula.

Key words:

- customary system
- metric system
- conversion table
- Celsius
- Fahrenheit

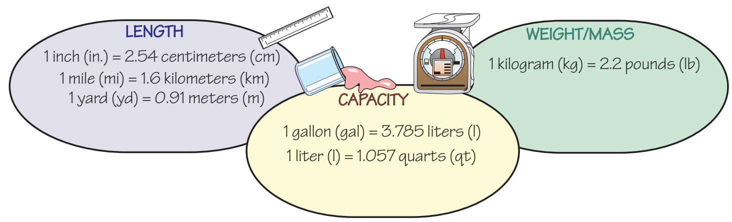
Check it out:

This is exactly the same method that you used in the previous Lesson to convert measures within either the customary or metric system.

Lots of countries use the metric system as their standard system of measurement — for example, distances on European road signs are often given in kilometers. So if you ever go abroad you'll find it useful to be able to convert the metric measures into the customary units that you're more familiar with.

Convert Between the Customary and Metric Systems

Here's a conversion table that tells you **approximately how many customary units make a metric unit.**



You can convert between customary and metric units by **setting up** and **solving proportions**.

Example 1

How many gallons are equivalent to 29 liters?

Solution

The ratio of gallons to liters is 1 : 3.785 or $\frac{1}{3.785}$.

Write a proportion where there are x gallons in 29 liters:

$$\frac{1}{3.785} = \frac{x}{29}$$

Cross-multiply and solve for x:

$$29 \times 1 = 3.785 \times x$$

$$29 = 3.785x$$

$$x = 29 \div 3.785 = 7.661...$$

So there are approximately 7.66 gallons in 29 liters.

Check the reasonableness: 1 gallon is around 4 liters.
So 8 gallons is about $8 \times 4 = 32$ liters.
This estimation is close to the answer, so it is **reasonable.**

- **Strategic Learners**
  Review what has been learned using "measurement system snap." Make a set of cards for each group of students. On half the cards, put customary and metric units for length, mass, and capacity (for example, 1 inch, 1 gallon, 1 kg). On the other half, put their equivalents in the other system (for example, 2.54 cm, 3.785 liters, 2.2 lb). Each group plays according to the usual rules of snap.

- **English Language Learners**
  Put students into pairs. Ask each student to write three multiple-choice questions involving conversions between measurement systems — one about length, one about capacity, and one about mass. Each question should have three wrong answers and one correct answer. For example, "Is 72 inches equivalent to A) 2 feet, B) 2 yards, C) 6 yards, or D) 2 meters?" Have them try to answer their partner's questions.

Guided Practice

In Exercises 1–6, find the missing value.

1. 235 lb = ? kg 107   **2.** 9.3 mi = ? km 15   **3.** 500 cm = ? in. 197
4. 5.7 m = ? yd 6.26   **5.** 7.32 kg = ? lb 16.1   **6.** 76 qt = ? l 72

Convert Twice to Get to Units That Aren't in the Table

Only the **most common** conversions are in the conversion table. There are **lots more** you could make. For example, you might want to convert centimeters into feet, or liters into cups.

Example 2

Find how many feet are in 30 centimeters.

Solution

There's **no direct conversion** from centimeters to feet listed in the table. But there are conversions from **centimeters to inches**, and **inches to feet**.

Step 1: Convert 30 centimeters to inches.
The ratio of inches to centimeters is **1 : 2.54**. So set up a proportion and solve it to find x, the number of inches in 30 cm.

Set up a proportion... $\frac{1}{2.54} = \frac{x}{30}$

$1 \times 30 = x \times 2.54$ **...cross-multiply to solve**

$x = 30 \div 2.54 = 11.81102... \approx 11.8$

So there are approximately **11.8 inches** in 30 centimeters.

Step 2: Now convert 11.8 inches to feet.
The ratio of feet to inches is 1 : 12. Set up a second proportion and solve it to find y, the number of feet in 11.8 inches.

Set up a proportion... $\frac{1}{12} = \frac{y}{11.8}$

$1 \times 11.8 = y \times 12$ **...cross-multiply to solve**

$y = 11.8 \div 12 = 0.98 \approx 1$

This means that there's approximately 1 foot in 30 centimeters.

Check it out:
For conversions like these, you need to use the metric and customary conversion tables from the last Lesson too.

Guided Practice

In Exercises 7–12, find the missing value.

7. 235 kg = ? tons 0.26   **8.** 0.08 mi = ? cm 12,800   **9.** 500 cm = ? yd 5.47
10. 12.3 m = ? ft 40.55   **11.** 7.32 kg = ? oz 258   **12.** 0.05 qt = ? ml 47.3

2 Teach (cont)

Guided practice
Level 1: q1–3
Level 2: q1–5
Level 3: q1–6

Additional examples
1) How many liters are equivalent to 8 quarts? **About 7.57 liters**
2) How many pounds are equivalent to 500 g? **About 1.1 pounds**

Concept question
"How many centimeters are there in x inches?"
2.54x centimeters

Concept question
"How many meters are there in x inches?"
0.0254x meters

Guided practice
Level 1: q7–9
Level 2: q7–11
Level 3: q7–12

Solutions
For worked solutions see the Solution Guide

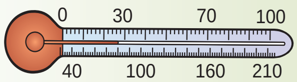
- **Advanced Learners**
  Give students a simple baking recipe, such as for cake or fruit scones, with all the measurements in customary units. Ask them to rewrite the recipe, giving the amount of each ingredient needed to make a double batch. Tell them that you want everything to be in metric units, so they must convert and round as they go. Make sure the recipe includes some dry mass measures, some liquid measures, and a length measure (for example, the size of a tin or cookie cutter). Let them take their new recipe home to try making if they wish.

2 Teach (cont)

Universal access

Most thermometers have both the Celsius and the Fahrenheit scale on them. Bring in such a thermometer for students to pass around and have a look at.

Then get them to make a diagram of the thermometer, like the one shown below, with both scales on.



When they do a temperature conversion, they could have their diagram to hand, and use it to check whether or not their answer looks reasonable.

For example, if they converted 90 °F and came up with the answer 70 °C, it would not be likely to be correct, as 70 °C is nearer 160 °F on the diagram.

Common error

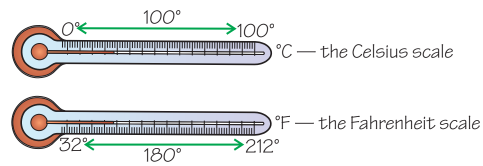
When trying to recall the formula for temperature conversion, students often forget which way around the expression goes. For example, they may incorrectly write $C = \frac{9}{5}F + 32$ instead of the correct $F = \frac{9}{5}C + 32$.

There's a Formula to Convert Temperatures

There are two common units for temperature — **degrees Fahrenheit** (**°F**) and **degrees Celsius** (**°C**).

Converting from one to the other is more complicated than other conversions because the scales don't have the same **zero point**. 0 degrees Celsius is the same as 32 degrees Fahrenheit.

Check it out:
Water freezes at 0 °C or 32 °F. It boils at 100 °C or 212 °F.



So to convert **from degrees Celsius to degrees Fahrenheit**, you have to use this formula:

$$\boldsymbol{F = \frac{9}{5}C + 32}$$

Where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

Example 3

What is 30 °C in degrees Fahrenheit?

Solution

Use the formula: $F = \frac{9}{5}C + 32$

You know C, so put this into the formula and work out F.

$F = \frac{9}{5} \times 30 + 32 = 54 + 32 =$ **86 °F**

Don't forget:
Remember the order of operations, PEMDAS. You've got to do the $\frac{9}{5}C$ bit first and then add 32 to the result — or else you'll get the wrong answer.

Rearrange the Formula to Convert °F to °C

You can **rearrange** the formula so that you can convert **from degrees Fahrenheit to degrees Celsius**:

1. **Take 32 from both sides:** $F - 32 = \frac{9}{5}C$
2. **Multiply both sides by 5:** $5 \times (F - 32) = 9C$
3. **Divide both sides by 9:** $\frac{5}{9} \times (F - 32) = C$

$$\boldsymbol{C = \frac{5}{9}(F - 32)}$$

Where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

Example 4

What is 52 °F in degrees Celsius?

Solution

Use this formula: $C = \frac{5}{9}(F - 32)$

You know F, so put this into the formula and work out C.

$C = \frac{5}{9} \times (52 - 32) = \frac{5}{9} \times 20 = 11.11111... \approx$ **11 °C**.

Guided Practice

In Exercises 13–18, find the missing value.

13. 212 °F = ? °C **100** **14.** 0 °C = ? °F **32** **15.** 88 °F = ? °C **31.1**

16. 132 °C = ? °F **269.6** **17.** –273 °C = ? °F **–459.4** **18.** –15 °F = ? °C **–26.1**

Independent Practice

In Exercises 1–3, find the missing value.

1. 128 ft = ? m **39.0** **2.** 340 miles = ? km **544** **3.** 75 kg = ? lb **165**

4. The weight limit for an airplane carry-on is 18 kilograms. The weight of Joe's carry-on bag is 33 pounds. Will Joe be able to take his carry-on on the airplane? **Yes. 33 pounds ≈ 15 kg**

5. A car has a 10-gallon tank for gasoline. How many liters of gasoline are needed to fill the tank? **37.85 L**

6. Javine has set up the following proportion to convert 70 km to mi:

$$\frac{1}{1.6} = \frac{70}{x}$$

no, the x should be on the top of the second ratio.

Explain whether Javine has set up the proportion correctly.

7. A car is traveling at 50 miles per hour. How fast is this in kilometers per hour? **80 km/h**

8. A recipe needs 180 ounces of apples. What is this in kilograms? **5.1 kg**

In Exercises 9–11, find the missing value.

9. 45 °C = ? °F **113** **10.** 108 °F = ? °C **42.2** **11.** 5727 °C = ? °F **10,341**

12. Josie has a new baby. She reads that the ideal temperature of a baby's bath is between 36 °C and 38 °C, but her thermometer only shows the Fahrenheit temperature scale. Advise Josie on the ideal temperature for her baby's bath in degrees Fahrenheit. **96.8 °F to 100.4 °F**

Now try these:
Lesson 4.3.2 additional questions — p454

Round Up

Conversions between different systems of length, mass, and capacity don't need a formula because they all start at the same point — 0 kg = 0 lb, etc. The Fahrenheit and Celsius scales start at different places — 0 °C = 32 °F, so you need to use a formula for these conversions.

2 Teach (cont)

Concept question

"What is $\frac{5}{9}x$ °C in degrees Fahrenheit?"
$(x + 32)$°F

Guided practice

Level 1: q13–14
Level 2: q13–16
Level 3: q13–18

Independent practice

Level 1: q1–2, 4–6, 9–10
Level 2: q1–7, 9–11
Level 3: q1–12

Additional questions

Level 1: p454 q1–6, 12
Level 2: p454 q1–8, 11–12
Level 3: p454 q1–12

3 Homework

Homework Book — Lesson 4.3.2

Level 1: q1, 2, 4–7
Level 2: q1–10
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 30 — Measuring

Solutions

For worked solutions see the Solution Guide

Lesson 4.3.3

Dimensional Analysis

This Lesson introduces students to the concept of dimensional analysis. Students learn to cancel units as they do a calculation, so that they can make sure that the unit of their answer is correct and appropriate to the question.

Previous Study: From grade 3 onwards students have been taught the importance of giving the correct units for an answer. Dimensional analysis was also touched upon in Section 1.2.

Future Study: In Algebra I, Geometry, and in all science subjects, students will be expected to be competent at manipulating units, and at giving the correct unit for a measurement or answer.

1 Get started

Resources:

- worked calculation lists and unit card sets (see Universal access activity below)

Warm-up questions:

- Lesson 4.3.3 sheet

2 Teach

Universal access

Split the class into groups, and give each group a list of worked calculations, like the one below.
The units of the answers should be missing. Also, give them a set of cards with the correct units on.
Ask them to match up the units with the answers to the calculations.

Speed = 80 m ÷ 10 s = 8? ⇐ $\boxed{\text{m/s}}$
Area = 3 cm × 3 cm = 9? ⇐ $\boxed{\text{cm}^2}$
Cost = 3 lb × 2 \$/lb = 6? ⇐ $\boxed{\text{dollars}}$
Perimeter = 4 × 3 cm = 12? ⇐ $\boxed{\text{cm}}$
Time = 6 km ÷ 2 km/h = 3? ⇐ $\boxed{\text{hours}}$

Common error

When doing dimensional analysis calculations involving rates, students sometimes put a rate's units the wrong way up. For example:
"A sign at the store says you can buy an ounce of granola for \$2. How much will 3 ounces of granola cost you?"

$$3\text{ ounces} \times 2\,\frac{\text{ounces}}{\text{dollars}} = 6\,\frac{\text{ounces}^2}{\text{dollars}}$$ ✘

Remind students that the end unit must be sensible in the context of the question. In this example they are being asked to find a cost in dollars, so the unit is not sensible. They would need to take another look at their work and try to find the error.

Lesson 4.3.3

Dimensional Analysis

California Standards:

Algebra and Functions 4.2
Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

Measurement and Geometry 1.3
Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

What it means for you:
You'll learn how to use dimensional analysis to check the units of your answers to rate problems.

Key words:
- dimensional analysis
- formulas
- canceling

Check it out:
A person-day is a unit that means the amount of work done by 1 person working for 1 day. Units separated by hyphens are products.

This Lesson is about dimensional analysis. Dimensional analysis is a neat way of checking the units in a calculation. It shows whether or not your answer is reasonable.

Dimensional Analysis — Check Your Units

You can use **dimensional analysis** to check the **units** for an answer to a calculation.

For example, if you were trying to calculate a **distance** and dimensional analysis showed that the units should be **seconds**, you know something has **gone wrong**.

You can **cancel units** in the same way that you can cancel numbers.
For example,

$$\frac{70\text{ miles}}{2\cancel{\text{ hours}}} \times 6\cancel{\text{ hours}} = 210\textbf{ miles}$$ ⇐ The units on both sides of the calculation must balance — so the answer must be a distance in miles.

Example 1

Jonathan earns 10 dollars per hour.
How much does he earn for 40 hours' work?

Solution

You need to multiply Jonathan's hourly rate by the number of hours he works.

Earnings = 10 dollars per hour × 40 hours
= **400 dollars**

You can use dimensional analysis to check your answer is reasonable:

$$10\,\frac{\text{dollars}}{\cancel{\text{hour}}} \times 40\cancel{\text{ hours}} = 400\text{ dollars}$$

Example 2

It takes 12 person-days to tile a large roof. If there are three workers working on the roof, how many days will it take them to tile it?

Solution

Number of days = total person-days ÷ number of persons
= 12 ÷ 3 = **4 days**

You can use dimensional analysis to check your answer is reasonable:
3 persons × 4 days = 12 person-days

- **Strategic Learners**
  Review the process of canceling in fraction multiplication. Write on the board the expressions $\frac{3}{4} \times \frac{4}{7} = \frac{3}{7}$, $\frac{5}{3} \times \frac{4}{10} = \frac{2}{3}$, and $\frac{7}{3} \times \frac{2}{7} \times \frac{3}{2} = 1$.
  Have students think about how they would cancel each one before multiplying. Do the same thing with fractions made up of units, like $\frac{\text{meters}}{\text{seconds}} \times \text{seconds} = \text{meters}$, and $\frac{\text{miles}}{\text{dollars}} \times \frac{\text{dollars}}{\text{gallon}} = \frac{\text{miles}}{\text{gallon}}$.
- **English Language Learners**
  Students can be confused by the many different ways used to write fractional and compound units. For instance, $\frac{\text{miles}}{\text{hour}}$ is often written miles per hour or miles/hour, and persons × days is often written person-days. Have them copy these examples into their notebooks as a reminder.

Example 3

You are organizing a three-legged race. You need 2.5 feet of ribbon for every two people. You have 660 inches of ribbon.
How many people can join in the race?

Solution

First convert the length of the ribbon from inches to feet.
You need to set up a proportion. 12 inches = 1 foot, so:

$$\frac{12}{1} = \frac{660}{x} \quad \leftarrow x = \text{length in feet}$$

$12x = 660 \times 1 \quad \Rightarrow \quad x = 55$ **feet**

So 660 inches is equivalent to 55 feet.

You can check this by **dimensional analysis**:

$$x = \frac{660\,\cancel{\text{inches}} \times 1\text{ foot}}{12\,\cancel{\text{inches}}} = 55\text{ feet}$$

Now divide the length of ribbon by the amount you need per person:

$$55\text{ feet} \div \frac{2.5\text{ feet}}{2\text{ people}} = 55\,\cancel{\text{feet}} \times \frac{2\text{ people}}{2.5\,\cancel{\text{feet}}} = \mathbf{44\ people}$$

Guided Practice

In Exercises 1–2, find the missing unit.

1. $4\text{ miles} \times \frac{1.6\text{ km}}{1\text{ mile}} = 6.4\ ?$ km **2.** $26.5\text{ inches} \times \frac{2.54\text{ cm}}{1\text{ inch}} = 67.31\ ?$ cm

Checking Formulas with Dimensional Analysis

Dimensional analysis is useful for **checking whether a formula is reasonable**. The units on each side of a formula must balance.

Example 4

A formula says that speed (in meters per second) multiplied by time (in seconds) is equal to the distance traveled (in meters).
Use dimensional analysis to check the reasonableness of the formula.

Solution

Write out the formula suggested and include the units.

$$\text{speed}\left(\frac{\text{m}}{\cancel{\text{s}}}\right) \times \text{time}\ (\cancel{\text{s}}) = \text{distance}\ (\text{m}\,?)$$

So the seconds cancel and leave units of meters.
This means that **the formula is reasonable** since the units on each side of the equation match.

Check it out:
A unit of meters per second means "meters ÷ 1 second." This is the number of meters traveled in 1 second. When you get a unit with the word "per" in it, you know that you can write the unit as a fraction.

2 Teach (cont)

Application
Dimensional analysis is used all the time in scientific calculations.

This is a good opportunity to tie in with the students' science classes. Set them some dimensional analysis problems involving commonly used scientific units. For example:

1) The formula for power is Power = Energy ÷ Time. What power is output by a light bulb converting 600 joules of energy over 10 seconds? **60 J/s**
(1 J/s is commonly called a watt, W)

2) The formula for pressure is Pressure = Force ÷ Area. What pressure is exerted by a force of 80 newtons applied over 2 m^2?
40 N/m^2 (also called pascals, Pa)

Guided practice
Level 1: q1
Level 2: q1–2
Level 3: q1–2

Additional example
A formula says that the volume of a pyramid is equal to the area of its base (in cm^2), multiplied by one-third, multiplied by its height (in cm). Use dimensional analysis to show whether the formula is reasonable.

cm^2 × cm = cm^3. So the formula

area (cm^2) × $\frac{1}{3}$ × height (cm) = volume (cm^3)

is reasonable because it gives the answer in cm^3, which is a unit of volume.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Give students the average distance from Earth to the Moon (384,400 kilometers), and the height of the Sears Tower in feet and inches (1450 ft 7 in.). Ask students to use all they know about conversion factors and dimensional analysis to find how many Sears Towers you would need to stack on top of each other to reach the Moon. Ask for the units of the answer to be "Sears Towers." Their answers should be in the region of 870,000 to 880,000 Sears Towers, but allow plenty of leeway when marking since using approximate conversion factors introduces errors.

2 Teach (cont)

Math background
The density of an object is the ratio of its mass to its volume.

Concept question
"If I graph the distance I traveled in kilometers on the y-axis, against the time it took in hours on the x-axis, what does the slope of my line represent?"
My (average) speed in kilometers per hour.

Guided practice
Level 1: q3
Level 2: q3–4
Level 3: q3–4

Independent practice
Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

Additional questions
Level 1: p455 q1–7
Level 2: p455 q1–10
Level 3: p455 q1–11

3 Homework

Homework Book — Lesson 4.3.3
Level 1: q1–4, 6, 8
Level 2: q1–4, 6–10
Level 3: q1–10

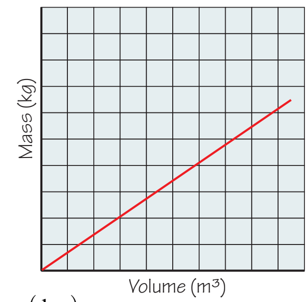
4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 28 — Graphing Linear Equations
- Worksheet 30 — Measuring

Example 5

It is suggested that the slope of this graph is equal to density, which is measured in kg per m³.

Is this a reasonable suggestion?



Solution

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

$$\text{Slope} = \frac{\text{change in mass (kg)}}{\text{change in volume (m}^3\text{)}} = \text{density}\left(\frac{\text{kg}}{\text{m}^3}\right)$$

So the unit of the slope of the graph is $\left(\frac{\mathbf{kg}}{\mathbf{m}^3}\right)$.

This is the same as kg per m³.

So it's reasonable that the slope is equal to density, since it has the right units.

Guided Practice

3. Find eight different units of speed, if speed = distance ÷ time. **see below**

4. The formula for acceleration is "change in speed ÷ time." Which of the following could be a unit for acceleration: **A and D**

A. $\frac{\text{in.}}{\text{h}^2}$ **B.** $\frac{\text{cm}^2}{\text{min}}$ **C.** $\frac{\text{s}}{\text{m}^2}$ **D.** $\frac{\text{km}}{\text{s}^2}$

Independent Practice

In Exercises 1–4, find the missing unit.

1. $15 \text{ tomatoes} \times \frac{\$0.20}{1 \text{ tomato}} = 3\ ?$ **\$**

2. $\$56 \times \frac{1.3 \text{ lb}}{\$1} = 72.8\ ?$ **lb**

3. $54.3 \text{ yd} \times \frac{0.9144 \text{ m}}{1 \text{ yd}} = 49.65...\ ?$ **m**

4. $\$500 \times \frac{£0.53}{\$1} = 265\ ?$ **pounds**

5. Use dimensional analysis to check that this expression is reasonable. Use it to find the number of seconds in an hour.

$$1 \text{ hour} = 1\ \cancel{\text{hour}} \times \frac{60\ \cancel{\text{minutes}}}{1\ \cancel{\text{hour}}} \times \frac{60 \text{ seconds}}{1\ \cancel{\text{minute}}}$$

3600 seconds. This is reasonable: the answer is in seconds.

6. You need to save up \$240 for a ski trip. You earn six dollars per hour babysitting. How many hours do you need to work to pay for the trip? Check your answer using dimensional analysis.

40 hours: 40 hours × 6 $\frac{\text{dollars}}{\text{hour}}$ = 240 dollars

Now try these:
Lesson 4.3.3 additional questions — p455

Round Up

So dimensional analysis is basically making sure your units balance — it's useful for checking you've worked out what you think you have. Try to get into the habit of using it for all types of problems.

Solutions
For worked solutions see the Solution Guide

Guided Practice
3. Accept any eight valid units. For example: **miles/hour**, **km/hour**, **meters/second**, **miles/minute**, **cm/second**, **meters/year**, **inches/day**, **km/second**, **yards/month**, **miles/nanosecond**, **feet/year**.

Lesson 4.3.4

Converting Between Units of Speed

In this Lesson, students learn how to convert between units of speed, which often involves going through an intermediate step. They are taught to do this by setting up conversion fractions equal to 1 and multiplying by them.

Previous Study: At grade 6 students learned how to solve problems involving rate, average speed, distance, and time. As part of this study, they were introduced to commonly used units of speed.

Future Study: In Algebra I students will learn how to solve complex rate problems using algebraic techniques. As part of this process they will be expected to be able to find appropriate units for their answers.

Lesson 4.3.4

Converting Between Units of Speed

California Standards:

Measurement and Geometry 1.1

Compare weights, capacities, **geometric measures, times,** and temperatures **within and between measurement systems (e.g., miles per hour and feet per second,** cubic inches to cubic centimeters**).**

What it means for you:

You'll learn how to convert from one unit of speed to another unit of speed.

Key words:

- conversion
- dimensional analysis

There are a lot of units that can be used for speed — kilometers per hour, miles per hour, meters per second, inches per minute.

Speed units are all made up of a distance unit divided by a time unit. This makes them a bit tougher to convert than other units.

You Can Set Up Conversion Fractions Equal to 1

When you multiply something by **1**, it **doesn't change**. Fractions with the same thing on the top and the bottom, such as $\frac{5}{5}$, are equal to 1, so whatever you multiply by them doesn't change either.

60 seconds = 1 minute, so the fraction $\frac{\textbf{60 seconds}}{\textbf{1 minute}}$ has the same on the top and the bottom, so it's **equal to 1** too — this is called a **conversion fraction**.

Use Conversion Fractions to Convert Units

You can use a conversion fraction equal to 1 to convert from one unit to another. Here's how a time can be converted:

Example 1

A ship takes 1.75 days to reach its destination. How many hours is this?

Solution

24 hours = 1 day **Start with a conversion equation**

$\frac{24 \text{ hours}}{1 \text{ day}} = \frac{1 \text{ day}}{1 \text{ day}}$ **Divide both sides by 1 day**

$\frac{24 \text{ hours}}{1 \text{ day}} = 1$ You now have a fraction that is equal to 1.

So, whatever you multiply by the fraction $\frac{24 \text{ hours}}{1 \text{ day}}$ won't change.

$1.75 \cancel{\text{days}} \times \frac{24 \text{ hours}}{1 \cancel{\text{day}}} = 1.75 \times 24 \text{ hours} = \textbf{42 hours}$ **Cancel the units**

1.75 days is equivalent to 42 hours.

Don't forget:

1 is the multiplicative identity. See Lesson 1.1.4.

Check it out:

If you don't end up with the units you wanted, your conversion fraction may be upside down.

Guided Practice

Convert each of the following by multiplying by a conversion fraction.

1. 6 inches to centimeters 15.24 cm **2.** 45 minutes to seconds 2700 seconds

3. 12 miles to kilometers 19.2 km **4.** 6 liters to quarts 6.34 quarts

Check it out:

You'll need to look back at the conversion tables in Lessons 4.3.1 and 4.3.2 for these.

1 Get started

Resources:
- stopwatches

Warm-up questions:
- Lesson 4.3.4 sheet

2 Teach

Additional examples

Write conversion fractions from the following equations:

1) 60 minutes = 1 hour

$\frac{60 \text{ minutes}}{1 \text{ hour}}$ or $\frac{1 \text{ hour}}{60 \text{ minutes}}$

2) 100 cm = 1 m

$\frac{100 \text{ cm}}{1 \text{ m}}$ or $\frac{1 \text{ m}}{100 \text{ cm}}$

3) 1 mile = 1760 yards

$\frac{1 \text{ mile}}{1760 \text{ yards}}$ or $\frac{1760 \text{ yards}}{1 \text{ mile}}$

4) 10 miles ≈ 16 km

$\frac{10 \text{ miles}}{16 \text{ km}}$ or $\frac{16 \text{ km}}{10 \text{ miles}}$

Guided practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1–4

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  Give the students a simple word problem to solve, like the following: "Jen and Leroy had a race over a 10-mile course. Leroy's average speed was 8 miles/hour. Jen's average speed was 16 km/hour. Who won the race? How long did they each take to get to the finish line?" Encourage students to draw a diagram to help clarify the problem.

- **English Language Learners**
  Have students split a page in their notebook into two columns. Tell them to head the first column "Units of distance" and the second "Units of time." Ask them to list as many units of distance and time as they can, with equivalents in their first language if it's useful to them. Remind them that a unit of speed could be any distance unit from the first column divided by any time unit from the second.

2 Teach (cont)

Universal access

Ask the class to think about why we might need to convert units of speed. Here are some examples to discuss:

1) **Comparing how fast two things can move if you are given their speeds in different units.** If you are told that a lion can run at 50 mi/h, while a zebra can run at 65 km/h, how do you know which is faster?

2) **Working out what speed limits in other countries mean when you go there.** My speedometer is marked in mi/h. On holiday in Canada I am driving on the highway, where the speed limit is 80 km/h. How can I tell what the legal speed limit is equivalent to on my speedometer?

Concept question

"Runner A and Runner B had a sprint race. Runner A's average speed was 10 mi/h. Runner B's average speed was 16 km/hour. Who won the race?"

10 mi/h and 16 km/hour are equivalent speeds, so the race was a tie.

Common error

Students sometimes forget to convert the time part of a speed unit.

Remind them that speed units are always compound units. Suggest that when they start a speed unit conversion question, they should pause, look at the units, and ask themselves, "What do I need to change the distance unit to? What do I need to change the time unit to?" They should plan out the stages of their unit conversion before starting.

Speed Units May Have Two Parts to Convert

A speed unit is always a **distance unit** divided by a **time unit**.

If you want to change both of these parts, you need to do **two separate conversions**. For instance, if you were converting **centimeters per minute** to **inches per second**, you might do the following conversions:

centimeters per minute ⇨ **inches per minute** ⇨ **inches per second**

Check it out:
You could also convert centimeters per minute to **centimeters per second**, before finally converting this to inches per second.

Example 2

A train travels 1.2 miles per minute.
What is the speed of the train in kilometers per hour?

Solution

Break this down into **two stages** —

Stage 1: Convert from miles per minute to miles per hour.

First, you have to write a conversion fraction:

$60 \text{ minutes} = 1 \text{ hour}$ — **Start with a conversion equation**

$\frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{1 \cancel{\text{hour}}}{1 \cancel{\text{hour}}}$ — **Divide both sides by 1 hour**

$\frac{60 \text{ minutes}}{1 \text{ hour}} = 1$ — **You've now got a fraction equal to 1**

So, whatever you multiply by the fraction $\frac{60 \text{ minutes}}{1 \text{ hour}}$ won't change.

$\frac{1.2 \text{ miles}}{1 \cancel{\text{minute}}} \times \frac{60 \cancel{\text{minutes}}}{1 \text{ hour}} = (1.2 \times 60)\frac{\text{miles}}{\text{hour}}$ — **Cancel the units**

$= 72 \text{ miles per hour}$

Check it out:
Dimensional analysis shows that the product should have units of miles per hour — that's the unit we are aiming for at this stage.

Stage 2: Convert from miles per hour to kilometers per hour.

Write another conversion fraction:

$1 \text{ mile} = 1.6 \text{ kilometers}$ — **Start with a conversion equation**

$\frac{1 \cancel{\text{mile}}}{1 \cancel{\text{mile}}} = \frac{1.6 \text{ kilometers}}{1 \text{ mile}}$ — **Divide both sides by 1 mile**

$1 = \frac{1.6 \text{ kilometers}}{1 \text{ mile}}$ — **You've now got a fraction equal to 1**

So, whatever you multiply by the fraction $\frac{1.6 \text{ kilometers}}{1 \text{ mile}}$ won't change.

$\frac{72 \cancel{\text{miles}}}{1 \text{ hour}} \times \frac{1.6 \text{ kilometers}}{1 \cancel{\text{mile}}} = (72 \times 1.6)\frac{\text{kilometers}}{\text{hour}}$ — **Cancel the units**

$\approx 115 \text{ kilometers per hour}$

Check it out:
Using dimensional analysis here shows that the product should have units of kilometers per hour — that's the unit we want.

- **Advanced Learners**
  Put students into pairs. Ask each student to cover a measured distance of, say, 100 m or 10 m. This could be on a school running track or in the schoolyard. Ask students to cover the measured distance any way they choose (walking, running, hopping) while their partner times them using a stopwatch. This will allow them to calculate the speed of their activity in m/s. Ask them to convert this value, and discover how fast they were moving in miles/hour.

Guided Practice

5. Convert 18 miles per hour into kilometers per hour. **29 kilometers per hour**

6. Which is faster — 56 miles per hour or 83 kilometers per hour? **56 miles per hour**

7. Convert 14 inches per minute into the unit feet per second. **0.019 feet per second**

8. Which is faster — 22 centimeters per minute or 500 inches per hour? **22 centimeters per minute**

Independent Practice

Write conversion fractions from the equations given below.

1. 3 feet = 1 yard $\frac{\text{3 feet}}{\text{1 yard}}$ or $\frac{\text{1 yard}}{\text{3 feet}}$

2. 1 kilometer = 1000 meters $\frac{\text{1 kilometer}}{\text{1000 meters}}$ or $\frac{\text{1000 meters}}{\text{1 kilometer}}$

3. 3600 seconds = 1 hour $\frac{\text{3600 seconds}}{\text{1 hour}}$ or $\frac{\text{1 hour}}{\text{3600 seconds}}$

Convert the following by multiplying by a conversion fraction.

4. 3 feet into inches **36 inches**

5. 4.5 hours into seconds **16,200 seconds**

6. 36 miles into kilometers **57.6 kilometers**

7. Jonah and Ken were both running separate races.
Jonah ran 100 meters in 12 seconds, and Ken ran 100 yards in 0.18 minutes. Who ran faster? **Ken**

8. A snail managed to crawl 50 centimeters in 14 minutes.
A slug crawled 40 inches in 0.5 minutes. Which was faster? **The slug.**

Convert the following speeds into the units given.

9. 25.6 kilometers per hour into miles per hour **16 miles per hour**

10. 36 inches per hour into centimeters per minute **1.52 cm/minute**

11. 7 feet per minute into yards per hour **140 yards per hour**

12. A boat travels 20 miles in 4 hours.
How fast is this in kilometers per hour? **8 kilometers per hour**

A car travels 0.6 miles in two minutes.

13. How fast is this in kilometers per hour? **28.8 kilometers per hour**

14. How fast is this in meters per hour? **28,800 meters per hour**

Now try these:
Lesson 4.3.4 additional questions — p455

Round Up

Speed units have two parts — a distance part and a time part. That's why you often have to do two separate conversions to convert a speed into different units. Multiplying by a conversion fraction is a useful way of converting any units. But it's real important that you always check your work using dimensional analysis.

Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Guided practice
Level 1: q5–6
Level 2: q5–7
Level 3: q5–8

Independent practice
Level 1: q1–9
Level 2: q1–12
Level 3: q1–14

Additional questions
Level 1: p455 q1–8
Level 2: p455 q1–11
Level 3: p455 q1–12

3 Homework

Homework Book — Lesson 4.3.4
Level 1: q1–6
Level 2: q3–9
Level 3: q3–10

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 30 — Measuring

Lesson 4.4.1

Linear Inequalities

In this Lesson, students review inequalities, and how to graph them on a number line. They move on to learn the basic techniques for solving inequalities, and how to write systems of inequalities.

Previous Study: In Section 1.3 students learned how to write one- and two-step inequalities, and how to graph one-step inequalities on a number line. They were introduced to systems of equations in Section 4.1.

Future Study: In Algebra I students will learn how to solve inequalities and systems of inequalities in one and two variables, including those containing absolute value expressions, and graph their solution sets.

1 Get started

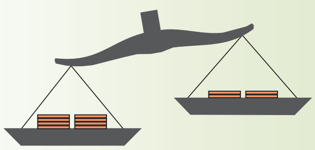
Resources:
- red and blue pens

Warm-up questions:
- Lesson 4.4.1 sheet

2 Teach

Concept question

Copy the diagram below onto the board:



"If I remove two pennies from each side of the scales, will it balance?"

No — if you remove the same number of pennies from each side, the scales will remain unbalanced.

Math background

The arrow at the end of the "ray" tells you that the solution set of the inequality goes on along the number line **forever** in that direction.

Lesson 4.4.1

California Standards:

Algebra and Functions 1.1

Use variables and appropriate operations to write an expression, an equation, an inequality, or **a system of** equations or **inequalities that represents a verbal description** (e.g., three less than a number, half as large as area A).

Algebra and Functions 4.1

Solve two-step **linear** equations and **inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.**

What it means for you:

You'll solve inequalities by addition and subtraction using the same methods that you used for solving equations.

Key words:
- inequality
- addition
- subtraction

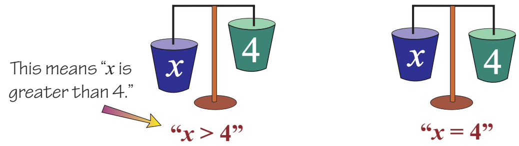
Section 4.4

Linear Inequalities

In Chapter 1 you learned what inequalities were and how to write them. In this Lesson, you'll review some of the things you've already seen, and learn how to solve inequalities using the same kind of method that you used to solve equations.

Inequalities Have an Infinite Number of Solutions

Inequalities have **more than one solution**. The inequality $x > 4$ tells you that x could take **any value greater than 4**, whereas the equation $x = 4$ tells you x can **only** take the value of 4.



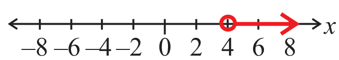
Remember the four **inequality symbols** you learned in Chapter 1:

The Inequality Symbols

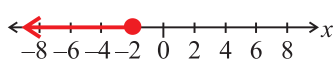
"$>$" means "greater than," "more than," or "over"
"$<$" means "less than" or "under"
"$\geq$" means "greater than or equal to," "minimum," or "at least"
"$\leq$" means "less than or equal to," "maximum," or "at most"

Show Solutions to an Inequality on a Number Line

The solution to an inequality with one variable can be shown on a **number line**. A **ray** is drawn **in the direction of all the numbers in the solution set**. So for $x > 4$, the ray should go through all numbers greater than 4.



This number line shows "$x > 4$."



This number line shows "$x \leq -2$."

An **open circle** ○ means the number **is not** included in the solution; a **closed circle** ● means the number **is** included in the solution.

- **Strategic Learners**
  Put the equation $x - 1 = 4$ on the board. Ask the students what operation is on the left side of the equation, and what they could do to reverse it. When they have found the solution ($x = 5$), have them check it by substituting it back into the equation. Now change the equation to the inequality $x - 1 > 4$. Ask the same questions, and get students to check the solution ($x > 5$) by testing possible values of x.

- **English Language Learners**
  The phrase "more than" is used to indicate addition as well as an inequality symbol. For example, 3 + 4 = 7 may be read "4 more than 3 is 7." This can cause confusion for students. Counteract this by writing some inequalities on the board, and have the class practice reading them aloud. When reading inequalities involving addition, use "plus" or "added to" and never "more than."

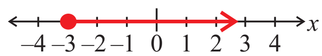
Don't forget:
You've done problems like this before in Section 1.3. To write inequalities that are given in words, look for the key words given in the table of inequality symbols on the previous page. Then think about which way around the expression should go.

Example 1

Write and plot an inequality to say that y must be a minimum of –3.

Solution

The word **"minimum"** tells you that $y \geq -3$.
You need to use a **closed circle**, because –3 is included in the solution set.



The ray goes in the direction of all values greater than –3.

Guided Practice

In Exercises 1–4, write the inequality in words.

1. $z < 7$ **z is less than seven.**
2. $y > -10$ **y is greater than negative ten.**
3. $x \leq -1$ **x is less than or equal to negative one.**
4. $n \geq 89$ **n is greater than or equal to eighty-nine.**

In Exercises 5–8, plot the inequality on a number line.

5. $k > 8$ **see below**
6. $j \geq 2.5$ **see below**
7. $a \leq -4$ **see below**
8. $d < -50$ **see below**

9. To go on Ride A, children must be at least 1 m tall. Write this as an inequality, and plot the inequality on a number line. **see below**

You Can Write Systems of Inequalities

Don't forget:
You wrote systems of equations in Lesson 4.1.2. Systems of inequalities are similar, but are likely to have a range of solutions.

A system of inequalities is a **set** of two or more inequalities in the **same variables**. The inequalities $x > 2$ and $x - 1 \leq 6$ make a system of inequalities in the variable x.

The **solutions** to a system of inequalities have to satisfy **all** the inequalities at the same time.

So if **x is an integer**, the solution set of the system of inequalities $x > 2$ and $x - 1 \leq 6$, must be **{3, 4, 5, 6, 7}**. These values make **both** inequalities true.

Example 2

Write a system of inequalities to represent the following statement:
"**3 times y is greater than 5**, and **2 plus y is less than or equal to 7**."

Solution

You need to write **two inequalities** that **both** need to be true for the statement to be true.

The first part says "3 times y is greater than 5," so $3y > 5$.
The second part says "2 plus y is less than or equal to 7," so $2 + y \leq 7$.

These two equations form a **system of inequalities**.

2 Teach (cont)

Additional examples

Decide whether the circle would be open or closed if these inequalities were plotted on a number line.

1) $w < 5$ **Open**
2) $x \geq -2$ **Closed**
3) $y > 0$ **Open**
4) $z \leq -7$ **Closed**

Guided practice

Level 1: q1–2, 5–6, 9
Level 2: q1–3, 5–7, 9
Level 3: q1–9

Universal access

A system of inequalities relates two sets of information that you have to deal with at the same time.

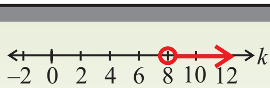
Ask students to come up with two real-life word inequalities that are both true for a variable at the same time.

For example, the system of inequalities "To attend our school, your age must be less than 15," and "To attend our school, your age must be greater than 10," form the restriction "To attend our school, you must be between the ages of 11 and 14."

Then ask them to write their inequalities as math sentences, for example, $x > 10$ and $x < 15$. Have them write out all the integer values of x that would make their system true — for example, {11, 12, 13, 14}.

Solutions

For worked solutions see the Solution Guide

5.


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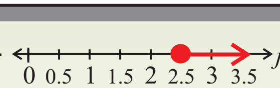

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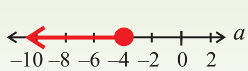

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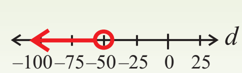
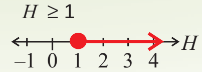


9. Let H = height
$H \geq 1$



● **Advanced Learners**
Introduce students to linear inequalities with one variable that also contain an absolute value symbol. Give them the inequality $|x| < 8$. Ask students to try to find two different values of x that would make the inequality true, one positive, and one negative. When they have done this, ask them to think carefully about the values of x that they found, and challenge them to write down all the possible integer solutions of $|x| < 8$ (which are 7, 6, 5, 4, 3, 2, 1, 0, –1, –2, –3, –4, –5, –6, and –7).

2 Teach (cont)

Guided practice

Level 1: q10
Level 2: q10–11
Level 3: q10–12

Math background

To solve an inequality, you use the same techniques that you would when solving an equation. Solving one- and two-step equations was covered in Section 1.2. The key thing that must be done differently when solving inequalities is reversing the sign when you multiply or divide by a negative number. This is covered in the next Lesson.

The aim is to isolate the variable on one side of the inequality symbol. To do this you need to reverse all the operations that have been applied to it.

Whatever you do, you must do to both sides of the inequality. This means that in each step you create an equivalent inequality — one in which the variable can take the same values as in your starting inequality.

Universal access

Put students into pairs and ask one student in each pair to write and read aloud a sentence describing some operations being performed on a variable. For example, they may say, "I multiplied x by five, then added two to the product." They should then write down the expression they produced in red.

Then ask the second student to write and read aloud a sentence describing how they would get back to the variable if given that expression. For the example given above, they would say, "I would subtract two from the expression, and then divide it by five." They should then write this underneath the expression in blue.

They should then substitute some simple values for x to check the inverse statement is correct.

Guided Practice

10. d is an integer, and $d > -1$, and $d \le 4$.
What values of d would make this system of inequalities true? **{0, 1, 2, 3, 4}**

In Exercises 11–12, write a system of inequalities to represent each statement:

11. z is less than 0, and the sum of z and 4 is greater than –12.
$z < 0$, and $z + 4 > -12$

12. A third of p is less than or equal to 0, and the product of p and –3 is less than 30. **$p \div 3 \le 0$, and $-3p < 30$**

Solve Inequalities by Reversing Their Operations

To solve an inequality you need to get the variable **by itself** on one side — you do this by **"undoing"** the operations that are done to it. This means **doing the "opposite."**

So, if a variable has a number **subtracted** from it, you undo this by **adding** the same number to it. Remember — you have to do exactly the same to **each side** of the inequality.

$$\begin{aligned} x - 6 &> 16 \\ x &> 16 + 6 \\ x &> 22 \end{aligned}$$

+ 6   + 6

Check it out:
When you're solving inequalities by addition and subtraction, treat the expression like an equation, but with the inequality sign in the middle.

Example 3

Solve the inequality $y + 7 \le 21$.

Solution

$y + 7 - 7 \le 21 - 7$ **Subtract 7 from each side of the inequality**

$\mathbf{y \le 14}$ **Simplify**

Check it out:
You might see this solution set written as $\{y : y \le 14\}$ — it means the same.

You might get **word problems** that ask you to solve inequalities.

Example 4

A number increased by 3 is at most 9. Write and solve this inequality.

Solution

"A number increased by 3" means $x + 3$.

"at most 9" means "less than or equal to 9" so ≤ 9.

This means the inequality is $\mathbf{x + 3 \le 9}$.

Now solve the inequality by subtracting 3 from each side:

$x + 3 - 3 \le 9 - 3$

$\mathbf{x \le 6}$

Solutions
For worked solutions see the Solution Guide

Example 5

An elevator has a weight of 1250 pounds already in it. If the maximum load for the elevator is 2500 pounds, write and solve an inequality to find the amount of additional load that can be put in the elevator safely.

Solution

"maximum" load of 2500 pounds means ≤ 2500 pounds.

1250 pounds plus the additional load that can be added, x, must be less than or equal to 2500 pounds.

$$1250 + x \leq 2500$$

$$x + 1250 - 1250 \leq 2500 - 1250 \quad \text{Take 1250 from both sides}$$

$$x \leq 1250$$

The load that can be added is a maximum of 1250 pounds.

Guided Practice

In Exercises 13–20, solve the inequality for the unknown.

13. $z + 5 < 17$ $z < 12$ **14.** $y - 10 > -10$ $y > 0$ **15.** $2 + x \leq -1$ $x \leq -3$ **16.** $p + 45 \leq 76$ $p \leq 31$

17. $h - 6 > 3$ $h > 9$ **18.** $-6 + h > 3$ $h > 9$ **19.** $14 + x \geq 12$ $x \geq -2$ **20.** $1 + y < 1$ $y < 0$

21. A number decreased by 17 is at least 16. Write and solve an inequality to find the number. $x - 17 \geq 16,\ x \geq 33$

22. Sophia must complete at least 40 hours of training to qualify. She has already completed 32 hours of training. Write an inequality and solve it to find the remaining hours of training she must complete. $x + 32 \geq 40,\ x \geq 8$

Independent Practice

In Exercises 1–4, plot the inequality on a number line.

1. $x > 3$ see below **2.** $t \leq 14$ see below **3.** $n < 1$ see below **4.** $z \geq -2$ see below

5. An elevator has a safe maximum load of 2750 pounds. Write an inequality that shows the safe load for this elevator. load ≤ 2750

6. Write a system of inequalities to represent this statement: 4 plus f is greater than 14, and the product of f and 6 is less than 14. $4 + f > 14,\ 6f < 14$

In Exercises 7–14, solve the inequality for the unknown.

7. $p - 6 > 10$ $p > 16$ **8.** $z + 12 < 1$ $z < -11$ **9.** $c + 1 \leq -6$ $c \leq -7$ **10.** $13 + d \geq 12$ $d \geq -1$

11. $x + 7 \geq -7$ $x \geq -14$ **12.** $-12 + y > 6$ $y > 18$ **13.** $f - 100 \leq -2$ $f \leq 98$ **14.** $g + 130 < 12$ $g < -118$

15. A number, y, increased by 12 is larger than 12. Write and solve an inequality to find the solution set. Plot the solution on a number line. see below

16. The area of Portia's yard is 32 ft². The area of Gene's yard is at least 4 ft² larger than Portia's. Write and solve an inequality for the area of Gene's yard. $x - 4 \geq 32,\ x \geq 36$

Now try these:
Lesson 4.4.1 additional questions — p455

Round Up

Solving inequalities that involve addition and subtraction is exactly like solving equations. In the next Lesson you'll solve inequalities involving multiplication and division — this has an important difference.

2 Teach (cont)

Additional example

Mica is four years younger than her brother Martin. She has graduated university, so she must be at least 22 years old. Write and solve an inequality to describe how old Martin is.

Let Martin's age = M
Mica's age = $M - 4$
$M - 4 \geq 22$
$M \geq 26$

Guided practice

Level 1: q13–16, 21
Level 2: q13–18, 21
Level 3: q13–20, 22

Independent practice

Level 1: q1–2, 5–9, 15
Level 2: q1–3, 5–12, 15
Level 3: q1–4, 6–14, 16

Additional questions

Level 1: p455 q1–7, 9–12
Level 2: p455 q1–12, 14
Level 3: p455 q1–15

3 Homework

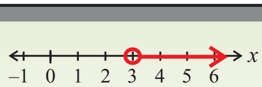
Homework Book — Lesson 4.4.1
Level 1: q1–8
Level 2: q1–10
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 27 — Inequalities

Solutions

For worked solutions see the Solution Guide

1.


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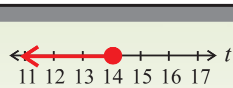
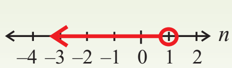


15. $y + 12 > 12$
$y > 0$

3.


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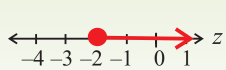
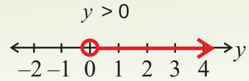




Lesson 4.4.2

More on Linear Inequalities

In this Lesson, students learn how to solve linear inequalities involving multiplication and division operations. They learn that multiplying or dividing by a negative number reverses the sign of the inequality.

Previous Study: In Section 1.3 students learned how to write one- and two-step inequalities, and how to graph one-step inequalities on a number line.

Future Study: In Algebra I students will learn how to solve linear inequalities in one and two variables, and to graph their solution sets on the coordinate plane.

1 Get started

Resources:
- scales
- pennies

Warm-up questions:
- Lesson 4.4.2 sheet

2 Teach

Universal access

Introduce the concept of multiplying and dividing inequalities using scales and some pennies.

Start the activity with nothing on either side of the scales, so that it is level. Now place 12 pennies on one side, and eight on the other, so that it is clearly unbalanced.

Ask students which inequality sign should be placed between the two scalepans (12 > 8). Write this inequality on the board.

Now ask students what will happen to the scales if half of the pennies are taken away from each side — will the scales remain the same or not? When they have figured out that it will, take half the pennies away from each side, to leave six on one and four on the other. Add to the board:

$$12 \div 2 > 8 \div 2$$
$$6 > 4$$

Repeat the activity, but this time double the number of pennies instead.

Common error

When multiplying by a whole number to get rid of a fractional coefficient, students often forget to apply the operation to all the terms in the inequality. For example:

$$\frac{1}{2}x + 5 \geq 17$$
$$2\left(\frac{1}{2}x\right) + 5 \geq 2 \times 17$$

Here the student applied the "×2" to the fraction coefficient of x, and the 17, but forgot to apply it to the 5.

Guided practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1–4

Lesson 4.4.2

More on Linear Inequalities

California Standards:

Algebra and Functions 4.1

Solve two-step **linear** equations and **inequalities in one variable over the rational numbers**, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.

What it means for you:

You'll get more practice at solving linear inequalities and learn how to multiply and divide inequalities by negative numbers.

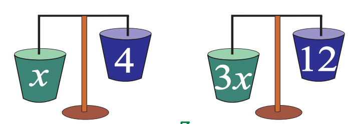
Key words:
- inequality
- multiplication
- division
- inequality symbol

So far you've set up and solved linear inequalities that use addition and subtraction. The next step is to use multiplication and division. This is a bit trickier, because you need to remember to swap the inequality symbol when you multiply or divide by a negative number.

Multiplying and Dividing by Positive Numbers

The **rules** for multiplying and dividing inequalities by positive numbers are **the same** as for multiplying and dividing equations. The **inequality symbol doesn't change.**

The main thing to remember is to **always do the same thing to both sides**.



$x > 4 \xrightarrow{\times 3} 3x > 12$

Example 1

Solve the inequality $4x < 32$.

Solution

$\frac{4x}{4} < \frac{32}{4}$ **Divide both sides of the inequality by 4**

$x < 8$

Example 2

Solve the inequality $\frac{x}{9} \geq 45$.

Solution

$\frac{x}{9} \times 9 \geq 45 \times 9$ **Multiply both sides of the equation by 9**

$x \geq 405$

Check it out:

You can't do this if you're multiplying or dividing by a negative number. There's a different rule for negative numbers, which you'll come to next.

Guided Practice

In Exercises 1–4, solve the inequality for the unknown.

1. $5c > 12.5$ $c > 2.5$ **2.** $3p \geq 63$ $p \geq 21$ **3.** $\frac{f}{12} < 9$ $f < 108$ **4.** $\frac{g}{60} \leq -3$ $g \leq -180$

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  Emphasize the idea that an inequality has direction. In the same way that a negative number is the opposite of a positive, “less than” is the opposite of “more than.” Tell students that when you multiply or divide a whole inequality by a negative, you must be sure to make **everything** opposite — numbers (turn negatives positive and vice versa) and inequality symbols (turn “less than “into” more than” and vice versa).

- **English Language Learners**
  See if students can discover for themselves the rule about reversing the inequality symbol when multiplying or dividing by a negative number. Give them an example using just integers to look at, such as $-8 < 24$. If they try multiplying both sides by –1, they might get $8 < -24$, which isn't true. After several examples, they should realize that you need to reverse the signs to make the inequalities true.

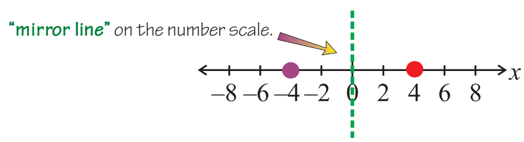
Multiplying and Dividing by Negative Numbers

You know that a number and its opposite are the **same distance** away from zero on a number line. So 4 and –4 are both 4 units from zero.

When you multiply a number by –1 you are effectively **“reflecting”** the number about **zero on the number line**.

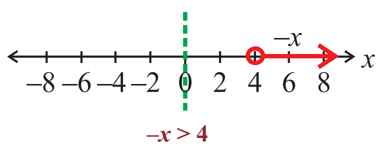
For example, $\mathbf{4 \times -1 = -4}$ and $\mathbf{-4 \times -1 = 4}$.

Check it out:
You could think of the negative scale as a reflection of the positive scale about zero on the number line.



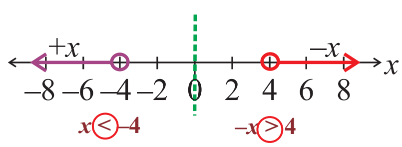
You can use this idea to understand what happens when you **multiply or divide an inequality by –1** on the number line.

For example, consider the inequality $-x > 4$. This is saying that some number, $-x$, can be anywhere in the region **greater than 4** on the number line.



You want to solve the inequality to find x, so you need to **divide both sides of the inequality by –1**.

Reflect the inequality about the **origin** of the number line to see what the solution looks like.



The inequality symbol is reflected too.

So if $-x$ is **greater than** 4, then x is **less than** –4.

This “reflection” idea works for all inequalities, so there's a rule:

> **When you multiply or divide by a negative number, always reverse the sign of the inequality.**

2 Teach (cont)

Universal access

The concept of reversing the sign when you multiply or divide by a negative number can be a hard one for students to understand.

If they are having trouble with it, get them to look at this example.

$$\begin{gathered} x > y \\ 0 > y - x \\ -y > -x \\ -x < -y \end{gathered}$$

It uses the rules of addition and subtraction of inequalities. But it has had the same effect as multiplying both sides by –1. So when you multiply by a negative, you need to reverse the sign.

If students are still having trouble remembering to reverse the sign, they can use this addition/subtraction method to solve inequalities instead. There's a worked example in the student margin on the next page.

Concept question

“Why don't we need to swap the sides of an equation over when we multiply both sides by a negative number?”

Unlike an inequality, both sides of an equation have the same value. That means it doesn't matter which side of the equation each expression is on. This is not the case when you're solving an inequality.

- **Advanced Learners**
  Challenge students to write a word problem that can be solved using a one-step inequality involving multiplication or division. For example, "Some bananas are shared between five people. Each person gets no more than six bananas. How many bananas are there altogether?" (If the original number of bananas is n, then $\frac{n}{5} \le 6 \Rightarrow n \le 30$.)
  Ask them to write a worked solution on a separate sheet. When they have done this, they should swap problems with a partner. Tell them to solve the problem they have been given, and then to check the answer with their partner.

2 Teach (cont)

Additional examples
Solve the following inequalities:

1) $-x > 5$ **$x < -5$**
2) $-2x < -4$ **$x > 2$**
3) $-\frac{1}{2}x > 5$ **$x < -10$**

Check it out:
If you're finding all this stuff really tricky, you can sometimes **avoid** multiplying and dividing by a negative altogether.
Take the inequality $-4x < 32$.
Add $+4x$ to both sides:
$-4x + 4x < 32 + 4x$.
So, $0 < 32 + 4x$
Now take 32 from both sides:
$-32 < 32 - 32 + 4x$
$-32 < 4x$
Now divide by 4:
$-8 < x$
This is your answer.
It's the same as $x > -8$.

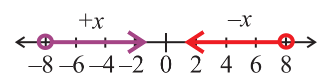
Example 3

Solve the inequality $-4x < 32$.

Solution

Divide both sides of the equation by -4. As you're dividing by a negative number, **reverse the symbol of inequality**.

$$-4x < 32$$
$$\div -4 \quad \frac{-4x}{-4} > \frac{32}{-4}$$
$$x > -8$$



$x > -8$ $\quad$ $-4x < 32$, $-x < 8$

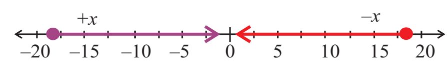
Example 4

Solve the inequality $\frac{x}{-6} \le 3$.

Solution

You need to **multiply** both sides of the equation by -6. As you're multiplying by a negative number, **reverse the symbol of inequality**.

$$\frac{x}{-6} \le 3$$
$$x \ge 3 \times -6$$
$$x \ge -18$$



$x \ge -18$ $\quad$ $-x \le 18$

Guided practice
Level 1: q5–8
Level 2: q5–10
Level 3: q5–12

Guided Practice

In Exercises 5–12, solve the inequality for the unknown.

5. $-4h > 24$ **$h < -6$** **6.** $-10y \ge -20$ **$y \le 2$** **7.** $\frac{r}{-5} < 25$ **$r > -125$** **8.** $\frac{z}{-32} \le -1$ **$z \ge 32$**

9. $\frac{x}{-2} > -6$ **$x < 12$** **10.** $-100x < -25$ **$x > 0.25$** **11.** $\frac{k}{-7} \le 3$ **$k \ge -21$** **12.** $-17d > 34$ **$d < -2$**

Independent practice
Level 1: q1–4
Level 2: q1–6, 9
Level 3: q1–9

Additional questions
Level 1: p456 q1–11, 14–15
Level 2: p456 q1–15
Level 3: p456 q1–17

Independent Practice

In Exercises 1–8, solve the inequality for the unknown.

1. $2g > 4$ **$g > 2$** **2.** $56b < 112$ **$b < 2$** **3.** $-5h < -15$ **$h > 3$** **4.** $-8x > 12$ **$x < -1.5$**

5. $2k \ge -6$ **$k \ge -3$** **6.** $-13p \le 39$ **$p \ge -3$** **7.** $\frac{x}{-4} \le 1.5$ **$x \ge -6$** **8.** $\frac{y}{-3} \ge 3.3$ **$y \le -9.9$**

9. Write and solve the inequality that says that twice the negative of a number is no more than 14. **$-2x \le 14$, $x \ge -7$**

Now try these:
Lesson 4.4.2 additional questions — p456

3 Homework

Homework Book — Lesson 4.4.2
Level 1: q1–3, 5–9
Level 2: q3–11
Level 3: q3–11

Round Up

So that's how to solve inequalities that involve multiplication and division. You've got to remember to switch the direction of the inequality symbol each time you multiply or divide by a negative number.

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
- Worksheet 27 — Inequalities

Solutions
For worked solutions see the Solution Guide

Lesson 4.4.3

Solving Two-Step Inequalities

In this Lesson, students review what two-step inequalities are, and learn how to solve them. This includes setting up and solving two-step inequalities from word problems.

Previous Study: In Section 1.3 students learned how to write one- and two-step inequalities. Earlier in this Section, students learned how to solve one-step inequalities.

Future Study: In Algebra I students will learn how to solve multistep problems involving linear inequalities in one variable, and provide justification for each step of their work.

Lesson 4.4.3

Solving Two-Step Inequalities

California Standards:
Algebra and Functions 4.1
Solve two-step linear equations and **inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.**

What it means for you:
You'll learn how to solve two-step linear inequalities.

Key words:
- inequality
- system of inequalities

Don't forget:
You learned how to set up two-step inequalities in Section 1.3. Now you're going to solve them too.

Don't forget:
You've used PEMDAS to remember the order of operations when evaluating expressions. Don't be tempted to use this when solving equations and inequalities. The calculation is often simpler if you add and subtract first so that you only have one term on each side of the inequality before multiplying and dividing.

So far in this Section you've learned how to solve one-step linear inequalities, and why you have to reverse the inequality whenever you multiply or divide by a negative number. Two-step inequalities follow the same rules, but you need to do two steps to solve them.

Two-Step Inequalities Have Two Different Operations

A two-step inequality contains **two different operations**. So you need to do **two steps** to **solve** the inequality.

$$\underbrace{2 \times x}_{\text{first operation}} \underbrace{+ 12}_{\text{second operation}} > 10$$

You need to get the variable by itself on one side of the inequality, so you must undo whatever has been done to it. It's usually best to undo **additions and subtractions first**, and **multiplications and divisions second**. That way, you only have to multiply or divide one term.

Example 1

Solve the inequality $2x + 12 > 10$.

Solution

- First **subtract** 12 from both sides of the inequality:

$$2x + 12 - 12 > 10 - 12$$
$$2x > -2$$

- Then **divide** both sides by 2:

$$2x \div 2 > -2 \div 2$$
$$\boldsymbol{x > -1}$$

Don't forget to **reverse the sign** when you multiply or divide by a **negative**.

Example 2

Solve the inequality $\frac{x}{-4} - 2 < 14$.

Solution

- First **add** 2 to both sides of the inequality:

$$\frac{x}{-4} < 16$$

- Then **multiply** both sides by –4, remembering to reverse the sign:

$$\boldsymbol{x > -64}$$

1 Get started

Resources:
- photocopies of the graph $y > 2x + 1$
- red and green pens

Warm-up questions:
- Lesson 4.4.3 sheet

2 Teach

Concept question
"If I'm going to solve the inequality $2x + 4 > 10$, what order would it be easiest to undo the operations in?"
Subtract 4, then divide by 2.

Concept question
"If I'm going to solve the inequality $2(x + 4) > 10$, what order would it be easiest to undo the operations in?"
Divide by 2, then subtract 4.

Common error
Students sometimes forget to reverse the inequality symbol when they multiply or divide by a negative number.

Remind them that whether it is a pure math problem or a real-life word problem they are answering, this is something that they always need to remember to do — or they won't get the right answer.

- **Strategic Learners**
  Ask everyone to write a sentence that describes a simple two-step inequality, such as, "I'm thinking of a number. If you double it and add one, the sum will be more than five." Write one yourself. Give out individual whiteboards. Read out your sentence. Ask students to write the inequality it describes and solve it. The first one to hold up a correct answer (in this case $x > 2$) gets to read out their sentence.

- **English Language Learners**
  Review the process of solving a two-step equation with the example $-3x + 5 = 17$. Ask students to identify the operations that have been applied to the variable, and choose the inverse operations they need to isolate it. Then give them the inequality $-3x + 5 > 17$. Have them solve it. Ask students to note the key differences between solving equations and solving inequalities in their notebooks.

2 Teach (cont)

Guided practice
Level 1: q1–3
Level 2: q1–4
Level 3: q1–6

Universal access
Give students a word problem involving a two-step inequality to solve. You could use the additional examples below.

Ask them to read through it carefully once. Then ask them to read through it carefully again, underlining important information in red, and circling important numbers.

Now have them use the information they have highlighted to set up an inequality, and solve it.

Remind them that this is a useful sifting strategy to adopt when approaching any word problem.

Additional example
A mechanic quoted the cost for repairing a car as a maximum of \$1200. The parts would cost her \$900, and the rate for labor would be \$30 an hour. Write an inequality that you could use to work out the maximum amount of time she needed to fix the car. Then solve it.

Let the number of hours = h
$900 + 30h \le 1200$
$30h \le 300$
$h \le 10$
So the mechanic needed a maximum of 10 hours to fix the car.

Don't forget:
You're reversing the sign of the inequality because you're dividing both sides by a negative number. Remember you need to reverse the sign whenever you multiply or divide by a negative number.

Example 3

Solve the inequality $-5x - 2 > 103$.

Solution

• First **add** 2 to both sides of the inequality:

$$-5x - 2 + 2 \le 103 + 2$$
$$-5x \le 105$$

• Then **divide** both sides by –5, remembering to reverse the sign:

$$x \ge 105 \div -5$$
$$\mathbf{x \ge -21}$$

Guided Practice

In Exercises 1–6, solve the inequality for the unknown.

1. $4c - 2 > 6$ $c > 2$ **2.** $-6z - 14 < -36$ $z > \frac{11}{3}$ **3.** $3x + 3 \le -18$ $x \le -7$

4. $\frac{z}{-32} + 18 \ge 2$ $z \le 512$ **5.** $\frac{r}{-5} - 12 > -6$ $r < -30$ **6.** $-12g + 4 < 12$ $g > -\frac{2}{3}$

Solve Real Problems with Inequalities

There are lots of **real-life problems** that involve inequalities. The key is in interpreting the question and coming up with a **sensible answer** in the **context of the question**.

Example 4

Two students decide to go to a restaurant for lunch. They order two drinks at \$2 each, then realize they only have a maximum of \$20 to spend between them.

If they want one meal each, what is the maximum price they can spend on each meal? Assume their meals cost the same amount.

Solution

First you have to **write this as an inequality**.

Call the price of each meal x. They want two equally priced meals, which is $2x$. The price of the meals plus the two drinks they have already bought must be no more than \$20.

So, $\mathbf{2x + 4 \le 20}$. This is your inequality.

Now you have to solve the inequality to find x, the price of each meal.

$2x + 4 \le 20$ **Subtract 4 from both sides**

$2x \le 16$ **Divide both sides by 2**

$\mathbf{x \le 8}$

So the maximum price of each meal is \$8.

Don't forget:
You've got to interpret your answer in the context of the question here. Look back at what x represents and what units you need to use in your answer.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Give students a two-step inequality involving two variables, like $y \geq 2x + 1$. Remind them that when they had an inequality with one variable, they could show its solutions on a number line. Ask them to discuss how they could show the solution set of an inequality with two variables. When they have done this, give everyone a copy of the graph of $y = 2x + 1$. Ask them to find four ordered pairs (x, y) that make the inequality $y \geq 2x + 1$ true, and plot them. Then point out that any point on or above the line will make the inequality true. Ask them to finish the graph by shading the area above the line, and writing that this is the solution set of the inequality $y \geq 2x + 1$.

Example 5

Joaquin goes to a fair. He buys an unlimited ticket that costs \$30 and allows him entry to all the rides that normally cost \$4 each.
The ticket also gives him one go on the coconut shy, which normally costs \$2. How many rides does Joaquin need to go on in order to have made buying the unlimited ticket worthwhile?

Solution

First you have to **write this as an inequality**.

Call the number of rides that Joaquin goes on x. So the amount that Joaquin would normally spend on the rides is \$4 × x, or $4x$.

For buying the ticket to have been worthwhile, the total value of the rides plus the value of the go on the coconut shy must be at least the price of the unlimited ticket.

So, $\mathbf{4x + 2 \geq 30}$.

Now **solve the inequality** to find the number of rides, x.

$4x \geq 28$ — **Subtract 2 from both sides**

$x \geq 7$ — **Divide both sides by 4**

Joaquin needs to go on at least 7 rides in order to get his money's worth.

Guided Practice

7. Anne-Marie is saving up to buy a concert ticket by babysitting for \$5 an hour. Anne-Marie owes \$15 to her mother already, and the concert ticket costs \$25. How many hours does she need to work in order to be able to buy the ticket and pay her mother?
Show how you reached your solution using an inequality. **At least 8 — see below.**

Independent Practice

In Exercises 1–6, solve the inequality for the unknown.

1. $4x - 3 > 5$ — $x > 2$
2. $7x + 12 < 19$ — $x < 1$
3. $-6x - 6 > 6$ — $x < -2$
4. $0.5g + 3 \geq 6$ — $g \geq 6$
5. $\frac{x}{-5} - 5 < 15$ — $x > -100$
6. $\frac{x}{8} + 14 \leq -2$ — $x \leq -128$

7. Juan runs a salsa class on a Wednesday night. Entrance to his class is \$3 each. If the venue costs \$50 and the music equipment costs \$10 to hire, what is the minimum number of people needed to attend the class in order for Juan to make his money back on the night? **A minimum of 20 people.**

Don't forget:
Remember that multiplying by 0.5 is the same as dividing by 2. If in doubt, convert 0.5 into a fraction first.

Now try these:
Lesson 4.4.3 additional questions — p456

Round Up

The trick with real-life inequality problems is understanding what the question is telling you. Try to break the question down into parts, and work out what each part means in math terms.

2 Teach (cont)

Common error
For every real-life question they do, students need to think about whether their answer is sensible in the context of the question, and whether it has the right units attached to it.

If they forget, remind them that they need to check these things after every question they answer.

Guided practice
Level 1: q7
Level 2: q7
Level 3: q7

Independent practice
Level 1: q1–2, 7
Level 2: q1–4, 7
Level 3: q1–7

Additional questions
Level 1: p456 q1–7, 9–10
Level 2: p456 q1–12
Level 3: p456 q1–15

3 Homework

Homework Book — Lesson 4.4.3
Level 1: q1–5, 9, 10
Level 2: q1–11
Level 3: q1–11

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
- Worksheet 27 — Inequalities

Solutions
For worked solutions see the Solution Guide

Guided Practice

7. Let h = number of hours Anne-Marie needs to work.

$5h - 15 \geq 25$
$5h \geq 40$
$h \geq 8$ She must work for at least 8 hours.

Investigation — Choosing a Route

Purpose of the Investigation

This Investigation provides students with the opportunity to use their knowledge of rates, dimensional analysis, measurements and the speed-distance-time formula to make decisions. Students are asked to compare several routes and to pick the route that they feel is the most suitable in terms of time, distance traveled, and fuel consumption.

Resources

• local town/city maps

Strategic & EL Learners

Strategic learners may have difficulty analyzing all of the routes.
To scale down the Investigation, give students just one or two routes to examine.

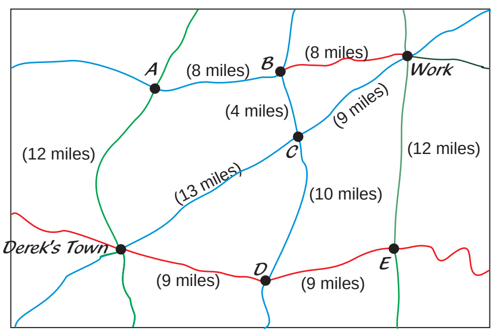
This is also a good project for small groups. Each student can take part of the task, and then the group members can compare results.
This approach provides support for both strategic learners and English language learners.

Investigation Notes on p264 B-C

Chapter 4 Investigation
Choosing a Route

Rates compare one quantity to another. There are rates involved in driving to places — roads have speed limits in miles per hour, and when driving at a steady speed, you travel a particular number of miles per gallon of gas. "Per" just means "for each."

Derek is starting a new job next week in a different town. He wants to get to work **quickly**, but also doesn't want to put a lot of miles on his car. He looks at a map to examine all of the possible routes.



Scale:

0 5 miles

Key:

55 mph road (18 miles per gallon)

40 mph road (20 miles per gallon)

30 mph road (25 miles per gallon)

Find the **shortest** and the **quickest** routes that Derek could take.
Decide which route you think is the **best overall**. Explain your reasoning.

Things to think about:
The best route should be a **balance** between time and distance. A route that is 15 miles long and takes 15 minutes might be less desirable than a route that is 10 miles long and takes 20 minutes. Compare different scenarios before picking the best route.

Extensions

1) Derek is concerned about the effect of burning gas on climate change.
Determine **how many gallons of gas** are used driving the route you have chosen.
Are there any other routes that would **reduce** the amount of gas he uses?

2) Determine the **minimum** yearly cost of Derek's commute.
Assume he drives to work and back home again on 250 days each year, and that the average price for a gallon of gas is $2.80.

Open-ended Extensions

1) Using a map of your town or city, examine different routes from your school to a major town landmark. Which do you think is the best route? Consider factors such as distance, speed limit and time of travel.

2) Convert the scale, distances, and speed limits on this map to the metric system.

Round Up

Real-life decisions are often not straightforward. There's often no perfect right answer — you have to decide what's most important to you, or find the best compromise.

Investigation — Choosing a Route

Mathematical Background

The formula $\text{speed} = \frac{\text{distance}}{\text{time}}$ can be used to calculate speed if you know the distance traveled and the time taken.

The formula can also be rearranged to distance = speed × time, or, $\text{time} = \frac{\text{distance}}{\text{speed}}$.

Speed measurements are **rates**. They tell you **how far** something travels **per unit of time**. They can have a variety of units — miles per hour and kilometers per hour are the ones most commonly used to describe driving speeds.

You can convert from one unit to another by setting up a proportion (see p242) or by using a conversion fraction (see p251). When you are converting between speed units, you often have to do the conversion in two steps — one to convert the distance part of the unit, and another to convert the time part of the unit. However, when you're converting between kilometers per hour and miles per hour, only one conversion is needed because the time part of the unit is hours in both cases.

Fuel efficiency is a rate which is often given in miles per gallon. This is the distance that can be driven on each gallon of gas. It usually varies according to how fast you drive, so different fuel efficiencies are given for different speeds. There are other factors that affect fuel efficiency, such as the engine size, road surface and wind speed and direction.

The reasonableness of conversions and other calculations can be checked using dimensional analysis. See page 248.

Approaching the Investigation

Students need to determine the shortest route from Derek's home to his work. Each possible route should be identified, and its length calculated (the length of each section of road is marked on the map). This is shown in the table below left.

Once the distance has been calculated, then the speed-distance-time formula can be used to calculate the time needed for each route. Students should assume that Derek travels at the speed limit on each road. It's easier to calculate the time needed to drive each section of road, then add up the necessary times for each route.

Route	Route description	Distance (miles)	Time (minutes)
1	Home — A — B — Work	12 + 8 + 8 = **28**	13 + 16 + 12 = **41**
2	Home — C — B — Work	13 + 4 + 8 = **25**	26 + 8 + 12 = **46**
3	Home — C — Work	13 + 9 = **22**	26 + 18 = **44**
4	Home — D — C — Work	9 + 10 + 9 = **28**	13.5 + 20 + 18 = **51.5**
5	Home — D — E — Work	9 + 9 + 12 = **30**	13.5 + 13.5 + 13 = **40**

Road	Distance (miles)	Speed (mph)	Time [= distance (miles) ÷ speed (mph)]
Home — A	12	55	0.22 hours = 13 minutes
A — B	8	30	0.27 hours = 16 minutes
B — Work	8	40	0.20 hours = 12 minutes
Home — C	13	30	0.43 hours = 26 minutes
C — B	4	30	0.13 hours = 8 minutes
C — Work	9	30	0.3 hours = 18 minutes
Home — D	9	40	0.225 hours = 13.5 minutes
D — C	10	30	0.33 hours = 20 minutes
C — Work	9	30	0.3 hours = 18 minutes
D — E	9	40	0.225 hours = 13.5 minutes
E — Work	12	55	0.22 hours = 13 minutes

The students will get a decimal time in hours. This will be easier to work with when it's converted to minutes. To convert the time in hours to a time in minutes, multiply it by 60.

The proportion below shows why you multiply by 60. It uses the conversion ratio 60 minutes : 1 hour.

To convert 0.2 hours to minutes: $\frac{60}{1} = \frac{x}{0.2}$

$\Rightarrow 60 \times 0.2 = x \times 1$

$\Rightarrow x = 12$ minutes

Route 3 is the shortest at 22 miles. Route 5 is the quickest at 40 minutes.

Students then need to weigh up which route they think is best. Different answers are possible, but students should be able to justify the route they have chosen. For example, they may feel that Route 3 is best because it will **save 3 miles**, and only add on an **extra 4 minutes** to the shortest time possible.

Investigation — Choosing a Route

Extensions

1) In this Extension, students must work out how many gallons of gas will be used each day driving their chosen route. They must also check whether a different route would require less fuel. Once again, a good approach is to calculate the number of gallons used for each section of road, then to add the fuel consumption for the relevant sections together.

   To find the number of gallons used to drive each road section, divide the distance (in miles) by the appropriate miles per gallon rate. Students should be encouraged to check their calculations using dimensional analysis.
   For example:

$$12 \text{ miles} \div \frac{18 \text{ miles}}{1 \text{ gallon}} = 12 \text{ miles} \times \frac{1 \text{ gallon}}{18 \text{ miles}} = \frac{12 \cancel{\text{ miles}} \times 1 \text{ gallon}}{18 \cancel{\text{ miles}}} = \frac{12}{18} \times 1 \text{ gallon} = \mathbf{0.67 \text{ gallons}}$$

Road	Distance (miles)	Miles per gallon	Gallons used
Home — A	12	18	0.67
A — B	8	25	0.32
B — Work	8	20	0.4
Home — C	13	25	0.52
C — B	4	25	0.16
C — Work	9	25	0.36
Home — D	9	20	0.45
D — C	10	25	0.4
C — Work	9	25	0.36
D — E	9	20	0.45
E — Work	12	18	0.67

Route	Route description	Gallons used (one way)
1	Home — A — B — Work	0.67 + 0.32 + 0.4 = **1.39**
2	Home — C — B — Work	0.52 + 0.16 + 0.4 = **1.08**
3	Home — C — Work	0.52 + 0.36 = **0.88**
4	Home — D — C — Work	0.45 + 0.4 + 0.36 = **1.21**
5	Home — D — E — Work	0.45 + 0.45 + 0.67 = **1.57**

Route 3 uses the least gas. It takes 0.88 gallons.

2) Derek uses 0.88 × 2 = 1.76 gallons each day on his commute to and from work.
   Over 250 days this will be 1.76 × 250 = 440 gallons.
   If the cost per gallon is $2.80, Derek will spend 440 × $2.80 = **$1232 per year** on gas for commuting.

Open-Ended Extensions

1) This open-ended Extension is intended to encourage students to take their knowledge and apply it to their everyday life. Students will have the benefit of knowing their town and what special circumstances are involved in getting from one place to another. For example, in a congested area, the actual speeds cars travel might be much lower than the posted speed limits. Also, certain routes may be best avoided at certain times of day.

2) In the second open-ended Extension, students review how to convert units of distance and speed. They should use the methods presented in Chapter 4. They should check their conversions using dimensional analysis.

Chapter 5

Powers

How Chapter 5 fits into the K-12 curriculum

Section 5.1 — Operations on Powers		
Section 5.1 covers Number Sense 2.1, 2.3, Algebra and Functions 2.1 **Objective: To multiply and divide with powers, and to apply exponent rules to fractions**		
Previous Study In Section 2.5 students learned that powers are repeated multiplications. They were introduced to base and exponent notation for representing powers.	**This Section** Students begin by learning how to multiply powers, and then go on to divide powers. Finally, students extend the use of the exponent rules for multiplication and division of fractions.	**Future Study** In Section 5.2, students will learn the meaning of negative exponents and will manipulate expressions containing them. In Algebra I students will simplify expressions involving fractional powers.

Section 5.2 — Negative Powers and Scientific Notation		
Section 5.2 covers Number Sense 1.1, 1.2, 2.1, Algebra and Functions 2.1 **Objective: To understand negative exponents and to understand and write numbers in scientific notation**		
Previous Study In Sections 2.4 and 5.1, students simplified expressions with positive integer exponents and rational bases. Also in Section 2.4, they learned how to write large and small numbers using scientific notation.	**This Section** Students learn the meaning of zero and negative exponents, and then apply exponents rules to negative exponents. Then they go on to review scientific notation, and use this to compare very small and very large numbers.	**Future Study** In Algebra I students will extend their knowledge of powers to fractional exponents and simplify expression involving them. Students will encounter scientific notation at various stages through their study of science, particularly in Physics.

Section 5.3 — Monomials		
Section 5.3 covers Algebra and Functions 1.4, 2.2 **Objective: To multiply and divide monomials and to calculate powers and square roots of monomials**		
Previous Study In Section 5.1, students learned the rules for multiplying and dividing exponents. They were introduced to the concepts of square roots and absolute value in Chapter 2.	**This Section** Students are introduced to the terms monomial and coefficient, and then learn to multiply and divide monomials. They then learn the rule for raising a power to a power, and are finally shown how to find the square root of a monomial.	**Future Study** In Algebra I, students will be introduced to polynomials, and will learn to add, subtract, multiply, and divide polynomials. They will also learn the multiplicative and division properties of square roots.

Section 5.4 — Graphing Non-Linear Functions		
Section 5.4 covers Algebra and Function 3.1, Mathematical Reasoning 2.3, 2.5 **Objective: To draw graphs of the form nx^2 and nx^3**		
Previous Study In Section 4.1, students were introduced to graphing linear equations. They also solved systems of equations by finding the intersection point, and computed the slopes of lines.	**This Section** Students are introduced to graphs of the form nx^2 for positive and then negative values of n, and then plot corresponding graphs. They then go on to deal with graphs of the form nx^3.	**Future Study** In Algebra I, students learn methods for solving quadratic equations algebraically. If students go on to study Algebra II, they will investigate the equations and graphs of circles, ellipses, and hyperbolas.

Pacing Guide – Chapter 5

40- to 50-Minute Class Periods

If your class periods are 40-50 minutes, we recommend allowing **19 days** for teaching Chapter 5.
As well as the **14 days of basic teaching**, you have **5 days** remaining to allocate 5 of the 7 optional activities (to be delivered at any appropriate point during the Chapter).

The table shows the 14 teaching days as well as all of the **optional days** you may choose for Chapter 5, in the order we recommend.

Day	Lesson	Description
Section 5.1 — Operations on Powers		
1	5.1.1	Multiplying With Powers
2	5.1.2	Dividing With Powers
3	5.1.3	Fractions With Powers
Optional		*Assessment Test — Section 5.1*
Section 5.2 — Negative Powers and Scientific Notation		
4	5.2.1	Negative and Zero Exponents
5	5.2.2	Using Negative Exponents
6	5.2.3	Scientific Notation
7	5.2.4	Comparing Numbers in Scientific Notation
Optional		*Assessment Test — Section 5.2*
Section 5.3 — Monomials		
Optional		*Exploration — Monomials*
8	5.3.1	Multiplying Monomials
9	5.3.2	Dividing Monomials
10	5.3.3	Powers of Monomials
11	5.3.4	Square Roots of Monomials
Optional		*Assessment Test — Section 5.3*
Section 5.4 — Graphing Non-Linear Functions		
Optional		*Exploration — The Pendulum*
12	5.4.1	Graphing $y = nx^2$
13	5.4.2	More Graphs of $y = nx^2$
14	5.4.3	Graphing $y = nx^3$
Optional		*Assessment Test — Section 5.4*
Chapter Investigation		
Optional		*Investigation — The Solar System*

Accelerating and Decelerating

- To **accelerate** Chapter 5, allocate fewer than 5 days to the optional material, or cover the core Lessons in less than 14 days, as students may be familiar with much of the content from earlier grades. This will give you extra days to allocate to other Chapters.
  Note that you may use the remaining optional days at the end of the 160-day course.
- To **decelerate** Chapter 5, consider allocating more than 5 days to the optional Assessment Tests, Section Explorations, or Chapter Investigation, or spend longer teaching some Lessons. Also consider preparing students for difficult Lessons by reviewing previous coverage of math topics on related Skills Review Worksheets.
  Note that decelerating Chapter 5 will result in fewer days being available for teaching other Chapters.

90-Minute Class Periods

If you are following a block schedule with 90-minute class periods, we recommend allowing **9.5 days** for teaching Chapter 5.
The basic teaching material will take up **7 days**, and you can allocate the remaining **2.5 days** to the **optional material**.
To accelerate or decelerate a block schedule, follow the same advice as given above.

Lesson 5.1.1

Multiplying with Powers

In this Lesson, students learn that powers with the same base can be multiplied by adding the exponents. They use the rule to solve multiplication problems mentally.

Previous Study: In Section 2.5 students learned that powers are repeated multiplications. They were introduced to base and exponent notation for representing powers.

Future Study: In Algebra I students will be expected to know and use all the rules for exponents. They will apply the exponent rules to any rational numbers.

1 Get started

Resources:
- vocabulary cards for English language learners

Teacher Resources CD-ROM
- Number and Operation Tiles

Warm-up questions:
- Lesson 5.1.1 sheet

2 Teach

Universal access

Review base and exponent form by giving examples on the board. Ask students to identify the base and exponent in each case.

For example,

$5^2 = 5 \times 5$ — base is 5, exponent is 2
$8^3 = 8 \times 8 \times 8$ — base 8, exponent 3
$3 \times 3 \times 3 \times 3 \times 3 = 3^5$
$3 \times 3 \times 3 \times 5 \times 5 = 3^3 \times 5^2$

Give negative number base examples to show the use of parentheses.

$(–8)^2 = –8 \times –8 = 64$
$–8^2 = –(8^2) = –(8 \times 8) = –64$

Point out to students that the exponent is "attached" only to the term it comes directly after.

Guided practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1–4

Universal access

Use repeated multiplication expressions of powers to "discover" the rule for multiplying powers. Go through several worked examples and ask the students if they can spot a pattern. Direct their attention to the exponents if necessary.

$4^3 \times 4^2$
$= (4 \times 4 \times 4) \times (4 \times 4)$
$= (4 \times 4 \times 4 \times 4 \times 4)$
$= 4^5$ (exponents: 3 + 2 = 5)

$3^2 \times 3^5$
$= (3 \times 3) \times (3 \times 3 \times 3 \times 3 \times 3)$
$= (3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3)$
$= 3^7$ (exponents: 2 + 5 = 7)

$6^2 \times 6^2$
$= (6 \times 6) \times (6 \times 6)$
$= (6 \times 6 \times 6 \times 6)$
$= 6^4$ (exponents: 2 + 2 = 4)

Lesson 5.1.1

California Standards:

Number Sense 2.1
Understand negative whole-number exponents. **Multiply** and divide **expressions involving exponents with a common base.**

Number Sense 2.3
Multiply, divide, **and simplify rational numbers by using exponent rules.**

Algebra and Functions 2.1
Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. **Simplify and evaluate expressions that include exponents.**

What it means for you:
You'll see how to use a rule that makes multiplying powers easier.

Key words:
- power
- base
- exponent

Check it out:
The variables a, m, and n could stand for any numbers at all here. Whatever numbers they are, the equation will still be true.

Section 5.1
Multiplying with Powers

When you have to multiply powers together, like $5^3 \cdot 5^4$, there's a rule you can use to simplify the calculation. But it only works with powers that have the same base.

A Power is a Repeated Multiplication

In Chapter 2 you saw that a power is a **product** that results from **repeatedly multiplying** a number by itself.

You can write a power in **base and exponent form**.

The base is the number that is being repeated as a factor in the multiplication. $\Rightarrow 5^2 \Leftarrow$ The exponent tells you how many times the base is repeated as a factor in the multiplication.

For example $7 \cdot 7 = 7^2$, and $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$.

Guided Practice

Write the expressions in Exercises 1–4 in base and exponent form.

1. $3 \cdot 3$ 3^2
2. $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$ 11^6
3. $k \cdot k \cdot k \cdot k$ k^4
4. $(–5) \cdot (–5) \cdot (–5) \cdot (–5)$ $(–5)^4$

The Multiplication of Powers Rule

Look at the multiplication $2^2 \cdot 2^4$. When you write it out it looks like this:

$$\underbrace{2 \cdot 2}_{\text{2 factors}} \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{\text{4 factors}} = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{6 factors}}$$

The solution is also a **repeated multiplication**. 2 is repeated as a factor six times, so it's the same as writing 2^6.

That means you can write $2^2 \cdot 2^4 = 2^6$.

In the multiplication expression the two exponents are 2 and 4, and in the solution the exponent is 6. If you **add** the exponents of the original powers together you get the exponent of the solution — the base stays the same.

Multiplication of Powers Rule:

When you are multiplying two powers with the same base, add their exponents to give you the exponent of the answer.

$$a^m \cdot a^n = a^{(m+n)}$$

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
The Universal access activity on the previous page helps students understand why the multiplying powers rule works. This activity could be adapted by using Number and Operation Tiles from the **Teacher Resources CD-ROM** to represent the factors of each power. For example, $5^2 \times 5^3$ would involve combining a two-card chain and three-card chain to make a five-card chain. This could also be used to show why the rule doesn't work when the bases are different.

- **English Language Learners**
Make vocabulary cards to cover the vocabulary of the Lesson — power, base, exponent, factor — and the rule for multiplying powers. Cards could be made for both home study and in-class review.

Only Use the Rule If the Bases are the Same

It's important to remember that this rule **only** works with powers that have the same base. You **can't** use it on two powers with different bases.

- You **could** use the rule to simplify $5^3 \cdot 5^4$ as the bases are the same.
- You **couldn't** use it to simplify $3^5 \cdot 4^5$ because the bases are different.

Example 1

What is $3^2 \cdot 3^6$? Give your answer in base and exponent form.

Solution

You could write the multiplication out in full

$3^2 \cdot 3^6$

$= (3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$ 3 is repeated as a factor 8 times in the multiplication.

$= 3^8$

But these two powers have the same base. So you can use the multiplication of powers rule.

$3^2 \cdot 3^6$

$= 3^{(2+6)} = 3^8$

Don't forget:
$(-5)^2$ is not the same as $-(5^2)$.
$(-5)^2 = (-5) \cdot (-5) = 25$
$-(5^2) = -(5 \cdot 5) = -25$

Example 2

What is $(-5)^{12} \cdot (-5)^{14}$? Give your answer in base and exponent form.

Solution

The powers have the same base. Use the multiplication of powers rule.

$(-5)^{12} \cdot (-5)^{14}$

$= (-5)^{(12+14)} = (-5)^{26}$

(This is the same as 5^{26}, since the exponent is even.)

Guided Practice

Evaluate the expressions in Exercises 5–12. Use the multiplication of powers rule and give your answers in base and exponent form.

5. $2^2 \cdot 2^2$ 2^4 **6.** $9^{10} \cdot 9^8$ 9^{18}

7. $6^{104} \cdot 6^{62}$ 6^{166} **8.** $(-7)^7 \cdot (-7)^3$ $(-7)^{10}$

9. $10^8 \cdot 10^1$ 10^9 **10.** $5^6 \cdot 5$ 5^7

11. $k^8 \cdot k^5$ k^{13} **12.** $(-t)^{14} \cdot (-t)^{17}$ $(-t)^{31}$

Don't forget:
Any number to the power 1 is just the number itself. So an expression like $2^5 \cdot 2$ can also be written as $2^5 \cdot 2^1$.

2 Teach (cont)

Common errors

For students who simply memorize the rule there are two common mistakes.

1. On seeing the multiplication sign, students tend to multiply the exponents, rather than add them. For example, $5^4 \times 5^2 = 5^8$

2. Students may also apply operations to the base, rather than just the exponents. For example, $5^4 \times 5^2 = 10^6$ or $5^4 \times 5^2 = 25^8$

Concept question

"Explain why each statement below is incorrect."

1. $5^3 = 5 \times 3 = 15$
5^3 means $5 \times 5 \times 5$, not 5×3.

2. $-3^2 = (-3) \times (-3) = 9$
-3^2 means $-(3^2)$, not $(-3)^2$.
So the correct answer is $-(3 \times 3) = -9$.

3. $-x^2 = 16$ when $x = -4$
$-x^2$ means $-(x^2)$, not $(-x)^2$.
Correct answer: $-x^2 = -(-4)^2$
$= -(-4 \times -4) = -(16) = -16$

4. $15 + (-3)^2 = 15 + -9 = 6$
$(-3)^2 = 9$, not -9. Answer is $15 + 9 = 24$.

5. $x^2 + x^5 = x^{(2+5)} = x^7$
The multiplication of powers rule does not apply here as the powers are being added, not multiplied.

Guided practice

Level 1: q5–7
Level 2: q5–10
Level 3: q5–12

Solutions

For worked solutions see the Solution Guide

● **Advanced Learners**
Many students may initially consider the mental method below too complicated to be worthwhile. Give students various integer multiplications to practice the method until they are more comfortable with it. Ask them to write out the common powers of 2, 3, 5, and so on, memorizing any they don't know. Also, ask students to consider whether the method will work for any integer multiplication.

2 Teach (cont)

Additional examples
Simplify the following.

1. $5^4 \times 5^2$
$= 5^{(4+2)} = 5^6$
2. $b^2 \times b^7$
$= b^{(2+7)} = b^9$
3. $12^c \times 12^d$
$= 12^{(c+d)}$
4. $(-3)^3 \times (-3)^3$
$= (-3)^{(3+3)} = (-3)^6 = 3^6$

Guided practice
Level 1: q13–14
Level 2: q13–15
Level 3: q13–16

Independent practice
Level 1: q1–7
Level 2: q1–11
Level 3: q4–15

Additional questions
Level 1: p457 q1–6, 11–13
Level 2: p457 q4–16
Level 3: p457 q4–16

3 Homework

Homework Book — Lesson 5.1.1
Level 1: q1a–c, 2a–c, 3, 5, 7
Level 2: q1–5, 7–10
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 13 — Powers

Multiplication of Powers Can Help with Mental Math

Sometimes changing numbers into base and exponent form and using the multiplication of powers rule can make **mental math** easier.

Check it out:
Write yourself out a table showing the first few powers of all the numbers up to ten. Use it to help with problems like Example 3. You'll begin to remember some of the more common powers — then you'll know them when you see them again.

Example 3

What is 16 • 8?

Solution
16 and 8 are both powers of 2, so you can rewrite the problem in base and exponent form:
$16 \bullet 8 = 2^4 \bullet 2^3$.

Now use the multiplication of powers rule:
$2^4 \bullet 2^3 = 2^{(4+3)} = 2^7$.
$2^7 = 128$. So $16 \bullet 8 =$ **128**.

If you knew lots of powers of 2, this could make your mental math faster.

Guided Practice

Evaluate the expressions in Exercises 13–16 using the multiplication of powers rule.

13. 9 • 27 **243 (3^5)**
14. 10 • 100,000 **1,000,000 (10^6)**
15. 4 • 64 **256 (4^4)**
16. 125 • 25 **3125 (5^5)**

Independent Practice

Write the expressions in Exercises 1–4 in base and exponent form.

1. 5 • 5 **5^2**
2. 17 • 17 • 17 • 17 • 17 • 17 **17^6**
3. $q \bullet q \bullet q \bullet q \bullet q$ **q^5**
4. $-y \bullet -y \bullet -y$ **$(-y)^3$**

5. Can you use the multiplication of powers rule to evaluate $8^3 \bullet 9^3$? Explain your answer. **No: you can't use the multiplication of powers rule because the bases are different.**

Evaluate the expressions in Exercises 6–13. Use the multiplication of powers rule and give your answers in base and exponent form.

6. $5^7 \bullet 5^7$ **5^{14}**
7. $11^{26} \bullet 11^9$ **11^{35}**
8. $(-15)^3 \bullet (-15)^5$ **$(-15)^8 = 15^8$**
9. $(-23)^{11} \bullet (-23)^{17}$ **$(-23)^{28} = 23^{28}$**
10. $9^5 \bullet 9$ **9^6**
11. $h^5 \bullet h^{10}$ **h^{15}**
12. $(-b)^9 \bullet (-b)^{11}$ **$(-b)^{20} = b^{20}$**
13. $a^x \bullet a^y$ **$a^{(x+y)}$**

14. A piece of land is 2^6 feet wide and 2^7 feet long. What is the area of the piece of land? Give your answer as a power in base and exponent form. Then evaluate the power. **2^{13} feet^2, 8192 feet^2.**

15. Evaluate 81 times 27 by converting the numbers to powers of three. **3^7 or 2187**

Now try these:
Lesson 5.1.1 additional questions — p457

Round Up

Using the multiplication of powers rule makes multiplying powers with the same base much easier. Just add the exponents together, and you'll get the exponent that goes with the answer.

Solutions
For worked solutions see the Solution Guide

Lesson 5.1.2

Dividing with Powers

In this Lesson, students learn how powers with the same base can be divided by subtracting the exponents. This is linked to simplifying fractions by canceling factors. Students then use the division of powers rule to solve division problems mentally.

Previous Study: In the previous Lesson students learned the exponent rule for multiplying powers. In grade 5 and Section 2.1 of this book, students reduced fractions by canceling factors in the numerator and denominator.

Future Study: In the next Lesson the rules for multiplying and dividing powers will be used on terms that have fractional bases.

Lesson 5.1.2

Dividing with Powers

California Standards:

Number Sense 2.1
Understand negative whole-number exponents. Multiply and **divide expressions involving exponents with a common base.**

Number Sense 2.3
Multiply, **divide, and simplify rational numbers by using exponent rules.**

What it means for you:

You'll learn about a rule that makes dividing powers easier.

Key words:

- power
- base
- exponent

Don't forget:

You can rewrite any division problem as a fraction.

$x \div y$ is the same as $\frac{x}{y}$.

So $x^n \div y^m$ is the same as $\frac{x^n}{y^m}$.

Check it out:

The variables m and n could stand for any numbers, and a could stand for any number except zero.
Whatever numbers they are, the equation will still be true.

In the last Lesson you saw how you can use the multiplication of powers rule to help simplify expressions with powers in them. There's a similar rule to use when you're dividing powers with the same base.

The Division of Powers Rule

Look at the division $2^7 \div 2^4$.
If you write it out as a **fraction** it looks like this: $\frac{2^7}{2^4}$

Now write out the powers in **expanded form**: $\frac{2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2}{2\cdot2\cdot2\cdot2}$

If you do the same thing to both the top and bottom of the fraction you don't change its value — you create an **equivalent fraction**. So you can cancel four twos from the numerator with four twos from the denominator.

$$\frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot2\cdot2\cdot2}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot\cancel{2}} = 2\cdot2\cdot2 = 2^3$$

Now you can say that $2^7 \div 2^4 = 2^3$.

In the division expression the two exponents are 7 and 4, and in the solution the exponent is 3.
When you **subtract** the exponent of the denominator from the exponent of the numerator, you get the exponent of the solution. The base stays the same.

This is called the **division of powers rule**:

> **Division of powers rule:**
> When you are dividing two powers with the same base, subtract the second exponent from the first to give you the exponent of the answer.
> $$a^m \div a^n = a^{(m-n)}$$

This Rule Only Works for the Same Bases

Just like with the multiplication of powers rule, it's important to remember that this rule **only** works with powers that have the same base. You **can't** use it on two powers with different bases.

- You **could** use the rule to simplify $5^3 \div 5^4$ as the bases are the same.
- You **couldn't** use it to simplify $3^5 \div 4^5$ because the bases are different.

1 Get started

Resources:
- vocabulary cards

Teacher Resources CD-ROM
- Number and Operation Tiles

Warm-up questions:
- Lesson 5.1.2 sheet

2 Teach

Universal access
Use repeated multiplication expressions of powers and division in fraction format to "discover" the rule for dividing with powers. Do several worked examples and ask students if they can spot a pattern. Direct them to the exponents if necessary.

$$5^4 \div 5^2 = \frac{5\times5\times\cancel{5}\times\cancel{5}}{\cancel{5}\times\cancel{5}} = 5^2$$
(exponents: 4 – 2 = 2)

$$8^7 \div 8^3 = \frac{8\times8\times8\times8\times\cancel{8}\times\cancel{8}\times\cancel{8}}{\cancel{8}\times\cancel{8}\times\cancel{8}} = 8^4$$
(exponents: 7 – 3 = 4)

$$4^3 \div 4^2 = \frac{4\times\cancel{4}\times\cancel{4}}{\cancel{4}\times\cancel{4}} = 4$$
(exponents: 3 – 2 = 1)

Universal access
Students may find the canceling process easier to remember if the fraction is modeled using digit cards and a ruler or pencil for the fraction bar. The fraction can be canceled down by removing matching cards from the top and bottom of the fraction.

With this approach, there are several misconceptions to avoid: it may not be clear to the students that the numbers on the top and bottom are multiplied together. Also, when all the factors are canceled from the denominator, students may believe that the denominator becomes 0, rather than 1. This can be dealt with by explaining that canceling is actually dividing the number from the top and bottom of the fraction.

- **Strategic Learners**
The Universal access activities on the previous page both provide useful learning strategies. The first activity helps students understand why the dividing rule actually works, which will make it easier for them to remember. The second activity involves the use of number cards to better visualize the process of canceling fractions.

- **English Language Learners**
Make vocabulary cards giving the rule for dividing powers with the same base. Include some worked examples, and on these label the key words from the Lesson — power, base, exponent, factors. Make personal cards for home study and matching cards for in-class review.

2 Teach (cont)

Concept questions

"Explain why each of these statements is wrong."

1. $6^8 \div 6^4 = 6^{(8 \div 4)} = 6^2$
The exponents should be subtracted, not divided.

2. $8^5 \div 2^3 = 8^{(5-3)} = 8^2$
The division of powers rule requires the bases to be the same. It doesn't apply here.

3. $6^3 \div 6^3 = 6^{(3-3)} = 6^0 = 0$
Any number raised to 0 is 1, not 0. So the answer should read "$= 6^0 = 1$."

4. $a^9 \div (a^5 \div a^3) = a^{(9-5-3)} = a^1 = a$
The parentheses should be evaluated first. Correct answer is "$= a^9 \div a^2 = a^7$."

Common error

The division sign can lead students to divide the exponents rather than subtract them.

For example, $5^{12} \div 5^4 = 5^3$

Encouraging students to visualize the expanded powers can help to avoid this error.

Additional examples

Simplify the following.

1. $3^6 \div 3^2$

$= 3^{(6-2)} = 3^4$

2. $g^8 \div g^3$

$= g^{(8-3)} = g^5$

3. $4^v \div 4^w$

$= 4^{(v-w)}$

4. $a^b \div a^{2b}$

$= a^{-b}$

Guided practice

Level 1: q1–3
Level 2: q1–6
Level 3: q1–8

Don't forget:

If you do the same thing to the top and bottom of a fraction its value stays the same. This means that if you have the same factor in the multiplication expressions on the top and bottom of a fraction you can cancel them with each other. It's the same as dividing the numerator and denominator by the same amount.

For example: $\frac{x \bullet y}{x \bullet z} = \frac{y}{z}$

In Example 1 you can cancel three fourteens from the top with three fourteens from the bottom.

Example 1

What is $14^6 \div 14^3$? Give your answer in base and exponent form.

Solution

You could write the division out in full

$$14^6 \div 14^3$$
$$= \frac{14 \bullet 14 \bullet 14 \bullet 14 \bullet 14 \bullet 14}{14 \bullet 14 \bullet 14}$$
$$= \frac{\cancel{14} \bullet \cancel{14} \bullet \cancel{14} \bullet 14 \bullet 14 \bullet 14}{\cancel{14} \bullet \cancel{14} \bullet \cancel{14}} = 14^3$$

But these two powers have the same base. So you can use the division of powers rule.

$14^6 \div 14^3$
$= 14^{(6-3)} = 14^3$

Example 2

What is $(-5)^{18} \div (-5)^{10}$? Give your answer in base and exponent form.

Solution

The powers have the same base. Use the division of powers rule.

$(-5)^{18} \div (-5)^{10}$
$= -5^{(18-10)} = (-5)^8$ or 5^8

Check it out:

The order that you do the subtraction in is very important. For example:

$5^5 \div 5^3 = 5^2 = 25$

$5^3 \div 5^5 = 5^{-2} = \frac{1}{25} = 0.04$

The answers are different.

You'll see more about what a negative power means in Section 5.2.

When you're using the division of powers rule you must **always** remember to subtract the exponent of the **second power** from the exponent of the **first power** — never the other way around.

The commutative property doesn't apply to subtraction problems. If you change the order of the numbers you'll get a **different** answer.

Guided Practice

Evaluate the expressions in Exercises 1–8. Use the division of powers rule and give your answer in base and exponent form.

1. $6^9 \div 6^4$ **6^5**
2. $15^{25} \div 15^{10}$ **15^{15}**
3. $4^{206} \div 4^{54}$ **4^{152}**
4. $(-3)^7 \div (-3)^2$ **$(-3)^5$**
5. $27^4 \div 27^1$ **27^3**
6. $7^5 \div 7$ **7^4**
7. $d^{10} \div d^7$ **d^3**
8. $(-w)^{14} \div (-w)^{-17}$ **$(-w)^{31}$**

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  In this Lesson, all the answers involve positive exponents. Give students some division problems that produce negative exponents, such as $5^3 \div 5^7$. Ask students to solve the problems both by applying the rule and by expanding the powers. From these problems, some students may be able to deduce the definition of a negative exponent.

Division of Powers Can Help with Mental Math

Like the multiplication of powers rule the **division of powers rule** can come in handy when you're doing **mental math**. Convert the numbers into **base and exponent form** and use the rule to **simplify** the problem.

Example 3

What is $1024 \div 64$?

Solution

1024 and 64 are both powers of 4. So you can rewrite the problem in base and exponent form:
$1024 \div 64 = 4^5 \div 4^3$.

Now use the division of powers rule:
$4^5 \div 4^3 = 4^{(5-3)} = 4^2$.
$4^2 = 16$. So $1024 \div 64 =$ **16**.

Guided Practice

Check it out:
343 and 49 are both powers of 7.
512 and 32 are both powers of 2.

Evaluate the expressions in Exercises 9–12.

9. $1024 \div 16$ **64 (4^3)**
10. $100{,}000 \div 100$ **1000 (10^3)**
11. $343 \div 49$ **7 (7^1)**
12. $512 \div 32$ **16 (2^4)**

Independent Practice

1. Evaluate $7^6 \div 7^4$ by writing it out in full as a fraction and canceling the numerator with the denominator. Check your answer using the division of powers rule. **7^2 or 49.**

Evaluate the expressions in Exercises 2–9. Use the division of powers rule and give your answer in base and exponent form.

2. $3^6 \div 3^2$ **3^4**
3. $23^{42} \div 23^{23}$ **23^{19}**
4. $(-8)^{20} \div (-8)^9$ **$(-8)^{11}$**
5. $(-41)^{112} \div (-41)^{52}$ **$(-41)^{60}$**
6. $4^8 \div 4$ **4^7**
7. $z^7 \div z^3$ **z^4**
8. $(-p)^{17} \div (-p)$ **$(-p)^{16}$**
9. $g^a \div g^b$ **$g^{(a-b)}$**

10. A research lab produces 10^7 placebos (sugar pills) for a medical experiment. It distributes the placebos evenly among 10^3 bottles. How many placebos are in each bottle? Give your answer as a power in base and exponent form, then evaluate the power. **10^4 placebos, 10,000 placebos.**

11. Evaluate 1296 divided by 216 by converting the numbers to powers of six. **6^1 or 6.**

12. What is half of 2^n? **$2^{(n-1)}$**

Now try these:
Lesson 5.1.2 additional questions — p457

Round Up

When you have two powers with the same base you can divide one by the other using the division of powers rule. Just subtract the exponent of the second power from the exponent of the first power, and you'll get the exponent that goes with the answer.

2 Teach (cont)

Concept question
"Russel made the conjecture that dividing any two powers of 2 results in a number divisible by 4.

For each of the following, explain whether it supports the conjecture, is a counterexample, or has no bearing on it."

1. $2^8 \div 2^5$
Supports the conjecture.
$2^8 \div 2^5 = 2^3 = 8$ and 8 is divisible by 4.

2. $3^4 \div 3^2$
Has no bearing on the conjecture.
Neither 3^4 nor 3^2 are powers of 2.

3. $2^7 \div 2^6$
It is a counterexample to the conjecture.
$2^7 \div 2^6 = 2$ and 2 is not divisible by 4.

Guided practice
Level 1: q9–10
Level 2: q9–12
Level 3: q9–12

Independent practice
Level 1: q1–5
Level 2: q1–9
Level 3: q1–12

Additional questions
Level 1: p457 q1–3, 8–10
Level 2: p457 q1–12, 14
Level 3: p457 q1–14

3 Homework

Homework Book — Lesson 5.1.2
Level 1: q1–3, 5–7
Level 2: q1–10
Level 3: q1–11

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 13 — Powers

Solutions
For worked solutions see the Solution Guide

Lesson 5.1.3

Fractions with Powers

In this Lesson, students extend the use of the exponent rules for multiplication and division to fractions. They use their knowledge of exponents to simplify complex fraction expressions.

Previous Study: In Section 2.5 students learned how to evaluate a fraction raised to a power. In the previous two Lessons, students learned the exponent rules for multiplication and division of powers.

Future Study: In the next Section, students learn the meaning of negative exponents and manipulate expressions containing them. In Algebra I students simplify expressions involving fractional powers.

1 Get started

Warm-up questions:
- Lesson 5.1.3 sheet

2 Teach

Universal access

To help students understand why the rule for raising a fraction to a power works, show them some examples where the power is converted to a repeated multiplication.

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3}\times\frac{2}{3} = \frac{2\times 2}{3\times 3} = \frac{2^2}{3^2}$$

$$\left(\frac{4}{6}\right)^3 = \frac{4}{6}\times\frac{4}{6}\times\frac{4}{6} = \frac{4\times 4\times 4}{6\times 6\times 6} = \frac{4^3}{6^3}$$

Encourage students to revert to repeated multiplication if they get confused by powers of fractions.

Guided practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1–4

Additional examples

Simplify.

1. $\frac{4^2}{5^3}\times 4^3$

$= \frac{4^2}{5^3}\times\frac{4^3}{1} = \frac{4^2\times 4^3}{5^3\times 1} = \frac{4^5}{5^3}$

2. $\frac{6^7}{7^2}\div 6^4$

$= \frac{6^7}{7^2}\cdot\frac{1}{6^4} = \frac{6^7}{6^4}\cdot\frac{1}{7^2} = 6^3\cdot\frac{1}{7^2} = \frac{6^3}{7^2}$

3. $\frac{4^8}{3^3}\div\frac{3^7}{4^2}$

$= \frac{4^8}{3^3}\cdot\frac{4^2}{3^7} = \frac{4^8\cdot 4^2}{3^3\cdot 3^7} = \frac{4^{10}}{3^{10}} = \left(\frac{4}{3}\right)^{10}$

4. $\left(\frac{4}{7}\right)^2\cdot\left(\frac{4}{5}\right)^2$

$= \frac{4^2}{7^2}\cdot\frac{4^2}{5^2} = \frac{4^2\cdot 4^2}{7^2\cdot 5^2}$

$= \frac{4^4}{7^2\cdot 5^2}$

Lesson 5.1.3

Fractions with Powers

California Standards:

Number Sense 2.1
Understand negative whole-number exponents. **Multiply and divide expressions involving exponents with a common base.**

Number Sense 2.3
Multiply, divide, and simplify rational numbers by using exponent rules.

What it means for you:
You'll see how to use the rules you saw in Lessons 5.1.1 and 5.1.2 to simplify expressions with fractions in.

Key words:
- power
- exponent
- base

Don't forget:
When you raise a fraction to a power it's the same as raising its numerator and denominator separately to the same power. For example:

$$\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3}$$

For a reminder see Section 2.5.

The multiplication and division of powers rules still work if the bases are fractions. But you have to remember that to raise a fraction to a power you must raise the numerator and denominator separately to the same power — you saw this in Chapter 2.

The Rules Apply to Fractions Too

When the **bases** of powers are **fractions**, the multiplication and division of powers rules still apply, just as they would for whole numbers.

For example:

$$\left(\frac{2}{3}\right)^6\cdot\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{(6+4)} = \left(\frac{2}{3}\right)^{10} \quad\text{and}\quad \left(\frac{2}{3}\right)^6\div\left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{(6-4)} = \left(\frac{2}{3}\right)^2$$

For the rules to work the bases must be **exactly the same** fractions.

Guided Practice

Simplify the expressions in Exercises 1–4.
Give your answers in base and exponent form.

1. $\left(\frac{1}{2}\right)^3\cdot\left(\frac{1}{2}\right)^2$ $\left(\frac{1}{2}\right)^5$

2. $\left(\frac{1}{3}\right)^7\div\left(\frac{1}{3}\right)^5$ $\left(\frac{1}{3}\right)^2$

3. $\left(-\frac{2}{5}\right)^{15}\cdot\left(-\frac{2}{5}\right)^{23}$ $\left(\frac{2}{5}\right)^{38}$

4. $\left(\frac{a}{b}\right)^{10}\div\left(\frac{a}{b}\right)^7$ $\left(\frac{a}{b}\right)^3$

Simplifying Fraction Expressions with Different Bases

If you have an expression with **different** fractions raised to powers, apply the powers to the **numerators** and **denominators** of the fractions **separately**. Then use the rules to **simplify** the expression.

Look at this expression. $\left(\frac{5}{3}\right)^2\cdot\left(\frac{2}{3}\right)^4$

1) You can't use the multiplication of powers rule to simplify the expression as it is, because the bases are two different fractions.

$$\underbrace{\left(\frac{5}{3}\right)^2}_{\text{fraction 1}}\cdot\underbrace{\left(\frac{2}{3}\right)^4}_{\text{fraction 2}}$$

2) So write out the fractions with the numerators and denominators raised separately to the powers.

$$\left(\frac{5}{3}\right)^2\cdot\left(\frac{2}{3}\right)^4 = \frac{5^2}{3^2}\cdot\frac{2^4}{3^4}$$

3) When you multiply two fractions together, you multiply their **numerators** and their **denominators**.

$$\frac{5^2}{3^2}\cdot\frac{2^4}{3^4} = \frac{5^2\cdot 2^4}{3^2\cdot 3^4}$$

4) Now you can apply the **multiplication of powers** rule to the denominator of the fraction.

$$\frac{5^2\cdot 2^4}{3^2\cdot 3^4} = \frac{5^2\cdot 2^4}{3^6}$$

5) The powers in the numerator have different bases. You can't simplify the fraction further without evaluating the exponents, so leave the answer in base and exponent form.

$$\frac{5^2\cdot 2^4}{3^2\cdot 3^4} = \frac{5^2\cdot 2^4}{3^6}$$

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
Most of the problems in this Lesson involve simplifying expressions in several stages. Students may struggle with the number of different rules being used. The Universal access activities have been designed to help students gain confidence with a particular rule before tackling the harder questions. As with previous Lessons, number cards can be used to visually demonstrate the examples.

- **English Language Learners**
For a selection of the exercises, ask students to justify each step of their work, as shown in Example 1 below. This gives the students practice in using the key words of the Lesson, such as exponent, reciprocal, numerator, and denominator.

Don't forget:

Dividing by a fraction is the same as multiplying by its reciprocal.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

To divide one fraction by another fraction, flip the second fraction upside down and multiply. There's more on this in Lesson 2.3.4.

Check it out:

The number 1 to any power is always 1. That's because $1 \cdot 1 = 1$, $1 \cdot 1 \cdot 1 = 1$, etc. So $1^3 = 1$, and $1^3 \cdot 3^4 = 3^4$.

Example 1

Simplify the expression $\left(\frac{1}{2}\right)^3 \div \left(\frac{2}{3}\right)^4$ as far as possible, leaving your answer in base and exponent form.

Solution

$$\left(\frac{1}{2}\right)^3 \div \left(\frac{2}{3}\right)^4$$

$= \frac{1^3}{2^3} \div \frac{2^4}{3^4}$ — **Apply the exponents to the numerators and denominators separately**

$= \frac{1^3}{2^3} \cdot \frac{3^4}{2^4}$ — **Multiply by the reciprocal of the second fraction**

$= \frac{1^3 \cdot 3^4}{2^3 \cdot 2^4}$ — The two powers on the bottom of the fraction have the same base. You can use the multiplication of powers rule.

$= \frac{3^4}{2^7}$ — **Multiply the numerators and denominators**

Guided Practice

Simplify the expressions in Exercises 5–10.
Give your answers in base and exponent form.

5. $\left(\frac{5}{2}\right)^3 \cdot \left(\frac{3}{2}\right)^2$ $\quad \frac{5^3 \cdot 3^2}{2^5}$
6. $\left(\frac{2}{9}\right)^2 \cdot \left(\frac{5}{9}\right)^{10}$ $\quad \frac{2^2 \cdot 5^{10}}{9^{12}}$
7. $\left(\frac{1}{4}\right)^{55} \cdot \left(\frac{3}{4}\right)^{72}$ $\quad \frac{3^{72}}{4^{127}}$
8. $\left(\frac{1}{2}\right)^4 \div \left(\frac{2}{7}\right)^8$ $\quad \frac{7^8}{2^{12}}$
9. $\left(\frac{1}{3}\right)^{15} \div \left(\frac{3}{2}\right)^{10}$ $\quad \frac{2^{10}}{3^{25}}$
10. $\left(\frac{3}{2}\right)^3 \div \left(\frac{2}{3}\right)^2$ $\quad \frac{3^5}{2^5}$ or $\left(\frac{3}{2}\right)^5$

You Can Use Powers to Simplify Fraction Expressions

Sometimes converting numbers into **base and exponent** form can help you to **simplify** an expression that has **fractions** in it.

For example: $\left(\frac{2}{3}\right)^3 \cdot \frac{1}{9}$

You can expand the $\left(\frac{2}{3}\right)^3$ to $\frac{2^3}{3^3}$. At first this doesn't seem to help because the other fraction doesn't contain any powers with a base of 2 or 3.

But 9 is a power of 3: $9 = 3^2$. If you change the 9 in the fraction into 3^2 then the expression becomes:

$$\left(\frac{2}{3}\right)^3 \cdot \frac{1}{9} = \frac{2^3}{3^3} \cdot \frac{1}{3^2}$$

Now you can use the multiplication of powers rule to simplify the denominator.

$$\frac{2^3}{3^3} \cdot \frac{1}{3^2} = \frac{2^3 \cdot 1}{3^3 \cdot 3^2} = \frac{2^3}{3^5}$$

2 Teach (cont)

Universal access

Students will struggle if they are not confident in their abilities to multiply and divide simple numeric fractions. For these students, it will be helpful to work through some simple numeric examples first.

$$\frac{2}{3} \times \frac{1}{7} = \frac{2 \times 1}{3 \times 7} = \frac{2}{21}$$

$$\frac{4}{5} \div \frac{3}{4} = \frac{4}{5} \times \frac{4}{3} = \frac{4 \times 4}{5 \times 3} = \frac{16}{15}$$

Guided practice

Level 1: q5–6
Level 2: q5–8
Level 3: q5–10

Universal access

The example opposite introduces the extra step of rewriting an integer as a power. For students struggling with this, provide some additional practice at this step.

For example:

9 can be expressed as 3^2.
27 can be expressed as 3^3.
8 can be expressed as 2^3.
16 can be expressed as 2^4 or 4^2.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Give students some more complex expressions to simplify such as ones involving several multiplications or divisions.
  For example: $\left(\frac{a}{b}\right)^2\times\left(\frac{a}{b}\right)^3\times\left(\frac{a}{b}\right)^4$, $\left(\frac{c}{d}\right)^3\div\left(\frac{c}{d}\right)^4\times\left(\frac{c}{d}\right)^2$, $\left(\frac{c}{d}\right)^3\div\left(\frac{c}{d}\right)^4\div\left(\frac{c}{d}\right)^2$

  Ask students to predict and then investigate whether placing parentheses around two of the fractions will change the expressions.

2 Teach (cont)

Concept questions

"For each of the following say whether the statement is true. If false, provide a correct statement."

1. $\frac{a}{b}\times\frac{1}{x}=a\div(b\div x)$

False. Answer is " $=\frac{a\times 1}{b\times x}=\frac{a}{bx}=a\div(b\times x)$ **or** $(a\div b)\div x$ **"**

2. $(a\div b)^c=a^c\div b^c$

True. This is the same statement as $\left(\frac{a}{b}\right)^c=\frac{a^c}{b^c}$

3. Half of 2^n is 1^n

False.
Answer is " $=2^n\div 2\ =\ 2^n\div 2^1\ =\ 2^{(n-1)}$ **"**

Guided practice

Level 1: q11–12
Level 2: q11–14
Level 3: q11–16

Independent practice

Level 1: q1–2, 5–6
Level 2: q3–12
Level 3: q3–16

Common error

A common problem with these simplification questions is that students don't realize they've finished the question and try to "oversimplify" the answer.

With an answer like $7^2\times 8^5$, they may try to further simplify it (incorrectly) to 56^7 or 56^{10}.

Additional questions

Level 1: p457 q1–3, 7
Level 2: p457 q1–8
Level 3: p457 q1–9

3 Homework

Homework Book — Lesson 5.1.3

Level 1: q1, 2, 3a–c, 8, 9
Level 2: q1–9
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 10 — Multiplying Fractions by Fractions
- Worksheet 11 — Dividing Fractions
- Worksheet 13 — Powers

Example 2

Simplify $\left(\frac{4}{3}\right)^2\div\frac{81}{64}$. Leave your answer in base and exponent form.

Solution

$\left(\frac{4}{3}\right)^2\div\frac{81}{64}$

$=\frac{4^2}{3^2}\div\frac{81}{64}$ — **Apply the exponent to the numerator and denominator separately**

$=\frac{4^2}{3^2}\cdot\frac{64}{81}$ — **Multiply by the reciprocal of the second fraction**

$=\frac{4^2}{3^2}\cdot\frac{4^3}{3^4}$ — **Convert the numbers in the second fraction into powers**

$=\frac{4^2\cdot 4^3}{3^2\cdot 3^4}=\frac{4^5}{3^6}$ — **Multiply the numerators and denominators**

Check it out:
1296 is a power of 6.
343 is a power of 7.
81 is a power of 3.
625 is a power of 5.
32 is a power of 2.

Guided Practice

Simplify the expressions in Exercises 11–16.

11. $\left(\frac{1}{2}\right)^{15}\cdot\frac{1}{4}$ **$\frac{1}{2^{17}}$**

12. $\left(\frac{4}{3}\right)^5\cdot\frac{2}{81}$ **$\frac{2^{11}}{3^9}$**

13. $\frac{64}{1296}\cdot\left(\frac{4}{6}\right)^2$ **$\frac{2^4}{3^6}$ or $\frac{4^2}{3^6}$**

14. $\left(\frac{2}{3}\right)^2\div\frac{5}{16}$ **$\frac{2^6}{3^2\cdot 5}$**

15. $\left(\frac{1}{7}\right)^8\div\frac{343}{15}$ **$\frac{15}{7^{11}}$**

16. $\frac{625}{32}\div\left(\frac{2}{5}\right)^{12}$ **$\frac{5^{16}}{2^{17}}$**

Independent Practice

Simplify the expressions in Exercises 1–4.

1. $\left(\frac{2}{3}\right)^2\cdot\left(\frac{2}{3}\right)^3$ **$\left(\frac{2}{3}\right)^5$**

2. $\left(\frac{3}{5}\right)^{10}\div\left(\frac{3}{5}\right)^7$ **$\left(\frac{3}{5}\right)^3$**

3. $\left(-\frac{1}{2}\right)^{19}\div\left(-\frac{1}{2}\right)^9$ **$\left(-\frac{1}{2}\right)^{10}$ or $\left(\frac{1}{2}\right)^{10}$**

4. $\left(\frac{x}{y}\right)^a\cdot\left(\frac{x}{y}\right)^b$ **$\left(\frac{x}{y}\right)^{(a+b)}$**

Simplify the expressions in Exercises 5–10. **Other equivalent answers are possible**

5. $\left(\frac{3}{4}\right)^2\cdot\left(\frac{7}{4}\right)^3$ **$\frac{3^2\cdot 7^3}{4^5}$**

6. $\left(\frac{5}{9}\right)^{15}\cdot\left(\frac{10}{9}\right)^{23}$ **$\frac{5^{15}\cdot 10^{23}}{9^{38}}$**

7. $\left(\frac{2}{x}\right)^4\cdot\left(\frac{3}{x}\right)^6$ **$\frac{2^4\cdot 3^6}{x^{10}}$**

8. $\left(\frac{7}{5}\right)^3\div\left(\frac{5}{3}\right)^4$ **$\frac{7^3\cdot 3^4}{5^7}$**

9. $\left(\frac{4}{11}\right)^{15}\div\left(\frac{11}{6}\right)^{16}$ **$\frac{4^{15}\cdot 6^{16}}{11^{31}}$**

10. $\left(\frac{4}{r}\right)^{10}\div\left(\frac{r}{16}\right)^4$ **$\frac{4^{10}\cdot 16^4}{r^{14}}=\frac{4^{18}}{r^{14}}$**

Now try these:
Lesson 5.1.3 additional questions — p457

Simplify the expressions in Exercises 11–16. **Other equivalent answers are possible**

11. $\left(\frac{1}{3}\right)^5\cdot\frac{7}{27}$ **$\frac{7}{3^8}$**

12. $\left(\frac{5}{6}\right)^4\cdot\frac{125}{10}$ **$\frac{5^7}{6^4\cdot 10}$**

13. $\frac{9}{512}\cdot\left(\frac{5}{8}\right)^7$ **$\frac{9\cdot 5^7}{8^{10}}=\frac{3^2\cdot 5^7}{2^{30}}$**

14. $\left(\frac{2}{9}\right)^2\div\frac{81}{5}$ **$\frac{2^2\cdot 5}{9^4}$**

15. $\left(\frac{4}{3}\right)^{10}\div\frac{47}{256}$ **$\frac{4^{14}}{3^{10}\cdot 47}$**

16. $\frac{1000}{49}\div\left(\frac{7}{10}\right)^5$ **$\frac{10^8}{7^7}$**

Check it out:
27 is a power of 3.
512 is a power of 8 (and 2).
256 is a power of 4 (and 2).
125 is a power of 5.
1000 is a power of 10.

Round Up

If you can spot powers of simple numbers you'll be able to recognize when you can simplify expressions using the multiplication and division of powers rules. And that's a useful thing to be able to do in math — whether the bases are whole numbers or fractions.

Solutions
For worked solutions see the Solution Guide

Lesson 5.2.1 Negative and Zero Exponents

In this Lesson, students learn the meaning of zero and negative exponents. They are shown how to derive their meanings from the division of powers rule.

Previous Study: In grade 5, students were introduced to the concept of powers as repeated multiplications. In Sections 2.5 and 5.1, they simplified expressions with positive integer exponents and rational bases.

Future Study: In the next Lesson, students will apply the multiplication and division of powers rules to negative exponents, and learn how to remove negative exponents from an expression.

Lesson 5.2.1

California Standards:

Number Sense 2.1

Understand negative whole-number exponents. Multiply and divide expressions using exponents with a common base.

Algebra and Functions 2.1

Interpret positive whole-number powers as repeated multiplication and **negative whole-number powers as repeated division** or multiplication by the multiplicative inverse. **Simplify and evaluate expressions that include exponents.**

What it means for you:

You'll learn what zero and negative powers mean, and simplify expressions involving them.

Key words:

- base
- exponent
- power

Don't forget:

The expression "$a \neq 0$" means a is not equal to zero.

Don't forget:

When you are dividing numbers with the same base, you can subtract the exponents.

Check it out:

a^0 doesn't change the value of whatever it's multiplied by. That's because it's equal to 1 — the multiplicative identity.

Section 5.2
Negative and Zero Exponents

Up to now you've worked with only positive whole-number exponents. These show the number of times a base is multiplied. As you've seen, they follow certain rules and patterns.

The effects of negative and zero exponents are trickier to picture. But you can make sense of them because they follow the same rules and patterns as positive exponents.

Any Number Raised to the Power 0 is 1

Any number that has an exponent of 0 is equal to **1**.

So, $2^0 = 1$, $3^0 = 1$, $10^0 = 1$, $\left(\frac{1}{2}\right)^0 = 1$.

For any number $a \neq 0$, $a^0 = 1$

You can show this using the **division of powers rule**.

If you start with 1000, and keep **dividing by 10**, you get this pattern:

$1000 = 10^3$

Now divide by 10: $10^3 \div 10^1 = 10^{(3-1)} = 10^2$

$100 = 10^2$

Now divide by 10: $10^2 \div 10^1 = 10^{(2-1)} = 10^1$

$10 = 10^1$

Now divide by 10: $10^1 \div 10^1 = 10^{(1-1)} = 10^0$

$1 = 10^0$

The most important row is the second to last one, shown in red.

When you **divide 10 by 10**, you have $10^1 \div 10^1 = 10^{(1-1)} = 10^0$.

You also know that 10 divided by 10 is 1. So you can see that $\mathbf{10^0 = 1}$.

This pattern works for any base.

For instance, $6^1 \div 6^1 = 6^{(1-1)} = 6^0$, and 6 divided by 6 is 1. So $\mathbf{6^0 = 1}$.

You can use the fact that any number to the power 0 is 1 to simplify expressions.

Example 1

Simplify $3^4 \times 3^0$. Leave your answer in base and exponent form.

Solution

$3^4 \times 3^0 = 3^4 \times 1 = 3^4$

You can use the multiplication of powers rule to show this is right:

$3^4 \times 3^0 = 3^{(4+0)} = 3^4$ **Add the exponents of the powers**

You can see that being multiplied by 3^0 didn't change 3^4.

1 Get started

Resources:

- calculators
- exponent matching cards

Warm-up questions:

- Lesson 5.2.1 sheet

2 Teach

Universal access

Rather than stating the rule at the start of the Lesson, students could be led through the process of discovering the rule for themselves. Show them various worked examples, solving them both by converting to numbers and by use of the division of powers rule.

For example:
$5^2 \div 5^2$
Converting to numbers gives:
$25 \div 25$, which is 1.
The division of powers rule gives:
$5^2 \div 5^2 = 5^{(2-2)} = 5^0$.
Conclusion: 5^0 appears to be 1.

Give them examples with different bases and different exponents until students can spot a pattern and form a hypothesis.

Math background

While it is easy to picture natural numbers (1, 2, 3...) as a collection of objects, it is difficult to picture 0 and negative numbers in this way. Historically, people had difficulties "understanding" the number 0 and negative numbers. The earliest recorded uses of numbers were for counting, so only positive whole numbers were used. The number 0 was developed much later on and negative numbers were only widely accepted many centuries after that.

- **Strategic Learners**
  Present the students with two sets of cards, one set with numbers in base and exponent form (include examples of positive, negative, and zero exponents), and the other set with the matching values in decimal or fractional form. Ask students to play pairs with the cards.

- **English Language Learners**
  The Universal access activities below are useful for helping English language learners to understand the link between negative exponents and repeated division, and how the division of powers rule can result in negative exponents.

2 Teach (cont)

Guided practice
Level 1: q1–5
Level 2: q1–8
Level 3: q1–9

Universal access
As with the previous page's activity, students could be led into discovering the definition of negative exponents for themselves.
Give them divisions that will result in negative powers.

$5^2 \div 5^3$, $3^2 \div 3^4$, $7^3 \div 7^6$, ...

In each case, solve the problem by using the division of powers rule and then by writing it as a fraction and expanding the powers.

For example, method 1 gives:
$5^2 \div 5^3 = 5^{(2-3)} = 5^{-1}$

Method 2 gives:
$5^2 \div 5^3 = \frac{\cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times 5} = \frac{1}{5}$

Conclusion: $5^{-1} = \frac{1}{5}$

Universal access
Negative exponents can be thought of as "repeated division," matching the idea of positive exponents being "repeated multiplication."

For example, 5^{-2} can be thought of as dividing by 5 twice.

So the problem $5^5 \times 5^{-2}$ becomes:
5^5 divided by 5 gives 5^4,
then 5^4 divided by 5 gives 5^3.

Concept question
"Look at the following equation:
$(x-5)^0 = (x-3)^0$
Is this equation true when:"

a) $x = 1$?
Yes — both sides of the equation = 1

b) $x = 2$?
Yes — both sides of the equation = 1

Guided Practice

Evaluate the following.

1. 4^0 **1**	**2.** x^0 $(x \neq 0)$ **1**	**3.** $11^0 + 12^0$ **2**
4. $(7+6)^0$ **1**	**5.** $4^3 \div 4^3$ **1**	**6.** $y^2 \div y^2$ $(y \neq 0)$ **1**
7. $3^2 \times 3^0$ **3^2 or 9**	**8.** $2^4 \times 2^0$ **2^4 or 16**	**9.** $a^8 \div a^0$ $(a \neq 0)$ **a^8**

You Can Justify Negative Exponents in the Same Way

By continuing the pattern from the previous page you can begin to understand the meaning of **negative exponents**.
Carry on dividing each power of 10 by 10:

$1000 = 10^3$
Now divide by 10: $10^3 \div 10^1 = 10^{(3-1)} = 10^2$
$100 = 10^2$
Now divide by 10: $10^2 \div 10^1 = 10^{(2-1)} = 10^1$
$10 = 10^1$
Now divide by 10: $10^1 \div 10^1 = 10^{(1-1)} = 10^0$
$1 = 10^0$
Now divide by 10: $10^0 \div 10^1 = 10^{(0-1)} = 10^{-1}$
$\frac{1}{10} = 10^{-1}$
Now divide by 10: $10^{-1} \div 10^1 = 10^{(-1-1)} = 10^{-2}$
$\frac{1}{100} = 10^{-2}$
Now divide by 10: $10^{-2} \div 10^1 = 10^{(-2-1)} = 10^{-3}$
$\frac{1}{1000} = 10^{-3}$

Look at the last rows, shown in red, to see the pattern:

One-tenth, which is $\frac{1}{10}$, can be rewritten as $\frac{1}{10^1} = 10^{-1}$.

One-hundredth, which is $\frac{1}{100}$, can be rewritten as $\frac{1}{10^2} = 10^{-2}$.

One-thousandth, which is $\frac{1}{1000}$, can be rewritten as $\frac{1}{10^3} = 10^{-3}$.

This works with any number, not just with 10.
For example:
$6^0 = 1$ $\quad 6^0 \div 6^1 = 6^{-1}$ and $1 \div 6 = \frac{1}{6}$, so $6^{-1} = \frac{1}{6}$.

$6^{-1} \div 6^1 = 6^{-2}$ and $\frac{1}{6} \div 6 = \frac{1}{6} \bullet \frac{1}{6} = \frac{1}{36} = \frac{1}{6^2}$, so $6^{-2} = \frac{1}{6^2}$.

This pattern illustrates the **general definition for negative exponents**.

For any number $a \neq 0$, $a^{-n} = \frac{1}{a^n}$

Check it out:
Two numbers that multiply together to give 1 are multiplicative inverses.

$6^{-1} = 1 \div 6^1$, so $6^1 \times 6^{-1} = 1$, meaning 6^{-1} is the multiplicative inverse or reciprocal of 6^1.

But also, $6^{-1} \times \frac{1}{6^{-1}} = \frac{6^{-1}}{6^{-1}} = 1$.

So, $\frac{1}{6^{-1}}$ is the multiplicative inverse of 6^{-1} too.

You can multiply 6^{-1} by either 6^1 or $\frac{1}{6^{-1}}$ to get 1, so $6^1 = \frac{1}{6^{-1}}$.

Solutions
For worked solutions see the Solution Guide

Advanced Learners

Remind students that they've only been told that $a^{-n} = \frac{1}{a^n}$ for nonzero values of a. Ask students to investigate why $a = 0$ is excluded from this definition. $\frac{1}{0^n} = \frac{1}{0}$, **and division by zero is undefined.**

Example 2

Rewrite 5^{-3} without a negative exponent.

Solution

$5^{-3} = \frac{1}{5^3}$ or $\frac{1}{125}$ **Using the definition of negative exponents**

Example 3

Rewrite $\frac{1}{7^5}$ using a negative exponent.

Solution

$\frac{1}{7^5} = 7^{-5}$ **Using the definition of negative exponents**

Guided Practice

Rewrite each of the following without a negative exponent.

10. 7^{-3} $\frac{1}{7^3}$ **11.** 5^{-m} $\frac{1}{5^m}$ **12.** x^{-2} $(x \neq 0)$ $\frac{1}{x^2}$

Rewrite each of the following using a negative exponent.

13. $\frac{1}{3^3}$ 3^{-3} **14.** $\frac{1}{6^4}$ 6^{-4} **15.** $\frac{1}{q \times q \times q}$ $(q \neq 0)$ q^{-3}

Independent Practice

Evaluate the expressions in Exercises 1–3.

1. 8702^0 1 **2.** g^0 $(g \neq 0)$ 1 **3.** $2^0 - 3^0$ 0

Write the expressions in Exercises 4–6 without negative exponents.

4. 45^{-1} $\frac{1}{45}$ **5.** x^{-6} $(x \neq 0)$ $\frac{1}{x^6}$ **6.** $y^{-3} - z^{-3}$ $(y \neq 0, z \neq 0)$ $\frac{1}{y^3} - \frac{1}{z^3}$

Write the expressions in Exercises 7–9 using negative exponents.

7. $\frac{1}{8^2}$ 8^{-2} **8.** $\frac{1}{r^6}$ $(r \neq 0)$ r^{-6} **9.** $\frac{1}{(p+q)^v}$ $(p + q \neq 0)$ $(p+q)^{-v}$

In Exercises 10–12, simplify the expression given.

10. $5^4 \times 5^0$ 5^4 **11.** $c^5 \times c^0$ $(c \neq 0)$ c^5 **12.** $f^3 \div f^0$ $(f \neq 0)$ f^3

13. The number of bacteria in a petri dish doubles every hour. The numbers of bacteria after each hour are 1, 2, 4, 8, 16, ... Rewrite these numbers as powers of 2. $2^0, 2^1, 2^2, 2^3, 2^4$

14. Rewrite the numbers 1, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ as powers of 2. $2^0, 2^{-1}, 2^{-2}$, and 2^{-3}

Now try these:
Lesson 5.2.1 additional questions — p458

Round Up

So remember — any number (except 0) to the power of 0 is equal to 1. This is useful when you're simplifying expressions and equations. Later in this Section, you'll see how negative powers are used in scientific notation for writing very small numbers efficiently.

2 Teach (cont)

Common errors

Students often incorrectly believe that:

1. $a^0 = 0$ for any base.
When students see 0 in this context, they want to multiply by 0 to get 0.

2. $a^{-n} = -(a^n)$
For example, $6^{-3} = -6^3$ or -216.

Also, when using the "repeated division" meaning of negative exponents, students may start with the base instead of 1. So for 5^{-3}, they may start with 5 and divide this by 5 three times, giving $\frac{1}{5^2}$ rather than $\frac{1}{5^3}$.

Guided practice
Level 1: q10–14
Level 2: q10–15
Level 3: q10–15

Independent practice
Level 1: q1–8
Level 2: q1–12
Level 3: q1–14

Additional questions
Level 1: p458 q1–7, 11–12, 15
Level 2: p458 q1–12, 15
Level 3: p458 q1–15

3 Homework

Homework Book — Lesson 5.2.1
Level 1: q1–5, 7
Level 2: q2–10
Level 3: q2–11

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 13 — Powers

Solutions
For worked solutions see the Solution Guide

Lesson 5.2.2

Using Negative Exponents

In this Lesson, students apply the multiplication and division of powers rules to powers with negative exponents. They use these rules to simplify expressions.

Previous Study: In the previous Section, students learned the multiplication and division of powers rules. In the previous Lesson, students learned the meaning of zero and negative exponents.

Future Study: In Algebra I students will extend their knowledge of powers to fractional exponents, and simplify expressions involving them.

1 Get started

Resources:
- calculators

Warm-up questions:
- Lesson 5.2.2 sheet

2 Teach

Universal access

When expressions with variables and negative powers are complicated, encourage students to substitute easy whole numbers for the variables to check their solution using a calculator.

For example:

$$\left(\frac{n}{y}\right)^{-3} = \left(\frac{y}{n}\right)^{3} = \frac{y^3}{n^3}$$

Use $n = 2$, $y = 5$ to check:

$$\left(\frac{n}{y}\right)^{-3} = \left(\frac{2}{5}\right)^{-3} = 15.625$$

$$\frac{y^3}{n^3} = \frac{5^3}{2^3} = 15.625$$

Universal access

When simplifying an expression with negative exponents, students may reach a "dead end." A common example is a double fraction like below, where it's not obvious how to proceed further:

$$\frac{1}{5^{-3}} = \frac{1}{\frac{1}{5^3}}$$

Students should be advised to start again and try a different approach if they get stuck. In this case, an alternative way to proceed would be:

$$= \left(5^{-3}\right)^{-1} = \left(\frac{1}{5^3}\right)^{-1} = 5^3$$

Guided practice

Level 1: q1–5
Level 2: q1–7
Level 3: q1–8

Lesson 5.2.2

Using Negative Exponents

California Standards:

Number Sense 2.1
Understand negative whole-number exponents. **Multiply and divide expressions using exponents with a common base.**

Algebra and Functions 2.1
Interpret positive whole-number powers as repeated multiplication and **negative whole-number powers as** repeated division or **multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.**

What it means for you:
You'll multiply and divide expressions that share the same base. This includes expressions containing negative exponents.

Key words:
- base
- denominator
- exponent
- numerator
- power

Don't forget:
As with regular addition and subtraction of integers, you need to be very careful with your positive and negative signs.

- Adding a negative can be thought of as a subtraction. For example, 7 + (–3) = 7 – 3.
- Subtracting a number is the same as adding its opposite. For example, the subtraction 6 – (–2) can be rewritten as 6 + 2.

Don't forget:
10 can be rewritten as 10^1.

Negative exponents might seem a bit tricky at first. But the rules for positive exponents work with negative exponents in exactly the same way. This Lesson gives you plenty of practice at using the rules with negative exponents.

Simplifying Expressions with Integer Exponents

Both the **multiplication of powers rule** ($a^m \times a^n = a^{m+n}$) and the **division of powers rule** ($a^m \div a^n = a^{m-n}$) work with **any** rational exponents — it doesn't matter if they are positive or negative.

The examples below apply the multiplication and division of powers rules to numbers with **negative** exponents.

Example 1

Simplify $5^{-4} \times 5^{-3}$.

Solution

The bases are the same, so the multiplication of powers rule can be applied.

$$\begin{aligned} 5^{-4} \times 5^{-3} &= 5^{(-4+(-3))} && \text{Use the multiplication of powers rule} \\ &= 5^{(-4-3)} \\ &= 5^{-7} \end{aligned}$$

Example 2

Simplify $7^6 \div 7^{-2}$.

Solution

The bases are the same, so the division of powers rule can be applied.

$$\begin{aligned} 7^6 \div 7^{-2} &= 7^{(6-(-2))} && \text{Use the division of powers rule} \\ &= 7^{(6+2)} \\ &= 7^8 \end{aligned}$$

Guided Practice

Simplify the expressions in Exercises 1–8.

1. $5^8 \times 5^{-2}$ 5^6 **2.** $6^{-7} \times 6^3$ 6^{-4} **3.** $10^{-4} \times 10$ 10^{-3} **4.** $7^{-3} \times 7^{-9}$ 7^{-12}
5. $7^5 \div 7^{-4}$ 7^9 **6.** $11^{-3} \div 11^7$ 11^{-10} **7.** $10 \div 10^6$ 10^{-5} **8.** $2^{-7} \div 2^{-5}$ 2^{-2}

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  All of the expressions to the right represent the same thing. Explain this to the students and ask them to think of as many different ways of writing 2 ÷ 3 as they can.

  $a \div b,\ \frac{a}{b},\ a \times \frac{1}{b},\ ab^{-1},\ \left(\frac{b}{a}\right)^{-1},\ \frac{b^{-1}}{a^{-1}}$

- **English Language Learners**
  For some of the practice exercises, ask the students to write down justification of the key steps in their work, as has been done in Examples 3 and 4 below. This will reinforce their learning of the key vocabulary of the Lesson.

You Can Turn Negative Exponents into Positive Ones

This is a different way to tackle the problems on the previous page. You can **decide** which method you prefer to use.

Some people like to convert negative exponents into **positive exponents** before doing any problems involving them.
So when they see a number like 4^{-6}, they rewrite it as $\frac{1}{4^6}$.

The examples on the previous page are repeated below using this method.

Check it out:
Another way to write x^{-n} is $\left(\frac{1}{x}\right)^n$.

Example 3

Simplify $5^{-4} \times 5^{-3}$.

Solution

$$5^{-4} \times 5^{-3} = \frac{1}{5^4} \times \frac{1}{5^3}$$ First convert to positive exponents

$$= \frac{1 \times 1}{5^4 \times 5^3}$$ Multiply the fractions

$$= \frac{1}{5^{(4+3)}}$$ Use the multiplication of powers rule

$$= \frac{1}{5^7} \text{ or } 5^{-7}$$

Example 4

Simplify $7^6 \div 7^{-2}$.

Solution

$$7^6 \div 7^{-2} = 7^6 \div \frac{1}{7^2}$$ First convert to positive exponents

$$= 7^6 \times \frac{7^2}{1}$$ To divide by $\frac{1}{7^2}$, multiply by its reciprocal

$$= 7^6 \times 7^2$$

$$= 7^{(6+2)}$$ Use the multiplication of powers rule

$$= 7^8$$

Don't forget:
To divide by a fraction, you multiply by its reciprocal. You find the reciprocal by flipping the fraction.
$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

Guided Practice

Simplify the expressions in Exercises 9–16 by first converting any negative exponents to positive exponents.

9. $6^3 \times 6^{-9}$ 6^{-6} or $\frac{1}{6^6}$ **10.** $4^2 \times 4^{-5}$ 4^{-3} or $\frac{1}{4^3}$ **11.** $7^8 \times 7^{-4}$ 7^4 **12.** $12^{-7} \times 12^{-2}$ 12^{-9} or $\frac{1}{12^9}$
13. $3^{-4} \div 3$ 3^{-5} or $\frac{1}{3^5}$ **14.** $2^3 \div 2^{-6}$ 2^9 **15.** $10^5 \div 10^{-6}$ 10^{11} **16.** $11^{-8} \div 11^{-3}$ 11^{-5} or $\frac{1}{11^5}$

2 Teach (cont)

Additional examples

Simplify.

1. $9^5 \times 9^{-2}$
$= 9^{(5+(-2))} = 9^3$

2. $6^{-3} \div 6^{-2}$
$= 6^{(-3-(-2))} = 6^{(-3+2)} = 6^{-1}$

Concept question

"Say whether each of the following is true or false. If false, write a correct statement."

a) $\frac{1}{a^{-1}} = \frac{a^{-1}}{1}$
False, a correct statement is $\frac{1}{a^{-1}} = 1 \div \frac{1}{a} = 1 \times a = a$

b) $x^{-3} = -x^3$
False, a correct statement is $x^{-3} = \frac{1}{x^3}$

c) $3^{a-b} = \frac{3^a}{3^b}$
True: $3^{a-b} = 3^a \cdot 3^{-b} = 3^a \cdot \frac{1}{3^b} = \frac{3^a}{3^b}$

Guided practice

Level 1: q9–12
Level 2: q9–14
Level 3: q9–16

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
The Universal access activity below suggests an alternative approach to these problems that involves converting the expression into a string of multiplications. Ask students to redo Guided Practice Exercises 17–20 using this approach. With this method, particular care must be taken to avoid errors with the order of operations. For example, $\frac{a}{b} \div c = (a \div b) \div c = ab^{-1}c^{-1}$, not "$= a \div (b \div c) = ab^{-1}c$."

2 Teach (cont)

Universal access
The examples opposite show the approach of removing all negative exponents as the first step of simplification.
For students who are comfortable with negative exponents, a different approach is to remove any fractions and convert any divisions into multiplications. The problem should eventually reduce to a string of multiplications.

Examples:

$$\frac{5^3}{4^4} \times 5^{-5}$$
$$= 5^3 \times 4^{-4} \times 5^{-5}$$
$$= 5^{3-5} \bullet 4^{-4} = 5^{-2} \bullet 4^{-4}$$

$$\frac{2^3}{3^4} \div \frac{1}{3^6} =$$
$$= 2^3 \bullet 3^{-4} \div 3^{-6}$$
$$= 2^3 \bullet 3^{-4} \bullet 3^{6}$$
$$= 2^3 \bullet 3^{-4+6}$$
$$= 2^3 \bullet 3^{2}$$

Guided practice
Level 1: q17–18
Level 2: q17–20
Level 3: q17–20

Independent practice
Level 1: q1–6
Level 2: q1–8
Level 3: q1–8

Additional questions
Level 1: p458 q1–6
Level 2: p458 q1–8
Level 3: p458 q3–11

3 Homework

Homework Book — Lesson 5.2.2
Level 1: q1–4, 7, 9a–b
Level 2: q1–10
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
- Worksheet 13 — Powers

Getting Rid of Negative Exponents in Fractions

You might have to deal with fractions that have **negative exponents** in the numerator and denominator, like $\frac{2^{-4}}{3^{-7}}$. It's useful to be able to change them into fractions with only **positive exponents** — because it's a simpler form.

A number with a **negative exponent** in the numerator is equivalent to the same number with a **positive exponent** in the denominator ⇨ $2^{-4} = \frac{2^{-4}}{1} = \frac{1}{2^4}$.

A number with a **negative exponent** in the denominator is equivalent to the same number with a **positive exponent** in the numerator ⇨ $\frac{1}{3^{-7}} = \frac{3^7}{1} = 3^7$.

So:

2^{-4} gets moved from the numerator to the denominator, where it is written as 2^4.

$$\frac{2^{-4}}{3^{-7}} = \frac{3^7}{2^4}$$

3^{-7} moves from the denominator and becomes 3^7 in the numerator.

Don't forget:
You could also do this sort of calculation by converting the negative exponents into positive exponents before multiplying.

$$\frac{7^3}{8^{-4}} \times \frac{7^{-6}}{8^2} = \frac{8^4 \times 7^3}{1} \times \frac{1}{8^2 \times 7^6}$$

$$\frac{8^4 \times 7^3}{1} \times \frac{1}{8^2 \times 7^6} = \frac{8^4 \times 7^3}{8^2 \times 7^6}$$

$$\frac{8^4 \times 7^3}{8^2 \times 7^6} = 8^{(4-2)} \times 7^{(3-6)}$$

$$8^{(4-2)} \times 7^{(3-6)} = 8^2 \times 7^{-3} \text{ or } \frac{8^2}{7^3}$$

Example 5

Simplify $\frac{7^3}{8^{-4}} \times \frac{7^{-6}}{8^2}$.

Solution

$$\frac{7^3}{8^{-4}} \times \frac{7^{-6}}{8^2} = \frac{7^3 \times 7^{-6}}{8^{-4} \times 8^2}$$ **Multiply the fractions**

$$= \frac{7^{(3+(-6))}}{8^{(-4+2)}}$$ **Use the multiplication of powers rule**

$$= \frac{7^{-3}}{8^{-2}} = \frac{8^2}{7^3}$$ **Convert to positive exponents**

Guided Practice

Rewrite the expressions in Exercises 17–20 without negative exponents.

17. $\frac{3^{-2}}{8^{-3}}$ $\frac{8^3}{3^2}$ **18.** $\frac{2^4}{3^{-6}}$ $2^4 \times 3^6$ **19.** $2^4 \times 5^{-3}$ $\frac{2^4}{5^3}$ **20.** $\frac{11^{-3}}{7^2}$ $\frac{1}{7^2 \times 11^3}$

Independent Practice

Simplify the expressions in Exercises 1–3.

1. $10^4 \div 10^{-3}$ 10^7 **2.** $5^{-2} \times 5^5$ 5^3 **3.** $7^{-3} \div 7^{-9}$ 7^6

Rewrite the expressions in Exercises 4–6 using only positive exponents.

4. $\frac{5^{-2}}{2^{-7}}$ $\frac{2^7}{5^2}$ **5.** $\frac{6^3}{11^{-2}}$ $6^3 \times 11^2$ **6.** $7^3 \times 4^{-9}$ $\frac{7^3}{4^9}$

Multiply the fractions in Exercises 7–8 and write the answers using only positive exponents.

7. $\frac{4^{-5}}{6^7} \times \frac{6^{-2}}{4^{-3}}$ $\frac{1}{4^2 \times 6^9}$ **8.** $\frac{2^{-5}}{11^3} \times \frac{2^{-4}}{11^{-7}}$ $\frac{11^4}{2^9}$

Now try these:
Lesson 5.2.2 additional questions — p458

Round Up

After this Lesson you should be comfortable with multiplying and dividing expressions with negative exponents. Remember — you can only use these rules if the bases are equal.

Solutions
For worked solutions see the Solution Guide

Lesson 5.2.3

Scientific Notation

In this Lesson, students are reminded how to write large numbers using scientific notation, and learn how to write very small numbers in this way. They also learn to interpret numbers written in scientific notation, and convert them into ordinary numbers.

Previous Study: In Section 2.5, students learned how to write large numbers using scientific notation. Previously in this Section, students learned the meaning of negative exponents.

Future Study: In the next Lesson, students will use their understanding of scientific notation to order numbers written in this form.

Lesson 5.2.3

Scientific Notation

California Standards:

Number Sense 1.1

Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), compare rational numbers in general.

What it means for you:

You'll see how you can use powers of 10 to make very big or very small numbers easier to work with.

Key words:

- scientific notation
- numeric form
- power
- decimal
- base
- exponent

Don't forget:

For more about writing large numbers in scientific notation, see Lesson 2.5.3.

Check it out:

You'll come across lots of very large and very small numbers written using scientific notation outside of math. For example, large numbers like distances between planets or populations of countries, and small numbers like the length of a molecule or the weight of a speck of dust.

Scientific notation is a handy way of writing very large and very small numbers. Earlier in the book, you practiced using powers of ten to write out large numbers. In this Lesson, you'll get a reminder of how to do that. Then you'll see that with negative powers, you can do the same thing for very small numbers.

You Can Use Powers of 10 to Write Large Numbers

In Chapter 2 you saw how to write **large** numbers as a product of two factors using **scientific notation**.

The first factor is a number that is at least 1 but less than 10.

The second factor is a power of ten. The exponent tells you how many places to move the decimal point to get the number.

$$1{,}200{,}000 = 1.2 \times 10^6$$

Example 1

The planet Saturn is about 880,000,000 miles away from the Sun. Write this number in scientific notation.

Solution

$880{,}000{,}000 = 8.8 \times 100{,}000{,}000$ Split the number into the appropriate factors.

$= \mathbf{8.8 \times 10^8}$ **miles** Write the power of ten in base and exponent form.

Guided Practice

Write the numbers in Exercises 1–6 in scientific notation.

1. 487,000,000,000 4.87×10^{11}

2. 6000 6×10^3

3. 93,840,000 9.384×10^7

4. –1,630,000,000,000 -1.63×10^{12}

5. 28,410,000,000,000 2.841×10^{13}

6. –3,854,000,000 -3.854×10^9

You Can Write Small Numbers in Scientific Notation

Scientific notation is also a useful way to write very **small numbers**. A number like 0.0000054 can be rewritten as a **division**.

$$0.0000054 = 5.4 \div 1{,}000{,}000$$

Using powers of 10 you can write this as

$$0.0000054 = 5.4 \div 10^6$$

And remember that $1 \div 10^6 = \frac{1}{10^6} = 10^{-6}$, so you can write

$$0.0000054 = 5.4 \times 10^{-6}$$

5.4×10^{-6} is 0.0000054 written in scientific notation.

1 Get started

Resources:

- Scientific notation matching cards

Warm-up questions:

- Lesson 5.2.3 sheet

2 Teach

Universal access

A good way to open the Lesson is to look at the powers of 10:

$10^1 = 10$
$10^2 = 100$
$10^3 = 1000$
$10^4 = 10{,}000$
$10^5 = 100{,}000$
$10^6 = 1{,}000{,}000$
$10^7 = 10{,}000{,}000$
$10^8 = 100{,}000{,}000$
$10^9 = 1{,}000{,}000{,}000$
$10^{10} = 10{,}000{,}000{,}000$
$10^{11} = 100{,}000{,}000{,}000$
$10^{12} = 1{,}000{,}000{,}000{,}000$

Ask students what they notice about this chart and its patterns.

There are two points to note:
1. The number of 0s to the right of the 1 equals the exponent.
2. The numeric form has 1 more digit than the value of the exponent.

The powers of 10 chart can be referred to throughout the Lesson.

Guided practice

Level 1: q1–3
Level 2: q1–5
Level 3: q1–6

Universal access

Another approach is simply to count the number of places that the decimal point moves. If the decimal point moves left, the exponent is positive. If the decimal point moves right, the exponent is negative.

To convert 880,000,000 to 8.8, the decimal point moves 8 places left.

So $880{,}000{,}000 = 8.8 \times 10^8$

To convert 0.0000054 to 5.4, the decimal point moves 6 places right.

So $0.0000054 = 5.4 \times 10^{-6}$

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  A matching cards activity could be used to reinforce learning. Use three sets of cards — one set with large and small numbers ("500,000 =," "2,530,000 =," "0.0032 =,"...), one set with the corresponding numbers between 1 and 10 ("5," "2.53," "3.2,"...), and a set with the corresponding powers of ten ("$\times 10^5$," "$\times 10^6$," ..., "$\times 10^{-3}$," ...). Cards can then be matched to form correct statements such as "500,000 = 5×10^5."

- **English Language Learners**
  Use the "take notes, make notes" approach with several examples so students can come back to their notes to review how to convert numbers to and from scientific notation. Ask the students to share their notes with a partner.

2 Teach (cont)

Universal access

Like the activity on the previous page, a chart can be used to represent negative powers of ten.

$10^{-1} = 0.1$
$10^{-2} = 0.01$
$10^{-3} = 0.001$
$10^{-4} = 0.0001$
$10^{-5} = 0.00001$
$10^{-6} = 0.000001$
$10^{-7} = 0.0000001$
...
...

Again, ask the students if they can spot any patterns — for example, "the number of zeros after the decimal point is one less than the absolute value of the exponent."

Guided practice

Level 1: q7–9
Level 2: q7–12
Level 3: q7–12

Common errors

1. Students often miscount places. The most common counting error is starting at the beginning of the number rather than the decimal point. For example:

$3 \times 10^4 = 3000$, instead of 30,000

2. A very common error is for students to move the decimal point in the wrong direction. For example:

$5 \times 10^{-3} = 5000$, instead of 0.005

3. Students often forget the condition that the number must be at least 1 and less than 10. So they will incorrectly believe numbers such as 53×10^5 or 0.5×10^6 to be in scientific notation.

Example 2

A red blood cell has a diameter of 0.000007 m. Write this number in scientific notation.

Solution

$0.000007 = 7 \div 1{,}000{,}000$ — Split the number into a decimal and a power of ten.

$= 7 \div 10^6$ — Write the power of ten in base and exponent form.

$= 7 \times 10^{-6}$ m — Change division by a positive power to multiplication by a negative power.

Check it out:
Remember to include the units — they'll be the same as in the original number.

Guided Practice

Write the numbers in Exercises 7–12 in scientific notation.

7. 0.000419 4.19×10^{-4}
8. 0.000000000015 1.5×10^{-11}
9. 0.00000007 7×10^{-8}
10. 0.000030024 3.0024×10^{-5}
11. 0.00008946 8.946×10^{-5}
12. 0.00000004645 4.645×10^{-8}

You Can Convert Numbers from Scientific Notation

Sometimes you might need to take a number that's in scientific notation, and write it as an ordinary number.

When you **multiply by 10**, the decimal point moves one place to the **right**.

When you **divide by 10**, the decimal point moves one place to the **left**.

You can use these facts to **convert** a number from scientific notation back to **numeric form**.

Check it out:
Numeric form means the number written out in full.

Example 3

Write 3.0×10^{11} in numeric form.

Solution

"3.0×10^{11}" means "multiply 3.0 by 10, 11 times."

To multiply 3.0 by 10^{11}, all you need to do is move the decimal point **11 places** to the **right**. It might help to write out the 3.0 with extra 0s — then you can see how the decimal point is moving.

3.0×10^{11}

$= 3.00000000000 \times 10^{11}$

$= 300{,}000{,}000{,}000$

The green line shows the decimal point moving 11 places to the right.

Solutions
For worked solutions see the Solution Guide

● **Advanced Learners**
"Engineering notation" uses 10 raised to the power of multiples of 3. For example, "kilo" means 10^3 and "mega" means 10^6. Ask students to find the meaning of the terms tera, giga, mega, milli, micro, nano, pico, femto, atto. Make comparison statements involving them, such as 5,000,000,000 Hz = 5×10^9 Hz = 5×10^3 MHz = 5 GHz.

Example 4

Write 4.2×10^{-10} in numeric form.

Solution

"4.2×10^{-10}" means "divide 4.2 by 10, 10 times."

You need to move the decimal point **10 places** to the **left**.
You can write in extra 0s in front of the 4 to help you:

$$00000000004.2 \times 10^{-10}$$
$$= \mathbf{0.00000000042}$$

Guided Practice

In Exercises 13–20, rewrite each number in numerical form.

13. 5.91×10^6 5,910,000 **14.** 5.91×10^{-6} 0.00000591 **15.** 2.2×10^3 2200
16. 4.85×10^{-8} 0.0000000485 **17.** 9.023×10^7 90,230,000 **18.** 6.006×10^{-2} 0.06006
19. 8.17×10^{10} 81,700,000,000 **20.** 7.101×10^{-5} 0.00007101

Independent Practice

Write the numbers in Exercises 1–6 in scientific notation.
1. 78,000 7.8×10^4 **2.** 0.00000091 9.1×10^{-7} **3.** 843,000,000,000 8.43×10^{11}
4. 0.00000000000416 4.16×10^{-12} **5.** 20,057,000,000,000 2.0057×10^{13}
6. 0.00000000000000000000100801 1.00801×10^{-22}

Write the numbers in Exercises 7–12 in numerical form.
7. 8.0×10^4 80,000 **8.** 6.2×10^{-5} 0.000062 **9.** 2.18×10^6 2,180,000
10. 3.03×10^{-10} 0.000000000303 **11.** 5.0505×10^9 5,050,500,000 **12.** 9.64×10^{-3} 0.00964

13. The planet Uranus is approximately 1,800,000,000 miles away from the Sun. What is this distance in scientific notation? 1.8×10^9 miles

14. An inch is approximately equal to 0.0000158 miles. Write this distance in scientific notation. 1.58×10^{-5} miles

15. The volume of the Earth is approximately 7.67×10^{-7} times the volume of the Sun. Express this figure in numeric form. 0.000000767

16. An electron's mass is approximately $9.1093826 \times 10^{-31}$ kilograms. What is this mass in numeric form?
0.00000000000000000000000000000091093826 kg

17. In 2006, Congress approved a 69 billion dollar tax cut. What is 69 billion dollars written in scientific notation?
\$$6.9 \times 10^{10}$

18. At the end of the 20th century, the world population was approximately 6.1×10^9 people. Express this population in numeric form. How would you say this number in words?
6,100,000,000 people; six billion, one hundred million

Don't forget:
Include the correct units in your answer. So 0.000028 cm written in scientific notation is 2.8×10^{-5} cm — not just 2.8×10^{-5}.

Now try these:
Lesson 5.2.3 additional questions — p458

Check it out:
1 billion is a thousand million. Written out as a number it looks like this:
1,000,000,000

Round Up

Scientific notation is an important real-life use for powers — it's called scientific notation because scientists use it all the time to save them having to write out really long numbers.

2 Teach (cont)

Additional examples

1. Write 330,000,000 in scientific notation.
Rewrite as 3.3 × 100,000,000

Now, to convert 100,000,000 to 1, the decimal point moves 8 places.

So the answer is 3.3×10^8

2. Convert 2.36×10^{10} to numeric form.
Step 1: Move the decimal point 10 places right, filling in with zeros: 23600000000
Step 2: add in commas: 23,600,000,000

3. Write 0.0000574 in scientific notation.
0.0000574 = 5.74 ÷ 100,000
(decimal point moves 5 places)
= $5.74 \div 10^5$
= 5.74×10^{-5}

4. Write 4.21×10^{-9} in numeric form.
This means divide 4.21 by 10, 9 times.
So move the decimal point 9 places left.
Answer = 0.00000000421

Guided practice
Level 1: q13–15
Level 2: q13–18
Level 3: q13–20

Independent practice
Level 1: q1–3, 7–9
Level 2: q1–15
Level 3: q1–18

Additional questions
Level 1: p458 q1–4, 9–10, 13–14
Level 2: p458 q1–6, 9–11, 13–18
Level 3: p458 q3–18

3 Homework

Homework Book — Lesson 5.2.3
Level 1: q1–5, 7a, 8a, 9a
Level 2: q1–7, 8a, 9a, 10
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 13 — Powers

Solutions
For worked solutions see the Solution Guide

Lesson 5.2.4

Comparing Numbers in Scientific Notation

In this Lesson, students learn how to order numbers written in scientific notation. They then practice doing this within contexts in which scientific notation is likely to be used, such as describing the masses of atoms, and the distances to stars.

Previous Study: In grade 6, students placed fractions, decimals, and mixed numbers on a number line. In the previous Lesson, students expressed very large and very small numbers using scientific notation.

Future Study: Students will encounter scientific notation at various stages throughout their study of science, particularly in areas of physics.

1 Get started

Resources:
- internet computers/reference books

Warm-up questions:
- Lesson 5.2.4 sheet

2 Teach

Universal access

To check for understanding, show a very large number or a very small number on the board, and ask students for a hand signal indicating whether the exponent will be positive or negative when it is written in scientific notation.

Universal access

It is important that students are able to correctly enter scientific notation into a calculator, and interpret the calculator's display.

Show the class how to enter numbers in scientific notation using the scientific notation button, rather than by multiplying by ten raised to a power. This is often the EXP button — so to enter 5×10^6, you'd input 5 EXP 6.

Give the students a list of numbers to enter into their calculator, and check that they get the correct results.

$5 \times 10^3 = 5000$
$3.23 \times 10^8 = 323{,}000{,}000$
$1.1 \times 10^{-5} = 0.000011$

Show students how to set up their calculator to display answers in scientific notation. (This is likely to involve changing the mode to "Sci.") With the calculator on this setting, practice reading numbers in scientific notation from the calculator display.

Lesson 5.2.4

Comparing Numbers in Scientific Notation

California Standards:

Number Sense 1.1

Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), compare rational numbers in general.

What it means for you:

You'll learn how to tell which is the larger of any two numbers written in scientific notation.

Key words:
- scientific notation
- exponent
- coefficient
- power

Don't forget:

The expression "$a > b$" means "a is greater than b."

When you look at two numbers, you can usually tell straightaway which is larger. If the two numbers are in scientific notation, you might need to think a bit harder. But once you know what part of the number to look at first, it becomes much more straightforward.

Look at the Exponent First, Then the Other Factor

The problem with comparing numbers written in scientific notation is that each number has two parts to look at.

There's the **number between 1 and 10**... **2.89** $\times 10^6$

... and there's the **power of 10**. $2.89 \times \mathbf{10^6}$

The **first** thing to look at is the **power**. The number with the **greater** exponent is the **larger** number.

Example 1

Which of the following numbers is larger?

4.23×10^8 or 7.91×10^6

Solution

The **exponent** in 4.23×10^8 is **8**.
The **exponent** in 7.91×10^6 is **6**.

$8 > 6$, so **4.23×10^8 is the larger number**.

If two numbers have the **same power of 10**, then you need to look at the **other factor** — the number between 1 and 10.
The number with the **greater** factor is the **larger** number.

Example 2

Which of the following numbers is larger?

4.23×10^8 or 7.91×10^8

Solution

The **exponents** of the power of 10 in these two numbers are the same, so you need to compare the **other factors**.

$7.91 > 4.23$, so **7.91×10^8 is the larger number**.

- **Strategic Learners**
  The Application section below gives some examples of scientific notation used in real life. Split the students into small groups and ask them to think of other situations where people deal with very small and very large numbers. Ask students to use the library or internet to find actual figures — in each case, they should write the number in scientific notation and in full, including any units.

- **English Language Learners**
  During the class activities, get the students to read and explain math statements out loud. For example, for the comparison $7 \times 10^4 < 4 \times 10^5$, students could say, "seven times ten to the four is less than 4 times ten to the five because 4 is less than 5." Statements could be written down to further reinforce the language of the Lesson.

Guided Practice

In Exercises 1–6, say which of each pair of numbers is greater.

1. 9.1×10^6, 8.2×10^9 **8.2×10^9**

2. 4.61×10^5, 1.05×10^{10} **1.05×10^{10}**

3. 3.21×10^3, 6.8×10^3 **6.8×10^3**

4. 8.4×10^8, 5.75×10^7 **8.4×10^8**

5. 6.033×10^{12}, 2.46×10^{12} **6.033×10^{12}**

6. 2.6×10^4, 2.09×10^4 **2.6×10^4**

In Exercises 7 and 8, write out each set of numbers in order from smallest to largest:

7. 8.34×10^{10} 7.1×10^9 5.71×10^{10} 9.64×10^9
7.1×10^9, 9.64×10^9, 5.71×10^{10}, 8.34×10^{10}

8. 3.8×10^5 3.09×10^6 3.41×10^5 4.12×10^5
3.41×10^5, 3.8×10^5, 4.12×10^5, 3.09×10^6

Be Careful with Negative Exponents

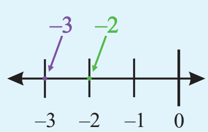
You need to take care when you compare numbers in scientific notation that have **negative powers**.

The number with the **greater** exponent is still **larger** — but remember that negative numbers can look like they're getting bigger, when they're actually getting smaller.

Check it out:

It might help to think about where the negative numbers would be on the number line. A number that is further to the right is always greater.

For example:
$-3 < -2$ or $-2 > -3$



Example 3

Which of the following numbers is larger?

4.23×10^{-5} or 7.91×10^{-7}

Solution

The **exponent** in 4.23×10^{-5} is **–5**.
The **exponent** in 7.91×10^{-7} is **–7**.

$-5 > -7$, so **4.23×10^{-5} is the larger number**.

If the negative exponents are the **same**, then the number with the greater **coefficient** is still larger.

Example 4

Which of the following numbers is larger?

4.23×10^{-9} or 7.91×10^{-9}

Solution

The **exponents** in these two numbers are the same, so you need to compare the **coefficients**.

$7.91 > 4.23$, so **7.91×10^{-9} is the larger number**.

2 Teach (cont)

Guided practice
Level 1: q1–4
Level 2: q1–7
Level 3: q1–8

Application
As the name suggests, scientific notation arises in many areas of science, where extremely large and extremely small measurements are commonly used. Scientific notation is a much more convenient way of writing and comparing these numbers at a glance.

Examples:
The speed of light is about 3×10^8 m/s (300,000,000 m/s).

The size of a hydrogen atom is about 1×10^{-10} m (0.0000000001 m).

The number of atoms in a grain of sand can be estimated to be about 7×10^{19} (70,000,000,000,000,000,000).

Common errors
When comparing numbers in scientific notation, students often compare the bases instead of first looking at the exponents.

For example,
"$5 \times 10^3 < 8 \times 10^2$ because $5 < 8$"

When negative exponents are involved, students often confuse the absolute values of the exponents with the real values.

For example,
"$10^{-6} > 10^{-5}$ because $6 > 5$"

Solutions
For worked solutions see the Solution Guide

Advanced Learners

The research activity on the previous page can be extended for advanced learners. As for the other activity, tell students to find very large and small numbers that occur in real life and express these numbers in both standard form and numeric form. Then ask them to use these figures to make comparisons. For example, "Mercury is on average 5.8×10^7 km from the Sun, and Pluto is on average 5.9×10^9 km away. Therefore Pluto is approximately 100 times further from the Sun than Mercury."

2 Teach (cont)

Guided practice
Level 1: q9–12
Level 2: q9–15
Level 3: q9–16

Independent practice
Level 1: q1–5
Level 2: q1–11
Level 3: q1–16

Concept question
"Given that a and b are integers and $a < b$, say which is the greater number in each case:"

1. $a \times 10^5$ or $b \times 10^3$
$a \times 10^5$

2. $a \times 10^3$ or $b \times 10^3$
$b \times 10^3$

3. 2.5×10^a or 2.1×10^b
2.1×10^b

4. 4×10^{-a} or 5×10^{-b}
4×10^{-a}

Additional questions
Level 1: p459 q1–4, 7–9
Level 2: p459 q1–4, 7–13
Level 3: p459 q3–14

3 Homework

Homework Book — Lesson 5.2.4
Level 1: q1–5, 7
Level 2: q1–9
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 13 — Powers

Guided Practice

In Exercises 9–14, say which of each pair of numbers is greater.

9. 1.4×10^{-4}, 2.3×10^{-6} **1.4×10^{-4}**
10. 5.0×10^{-6}, 4.8×10^{-6} **5.0×10^{-6}**
11. 7.42×10^{-33}, 3.89×10^{-23} **3.89×10^{-23}**
12. 1.57×10^{-4}, 9.31×10^{-5} **1.57×10^{-4}**
13. 6.04×10^{-86}, 6.2×10^{-86} **6.2×10^{-86}**
14. 9.99×10^{-40}, 1.45×10^{-17} **1.45×10^{-17}**

In Exercises 15 and 16, write out each set of numbers in order from smallest to largest:

15. 4.97×10^{-8} 4.52×10^{-7} 3.08×10^{-8} 3.18×10^{-7}
3.08×10^{-8}, 4.97×10^{-8}, 3.18×10^{-7}, 4.52×10^{-7}

16. 6.4×10^{-15} 6.04×10^{-13} 6.44×10^{-13} 6.14×10^{-15}
6.14×10^{-15}, 6.4×10^{-15}, 6.04×10^{-13}, 6.44×10^{-13}

Independent Practice

In Exercises 1–8, say which of each pair of numbers is greater.

1. 4.25×10^{18}, 3.85×10^{19} **3.85×10^{19}**
2. 9.16×10^{-12}, 6.4×10^{-10} **6.4×10^{-10}**
3. 2.051×10^{7}, 1.19×10^{4} **2.051×10^{7}**
4. 8.04×10^{-9}, 7.96×10^{-9} **8.04×10^{-9}**
5. 5.22×10^{45}, 7.01×10^{45} **7.01×10^{45}**
6. 6.861×10^{-22}, 4.0×10^{-21} **4.0×10^{-21}**
7. 7.89×10^{11}, 7.9×10^{11} **7.9×10^{11}**
8. 3.642×10^{-30}, 1.886×10^{-28} **1.886×10^{-28}**

This table shows the mass of one atom for five chemical elements. Use it to answer Exercises 9–11.

Element	Mass of atom (kg)
Titanium	7.95×10^{-26}
Lead	3.44×10^{-25}
Silver	1.79×10^{-25}
Lithium	1.15×10^{-26}
Hydrogen	1.674×10^{-27}

9. Which is the heaviest element? **Lead**

10. Which element is lighter, silver or titanium? **Titanium**

11. List all five elements in order from lightest to heaviest.
Hydrogen, Lithium, Titanium, Silver, Lead

This table shows the approximate distance from Earth of six stars. Use the table to answer Exercises 12–16.

Name of Star	Approx. Distance from Earth (miles)
Bellatrix	1.4×10^{15}
Sirius	5.04×10^{13}
Barnard's Star	3.50×10^{13}
Castor A	3.1×10^{14}
Peacock	1.1×10^{15}
Deneb	1.9×10^{16}

12. Which of these stars is nearest to Earth? **Barnard's Star**

13. Which of these stars is furthest from Earth? **Deneb**

14. How many of these stars are nearer than Castor A? **2**

15. Which is closer to Earth, Peacock or Sirius? **Sirius**

16. List all six stars in order from nearest to furthest.
Barnard's Star, Sirius, Castor A, Peacock, Bellatrix, Deneb

Now try these:
Lesson 5.2.4 additional questions — p459

Round Up

If you have two numbers written in scientific notation and you want to know which is larger, look at the two parts of the numbers. First compare their exponents, and then, if necessary, compare their other factors. And don't forget to watch out for negative exponents.

Solutions
For worked solutions see the Solution Guide

Section 5.3 Exploration — Monomials

Purpose of the Exploration

The purpose of the Exploration is for students to see how monomials can be multiplied and divided. The Exploration allows students to use algebra tiles to create rectangles of certain monomial dimensions. Their areas are then linked to the products of the monomials. This gives students a physical connection when they perform the operations without tiles.

Resources

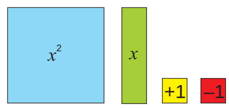
Teacher Resources CD-ROM

• Algebra Tiles

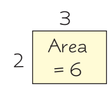
Section 5.3 introduction — an exploration into: Monomials

A monomial is a term that is a constant, a variable or a combination of both.
In this Exploration, you'll use Algebra Tiles to multiply and divide monomials.

The tiles you'll use are shown here:



When you **multiply** two numbers, the product is the same as the **area** of the rectangle that has the two numbers as the **length and width**.

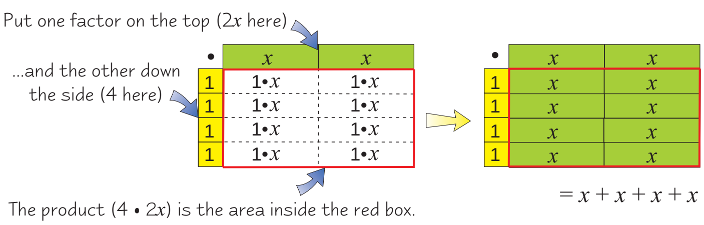


Monomial multiplication works in exactly the same way.

Example

Multiply the expression 4 • 2x using algebra tiles.

Solution



$= x + x + x + x + x + x + x + x = \mathbf{8x}$

You can use algebra tiles to **divide monomials**. You are given the **area** of the rectangle of tiles and one of the side lengths. The goal is to find the tiles that match the **other side length**.

Example

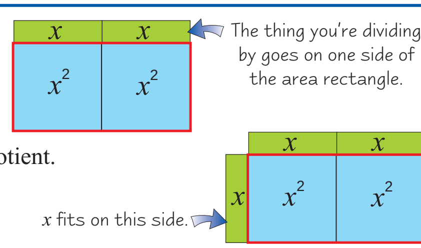
Divide the expression $2x^2 \div 2x$ using algebra tiles.

Solution

$2x^2$ is the area, and $2x$ is the length of one side.

The tiles that fit into the other side represent the quotient.

x fits on the other side. So $2x^2 \div 2x = x$



Exercises

1. Use algebra tiles to simplify the expressions.
   - **a.** 2 • 3x
   - **b.** 2x • 2x
   - **c.** 3 • 3x
   - **d.** x • 4 see below
   - **e.** $3x^2 \div x$
   - **f.** $5x \div 5$
   - **g.** $4x^2 \div 2x$
   - **h.** $2x^2 \div x$

Round Up

You know that the area of a rectangle is the length multiplied by the width. Well, if the length and width are monomials, then it's just the same — the area of the rectangle is their product.

Strategic & EL Learners

Strategic learners will benefit from starting off with examples that do not involve x or x^2. Show students easy problems such as 3 • 4 before getting into more difficult monomials.

EL learners, like other students, may be unfamiliar with the term monomial. A monomial can be a number, a letter (representing a variable) or both. The numbers and variables must be multiplied together, rather than added or subtracted.

Universal access

Remind students that the algebra tiles are representations of area.
An x tile has a width of 1 and an unknown length of x. This idea is extended to multiplying tiles.
The tiles they place inside the grid represent the area produced by multiplying the lengths outside the grid.

Common error

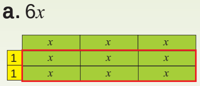
Students may encounter problems selecting the correct tiles. Allow students to undergo a series of trials and errors before stepping in to help. In addition, students may select the correct tiles but have difficulty identifying and writing the correct answer.

Math background

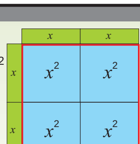
Students will need to be familiar with the concept of finding the areas of squares and rectangles. They will also need to be able to simplify like terms when they arrive at an answer.

Solutions

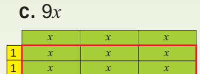
a. 6x



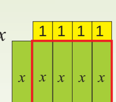
b. $4x^2$



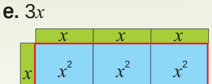
c. 9x



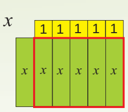
d. 4x



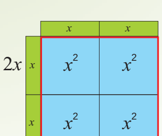
e. 3x



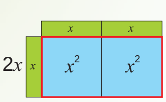
f. x



g. 2x



h. 2x



Lesson 5.3.1

Multiplying Monomials

In this Lesson, students are introduced to the terms monomial and coefficient. Students learn how to multiply two monomials using two different methods.

Previous Study: In Section 5.1, students learned the rule for multiplying powers by adding exponents.

Future Study: In Algebra I, students will be introduced to polynomials. They will learn to add, subtract, multiply, and divide polynomials.

1 Get started

Warm-up questions:
- Lesson 5.3.1 sheet

2 Teach

Universal access

Although the term is not required until Algebra I, you may wish to introduce students to polynomials while monomials are being taught.

Polynomials can have more than one term. The word comes from the Greek words "poly" — meaning many, and "nomos" meaning part or portion. So polynomial means "many terms." In "monomial," "mono" means one.

By learning the terms together and seeing their differences, students may find it easier to understand both concepts.

The examples and exercises can then be modified to: "say whether the expressions are monomials, polynomials, or neither."

Concept questions

"Say whether each of the following is a monomial."

1. $6x^8$ **Yes**
2. $7y^{-4}$ **No — exponent is negative**
3. $x^3 + x^6$ **No — two terms are added**
4. $3x^4y^4$ **Yes**
5. x^3 **Yes**

"Given that a and b are constants and x is a variable, find the coefficients of these monomials."

1. $5x^3$ **5**
2. $4ax^3$ **$4a$**
3. -3^2x^3 **-3^2 or -9**
4. $-8ab^2x^{-3}$ **$-8ab^2$**

Lesson 5.3.1

Section 5.3
Multiplying Monomials

California Standards:

Algebra and Functions 1.4
Use algebraic terminology (e.g., variable, equation, **term, coefficient,** inequality, **expression**, constant**) correctly.**

Algebra and Functions 2.2
Multiply and divide **monomials**; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

What it means for you:
You'll learn how to multiply together expressions that have only one term.

Key words:
- monomial
- coefficient
- constant

Don't forget:
The whole numbers are zero and all the positive integers: 0, 1, 2, 3, 4...

Don't forget:
$-9a^3$ means $-9 \times a^3$.

Check it out:
A monomial that is just a **number** is called a **constant**, since its value can't change (unlike the value of a monomial that contains a variable).

The monomials 2, 6, 3.75, and $\frac{1}{2}$ are all constants.

In Section 5.1 you saw how to do multiplications like $y \times y^2$, or $x^2 \times x^3$. Now you need to know how to multiply together terms that contain numbers or more than one variable, like $2x^2 \times 3x^3$, or $2xy \times y^2$. Things like $2x^2$ and $2xy$ are called monomials — and this Lesson is going to show you how to multiply them.

A Monomial Has Only One Term

A **monomial** is a type of expression that has only **one term** — meaning it has **no additions or subtractions**.
A monomial can include **numbers**, **fractions**, and **variables raised to whole number powers**, but they can only be **multiplied together**.

$\mathbf{8}$, y, y^3, $\frac{y}{2}$, and $5xy^2$ are monomials — expressions with just one term and only whole number powers.

$2 + y$, $a^2 + 4$, and $y^4 - 9$ aren't monomials — they all have more than one term.

x^{-1}, y^{-3}, and $m^{0.5}$ aren't monomials — they contain powers that aren't whole numbers.

In a Monomial the Number is the Coefficient

The number that the variable is multiplied by is called the **coefficient**.
- In the expression $5y$ the coefficient is **5**.
- In the expression $\frac{y}{2}$ the coefficient is $\frac{1}{2}$.
- In the expression x the coefficient is **1**. Multiplying by 1 **doesn't change** the value of a number, so writing $1x$ is **the same** as writing x.

Example 1

Which of the following expressions are monomials?
For those that are monomials, what is the coefficient?

a) $2x$, b) z^2, c) $a + b$, d) $5n^{-3}$ e) $\frac{y^4}{2}$

Solution

a) **$2x$ is a monomial**. The coefficient is **2**.
b) **z^2 is a monomial**. The coefficient is **1**. *The coefficient is 1 because $z^2 = 1 \times z^2$.*
c) **$a + b$ is not a monomial**. Two terms have been **added** together.
d) **$5n^{-3}$ is not a monomial**. -3 is not a **whole number power** because it is **negative**.
e) $\frac{y^4}{2}$ **is a monomial**. The coefficient is $\frac{1}{2}$ (since $\frac{y^4}{2} = \frac{1}{2}y^4$).

- **Strategic Learners**
Play a game with the class where you add extra numbers and variables to a monomial, one at a time. The class shouts "stop!" at the first point where the expression is no longer a monomial. For example, a sequence might go: "5," "$5x$," "$5x^2$," "$5x^2y$," "$5x^{-2}y$," "stop!" (The variable x is no longer raised to a whole-number power.) This activity could also be carried out in pairs or small groups.

- **English Language Learners**
Use the "take notes/make notes" approach to clarify and review these terms: monomials, exponents, coefficients, factors, combining like terms, powers, exponents, commutative property, associative property, identity property. Students should add cards to their personal vocabulary files as needed.

Guided Practice

In Exercises 1–8, state whether or not the expression is a monomial.

1. $3x^4$	monomial	**2.** $3x^{-4}$	not a monomial
3. $3 + x^4$	not a monomial	**4.** x^{-4}	not a monomial
5. $3xy^4$	monomial	**6.** $3x + y^4$	not a monomial
7. $x^4 - 3$	not a monomial	**8.** $\frac{x^4}{3}$	monomial

State the coefficient of each of the monomials in Exercises 9–12.

9. $11x^3$	11	**10.** $14xy^4$	14
11. $\frac{z^2}{8}$	$\frac{1}{8}$	**12.** $-9a^3$	-9

Treat Each Variable Separately When Multiplying

To multiply monomials you deal with the **coefficients** and **each different variable** separately. You'll often need to use the **multiplication of powers** rule too.

Example 2

Multiply together $4x^3y$ and $6x^4y^4$.

Solution

Multiply the coefficients and each different variable separately.

- Multiply the coefficients: $4 \times 6 = 24$
- Multiply the powers of x together: $x^3 \cdot x^4 = x^7$
- Multiply the powers of y together: $y \cdot y^4 = y^5$

Now **multiply** all these results together to form your final answer.
So $4x^3y$ multiplied by $6x^4y^4$ gives $\mathbf{24x^7y^5}$.

Sometimes, only one of the expressions contains a particular variable.

Example 3

Find $2ab^5 \cdot 4a^2c$.

Solution

Multiply the coefficients and each different variable separately.

- Multiply the coefficients: $2 \times 4 = 8$
- Multiply the powers of a together: $a \cdot a^2 = a^3$
- Multiply the powers of b together (only one monomial contains b so you just include that power of b in your answer): b^5
- Multiply the powers of c together (again, only one monomial contains c so just include that power of c in your answer): c

Now **multiply** all these results together to form your final answer.
So $2ab^5 \cdot 4a^2c = \mathbf{8a^3b^5c}$.

There's a quick way to work these out — **multiply the coefficients** and **add the exponents** of each different variable.

Don't forget:

Multiply powers together by adding the exponents:
$x^a \times x^b = x^{a+b}$
For example, $x^3 \times x^4 = x^7$.

This is the multiplication of powers rule. You saw it in Lesson 5.1.1.

Don't forget:

This method works because of the **commutative** and **associative** properties of multiplication (which say that things can be multiplied together in any order and you get the same results — see Lesson 1.1.5).

When you work out $4x^3y$ multiplied by $6x^4y^4$, you are really doing this:
$4 \cdot x^3 \cdot y \cdot 6 \cdot x^4 \cdot y^4$.

By using the method in Example 2, you are just multiplying them in the most convenient order — first the numbers, then the powers of x, and then the powers of y.

2 Teach (cont)

Guided practice
Level 1: q1–9
Level 2: q1–12
Level 3: q1–12

Universal access
An approach to multiplying monomials is to start with the multiplication of powers rule and slowly build up to monomial multiplication using known rules. Here is a sequence of problems to show the buildup (m and n are whole numbers):

1. $a^m \cdot a^n = a^{(m+n)}$

Multiplication of powers rule

2. $(6a^m) \cdot a^n = 6 \cdot (a^m \cdot a^n) = 6a^{(m+n)}$

Using the associative property and multiplication of powers property

3. $6a^m \cdot 3a^n = (6 \cdot 3) \cdot (a^m \cdot a^n)$
$= 18a^{(m+n)}$

Using the fact that the order of multiplication doesn't matter (obtained from the associative and commutative properties).

After this, you can then formalize the procedure for multiplication.

Common errors
Students may add the coefficients because they are adding the exponents. So $6a^2 \times 3a^4$ would become $9a^6$ instead of $18a^6$.

Have students write out a multiplication dot between the numbers and variables to avoid this:

$6a^2 \cdot 3a^4 = 6 \cdot a^2 \cdot 3 \cdot a^4$.

Students may also revert to old mistakes such as multiplying instead of adding the exponents. In this case, have the students expand the powers to show that a is multiplied 6 times:

$6 \cdot a \cdot a \cdot 3 \cdot a \cdot a \cdot a \cdot a$

Solutions
For worked solutions see the Solution Guide

Advanced Learners

Although they are not required to at this grade, students could explore some basic properties of polynomials. For instance, give students simple problems involving multiplying a monomial by a polynomial. For example: "Multiply $5x^3 + 5x^2$ by $2x^2$" or "Multiply $x^2 + 2xy$ by $3xy$." This requires the use of the distributive property.

2 Teach (cont)

Additional examples

1. $9r^5 \bullet 11r^{-2}$

$= (9 \bullet 11) \bullet (r^5 \bullet r^{-2}) = 99 \bullet r^{(5 + (-2))}$

$= 99r^3$

2. $\dfrac{a^2b^5}{3} \bullet 15a^2b$

$= \dfrac{1}{3} \bullet 15 \bullet (a^2 \bullet a^2) \bullet (b^5 \bullet b)$

$= \dfrac{15}{3} \bullet a^{(2+2)} \bullet b^{(5+1)}$

$= 5a^4b^6$

Guided practice

Level 1: q13–16
Level 2: q13–21
Level 3: q17–26

Independent practice

Level 1: q1–10
Level 2: q1–17
Level 3: q7–23

Additional questions

Level 1: p459 q1–10, 14
Level 2: p459 q1–16
Level 3: p459 q1–16

Example 4

Find $6x^5y^6 \bullet 2xy^2z$.

Solution

$6x^5y^6 \bullet 2xy^2z = (6 \times 2) \bullet x^{5+1} \bullet y^{6+2} \bullet z = \mathbf{12x^6y^8z}$

As always, be extra careful if there are **negative numbers** or **fractions**.

Don't forget:

When you are multiplying a fraction by an integer, just multiply the numerator of the fraction by the integer, and then simplify it if you can.

For example:

$3 \bullet \frac{1}{6} = \frac{3 \bullet 1}{6} = \frac{3}{6} = \frac{1}{2}$

Example 5

Find $-12p^2qr^4$ multiplied by $\frac{3}{4}prs^3$.

Solution

$-12p^2qr^4 \bullet \frac{3}{4}prs^3 = \left(-12 \times \frac{3}{4}\right)p^{2+1}qr^{4+1}s^3 = \mathbf{-9p^3qr^5s^3}$

Check it out:

You can multiply three (or more) monomials together in stages — first multiply two monomials together, and then multiply the result by the third.

Guided Practice

Find the results of each multiplication in Exercises 13–22.

13. $x^2 \bullet x^5$ x^7
14. $5y^8 \bullet 3y^2$ $15y^{10}$
15. $2z^2 \bullet x^4$ $2x^4z^2$
16. $-2a \bullet 3a$ $-6a^2$
17. $3a^2b^3c^2 \bullet a^4bc^4$ $3a^6b^4c^6$
18. $12a^3b^3c^3d^3 \bullet 3ab^2c^3d^4$ $36a^4b^5c^6d^7$
19. $5xy^4 \bullet x^2z$ $5x^3y^4z$
20. $0.5x^2 \bullet 3y^2z$ $1.5x^2y^2z$
21. $a^2b^4c^6$ multiplied by $a^2b^4c^6$ $a^4b^8c^{12}$
22. $\left(-\frac{2}{3}p^2qr\right) \bullet \left(\frac{3}{4}pq^5r^{10}s^2\right)$ $-\frac{1}{2}p^3q^6r^{11}s^2$

Write down the answers to Exercises 23–26.

23. $xy^2 \bullet x^2y \bullet xy$ x^4y^4
24. $ab^2 \bullet a^3b^4c^5 \bullet a^6b^7c^8$ $a^{10}b^{13}c^{13}$
25. $pqr^2 \bullet pr \bullet q^2r$ $p^2q^3r^4$
26. $mn^3 \bullet a^2b^7 \bullet abmn$ $a^3b^8m^2n^4$

Independent Practice

Which of the expressions in Exercises 1–6 are monomials?

1. $2bc$ monomial
2. $12a + 2$ not a monomial
3. xy^3 monomial
4. $3x^{-4}$ not a monomial
5. $-4x^3$ monomial
6. $a^2b^3c^4$ monomial

State the coefficient in each of the monomials in Exercises 7–12.

7. $5x$ 5
8. $8a^2b^3$ 8
9. p^2 1
10. $-3y^6$ –3
11. $0.3d^2$ 0.3
12. $3.142r^2$ 3.142

Now try these:

Lesson 5.3.1 additional questions — p459

Calculate the coefficient of each product in Exercises 13–14.

13. $5x$ multiplied by $2y$ 10
14. $-10a$ multiplied by $0.5b^3$ –5

Calculate each product in Exercises 15–20.

15. $5x \bullet 4x$ $20x^2$
16. $2xy \bullet 8x^2$ $16x^3y$
17. $3a^2b^2 \bullet 6ab^2c$ $18a^3b^4c$
18. $-12xy^3 \bullet 7xz$ $-84x^2y^3z$
19. $\frac{1}{2}p^2q^3 \bullet \frac{2}{3}p^3q^2$ $\frac{1}{3}p^5q^5$
20. $2f^{33}g^{11} \bullet 8f^{71}g^{12}$ $16f^{104}g^{23}$

Don't forget:

To "square" something means to multiply it by itself.

Square each monomial in Exercises 21–23.

21. x^2 x^4
22. $3y^3$ $9y^6$
23. $4a^2b^2$ $16a^4b^4$

Round Up

So that's how you multiply monomials — you deal with the numbers first, and then each of the variables in turn, and then multiply the results together. You'll get a lot of practice with this skill because you need to use it all the time in math.

3 Homework

Homework Book — Lesson 5.3.1

Level 1: q1, 2, 4a–c, 6a–b
Level 2: q1–4, 6–9
Level 3: q3–10

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:

- Worksheet 13 — Powers
- Worksheet 20 — Variables and Expressions

Solutions

For worked solutions see the Solution Guide

Lesson 5.3.2

Dividing Monomials

In this Lesson, students learn two different methods for dividing one monomial by another.

Previous Study: In Section 5.1, students learned the rule for dividing powers by subtracting the exponents. In the previous Lesson students learned to multiply two monomials.

Future Study: In the next Lesson, students will be shown how to raise monomials to a power. In Algebra I, students will learn how to divide polynomials.

Lesson 5.3.2

Dividing Monomials

California Standards:

Algebra and Functions 1.4

Use algebraic terminology (e.g., variable, equation, **term, coefficient**, inequality, **expression**, constant) **correctly.**

Algebra and Functions 2.2

Multiply and **divide monomials**; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

What it means for you:

You'll learn how to divide expressions that only have one term.

Key words:

- monomial
- coefficient

You saw how to multiply monomials in the previous Lesson. The next step is to learn how to divide monomials — and that's what this Lesson is all about.

Divide Monomials by Subtracting Exponents

Dividing monomials works in a very similar way to multiplying them.

You deal with coefficients and each different variable **in turn**. But when dividing monomials, you **subtract variables' exponents** rather than add them. This is because you're using the **division of powers rule**.

Example 1

Find $8a^8 \div 4a^6$.

Solution

Treat the coefficients and the variable separately.

- Divide the coefficients: $8 \div 4 = \mathbf{2}$
- Divide the powers of a using the division of powers rule: $a^8 \div a^6 = a^{8-6} = \boldsymbol{a^2}$

Now **multiply** these results together to form your final answer.
So $8a^8 \div 4a^6 = \mathbf{2}\boldsymbol{a^2}$.

Notice how you **divide** the coefficients and the variables, but then you **multiply** all the results together at the end.

So in the previous example, you **divided** the coefficients to get **2**, and you **divided** the powers of a to get $\boldsymbol{a^2}$ — but then you **multiplied** these together to get the final answer of $\mathbf{2}\boldsymbol{a^2}$.

Example 2

Find $10x^7y^4 \div 2x^2y$.

Solution

Treat the coefficients and each different variable separately.

- Divide the coefficients: $10 \div 2 = \mathbf{5}$
- Divide the powers of x using the division of powers rule: $x^7 \div x^2 = \boldsymbol{x^5}$
- Divide the powers of y using the division of powers rule: $y^4 \div y = \boldsymbol{y^3}$

Now **multiply** all these results together to form your final answer.
So $10x^7y^4 \div 2x^2y = \mathbf{5}\boldsymbol{x^5y^3}$.

Don't forget:

Any number or variable to the power 1 is just itself.
For example, $2^1 = 2$, $9^1 = 9$, $a^1 = a$, $x^1 = x$, and so on.
This means you can rewrite $2x^2y$ as $2x^2y^1$.

1 Get started

Warm-up questions:

- Lesson 5.3.2 sheet

2 Teach

Universal access

Algebra and Functions standard 2.1 mentions that you can think of negative exponents as multiplying by the multiplicative inverse. This approach can be used in dividing monomials. Each division problem can be converted into a multiplication problem as follows:

1. Turn the division sign into a multiplication sign, and change the second monomial according to steps 2 and 3 below.
2. Take the reciprocal of the coefficient of the second monomial.
3. For each variable power in the second monomial, switch the sign of the exponent.
4. Then follow the normal procedure for multiplying monomials.

Example:

$$2ab^5 \div 4a^2c = 2ab^5 \bullet \frac{1}{4}a^{-2}c^{-1}$$
$$= 2 \bullet \frac{1}{4} \bullet (a \bullet a^{-2}) \bullet b^5 \bullet c^{-1}$$
$$= \frac{1}{2} \bullet a^{(1+(-2))} \bullet b^5 \bullet c^{-1}$$
$$= \frac{1}{2}a^{-1}b^5c^{-1}$$

- **Strategic Learners**
  Show the students some examples to remind them that, unlike multiplication, the order of division does matter.
  For example, "3 ÷ (5 ÷ 6) = 3.6," "(3 ÷ 5) ÷ 6 = 0.1." Play a game where you give them an incomplete statement, and they have to insert parentheses in the correct place to make the statement true, such as, "16 ÷ 8 ÷ 6 ÷ 3 = 1," which should be "(16 ÷ 8) ÷ (6 ÷ 3) = 1."

- **English Language Learners**
  Review students' understanding of terms associated with division — including reciprocal, multiplicative inverse, factor, and canceling. Look at the connections between these terms. For example, "the multiplicative inverse of x and the reciprocal of x are the same thing."

2 Teach (cont)

Math background

Multiplying and dividing monomials is very different from adding and subtracting monomials. You can only add or subtract monomials if they are like terms — this means that they must contain exactly the same variables, raised to the same powers.

The adding and subtracting of like terms can be justified using the distributive property. For example:

$$4x^2 + 3x^2 = (4 + 3)x^2 = 7x^2$$

Guided practice

Level 1: q1–3
Level 2: q1–6
Level 3: q1–8

Universal access

Another way to approach division of monomials is to rewrite the division as a fraction and cancel common factors. For example.

"Find $8a^2b^2 \div 4a^4b^3$."

This could be done by expanding the powers:

$$\frac{8a^2b^2}{4a^4b^3} = \frac{2\cdot\cancel{4}\cdot\cancel{a}\cdot\cancel{a}\cdot\cancel{b}\cdot\cancel{b}}{\cancel{4}\cdot\cancel{a}\cdot\cancel{a}\cdot a\cdot a\cdot\cancel{b}\cdot\cancel{b}\cdot b} = \frac{2}{a^2b}$$

Or, if students are confident, the powers could be canceled directly.

$$\frac{8a^2b^2}{4a^4b^3} = \frac{\cancel{8}\,2\,\cancel{a^2}\,\cancel{b^2}}{\cancel{4}a^{\cancel{4}\,2}b^{\cancel{3}\,1}} = \frac{2}{a^2b}$$

You have to be very careful to get the **signs** of your **exponents** correct — especially if a variable only appears in the second monomial. It helps to remember that $x^0 = 1$ for **any** value of x. Example 3 shows why this is useful.

Check it out:
You can't change the order of division calculations. That means you must always subtract the exponents in the second monomial from those in the first, regardless of which is bigger. This might result in a negative exponent. If you need a reminder on negative exponents, see Section 5.2.

Example 3

Find $12a^4b^5 \div 6a^2c^4$.

Solution

First, rewrite the division — making sure that **all** the variables appear in **both** monomials. Do this using the fact that $b^0 = 1$ and $c^0 = 1$.
So you have to find $12a^4b^5c^0 \div 6a^2b^0c^4$.

Now divide the coefficients and each variable in turn in the normal way.

- Divide the coefficients: $12 \div 6 = 2$
- Divide the powers of a by subtracting the exponents: $a^4 \div a^2 = a^{4-2} = a^2$
- Divide the powers of b by subtracting the exponents: $b^5 \div b^0 = b^{5-0} = b^5$
- Divide the powers of c in exactly the same way: $c^0 \div c^4 = c^{0-4} = c^{-4}$

Then **multiply** all these results together to give your final answer.
So $12a^4b^5 \div 6a^2c^4 =$ $\mathbf{2a^2b^5c^{-4}}$.

Guided Practice

Find the results of each division in Exercises 1–8.

1. $3x^4 \div x$ $3x^3$
2. $10a^6 \div 5a^3$ $2a^3$
3. $6x^5y^4 \div 2x^2y^2$ $3x^3y^2$
4. $16p^{10}q^7r^2 \div 2p^4q^5$ $8p^6q^2r^2$
5. $-12m \div 2m^2n^5$ $-6m^{-1}n^{-5}$
6. $2xy^4 \div 5x^4y^5z^2$ $0.4x^{-3}y^{-1}z^{-2}$
7. $0.5xyz \div 2m^3n^8$ $0.25xyzm^{-3}n^{-8}$
8. $\frac{2}{3}p^2q^3r^2 \div \frac{4}{9}pq^2r^4s^6$ $\frac{3}{2}pqr^{-2}s^{-6}$

Dividing Monomials Doesn't Always Give a Monomial

When you **multiply** monomials, you **always** get another **monomial**.

However, when you **divide** one monomial by another the result **isn't** always a monomial. You may end up with an answer that contains a **negative exponent** — which is **not** a monomial. That's what happened in Example 3, above.

Don't forget:
Monomials only involve numbers and whole number powers of variables. The whole numbers are 0, 1, 2, 3, 4...
So $0.5x^{-1}$ isn't a monomial.

Example 4

Find $4x^2y \div 8x^3y$.

Solution

Treat the coefficients and the variables in turn, as usual.

$4 \div 8 = 0.5$, while $x^2 \div x^3 = x^{-1}$, and $y \div y = 1$.

So $4x^2y \div 8x^3y =$ $\mathbf{0.5x^{-1}}$.
But the exponent of x **isn't a whole number**, so this **isn't** a monomial.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Three different approaches to monomial division are suggested for this Lesson: 1) subtracting the exponents, 2) converting the divisions into multiplications, and 3) writing the division as a fraction and canceling factors. Ask students to do a selection of the exercises on these pages using all three techniques and check that each gives the same answer.

Guided Practice

Find the result of each of the divisions in Exercises 9–14.
State whether each result is a monomial.

9. $7x^5 \div x^2$ $7x^3$, monomial
10. $8y^5 \div 2y^7$ $4y^{-2}$, not a monomial
11. $9a^2b^3 \div 3a^2b^3$ 3, monomial
12. $18p^8q^5 \div 6p^8q^6$ $3q^{-1}$, not a monomial
13. $-6m^{11}n^8 \div 3m^2n^5$ $-2m^9n^3$, monomial
14. $2x^5y^6z^7 \div 5x^2y^3z^4$ $0.4x^3y^3z^3$, monomial

Divide Coefficients and Subtract Exponents

In the previous Lesson, you saw that there was a quick way to multiply monomials — you multiplied the coefficients and added exponents.

You can do a similar thing when you divide monomials.
But this time you **divide coefficients** and **subtract exponents**.

Example 5

Find $6x^5y^6 \div 2xy^2z$.

Solution

It's best to rewrite this so that **both** monomials contain **all** the different variables. Use the fact that $z^0 = 1$.
So you need to find $6x^5y^6z^0 \div 2xy^2z$.

$6x^5y^6z^0 \div 2x^1y^2z^1 = (6 \div 2) \cdot x^{5-1} \cdot y^{6-2} \cdot z^{0-1} = 3x^4y^4z^{-1}$

Don't forget:
A variable z can be rewritten as z^1.

Guided Practice

Find the results of each division in Exercises 15–18.

15. $12a^4 \div 4a^2$ $3a^2$
16. $100b^6c^2 \div 25a^2b^4c$ $4a^{-2}b^2c$
17. $26p^3q^5 \div 4p^2q^2r^2$ $6.5pq^3r^{-2}$
18. $169m^{100}n^9 \div 13q^5$ $13m^{100}n^9q^{-5}$

Independent Practice

Evaluate the divisions in Exercises 1–6.

1. $x^8 \div x^3$ x^5
2. $35 \div 5y^{-4}$ $7y^4$
3. $11y^4 \div y^6$ $11y^{-2}$
4. $6y^2 \div 3x^4$ $2x^{-4}y^2$
5. $-4a^3 \div 0.5ab^2$ $-8a^2b^{-2}$
6. $\frac{1}{2}m^2n^5 \div \frac{4}{9}mn^7$ $\frac{9}{8}mn^{-2}$

7. Which of these expressions have the same value: $(1 \div a^n)$, $(a^0 \div a^n)$, a^{-n}? All three expressions are equal.
8. What is $5x^3$ divided by $-5x^5$? $-x^{-2}$

Find the quotients in Exercises 9–11.

9. $20x^2y^6 \div 5x^8y^2$ $4x^{-6}y^4$
10. $11ab^5 \div 121bc^4$ $\frac{1}{11}ab^4c^{-4}$
11. $\frac{w^7z^7}{8} \div w^2z^4$ $\frac{w^5z^3}{8}$

Now try these:
Lesson 5.3.2 additional questions — p459

Round Up

Dividing monomials isn't really any harder than multiplying them. But you do have to remember that you subtract exponents when dividing, which means that you could end up with negative exponents. You'll use these ideas in later Lessons, so make sure you remember the rules.

2 Teach (cont)

Guided practice
Level 1: q9–11
Level 2: q9–13
Level 3: q9–14

Additional examples

1. $21b^5 \div 7b^{-2}$
$= (21 \div 7) \cdot (b^5 \div b^{-2})$
$= 3 \cdot b^{(5-(-2))}$
$= 3b^7$

2. $2x^7 \div \frac{1}{5}xy^3$
$= (2 \div \frac{1}{5}) \cdot (x^7 \div x) \cdot (1 \div y^3)$
$= (2 \times 5) \cdot (x^{(7-1)}) \cdot y^{-3}$
$= 10x^6y^{-3}$

Guided practice
Level 1: q15–16
Level 2: q15–18
Level 3: q15–18

Independent practice
Level 1: q1–5
Level 2: q1–8
Level 3: q1–11

Additional questions
Level 1: p459 q1–4, 9
Level 2: p459 q1–12
Level 3: p459 q3–14

3 Homework

Homework Book — Lesson 5.3.2
Level 1: q1a–c, 2–4
Level 2: q1–7
Level 3: q1–7

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 13 — Powers
- Worksheet 20 — Variables and Expressions

Solutions
For worked solutions see the Solution Guide

Lesson 5.3.3

Powers of Monomials

In this Lesson, students learn the rule for raising a power to a power. They learn how to raise monomials to powers by applying this rule.

Previous Study: In Section 5.1, students learned the rules for multiplication and division of powers. In the previous two Lessons, students learned methods for multiplying and dividing monomials.

Future Study: In the next Lesson, students will learn how to find the square roots of monomials.

1 Get started

Resources:
- square cards (about 1 inch × 1 inch) — see Strategic Learners activity

Warm-up questions:
- Lesson 5.3.3 sheet

2 Teach

Universal access

The rule for finding the power of a power can be "discovered" by starting from the multiplication of powers rule.

$a^m \cdot a^n = a^{(m+n)}$

Make the powers you are multiplying together the same:

$a^m \cdot a^m = a^{(m+m)} = a^{2m}$

Now expand to make a table or pattern:

$a^m \cdot a^m = a^{(m+m)} = a^{2m}$

$a^m \cdot a^m \cdot a^m = a^{(m+m+m)} = a^{3m}$

$a^m \cdot a^m \cdot a^m \cdot a^m = a^{(m+m+m+m)} = a^{4m}$

$a^m \cdot a^m \cdot a^m \cdot a^m \cdot a^m = a^{(m+m+m+m+m)} = a^{5m}$

And now it's a simple step to relate this repeated multiplication to exponential notation.

$(a^m)^2 = a^m \cdot a^m = a^{(m+m)} = a^{2m}$

$(a^m)^3 = a^m \cdot a^m \cdot a^m = a^{(m+m+m)} = a^{3m}$

$(a^m)^4 = a^m \cdot a^m \cdot a^m \cdot a^m = a^{(m+m+m+m)} = a^{4m}$

$(a^m)^5 = a^m \cdot a^m \cdot a^m \cdot a^m \cdot a^m = a^{(m+m+m+m+m)} = a^{5m}$

Lesson 5.3.3

Powers of Monomials

California Standards:

Algebra and Functions 1.4
Use algebraic terminology (e.g., variable, equation, **term, coefficient,** inequality, **expression**, constant**) correctly.**

Algebra and Functions 2.2
Multiply and divide monomials; **extend the process of taking powers** and extracting roots **to monomials** when the latter results in a monomial with an integer exponent.

What it means for you:
You'll see how to raise simple expressions to powers.

Key words:
- monomial
- coefficient
- power

Don't forget:
The associative law of multiplication says that you can group things in any way when multiplying and get the same result:
$a \times (b \times c) = (a \times b) \times c$
The commutative law of multiplication says that you can multiply things in any order and get the same result:
$a \times b = b \times a$
Combining these properties, you can show that you can multiply any number of things together in any order and get the same result.
So you don't need parentheses in expressions containing only multiplications. For example:
$a \times ((b \times c) \times d) = a \times b \times c \times d$

Earlier in the book you found out about powers. They show repeated multiplication — for example, 2 × 2 × 2 = 2³. This Lesson is all about how to raise monomial expressions to powers.

Powers Can Be Raised to Other Powers

You might see expressions in which a **power** is **raised to another power**.

For example, you can write the expression $2^4 \times 2^4 \times 2^4$ as $(2^4)^3$.
You can **add the powers** using the multiplication of powers rule to find the result. So $(2^4)^3 = 2^4 \times 2^4 \times 2^4 = 2^{4+4+4} = 2^{12}$.

This gives exactly the same result as **multiplying the exponents** in $(2^4)^3$ together. So you can write $(2^4)^3 = 2^{4 \times 3} = 2^{12}$.

You can write this more generally as:

$$(a^m)^n = a^{m \cdot n}$$

This is called the **power of a power rule**.

Example 1

By writing the expression as a multiplication, show that $(y^3)^2 = y^6$.

Solution

You've got to show that the power of a power rule works for $(y^3)^2$.

As always, work out the parentheses first: $y^3 = y \cdot y \cdot y$
So $(y^3)^2 = (y \cdot y \cdot y) \cdot (y \cdot y \cdot y)$

But you can remove the parentheses here, because it doesn't matter how you group things in multiplications.
Therefore $(y^3)^2 = y \cdot y \cdot y \cdot y \cdot y \cdot y = y^6$.

Example 2

Write $(4^3)^6$ as a power of 4.

Solution

Use the power of a power rule — multiply the powers.

$(4^3)^6 = 4^{3 \times 6} = 4^{18}$

Example 3

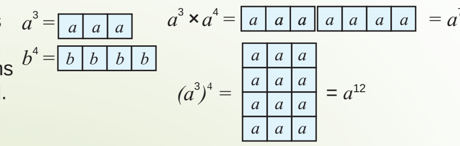
Simplify: **a)** $(x^5)^8$ **b)** $(x^5)^{-8}$

Solution

a) Multiply the powers together. $(x^5)^8 = x^{5 \times 8} = x^{40}$

b) The rule also works with negative powers. $(x^5)^{-8} = x^{5 \times (-8)} = x^{-40}$

- **Strategic Learners**
  The diagram to the right shows a way of modeling powers using square cards. The base of a power is written on each square. Counting the number of squares with a particular letter tells you the exponent. So three "a" squares represent the power a^3. Using square cards, ask the students to make expressions such as a^7, a^2, b^2, and a^2b. Explore how power multiplication and powers of powers can be represented.



- **English Language Learners**
  During the Lesson's worked examples, reinforce vocabulary by encouraging students to quote the new rules as they are used — for example, "raise a power to a power by multiplying the exponents." Also, get students to add the rules in symbol and written form to their vocabulary cards.

Guided Practice

Write the expressions in Exercises 1–9 using a single power.

1. $(2^3)^2$ 2^6 **2.** $(3^5)^4$ 3^{20} **3.** $(7^{99})^{10}$ 7^{990}
4. $(x^4)^8$ x^{32} **5.** $(a^8)^{-101}$ a^{-808} **6.** $(r^p)^q$ r^{pq}
7. $(5^5)^5$ 5^{25} **8.** $(s^{10})^{-10}$ s^{-100} **9.** $(a^{2m})^n$ a^{2mn}

Use the Same Rule to Find Powers of Monomials

All monomials can be raised to powers — even really complicated ones.

Just like with a power of a power, you can **simplify** this kind of expression by remembering that a power means **repeated multiplication**.

Example 4

Simplify $(3xy)^4$.

Solution

Everything inside the parentheses is raised to the power of 4.

$(3xy)^4 = (3xy) \bullet (3xy) \bullet (3xy) \bullet (3xy)$

You can simplify this by removing the parentheses.

$(3xy)^4 = 3 \bullet x \bullet y \bullet 3 \bullet x \bullet y \bullet 3 \bullet x \bullet y \bullet 3 \bullet x \bullet y$

Now you can rearrange this multiplication using the associative and commutative properties of multiplication.

$(3xy)^4 = 3 \bullet 3 \bullet 3 \bullet 3 \bullet x \bullet x \bullet x \bullet x \bullet y \bullet y \bullet y \bullet y$
$= 3^4 \bullet x^4 \bullet y^4$
$= \mathbf{81x^4y^4}$

You can see in Example 4 that **each part** of the original monomial is **raised to the 4th power** in the result.

To raise any monomial to a power, use the following rule.

> **Raising a monomial to a power**
> To take a monomial to the nth power, find the nth power of each part of the monomial, and multiply the results.

When you're raising **monomials** to **powers**, you'll often need to use the **power of a power rule**.

Example 5

Simplify $(5b^3)^2$.

Solution

$(5b^3)^2 = 5^2 \bullet (b^3)^2$
$= 5^2 \bullet b^6$
$= \mathbf{25b^6}$

Don't forget:
$(b^3)^2 = b^{3 \times 2} = b^6$

2 Teach (cont)

Guided practice

Level 1: q1–4
Level 2: q1–7
Level 3: q1–9

Concept question

"Locate and correct the errors in the following statements."

1. When $a = 3$ and $b = –2$,
$2ab^2 = 2 \times (3 \times –2)^2 = 2 \times (–6)^2 = 72$

Only "b" is squared. Answer should be:
$= 2 \times 3 \times (–2)^2 = 6 \times 4 = 24$

2. $(5x^3)^2 = 5x^{(3 \times 2)} = 5x^6$

"5" should also be squared.
Answer should be "$= 5^2x^{(3 \times 2)} = 25x^6$

Math background

The full name of the distributive property that students have previously encountered is the "distributive property of multiplication over addition." This property can be represented as:
$c(a + b) = ca + cb$

The rule given opposite for raising a monomial to a power is actually using the distributive property of exponentation over multiplication. This can be represented as:

$(ab)^c = a^c b^c$

Additional examples

1. Simplify $(5^{–2})^3$

$(5^{–2})^3 = 5^{(–2 \bullet 3)}$
$= 5^{–6}$

2. Simplify $(3xy^4)^3$

$(3xy^4)^3 = 3^3 \bullet x^3 \bullet (y^4)^3$
$= 27 \bullet x^3 \bullet y^{(4 \bullet 3)}$
$= 27x^3y^{12}$

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Allow advanced learners to "discover" the power of a power rule more independently. Ask them: "What is 2^2 raised to the power of 3?" Encourage them to try other examples, and then to try to arrive at a general rule.

2 Teach (cont)

Common errors

1. In finding a power of a monomial, students often forget to raise the power of the coefficient. So $(5b^3)^2$ would become $5b^6$ instead of $25b^6$. A way to avoid this problem is to remind students that the first thing to do is to address the coefficient.

2. Students may also try to add rather than multiply the powers. So $(5xy)^3$ might become $5^3 + x^3 + y^3$, rather than $5^3x^3y^3$. In this case, expand the power to show that only multiplication is involved: $5 \cdot x \cdot y \cdot 5 \cdot x \cdot y \cdot 5 \cdot x \cdot y$

3. Because the students are raising to a power, they may try to raise the first exponent to the power of the second. So they may write $(x^3)^2$ as x^9, rather than x^6.

Guided practice

Level 1: q10–12
Level 2: q10–14
Level 3: q10–18

Independent practice

Level 1: q1–7
Level 2: q1–11
Level 3: q6–15

Additional questions

Level 1: p460 q1–5, 10–12
Level 2: p460 q1–13, 16–17
Level 3: p460 q5–9, 12–15, 16–18

3 Homework

Homework Book — Lesson 5.3.3
Level 1: q1–5, 7, 8
Level 2: q1–10
Level 3: q1–11

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 13 — Powers
- Worksheet 20 — Variables and Expressions

There could be **any number** of variables in the monomial, but you always do exactly the same thing — raise each individual part of the monomial to the power outside the parentheses.

Example 6

Simplify $(2x^2yz^3)^5$.

Solution

$(2x^2yz^3)^5 = 2^5 \cdot (x^2)^5 \cdot y^5 \cdot (z^3)^5$
$= 2^5 \cdot x^{10} \cdot y^5 \cdot z^{15} = \mathbf{32x^{10}y^5z^{15}}$

The method stays the same if the expression contains negative exponents.

Example 7

Simplify $(a^2b)^{-4}$.

Solution

$(a^2b)^{-4} = a^{2 \times (-4)} \cdot b^{-4} = \mathbf{a^{-8}b^{-4}}$

Guided Practice

Simplify the powers of monomials in Exercises 10–18.

10. $(3x^3)^2$ $9x^6$ **11.** $(2x^2)^4$ $16x^8$ **12.** $(x^2y)^3$ x^6y^3

13. $(2a^4b^2)^4$ $16a^{16}b^8$ **14.** $(2p^4q^2r^3)^5$ $32p^{20}q^{10}r^{15}$ **15.** $(2^pq^r)^s$ $2^{ps}q^{rs}$

16. $\left(\frac{1}{2}ab\right)^2$ $\frac{1}{4}a^2b^2$ **17.** $\left(\frac{2}{3}x^2y\right)^3$ $\frac{8}{27}x^6y^3$ **18.** $(0.5p^3qr^4)^4$ $0.0625p^{12}q^4r^{16}$

Independent Practice

1. Use the multiplication of powers rule to simplify $a^m \cdot a^m \cdot a^m$. a^{3m}

2. Simplify $(7^2)^8$ by writing it in the form 7^a. 7^{16}

Simplify each of the expressions in Exercises 3–10.

3. $(y^3)^4$ y^{12} **4.** $(5x)^2$ $25x^2$ **5.** $(p^2)^q$ p^{2q}
6. $(8x^5)^2$ $64x^{10}$ **7.** $(10x^7y^3)^4$ $10{,}000x^{28}y^{12}$ **8.** $(z^9)^{-3}$ z^{-27}
9. $(5a^4b^3)^2$ $25a^8b^6$ **10.** $(x^6y^{11})^{-2}$ $x^{-12}y^{-22}$

11. Show that $(6^5)^3 = (6^3)^5$. $(6^5)^3 = 6^{5 \times 3} = 6^{15}$, and $(6^3)^5 = 6^{3 \times 5} = 6^{15}$

12. Show that $(a^m)^n = (a^n)^m$. $(a^m)^n = a^{m \times n} = a^{mn}$, and $(a^n)^m = a^{n \times m} = a^{mn}$

13. What number is equal to $((2^2)^2)^2$? $(2^{2 \times 2})^2 = 2^{2 \times 2 \times 2} = 2^8 = 256$

14 What is $((a^m)^n)^p$? a^{mnp}

15. A circular cross-section of an atom has a radius of 10^{-10} meters. Find the area of the cross-section. $\pi r^2 = \pi(10^{-10})^2 = \pi \times 10^{-20}$
$= 3.142 \times 10^{-20}$ m² (to 3 decimal places)

Now try these:
Lesson 5.3.3 additional questions — p460

Don't forget:
The formula you use to find the area of a circle is πr^2.

Round Up

So that's how you raise a monomial to a power — you just raise all the individual parts to the same power. You're likely to need to use the power of powers rule for this, so make sure you know it.

Solutions
For worked solutions see the Solution Guide

Lesson 5.3.4 Square Roots of Monomials

In this Lesson students are shown how to find the square roots of numbers and variables. They learn that positive numbers have two square roots, and how to give just the positive one. They then apply these ideas to give the square roots of monomials.

Previous Study: Students were introduced to the concepts of square roots and absolute value in Chapter 2. They used square roots when applying the Pythagorean theorem in Section 3.3.

Future Study: In Algebra I, students will learn the multiplication and division properties of square roots. They will also learn the connection between fractional powers and roots.

Lesson 5.3.4

Square Roots of Monomials

California Standards:

Algebra and Functions 2.2
Multiply and divide monomials; extend the process of taking powers, and **extracting roots to monomials when the latter results in a monomial with an integer exponent**.

What it means for you:
You'll learn how to find the square root of a whole expression.

Key words:
- monomial
- coefficient
- absolute value
- exponent
- square root
- perfect square

Taking the square root of a monomial is like the reverse of raising a monomial to the power of two. There's just one extra complication that you need to be aware of.

$\sqrt{x}$ Means the Positive Square Root of x

A **square root** of a number is a factor that can be **multiplied by itself** to give the number. All **positive** numbers have **one positive** and **one negative** square root. For example, 6 and –6 are both square roots of 36 because 6 • 6 = 36 and –6 • –6 = 36.

The **square root symbol**, $\sqrt{}$, always means the **positive** square root. If you are finding the **negative** square root, you must put a **minus sign** in front of the square root symbol.

So $\sqrt{4} = 2$ and the negative square root of 4 is $-\sqrt{4} = -2$.

Example 1

Find: **a)** the square roots of 81 **b)** $\sqrt{81}$ **c)** $-\sqrt{81}$

Solution

a) The square roots of 81 are **9 and –9**.

b) $\sqrt{81} = 9$ **c)** $-\sqrt{81} = -9$

Guided Practice

1. What are the square roots of 9? 3 and –3

2. What are the square roots of 25? 5 and –5

Evaluate the expressions in Exercises 3–6.

3. $\sqrt{1}$ 1 **4.** $-\sqrt{100}$ –10 **5.** $-\sqrt{196}$ –14 **6.** $\sqrt{9}$ 3

Use Absolute Value to Give the Positive Square Root

Don't forget:
Absolute value is covered in Section 2.2.

Check it out:
The negative square root of $\sqrt{x^2}$ is $-|x|$.

The square roots of x^2 are x and $-x$. But if you're asked to find $\sqrt{x^2}$, only the **positive square root** is correct.

x could be any value — positive **or** negative. So if you write $\sqrt{x^2} = x$, and it turns out that $x = -2$, then you **haven't** given the positive square root.

To get around this, you can write that $\sqrt{x^2} = |x|$ (the **absolute value** of x). This way, you know you've given the **positive answer**.

1 Get started

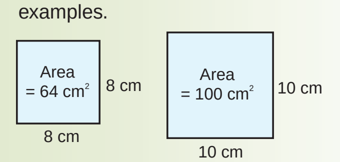
Resources:
- Sets of cards containing a collection of suitable numbers and symbols, such as "√," "x^6," " =," "|," "x^3," "|," "10," "100," "7," "49" (see Strategic Learners activity on p298)

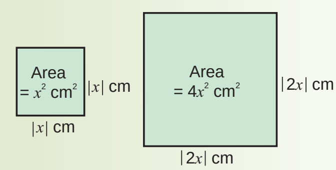
Warm-up questions:
- Lesson 5.3.4 sheet

2 Teach

Universal access

Explain that finding the square root is like finding the side length of a square when you know the area. Present the students with a series of examples. Start with simple numeric examples, and move on to algebraic examples.





For example:
The square with area $4x^2$ has side lengths of $|2x|$ because $|2x| \times |2x| = 4x^2$

This means that $\sqrt{4x^2} = |2x|$

Guided practice

Level 1: q1–4
Level 2: q1–6
Level 3: q1–6

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Split the class into groups, and give each a set of cards containing a combination of numbers and symbols. Ask each group to arrange their cards to form correct mathematical statements. For example, correct arrangements might be "√," "49," " =," "7." A more complicated example might be "√," "x^6," " =," "|," "x^3," "|."

- **English Language Learners**
  During worked examples, get the class to practice reading the math statements in words. So for "$\sqrt{z^2}$," the students should say together, "the square root of z squared." For "$|z^3|$," they should say, "the absolute value of z cubed."

2 Teach (cont)

Universal access

Using the concept of absolute value makes the square roots mathematically correct. However, it can sometimes be overwhelming for students. An alternative approach is to make the assumption that all variables are positive. The Lesson can then be carried out without having to address absolute value.

Most of the examples would then be greatly simplified, as follows:

Example 2: $\sqrt{z^2} = z$

Example 3b: $\sqrt{y^{10}} = y^5$

Example 4: $\sqrt{9x^2} = 3x$

Example 5: $\sqrt{36a^2b^2} = 6ab$

Example 6: $\sqrt{15b^6} = \sqrt{15}b^3$

Having covered the topic in this way, you then have the option of introducing negative variables and absolute value.

Concept question

"Are the following equations true for all values of x?"

1. $\sqrt{9x^6} = 3x^3$

No — the left-hand side will never be negative, whatever the value of x. The right-hand side is negative if $x < 0$.

2. $\sqrt{16x^8} = 4x^4$

Yes. $4x^4 \times 4x^4 = 16x^8$ and neither side of the equation can ever be negative, whatever the value of x.

Example 2

Find $\sqrt{z^2}$.

Solution

The square roots of z^2 are z and $-z$. You only want the positive value though. But without knowing anything about z, you can't say which of z or $-z$ is positive.

But you do know that $|z|$ (the **absolute value** of z) is **positive**.

So $\sqrt{z^2} = |z|$.

Divide Exponents by Two to Find the Square Root

The square roots of z^6 are z^3 and $-z^3$.
That's because $z^3 \cdot z^3 = z^6$ and $-z^3 \cdot -z^3 = z^6$.

$\sqrt{z^6}$ means just the **positive square root** of z^6 — so $\sqrt{z^6} = |z^3|$.

The absolute value signs are important because you can't say whether z^3 or $-z^3$ is positive — but you know that $|z^3|$ is definitely positive.

For instance, if z is -2, then $z^3 = -2 \cdot -2 \cdot -2 = -8$,
but $|z^3| = |-2 \cdot -2 \cdot -2| = |-8| = \mathbf{8}$.

It's a bit different if the square root has an **even exponent**.

For example, $\sqrt{z^8} = z^4$. You **don't** need absolute value signs here because z^4 is **always positive** — it doesn't matter if z is positive or negative.
Again, say $z = -2$:
$z^4 = -2 \cdot -2 \cdot -2 \cdot -2 = (-2 \cdot -2) \cdot (-2 \cdot -2) = 4 \cdot 4 = \mathbf{16}$.

So to find the positive square root of a variable:

1. Divide the exponent by **two**.
2. Put **absolute value** signs around any expression with an **odd** exponent.

Don't forget:
The multiplication of powers rule says that when you multiply two powers with the same base, you can add their exponents to give you the exponent of the answer.
$a^m \cdot a^n = a^{m+n}$

Don't forget:
$z^4 \cdot z^4 = z^{(4+4)} = z^8$

Example 3

Find **a)** $\sqrt{z^4}$, **b)** $\sqrt{y^{10}}$.

Solution

a) Divide the exponent by 2:

$\sqrt{z^4} = z^{4 \div 2} = z^2$ — You don't need to include absolute value signs because the exponent, 2, is even.

b) Divide the exponent by 2:

$\sqrt{y^{10}} = |y^5|$ — You do need to include absolute value signs here because the exponent, 5, is odd.

Don't forget:
Negative numbers with odd exponents are always negative.
Negative numbers with even exponents are always positive.

- **Advanced Learners**

Explain that square roots are actually equivalent to raising to the power $\frac{1}{2}$. Ask students to explore this idea by converting the roots in the examples and exercises into powers.

Challenge students to find the square roots of variables with negative exponents (such as $\sqrt{a^{-2}}$) by applying the division of square roots property: $\sqrt{\frac{m}{c}} = \frac{\sqrt{m}}{\sqrt{c}}$ (Note that variables with negative exponents are not monomials.)

Guided Practice

Evaluate the square roots in Exercises 7–12.

7. $\sqrt{t^2}$ $|t|$	**8.** $\sqrt{p^4}$ p^2	**9.** $\sqrt{r^{12}}$ r^6
10. $\sqrt{q^{14}}$ $|q^7|$	**11.** $\sqrt{s^{18}}$ $|s^9|$	**12.** $\sqrt{w^{30}}$ $|w^{15}|$

Taking the Square Root of a Monomial

Finding the square root of a **monomial** is very similar to raising a monomial to a **power**. You find the square root of each "individual part" of the monomial.

Check it out:
Raising a monomial to a power and finding a monomial's square root are so similar because finding a square root is raising to a power — the power of $\frac{1}{2}$.

Example 4

Find $\sqrt{9x^2}$.

Solution

First you need to find the positive square root of 9 and the positive square root of x^2. Then multiply the results together.

The positive square root of 9 is **3**.
The positive square root of x^2 is $|x|$.

So $\sqrt{9x^2} = 3|x|$ or $|3x|$.

The method is the same even if the monomial has **many parts**.

Example 5

Find $\sqrt{36a^2b^4}$.

Solution

Find the square root of each part, since $\sqrt{36a^2b^4} = \sqrt{36} \cdot \sqrt{a^2} \cdot \sqrt{b^4}$.

$\sqrt{36} = 6$, $\sqrt{a^2} = |a|$, and $\sqrt{b^4} = b^2$.

So $\sqrt{36a^2b^4} = 6|a|b^2$

Check it out:
$6|a|b^2$ could be written $6|ab^2|$, or $|6ab^2|$ — they are exactly the same. The important thing is that the expression is definitely positive.

Guided Practice

Find the square roots in Exercises 13–21.

13. $\sqrt{4x^2}$ $2|x|$	**14.** $\sqrt{16r^2}$ $4|r|$	**15.** $\sqrt{36s^4}$ $6s^2$
16. $\sqrt{100p^8}$ $10p^4$	**17.** $\sqrt{64x^2y^4}$ $8|x|y^2$	**18.** $\sqrt{25m^2n^6}$ $5|mn^3|$
19. $\sqrt{121m^2n^6p^2}$ $11|mn^3p|$	**20.** $\sqrt{x^{10}y^{12}z^{14}}$ $|x^5y^6z^7|$	**21.** $\sqrt{400p^{122}q^{246}r^{38}}$ $20|p^{61}q^{123}r^{19}|$

2 Teach (cont)

Guided practice
Level 1: q7–9
Level 2: q7–12
Level 3: q7–12

Universal access
In the method given for finding the square root of a monomial, students are actually using the multiplicative property of square roots:

$\sqrt{mc} = \sqrt{m} \bullet \sqrt{c}$

Students could be introduced to this rule. This might help them to understand the method being used for monomials.

Guided practice
Level 1: q13–15
Level 2: q13–18
Level 3: q13–21

Additional examples

Simplify.

1. $\sqrt{100c^4}$

$= \sqrt{\mathbf{100}} \cdot \sqrt{c^4}$
$= \mathbf{10}c^2$

2. $\sqrt{22d^2}$

$= \sqrt{\mathbf{22}} \cdot \sqrt{d^2}$
$= \sqrt{\mathbf{22}}\,|d|$

Guided practice
Level 1: q22–23
Level 2: q22–25
Level 3: q22–25

Independent practice
Level 1: q1–6
Level 2: q1–10
Level 3: q4–13

Additional questions
Level 1: p460 q1–6, 10–12
Level 2: p460 q3–9, 13–19
Level 3: p460 q3–9, 13–19

3 Homework

Homework Book — Lesson 5.3.4
Level 1: q1–4, 6a–b, 7a–b
Level 2: q1–10
Level 3: q1–11

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
• Worksheet 14 — Squares and Square Roots
• Worksheet 20 — Variables and Expressions

Don't forget:
A perfect square is the square of an integer.

Don't forget:
Irrational numbers can't be expressed as a fraction. They go on forever and have no repeating pattern — so there's no way to write them down exactly.

The Coefficient Might Not Always Be a Perfect Square

Every positive number has a square root. But if the number isn't a perfect square, then its square root will be a **decimal** — it may even be **irrational**. If you do get an irrational number, you should leave the **square root sign** in your answer.

Example 6

Find $\sqrt{15b^6}$.

Solution

$$\sqrt{15b^6} = \sqrt{15} \cdot \sqrt{b^6}$$
$$= \sqrt{15}\,|b^3|$$

15 is not a perfect square — so keep the square root sign in your answer.

Guided Practice

Find the square roots of the expressions in Exercises 22–25.

22. $\sqrt{3x^2}$ $\sqrt{3}\,|x|$ **23.** $\sqrt{7x^2y^6}$ $\sqrt{7}\,|xy^3|$ **24.** $\sqrt{22s^{10}t^{14}}$ $\sqrt{22}\,|s^5t^7|$ **25.** $\sqrt{55b^2c^{78}}$ $\sqrt{55}\,|bc^{39}|$

Independent Practice

1. What are the squares roots of 49? **7 and –7**
2. What is $\sqrt{49}$? **7**
3. Explain why your answers to Exercises 1 and 2 were different.
The square root sign just means the positive square root.
4. Stevie wrote this equation: $\sqrt{a^2} = a$, where a is an integer.
Explain why Stevie's equation is incorrect. Write a correct version.

4. a might be a negative number. The square root sign means the positive square root. Stevie should write $\sqrt{a^2} = |a|$.

In Exercises 5–12, simplify the expressions.

5. $\sqrt{x^6}$ $|x^3|$ **6.** $\sqrt{4x^2}$ $2|x|$ **7.** $\sqrt{9p^2q^{18}}$ $3|pq^9|$

8. $\sqrt{81r^4s^{22}}$ $9r^2|s^{11}|$ **9.** $\sqrt{19x^2}$ $\sqrt{19}\,|x|$ **10.** $\sqrt{2x^2y^2}$ $\sqrt{2}\,|xy|$

11. $\sqrt{5x^2y^8}$ $\sqrt{5}\,|x|\,y^4$ **12.** $\sqrt{169p^{16}q^8}$ $13p^8q^4$

13. Suppose you know that $\sqrt{q} = p$, and neither p nor q equals zero. Which of p and q are positive? Explain your answer.
p is positive, as it's the positive square root of q. And q must be positive, since negative numbers do not have square roots.

Now try these:
Lesson 5.3.4 additional questions — p460

Round Up

Don't forget — the square root sign means the positive square root. The trickiest thing about finding the positive square root is remembering to make sure that your answer is definitely positive. You need to remember to put absolute value bars around any variables with odd exponents.

Solutions
For worked solutions see the Solution Guide

Section 5.4

Exploration — The Pendulum

Purpose of the Exploration

The goal of the Exploration is to have students discover how some real-life situations follow a non-linear path.
The Exploration allows students to experiment, record, and display data, then make predictions based on their findings.

Resources

- graph paper
- rulers
- string and weights
- stopwatches

Section 5.4 introduction — an exploration into: The Pendulum

There are many real-life situations that can be modeled with graphs. In this Exploration, you'll be making pendulums of different lengths and recording the time they take to swing back and forth a certain number of times. You'll see that the graph you get isn't a linear (straight line) graph — it's a curve (or a non-linear graph).

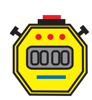
You should work with a partner to complete this Exploration. You'll make each pendulum by tying a weight to a piece of string — then you'll need to find a fixed hook to attach it to.

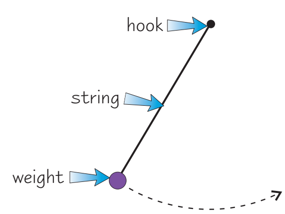
You need to make **four pendulums of different lengths** — 25 cm long, 50 cm long, 75 cm long, and 100 cm long.

You'll use a **stopwatch** to time how long each pendulum takes to complete **ten swings**. In one full swing, the pendulum moves from one side, to the other, and back again to its starting position. To make it a fair test, pull the weight out by the same amount each time.

Record the times in a copy of this table.

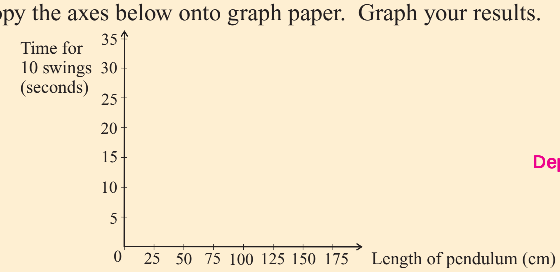
Length (cm)	Time (s) for 10 swings
25	
50	
75	
100	





Exercises

1. Copy the axes below onto graph paper. Graph your results.



Depends on students' results.

2. Do the points lie on a straight line or a curve?
Connect the points with an appropriate line or curve. The points should lie in a curve.

3. Use your graph to predict the amount of time it will take for a 150 cm pendulum to swing back and forth ten times. The theoretical figure for this would be about 24.6 seconds. Student results are likely to differ though.

4. Use your graph to predict the length of a pendulum which takes 17 seconds to swing back and forth ten times. The theoretical figure for this would be about 72 cm. Student results are likely to differ though.

Round Up

Some things don't have a linear (straight-line) relationship. So when you plot them on a graph the points don't lie in a straight line. They sometimes lie in a smooth curve — so you mustn't try to join them with a straight line. There's lots about non-straight line graphs later in this Section.

Strategic & EL Learners

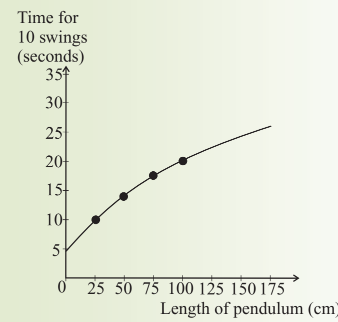
Strategic learners will benefit from being given a copy of the axes already drawn on graph paper.

EL Learners may be unfamiliar with the terms linear and non-linear. Show these students examples of linear and non-linear graphs.

Additional example

Here is some sample data that students could use to plot a graph.

Length (cm)	Time (s) for 10 swings
25	10.0
50	14.2
75	17.4
100	20.0



Common error

Inaccuracies in measuring may mean that students' points don't fall in a perfect curve. It's a good idea for students to repeat their measurements a few times with each pendulum, until they get several very similar measurements. Alternatively, find the average measurement for each pendulum from the whole class.

Some students may try to connect the points by drawing a straight line with a ruler. Remind students that the path of their graph will not be straight.

Math background

The time in seconds for a pendulum swing is directly proportional to the square root of the pendulum length. It's unaffected by the pendulum's weight, or the size of oscillation.

Lesson 5.4.1 Graphing $y = nx^2$

In this Lesson, students are introduced to graphs of the form $y = nx^2$ for positive values of n. They plot graphs of this form using a table of values, and use the graphs to solve equations.

Previous Study: In Section 4.1, students were introduced to graphing linear equations. They also solved systems of equations by finding the intersection point of their graphs, and computed the slopes of lines.

Future Study: In Algebra I, students will explore graphs of the form $y = nx^2 + c$. They will also learn to plot quadratic graphs by finding the roots of the corresponding quadratic equation or by completing the square.

1 Get started

Resources:
- graph paper
- graphing calculators or graphing computer software

Warm-up questions:
- Lesson 5.4.1 sheet

2 Teach

Universal access

Rather than begin the Lesson by showing the students a quadratic graph, you could let students discover the graph for themselves. With the textbook closed, give students the equation $y = x^2$, and get them to plot their own points. They may need to be guided toward drawing a table of values and translating this into points to plot. Once enough points have been plotted, students should try to draw a curve through the points.

Common errors

There are many common mistakes that students may make when drawing quadratic graphs.

1. Making the graph go vertical — explain that although the graph gets steeper and steeper, it never reaches vertical.

2. Students will often try to skew their graph to include an incorrect point. If a point looks out of place, it is wrong — they should go back and check it.

3. Students often find their points are going off the scale of the graph. Tell them to look at the extreme values in their table of values when deciding on a suitable scale for the graph.

Lesson 5.4.1

California Standards:

Algebra and Functions 3.1
Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

Mathematical Reasoning 2.3
Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

Mathematical Reasoning 2.5
Use a variety of methods, such as words, numbers, symbols, **charts, graphs, tables,** diagrams, and models, **to explain mathematical reasoning.**

What it means for you:
You'll learn how to plot graphs of equations with squared variables in them.

Key words:
- parabola
- vertex

Check it out:
The scales on the x- and y-axes are different, so be careful when you plot points or read off values.

Check it out:
The graph of $y = x^2$ is symmetrical about the y-axis. This means the y-axis is a "mirror line" for the graph.

Section 5.4
Graphing $y = nx^2$

Think about the monomial x^2. You can put any number in place of x and work out the result — different values of x give different results. The results you get form a pattern. And the best way to see the pattern is on a graph.

The Graph of $y = x^2$ is a Parabola

You can find out what the **graph** of $y = x^2$ looks like by **plotting** points.

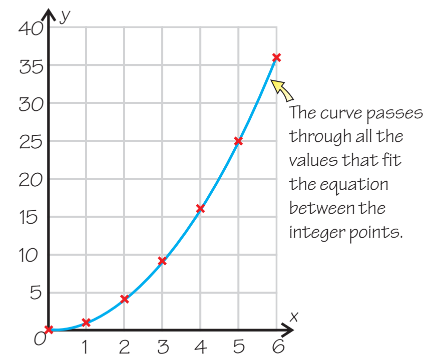
Example 1

Plot the graph of $y = x^2$ for values of x between 0 and 6.

Solution

The best thing to do first is to make a **table** for the integer values of x like the one below. Then you can **plot points** on a set of axes using the x- and y-values as **coordinates**, and join the points with a smooth curve.

x	y $(= x^2)$
0	0
1	1
2	4
3	9
4	16
5	25
6	36



To see what happens for **negative** values of x, you can extend the table.

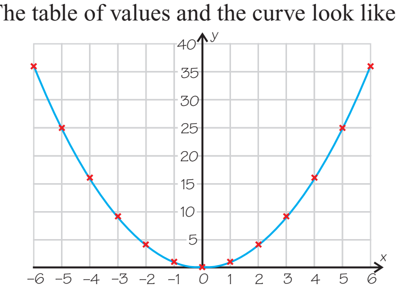
Example 2

Plot the graph of $y = x^2$ for values of x between –6 and 6.

Solution

The table of values and the curve look like this:

x	y $(= x^2)$
–6	36
–5	25
–4	16
–3	9
–2	4
–1	1



This kind of curve is called a **parabola**.

- **Strategic Learners**
  Discuss the properties of the $y = nx^2$ graphs. On the board, roughly sketch some quadratic graphs, such as $y = x^2$, $y = 3x^2$, $y = 10x^2$, and $y = \frac{1}{2}x^2$, and list their equations separately. Ask the students to match each graph to the correct equation (by looking at its steepness).

- **English Language Learners**
  Students may struggle with the new term "parabola." Play an interactive game with the class where you sketch a graph on the board, and the students have to shout out "parabola" or "not parabola." This could also be done using equations.

Check it out:

There are two ways to check if a point is on a graph.
You can either find the point on the coordinate plane and see if it lies on the graph.
Or you can put the x- and y-coordinates into the equation for the graph and see if the equation is true.
For instance, to test whether (2, 3) lies on the graph $y = x^2$, put $x = 2$ and $y = 3$ into the equation:
$y = x^2$
$3 \neq 2^2$
So the point (2, 3) doesn't lie on the graph.

Don't forget:

Always join points on graphs using a smooth curve.

Check it out:

Equations like $y = x^2$ represent functions.
A function is a rule that assigns each number to one other number. If you put a value for x into a function, you get one value for y out.

Check it out:

If $x^2 = 12$, then $x = \sqrt{12}$ or $x = -\sqrt{12}$.

Check it out:

The square roots of 12 are actually 3.464 and –3.464 (to 3 decimal places).

Guided Practice

1. Which of the following points are on the graph of $y = x^2$?
(1, 1), (–1, 1), (–2, –4), (2, 4), (3, 9), (–4, –16), (5, 25), (6, –36)
(1, 1), (–1, 1), (2, 4), (3, 9), and (5, 25)

In Exercises 2–5, calculate the y-coordinate of the point on the graph of $y = x^2$ whose x-coordinate is shown.

2. 6 36 **3.** –10 100 **4.** –2.5 6.25 **5.** $\frac{1}{3}$ $\frac{1}{9}$

In Exercises 6–9, calculate the two possible x-coordinates of the points on the graph of $y = x^2$ whose y-coordinate is shown.

6. 16 4 and –4 **7.** 25 5 and –5 **8.** 49 7 and –7 **9.** 30 $\sqrt{30}$ and $-\sqrt{30}$

You Can Use a Graph to Solve an Equation

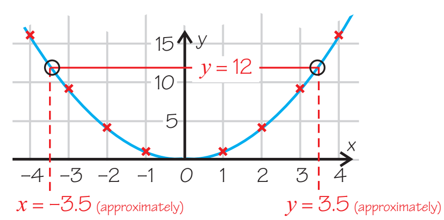
Graphs can be useful if you need to **solve an equation**. Using them means you don't have to do any tricky calculations — and they often show you **how many solutions** the equation has. The downside is that it can be impossible to get an **exact answer** by reading off a graph.

Example 3

Using the graph of $y = x^2$ in Example 2, solve $x^2 = 12$.

Solution

Since the graph shows $y = x^2$, you need to find where $y = 12$.
Then you can find the corresponding value (or values) of x.



You can see that there are **two** different values of x that correspond to $y = 12$, at approximately $\mathbf{x = 3.5}$ and $\mathbf{x = -3.5}$.

This is because 12 has two square roots — a positive one (3.5) and a negative one (–3.5). Or you can look at that another way, and say that the numbers 3.5 and –3.5 can both be squared to give 12 (approximately).

Guided Practice

Use the graph of $y = x^2$ to solve the equations in Exercises 10–13.

10. $x^2 = 16$ $x = 4$ or $x = -4$ **11.** $x^2 = 25$ $x = 5$ or $x = -5$
12. $x^2 = 10$ $x = 3.2$ or $x = -3.2$ (approximately) **13.** $x^2 = 30$ $x = 5.5$ or $x = -5.5$ (approximately)

2 Teach (cont)

Guided practice
Level 1: q1–3
Level 2: q1–6
Level 3: q1–9

Universal access

An alternative approach is to give the students the graph and see if they can figure out the equation from the graph. Simply giving the students the graph and asking for the equation would be too much of a leap. Instead, show students the graph and ask them to identify five points on the graph and record their x- and y-values.

Get the students to compare answers to see if others chose the same points. Then ask them to see if they can find a pattern in their points — or a rule linking the x- and y-values. Do they need to add some more points to make the pattern clear? In describing the pattern, some students will see that the y-value is the square of the x-value.

Guided practice
Level 1: q10–12
Level 2: q10–12
Level 3: q10–13

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to predict and then investigate the shape of the graphs with the following equations.
  $3y = x^2, x = y^2, \sqrt{y} = x, y = nx^2 + c$
  Confident students could be given some more challenging equations outside the scope of grade 7, such as $y = \frac{1}{x}$, $y = 2^x$, and $x^2 + y^2 = c$.

2 Teach (cont)

Universal access

An alternate approach to creating the graphs of the family $y = nx^2$ is to build up the table for the new graphs from the table of $y = x^2$. So start with the table(s) for $y = x^2$:

x	$y\ (= x^2)$
0	0
1	1
2	4
3	9
4	16
5	25
6	36

x	$y\ (= x^2)$
–1	1
–2	4
–3	9
–4	16
–5	25
–6	36

Now to graph $y = 2x^2$, multiply each y-value in the tables above by 2.
In this way, the students are literally doing what the equation asks.

x	$y\ (= 2x^2)$
0	0
1	2
2	8
3	18
4	32
5	50
6	72

x	$y\ (= 2x^2)$
–1	2
–2	8
–3	18
–4	32
–5	50
–6	72

Repeat the process for different values of n (including fractional values) and draw the graph each time on the same axis.

Discuss with the class which features of the graph change when the value of n is changed, and which features remain the same.

Universal access

This Lesson provides an excellent opportunity to use graphing calculators or graphing software to investigate curves of the form $y = nx^2$.

You may prefer to do this toward the end of the Lesson to discourage students from reaching straight for their calculator whenever they are presented with a graphing problem.

The Graph of $y = nx^2$ is Also a Parabola

The graph of $y = x^2$ is $y = nx^2$ where $n = 1$. It has the U shape of a parabola. Other values of n give graphs that look very similar.

Check it out:
Notice how $x = –1$ and $x = 1$ give the same value of $y = nx^2$. The same goes for any pair of positive and negative numbers with the same absolute value.

Example 4

Plot the graphs of the following equations for values of x between –5 and 5.

a) $y = 2x^2$ **b)** $y = 3x^2$ **c)** $y = 4x^2$ **d)** $y = \frac{1}{2}x^2$

Solution

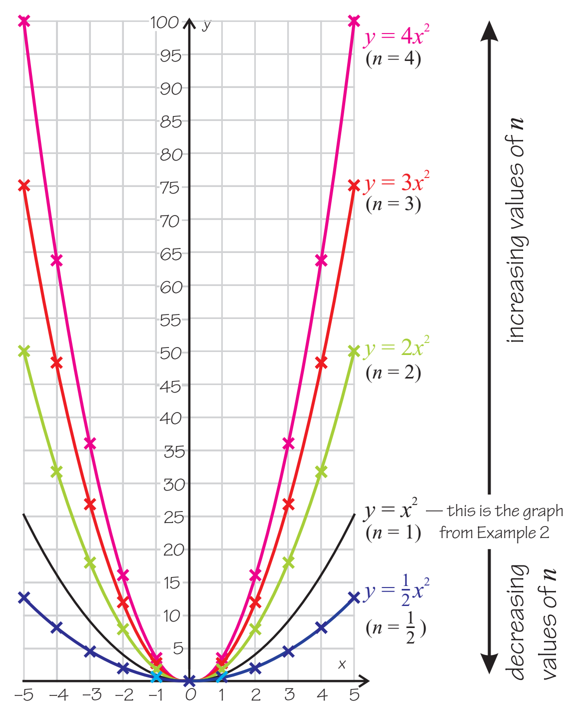
All these equations are of the form $y = nx^2$, for different values of n (2 then 3 then 4 then $\frac{1}{2}$).

The best place to start is with a table of values, just like before.

The table on the right shows values for parts **a)–d)**.

x	$2x^2$	$3x^2$	$4x^2$	$\frac{1}{2}x^2$
0	0	0	0	0
1 and –1	2	3	4	0.5
2 and –2	8	12	16	2
3 and –3	18	27	36	4.5
4 and –4	32	48	64	8
5 and –5	50	75	100	12.5

You then need to plot the ***y*-values** in each colored column against the ***x*-values** in the first column.



2 Teach (cont)

Notice how all the graphs are “u-shaped” parabolas. And all the graphs have their **vertex** (the lowest point) at the same place, the **origin**.

In fact, this is a general rule — if n is positive, the graph of $y = nx^2$ will always be a “u-shaped” parabola with its vertex at the origin.

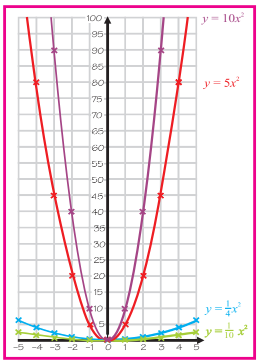
Also, the **greater** the value of $\boldsymbol{n}$, the **steeper** the parabola will be.
In Example 4, the graph of $y = 4x^2$ had the steepest parabola, while the graph of $y = \frac{1}{2}x^2$ was the least steep.

Check it out:

Values of n greater than 1 give parabolas steeper (or “narrower”) than $y = x^2$.

Values of n between 0 and 1 give parabolas less steep (or “wider”) than $y = x^2$.

14-17



Guided Practice

For Exercises 14–17, draw on the same axes the graph of each of the given equations. See left

14. $y = 5x^2$ **15.** $y = \frac{1}{4}x^2$ **16.** $y = 10x^2$ **17.** $y = \frac{1}{10}x^2$

In Exercises 18–23, use the graphs from Example 4 to solve the given equations.

18. $2x^2 = 20$ $x \approx 3.2$ or $x \approx -3.2$ **19.** $3x^2 = 25$ $x \approx 2.9$ or $x \approx -2.9$ **20.** $4x^2 = 15$ $x \approx 1.9$ or $x \approx -1.9$

21. $\frac{1}{2}x^2 = 10$ $x \approx 4.5$ or $x \approx -4.5$ **22.** $3x^2 = 70$ $x \approx 4.8$ or $x \approx -4.8$ **23.** $2x^2 = 42$ $x \approx 4.6$ or $x \approx -4.6$

Independent Practice

Using a table of values, plot the graphs of the equations in Exercises 1–3 for values of x between –4 and 4.

1. $y = 1.5x^2$ See right **2.** $y = 5x^2$ See right **3.** $y = \frac{1}{3}x^2$ See right

On the same set of axes as you used for Exercises 1–3, sketch the approximate graphs of the equations in Exercises 4–6.

4. $y = 2.5x^2$ See right **5.** $y = 6x^2$ See right **6.** $y = \frac{2}{3}x^2$ See right

7. If s is the length of a square’s sides, then a formula for its area, A, is $A = s^2$. Plot a graph of A against s, for values of s up to 10. See below

8. On a graph of $y = x^2$, what is the y-coordinate when $x = 10^3$? 10^6

For Exercises 9–12, find the y-coordinate of the point on the graph of $y = x^2$ for each given value of x.

9. $x = 10^{-1}$ 10^{-2} **10.** $x = 10^{-4}$ 10^{-8} **11.** $x = \frac{2}{3}$ $\frac{4}{9}$ **12.** $x = \frac{8}{5}$ $\frac{64}{25}$

For Exercises 13–15, find the x-coordinates of the point on the $y = x^2$ graph for each given value of y.

13. $y = 10^2$ 10 and –10 **14.** $y = 10^{-6}$ 10^{-3} and -10^{-3} **15.** $y = 2^8$ 2^4 and -2^4

Now try these:

Lesson 5.4.1 additional questions — p460

Don’t forget:

For Exercise 7, make sure all your values of s make sense as the length of a square’s sides.

Don’t forget:

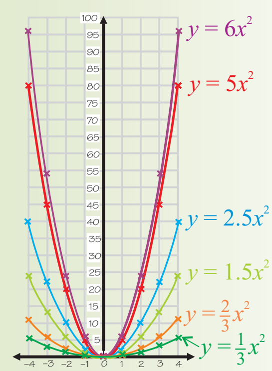
To take a square root of a power, you need to divide the exponent by 2. See Lesson 5.3.4 for more information.

Round Up

In this Lesson you’ve looked at graphs of the form $y = nx^2$, where n is positive. The basic message is that these graphs are all u-shaped. And the greater the value of n, the narrower and steeper the parabola is. Remember that, because in the next Lesson you’re going to look at graphs of the same form where n is negative.

Guided practice
Level 1: q14–16
Level 2: q14–20
Level 3: q14–23

Solutions
1–6.



Independent practice
Level 1: q1–3
Level 2: q1–8
Level 3: q1–15

Additional questions
Level 1: p460 q1–2, 4–5, 16
Level 2: p460 q1–6, 10–12, 16–17
Level 3: p460 q7–18

3 Homework

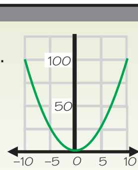
Homework Book — Lesson 5.4.1
Level 1: q1–3, 5, 6
Level 2: q1–8
Level 3: q1–8

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 28 — Graphing Linear Equations

Solutions
For worked solutions see the Solution Guide

7.



Lesson 5.4.2

More Graphs of $y = nx^2$

In this Lesson, students explore graphs of the form $y = nx^2$ for negative values of n. They plot these graphs using a table of values, and use the graphs to solve equations.

Previous Study: In the previous Lesson, students were introduced to graphs of the form $y = nx^2$ for positive values of n.

Future Study: In the next Lesson, students will extend their knowledge of nonlinear graphs to cubic graphs of the form $y = nx^3$. In Algebra I, students will learn methods for solving quadratic equations algebraically.

1 Get started

Resources:
- graph paper
- graphing calculators or graphing computer software

Warm-up questions:
- Lesson 5.4.2 sheet

2 Teach

Universal access

As was suggested for the previous Lesson, a good way to start the Lesson is to allow students to discover the new graphs for themselves.

Give the students the equation $y = -x^2$ and ask them to predict what the graph will look like. Then tell students to draw the graph using the table of values method they used in the previous Lesson.

Get them to compare this graph to the graph of $y = x^2$. For example, they might say, "it's upside down," "it's a mirror image," or "it is $y = x^2$ reflected in the x-axis."

The exercise could be extended by asking students to predict and then investigate the graphs of $y = -3x^2$, $y = -10x^2$, and $y = -\frac{1}{5}x^2$.

Math background

The symmetry of the graph is a powerful visual tool for analysis. For instance, you can see at a glance that if (a, b) is a point on the graph, then $(-a, b)$ is also on the graph.

In function notation, whenever you have a function such that $f(x) = f(-x)$, then the graph of that function will be symmetrical across the y-axis.

Lesson 5.4.2

More Graphs of $y = nx^2$

California Standards:

Algebra and Functions 3.1

Graph functions of the form $y = nx^2$ and $y = nx^3$ **and use in solving problems.**

Mathematical Reasoning 2.3

Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

What it means for you:

You'll learn more about how to plot graphs of equations with squared variables in them, and how to use the graphs to solve equations.

Key words:
- graph
- vertex

Don't forget:

$y = -x^2$ is just $y = nx^2$ with $n = -1$.

In the last Lesson you saw a lot of bucket-shaped graphs. These were all graphs of equations of the form $y = nx^2$, where n was positive. The obvious next thing to think about is what happens when n is negative.

The Graph of $y = nx^2$ is Still a Parabola if n is Negative

By plotting points, you can draw the graph of $y = -x^2$.

Example 1

Plot the graph of $y = -x^2$ for values of x from –5 to 5.

Solution

As always, first make a table of values, then plot the points.

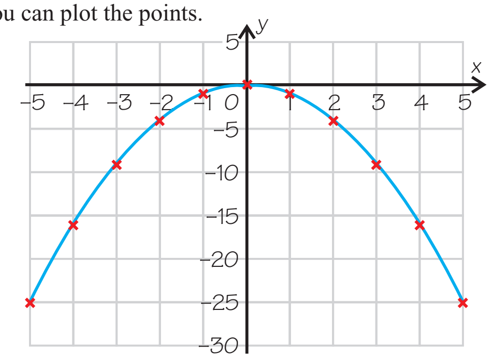
This time, the table of values is drawn horizontally, but it shows exactly the same information.

x	–5	–4	–3	–2	–1	0
x^2	25	16	9	4	1	0
$y = -x^2$	–25	–16	–9	–4	–1	0

You don't need a table for $x = 1, 2, 3, 4$, and 5, as it will contain the **same** values of y as above. However, if you find it easier to have all the values of x listed **separately**, then make a bigger table including the values below.

x	0	1	2	3	4	5
x^2	0	1	4	9	16	25
$y = -x^2$	0	–1	–4	–9	–16	–25

Now you can plot the points.



The graph of $y = -x^2$ is also a **parabola**.
But instead of being "u-shaped," it's "upside down u-shaped."

- **Strategic Learners**
  For students who are struggling with coordinates, tell them to write down the coordinates of the points after completing the table of values. Get them to write (x, y) over the tops of the coordinates to remind them that x comes first and y is second. The phrase "x is a-cross" can be used to help students remember that x is the horizontal axis.
- **English Language Learners**
  The Universal access activity suggested on this page will be useful to English language learners, as it provides lots of practice at using the vocabulary of the Lesson.

2 Teach (cont)

Nearly everything from the last Lesson about $y = nx^2$ for positive values of n also applies for negative values of n. However, for negative values of n, the graphs are **below** the x-axis.

Example 2

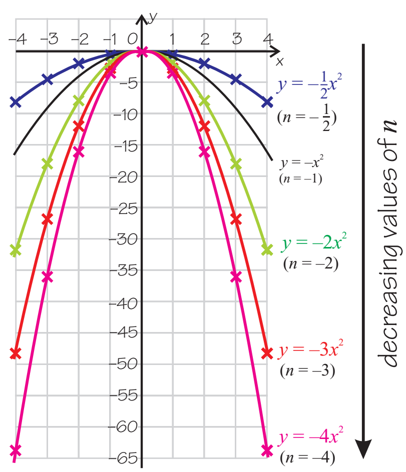
Plot the graphs of the following equations for values of x between –4 and 4.

a) $y = -2x^2$ **b)** $y = -3x^2$ **c)** $y = -4x^2$ **d)** $y = -\frac{1}{2}x^2$

Solution

As always, make a table and plot the points.

x	$-2x^2$	$-3x^2$	$-4x^2$	$-\frac{1}{2}x^2$
0	0	0	0	0
1	–2	–3	–4	–0.5
2	–8	–12	–16	–2
3	–18	–27	–36	–4.5
4	–32	–48	–64	–8



This time, since n is **negative**, all the graphs are "upside down u-shaped" parabolas. But all the graphs still have their **vertex** (the vertex is the **highest** point this time) at the same place, the **origin**.

Also, the **more negative** the value of n, the **steeper** and **narrower** the parabola will be.

Check it out:

This time, only positive values of x have been included in the table. The values for negative x will be identical.

Guided Practice

On which of the graphs in Example 2 do the points in Exercises 1–8 lie? Choose from $y = -x^2$, $y = -2x^2$, $y = -3x^2$, and $y = -\frac{1}{2}x^2$.

1. $(1, -3)$ $y = -3x^2$ **2.** $(-3, -4.5)$ $y = -\frac{1}{2}x^2$ **3.** $(4, -32)$ $y = -2x^2$ **4.** $(-5, -75)$ $y = -3x^2$

5. $(-3, -27)$ $y = -3x^2$ **6.** $(2, -2)$ $y = -\frac{1}{2}x^2$ **7.** $(5, -75)$ $y = -3x^2$ **8.** $(0, 0)$ all

Solve the equations in Exercises 9–14 using the graphs in Example 2. There are two possible answers in each case.

9. $-3x^2 = -12$ $x = 2$ or $x = -2$ **10.** $-\frac{1}{2}x^2 = -4.5$ $x = 3$ or $x = -3$ **11.** $-2x^2 = -32$ $x = 4$ or $x = -4$

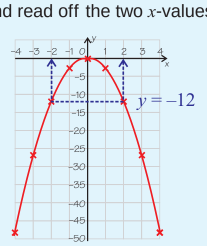
12. $-\frac{1}{2}x^2 = -2$ $x = 2$ or $x = -2$ **13.** $-3x^2 = -27$ $x = 3$ or $x = -3$ **14.** $-3x^2 = -40$ $x = 3.7$ or $x = -3.7$

Plot the graphs in Exercises 15–16 for x between –4 and 4.

15. $y = -5x^2$ See right **16.** $y = -\frac{1}{3}x^2$ See right

Check it out:

To solve the equations in Exercises 9–14, first find the correct graph — you need the one with the matching equation. So for Exercise 9 ($-12 = -3x^2$) you need the $y = -3x^2$ graph.
Then draw a line across the graph at the y-value you are given (in Exercise 9 it's –12) and read off the two x-values.



Universal access

Discuss the properties of parabolas with the class and address possible misconceptions.

For example: "As you extend the graph it gets steeper and steeper. Will anything special happen if you extend the graph far enough?"

Possible misconceptions here would be that the graph must eventually become vertical or even come back on itself.

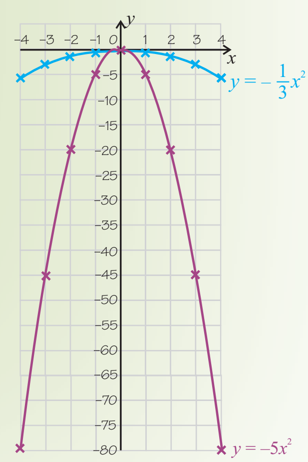
Discuss the symmetry of the graph and encourage the students to make observations such as:
"The graph is symmetrical across the y-axis," "if (3, –18) is on the graph, then (–3, –18) must be on the graph," and "$y = -nx^2$ is an exact reflection of $y = nx^2$ across the horizontal axis."

Guided practice

Level 1: q1–4, 15
Level 2: q1–12, 15
Level 3: q1–16

Solutions

15–16.



Solutions

For worked solutions see the Solution Guide

● **Advanced Learners**
Get students to investigate graphs of the form $y = (x - k)^2$ and $y = x^2 + k$. Or they could be given a function relationship. For example, define the function f as $f(x) = x^2$ and ask them to investigate the graphs of $y = f(x) + 3$, $y = f(3x)$, $y = f(x + 3)$, and $y = f(x - 3)$.

2 Teach (cont)

Additional examples

1. On the graph of $y = -3x^2$, what is the y-value when $x = -10$?
Substitute –10 for x in the equation $y = -3x^2$. So $y = -3 \times 10^2 = -3 \times 100 = -300$.

2. Describe in words the graph of $y = -\frac{1}{10}x^2$.

The graph is a wide parabola that opens down. Its vertex is (0, 0) and the y-axis is an axis of symmetry.

Common error

Students may confuse graphs with negative n with graphs where n is less than 1 but positive.

So they may interpret $y = -3x^2$ as being like $y = x^2$ but more shallow (something like $y = \frac{1}{3}x^2$).

Plotting some actual points should quickly reveal the true appearance of the graph.

Guided practice

Level 1: q17–18
Level 2: q17–18
Level 3: q17–18

Independent practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1–4

Additional questions

Level 1: p461 q1–2, 4–6
Level 2: p461 q1–6, 8–13
Level 3: p461 q1–13

3 Homework

Homework Book — Lesson 5.4.2

Level 1: q1, 2, 3a–d, 4a–b
Level 2: q1–7
Level 3: q1–7

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 28 — Graphing Linear Equations

Graphs of $y = nx^2$ for $n > 0$ and $n < 0$ are Reflections

The graphs you've seen in this Lesson (of $y = nx^2$ for **negative** n) and those you saw in the previous Lesson (of $y = nx^2$ for **positive** n) are very closely related.

Check it out:
k is just "any number." You could use n instead.

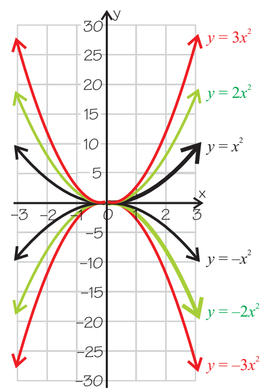
Example 3

By plotting the graphs of the following equations on the same set of axes for x between –3 and 3, describe the link between $y = kx^2$ and $y = -kx^2$.
$y = x^2$, $y = -x^2$, $y = 2x^2$, $y = -2x^2$, $y = 3x^2$, $y = -3x^2$.

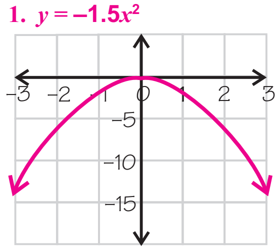
Solution

Plotting the graphs gives the diagram shown on the right.

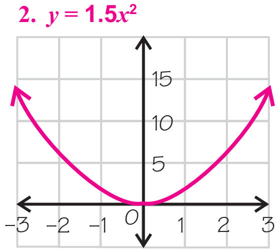
For a given value of k, the graphs of $y = kx^2$ and $y = -kx^2$ are **reflections** of each other. One is a "**u-shaped**" graph **above the x-axis**, while the other is an "**upside down u-shaped**" graph **below the x-axis**.



1. $y = -1.5x^2$



2. $y = 1.5x^2$



Guided Practice

17. The point (5, 100) lies on the graph of $y = 4x^2$. Without doing any calculations, state the y-coordinate of the point on the graph of $y = -4x^2$ with x-coordinate 5. **–100**

18. Without plotting any points, describe what the graphs of the equations $y = 100x^2$ and $y = -100x^2$ would look like. **See below**

Independent Practice

1. Draw the graph of $y = -1.5x^2$ for values of x between –3 and 3. **See left**

2. Without calculating any further y-values, draw the graph of $y = 1.5x^2$ for values of x between –3 and 3. **See left**

3. What are the coordinates of the vertex of the graph of $y = -\frac{1}{4}x^2$? **(0, 0)**

4. If a circle has radius r, its area A is given by $A = \pi r^2$.
Describe what a graph of A against r would look like.
Check your answer by plotting points for $r = 1, 2, 3$, and 4.
Half a u-shaped parabola above the x-axis with its vertex at (0, 0).

Now try these:
Lesson 5.4.2 additional questions — p461

Round Up

Well, there were lots of pretty graphs to look at in this Lesson. The graphs of $y = nx^2$ are important in math, and you'll meet them again next year. But next Lesson, it's something similar... but different.

Solutions

For worked solutions see the Solution Guide

18. The first is a steep u-shaped parabola above the x-axis with its vertex at (0, 0). The second is a reflection of this across the x-axis.

Lesson 5.4.3

Graphing $y = nx^3$

In this Lesson, students are introduced to graphs of the form $y = nx^3$. They explore how these graphs change for different values of n, including negative values.

Previous Study: Earlier in this Section, students learned how to plot graphs of equations of the form $y = nx^2$, and how these graphs change as n is varied.

Future Study: If students go on to study Algebra II, they will investigate the equations and graphs of circles, ellipses, and hyperbolas.

Lesson 5.4.3

Graphing $y = nx^3$

California Standards:

Algebra and Functions 3.1

Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.

Mathematical Reasoning 2.3

Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

What it means for you:

You'll learn about how to plot graphs of equations with cubed variables in them, and how to use the graphs to solve equations.

Key words:

- parabola
- plot
- graph

For the last two Lessons, you've been drawing graphs of $y = nx^2$. Graphs of $y = nx^3$ are very different, but the method for actually drawing the graphs is exactly the same.

The Graph of $y = x^3$ is Not a Parabola

You can always draw a graph of an equation by **plotting points** in the normal way. First make a **table** of values, then **plot** the points.

Example 1

Draw the graph of $y = x^3$ for x between –4 and 4.

Solution

First make a table of values:

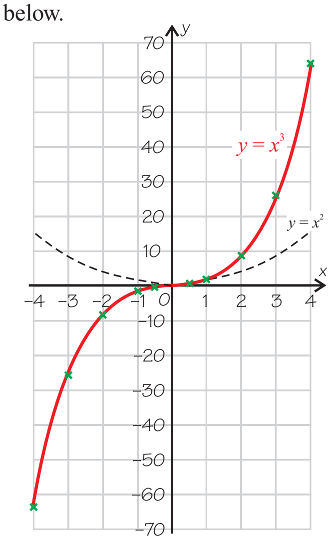
x	–4	–3	–2	–1	0	1	2	3	4
$y\ (= x^3)$	–64	–27	–8	–1	0	1	8	27	64

Then plot the points to get the graph below.

The graph of $y = x^3$ is completely different from the graph of $y = x^2$. It isn't "u-shaped" or "upside down u-shaped."

The graph still goes **steeply upward** as x gets more **positive**, but it goes **steeply downward** as x gets more **negative**.

The graph of $y = x^3$ passes through **all positive and negative** values of y.



The shape of the graph of $y = x^3$ is **not** a parabola — it is a curve that rises very quickly after $x = 1$, and falls very quickly below $x = –1$.

Check it out:

The graph of $y = x^3$ has a different kind of symmetry to that of $y = x^2$ — rotational symmetry.
If you rotate the graph 180° about the origin, it will look exactly the same.

Check it out:

Graphs of $y = nx^2$ pass through **either** all positive values of y **or** all negative values of y, depending on the value of n.

Check it out:

Try to figure out the shape of the curve before you plot it. Think about the value of x^3 if x is negative. How is this different from the value of x^3 if x is positive?

Guided Practice

1. Draw the graph of $y = –x^3$ by plotting points with x-coordinates –4, –3, –2, –1, –0.5, 0, 0.5, 1, 2, 3, and 4. See right

1 Get started

Resources:
- graph paper
- graphing calculators or graphing computer software
- 1 cm cubes

Warm-up questions:
- Lesson 5.4.3 sheet

2 Teach

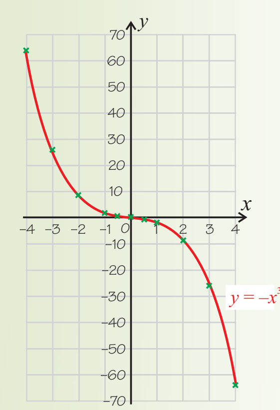
Universal access

As for the previous two Lessons, a good approach to the topic is to have the students draw their own graph before showing them what is in the textbook. Give graph paper to the students, and have them select points to graph for the equation $y = x^3$.

In making their own plots, students will soon run into the difficulty of scaling. The y-values increase very rapidly, so students need to examine the values in their table, and devise a suitable scale so that enough points can be plotted.

Solution

1.



Guided practice

Level 1: q1
Level 2: q1
Level 3: q1

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  Split the class into pairs to play a curve-sketching game. Start by asking one student from each pair to draw a large set of x- and y-axes. (No numbering is required on the axes — the purpose of the exercise is to compare graphs with each other, rather than accurately draw them.) One student then sketches and labels a graph of the form $y = nx^3$, such as $y = 5x^3$. Their partner then sketches and labels a different graph on the same axes. For example, they may sketch a steeper graph and label it $y = 10x^3$. The game continues like this. Different colors could be used for the different graphs.
- **English Language Learners**
  Throughout the Lesson, encourage students to describe patterns in words and to use correct math terminology. For example, "the graph has been reflected across the x-axis," and "the graph has a slope of 0 where it passes through the origin."

2 Teach (cont)

Universal access

Before drawing their own graph, students might find it useful to list the first 10 cubic numbers and to try to represent them. If you have 1 cm cubes, have students try to build cubes of side lengths 1–10 cm. They will quickly run out of cubes:

Side length	No. of cubes
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

Once students have produced a table, have them discuss it. Do they notice how fast the numbers increase? What will these points look like when graphed?

This activity prepares students to draw the graph of $y = x^3$.

Math background

The point where the graph "changes direction" is called the inflection point. There are several ways to define this point, such as, "the point at which the curvature changes sign." Positive curvature means the graph is curving upward; negative curvature means the graph is curving downward. Clearly, the inflection point of all graphs of the form $y = nx^3$ is (0, 0).

For these graphs, it is also a "saddle point" — a saddle point is a point of inflection with a slope of 0.

The Graph of $y = x^3$ Crosses the Graph of $y = x^2$

If you look really closely at the graphs of $y = x^3$ and $y = x^2$ you'll see that they **cross over** when $x = 1$.

Example 2

Draw the graph of $y = x^3$ for x values between 0 and 4.
Plot the points with x-values 0, 0.5, 1, 2, 3, and 4.

How does the curve of $y = x^3$ differ from that of $y = x^2$?

Solution

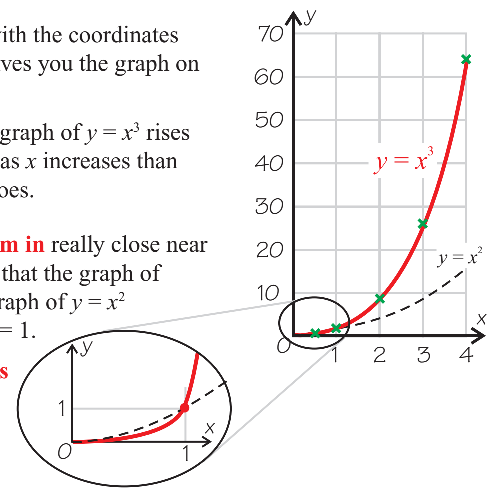
x	0	0.5	1	2	3	4
$y\ (= x^3)$	0	0.125	1	8	27	64

Plotting the points with the coordinates shown in the table gives you the graph on the right.

You can see that the graph of $y = x^3$ rises **much more steeply** as x increases than the graph of $y = x^2$ does.

But if you could **zoom in** really close near the origin, you'd see that the graph of $y = x^3$ is **below** the graph of $y = x^2$ between $x = 0$ and $x = 1$.

The two graphs **cross over** at the point (1, 1), and cross again at (0, 0).



Use Graphs of $y = x^3$ to Solve Equations

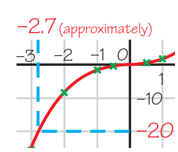
If you have an equation like $x^3 = 10$, you can **solve it** using a graph of $y = x^3$.

Example 3

Use the graph in Example 1 to solve the equation $x^3 = –20$.

Solution

First find –20 on the vertical axis.
Then find the corresponding value on the horizontal axis — this is the solution to the equation. So $x = –2.7$ (approximately).



Check it out:
There will only be one solution to equations like $x^3 = 10$ and $x^3 = –20$. This is because no two numbers can be cubed to give the same value.

- **Advanced Learners**
  Ask students to predict and then investigate the shape of the graphs $y = x^3 + k$ and $y = (x + k)^3$ for positive and negative values of k.

Guided Practice

Use the graph of $y = x^3$ to solve the equations in Exercises 2–7.

2. $x^3 = 64$ **$x = 4$** **3.** $x^3 = 1$ **$x = 1$** **4.** $x^3 = -1$ **$x = -1$**
5. $x^3 = -27$ **$x = -3$** **6.** $x^3 = 30$ **$x \approx 3.1$** **7.** $x^3 = -50$ **$x \approx -3.7$**

8. How many solutions are there to an equation of the form $x^3 = k$? Use the graph in Example 1 to justify your answer. **One — since the graph of $y = x^3$ takes each value of y just once.**

The Graph of $y = nx^3$ is Stretched or Squashed

The exact shape of the graph of $y = nx^3$ **depends on the value of n**.

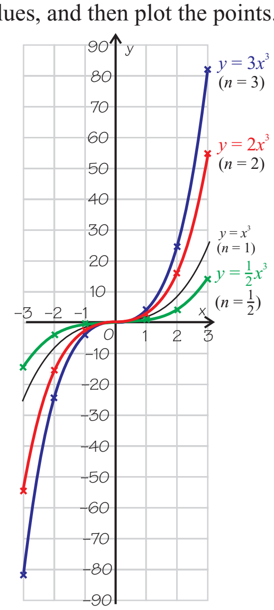
Example 4

Plot points to show how the graph of $y = nx^3$ changes as n takes the values 1, 2, 3, and $\frac{1}{2}$.

Solution

Using values of x between –3 and 3 should be enough for any patterns to emerge. So make a suitable table of values, and then plot the points.

x	$2x^3$	$3x^3$	$\frac{1}{2}x^3$
–3	–54	–81	–13.5
–2	–16	–24	–4
–1	–2	–3	–0.5
0	0	0	0
1	2	3	0.5
2	16	24	4
3	54	81	13.5



As **n increases**, the curves get **steeper and steeper**.

However, the basic shape remains the same. All the curves have rotational symmetry about the origin.

Guided Practice

Use the above graphs to solve the equations in Exercises 9–14.

9. $3x^3 = -60$ **$x \approx -2.7$** **10.** $2x^3 = 30$ **$x \approx 2.5$** **11.** $\frac{1}{2}x^3 = -10$ **$x \approx -2.7$**
12. $\frac{1}{2}x^3 = 10$ **$x \approx 2.7$** **13.** $3x^3 = 40$ **$x \approx 2.4$** **14.** $2x^3 = -35$ **$x \approx -2.6$**

15. How many solutions are there to an equation of the form $nx^3 = k$, where n and k are positive? **One**

Don't forget:
The graph of $y = x^3$ ($n = 1$) is also plotted on these axes for comparison.
The table of values is in Example 2 on the previous page.

2 Teach (cont)

Guided practice
Level 1: q2–4
Level 2: q2–7
Level 3: q2–8

Concept question
"Say whether each of these statements is true or false. If false, write a correct statement."

1. The graph of $y = 5x^2$ has two y-values corresponding to each x-value.
False — it has two x-values corresponding to each positive y-value.

2. The third quadrant of the graph of $y = x^3$ can be obtained by rotating the first quadrant through 180° about the origin.
True.

3. The graph of $y = -3x^3$ has only one y-value for each x-value and only one x-value for each y-value.
True.

4. The graph of $y = nx^2$ passes through all positive and negative values of y.
False — the graph of $y = nx^2$ passes through all positive values of y.

Guided practice
Level 1: q9–11
Level 2: q9–14
Level 3: q9–15

Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Common error
A common error is to draw the graph passing through (0, 0) at an angle, rather than horizontally.

Explain that the slope of the graph $y = nx^3$ at the point (0, 0) is always 0.

Universal access
Use graphing software or a graphing calculator with a zoom function to display the graph of $y = x^3$.

Use the zoom feature to zoom in on the origin so that students can see that the slope is always 0 through the origin.

Guided practice
Level 1: q16
Level 2: q16–18
Level 3: q16–18

Independent practice
Level 1: q1–2
Level 2: q1–4
Level 3: q1–4

Additional questions
Level 1: p461 q1–4, 9, 12–14
Level 2: p461 q1–10, 12–16
Level 3: p461 q9–17

3 Homework

Homework Book — Lesson 5.4.3
Level 1: q1–4, 5, 7
Level 2: q1–8
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 28 — Graphing Linear Equations

For $n < 0$, the Graph of $y = nx^3$ is Flipped Vertically

If n is **negative**, the graph of $y = nx^3$ is "upside down."

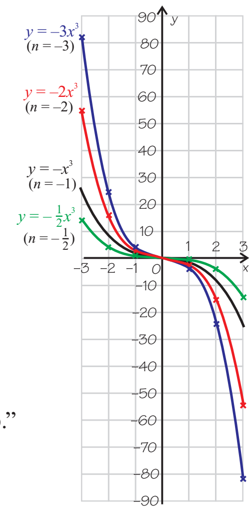
Example 5

Plot points to show how the graph of $y = nx^3$ changes as n takes the values –1, –2, –3, and $-\frac{1}{2}$.

Solution

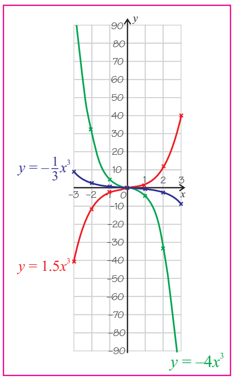
The table of values looks very similar to the one in Example 4. The only difference is that all the numbers **switch sign** — so all the positive numbers become negative, and vice versa.

x	$-2x^3$	$-3x^3$	$-\frac{1}{2}x^3$
–3	54	81	13.5
–2	16	24	4
–1	2	3	0.5
0	0	0	0
1	–2	–3	–0.5
2	–16	–24	–4
3	–54	–81	–13.5



This change in sign of all the values means the curves all do a "vertical flip."

1–3



Guided Practice

Use the above graphs to solve the equations in Exercises 16–18.

16. $-3x^3 = -50$ $x \approx 2.6$ **17.** $-3x^3 = 50$ $x \approx -2.6$ **18.** $-\frac{1}{2}x^3 = 10$ $x \approx -2.7$

Independent Practice

Using a table of values, plot the graphs of the equations in Exercises 1–3 for values of x between –3 and 3.

1. $y = 1.5x^3$ See left **2.** $y = -4x^3$ See left **3.** $y = -\frac{1}{3}x^3$ See left

4. If the graph of $y = 8x^3$ goes through the point (6, 1728), what are the coordinates of the point on the graph of $y = -8x^3$ with x-coordinate 6?
(6, –1728)

Now try these:
Lesson 5.4.3 additional questions — p461

Round Up

That's the end of this Section, and with it, the end of this Chapter. It's all useful information. You need to remember the general shapes of the graphs, and how they change when the n changes.

Solutions
For worked solutions see the Solution Guide

Chapter 5 Investigation — The Solar System

Purpose of the Investigation

This Investigation provides students with the opportunity to use scientific notation in a real-life application. The Investigation also deepens students' understanding of the motivation for using scientific notation, and of why scientists created the astronomical unit. The extensions bring together students' knowledge of scientific notation and scale drawings.

Resources

- internet computers, or reference books listing planet sizes
- rulers
- legal-size paper
- calculators

Strategic & EL Learners

Working cooperatively with other students will provide support for strategic learners, as well as for English Language Learners.

Investigation Notes on p313 B-C

Chapter 5 Investigation
The Solar System

Some numbers are really, really large — like distances in Space. It'd take a long time to write such numbers out in full, and then they'd be hard to compare and work with. So scientific notation is used — it makes things much simpler.

The eight planets of the Solar System travel around the Sun in paths called **orbits**. The orbits are actually elliptical, but for this Investigation, you'll treat them as **circles**. The **average radius** of each planet's orbit is given below in miles.

Part 1:
The data on the right is presented in alphabetical order, as you might find in a reference book.

Make a more useful table by presenting the data so that:
- the planets are **in order of distance** from the Sun
- the distances are **given in scientific notation**.

Planet	Approximate mean distance from Sun (in miles)
Earth	92,960,000
Jupiter	483,800,000
Mars	141,700,000
Mercury	35,980,000
Neptune	2,793,000,000
Saturn	886,700,000
Uranus	1,785,000,000
Venus	67,240,000

Things to think about:
- Is it easier to convert the numbers into scientific notation **before** ordering them, or to order the numbers and **then** convert them into scientific notation?

Part 2:
Using the scientific notation figure, **compute the approximate area** inside the Earth's orbit and present it in scientific notation.
(Remember that the area of a circle is given by $A = \pi r^2$, where A is the area and r is the radius. Use $\pi = 3.14$.)

Extensions

1) The mean distance from Earth to the Sun is called an astronomical unit (AU) and is about 92.96 million miles. Add a column to your table to show all the distances converted into AUs.

2) Make a scale drawing of the Solar System using the scale of 1 cm = 1 AU. Place the Sun at one edge of the paper. Use dots to represent the planets.

Open-ended Extensions

1) Research the diameters of the planets. Write the diameters in miles and then rewrite them in scientific notation.

2) Make scale drawings showing the size of the planets. What scale did you use?

3) If you drew the diameter of Mercury as 1 mm, how large a sheet of paper would you need to accurately draw the entire Solar System with planet sizes and distances to the Sun all to the same scale?

Round Up

When you're working with very big numbers, it's usually easier if you put them in scientific notation first. This way, you can tell which is the biggest by comparing just the exponents, rather than counting all the digits each time. It's a similar situation with very small numbers.

Investigation — The Solar System

Mathematical Background

It's more convenient to work with very big numbers if they're written in scientific notation.
To write a number in scientific notation, it is broken into two factors — the first must be at least 1, but less than 10. The second must be a power of 10. For example, $62{,}000{,}000 = 6.2 \times 10^7$.

When calculating with numbers in scientific notation, the multiplication and division of powers properties can be used to make the calculation easier.
For example:

$$(3.3 \times 10^6) \times (4.4 \times 10^4) = (3.3 \times 4.4) \times (10^6 \times 10^4) = 14.52 \times 10^{(6+4)} = 14.52 \times 10^{10}$$

Often, the answer won't be in scientific notation — it will need to be converted:

$$14.52 \times 10^{10} = (1.452 \times 10^1) \times 10^{10}$$

This isn't in scientific notation. 14.52 is more than 10.

$$= 1.452 \times (10^1 \times 10^{10}) = \mathbf{1.452 \times 10^{11}}$$

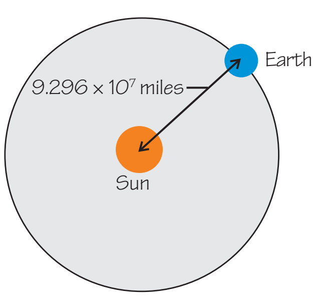
This is in scientific notation.

Approaching the Investigation

Part 1:
This part requires students to write the planets in order of their distance from the Sun. Some students may already know the order of the planets, but others will have to compare the numbers. Students should discover that it is much easier to order large numbers once they are written in scientific notation. Below is the table as required by the task:

Planet	Mean distance from Sun (in miles)
Mercury	3.598×10^7
Venus	6.724×10^7
Earth	9.296×10^7
Mars	1.417×10^8
Jupiter	4.838×10^8
Saturn	8.867×10^8
Uranus	1.785×10^9
Neptune	2.793×10^9

Part 2:
Students now have to calculate the approximate area inside the Earth's orbit, assuming it is circular. It may be useful to sketch a diagram to illustrate that the distance between the orbit and the Sun is the radius of a circle.



$$\text{Area} = \pi r^2$$
$$= 3.14 \times (9.296 \times 10^7)^2$$
$$= 3.14 \times (9.296)^2 \times (10^7)^2$$
$$= 271.3 \times 10^{14} \text{ square miles}$$

This isn't in scientific notation. 271.3 is more than 10.

Power of a power property

$$271.3 \times 10^{14} = 2.713 \times 10^2 \times 10^{14}$$
$$= \mathbf{2.713 \times 10^{16} \text{ square miles}}$$

Multiplication of powers property

Investigation — The Solar System

Extensions

1) By definition, Earth is exactly 1 astronomical unit (AU) away from the Sun. To convert the other planetary measures into AUs you need to divide each measure by the distance from Earth to the Sun. Encourage students to estimate the answer before calculating — then they'll know if their answer is about the right size. For instance, Mercury is about 36 million miles away while Earth is about 93 million miles away. 36 is a little more than one-third of 93, so its distance should be a little more than one-third of an AU.

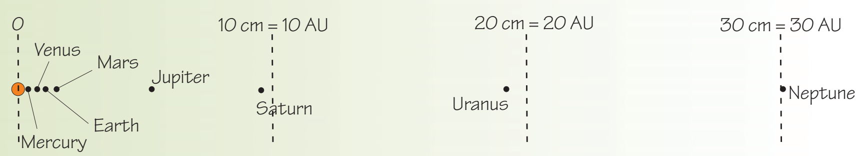
Here's the exact calculation: $(3.598 \times 10^7) \div (9.296 \times 10^7) = (3.598 \div 9.296) \times (10^7 \div 10^7)$
$= 0.387 \times 10^{(7-7)}$
$= 0.387 \times 10^0$
$= 0.387 \times 1 =$ **0.387 AU**

So Mercury is **0.387 AU** away from the Sun.

Dividing each planet's distance from the Sun by 9.296×10^7 gives the following table.

Planet	Astronomical Units from Sun
Mercury	0.387
Venus	0.723
Earth	1.00
Mars	1.52
Jupiter	5.20
Saturn	9.58
Uranus	19.2
Neptune	30.0

2) A scale drawing of the Solar System is more easily drawn on a sheet of legal-size paper (8.5" × 14") than a regular sheet of paper (8.5" × 11"). This allows students to use a scale of 1 cm : 1 AU. Students should produce something like the following:



Open-Ended Extensions

1) The diameters of the planets are summarized in this table. The figures will vary, depending on the sources used.

2) Students are given the opportunity to choose their own scale. They should pick this so that the largest planet will fit comfortably on a piece of paper. Two possible scales are included in the table — one where Earth is drawn with a diameter of 1 cm, and another where 10,000 miles = 1 cm.

Planet	Diameter in miles	Diameter in miles (scientific notation)	Scale drawing diameter, cm (1 cm = 7918 miles)	Scale drawing diameter, cm (1 cm = 10,000 miles)
Mercury	3032	3.032×10^3	0.4	0.3
Venus	7520	7.520×10^3	0.9	0.8
Earth	7918	7.918×10^3	1.0	0.8
Mars	4212	4.212×10^3	0.5	0.4
Jupiter	88,846	8.8846×10^4	11.2	8.9
Saturn	74,898	7.4898×10^4	9.5	7.5
Uranus	31,518	3.1518×10^4	4.0	3.2
Neptune	30,602	3.0602×10^4	3.9	3.1

3) If the diameter of Mercury is 1 mm, then 1 mm = 3.032×10^3 miles. The planet furthest from the Sun is Neptune, with an orbit radius of 2.793×10^9 miles.

At this scale, the radius of Neptune's orbit on the scale drawing will be:
$2.793 \times 10^9 \div 3.032 \times 10^3 = (2.793 \div 3.032) \times (10^9 \div 10^3) = 0.921 \times 10^6 = 9.21 \times 10^5$ mm.

Converting this to meters: $9.21 \times 10^5 \div 1000 = 9.21 \times 10^5 \div 10^3 = 9.21 \times (10^5 \div 10^3) = 9.21 \times 10^2$ m.

Assuming the Sun is at the edge of the paper, the paper would have to be 921 meters long — that's nearly a kilometer.

Chapter 6

The Basics of Statistics

How Chapter 6 fits into the K-12 curriculum

Section 6.1 — Analyzing Data		
Section 6.1 covers Statistics, Data Analysis, and Probability 1.1, 1.3, Mathematical Reasoning 2.6 **Objective: To understand median and range, and to draw box-and-whisker and stem-and-leaf plots**		
Previous Study In grades 4, 5, and 6 students compute the medians of data sets, and in grade 6 they calculated the ranges of data sets. In grade 5 they displayed single-variable data in appropriate graphs such as histograms and circle graphs.	**This Section** Students review median and range, and are then introduced to box-and-whisker and then stem-and-leaf plots, including how to construct each. They then answer real-life questions by organizing, displaying, and analyzing data.	**Future Study** In Section 6.2, students will display two related sets of data on scatterplots and investigate correlation.

Section 6.2 — Scatterplots		
Section 6.2 covers Statistics, Data Analysis, and Probability 1.2, Mathematical Reasoning 2.3 **Objective: To present data on scatterplots and to analyze scatterplots in terms of correlation**		
Previous Study In Section 6.1, and in earlier grades, students learned how to display data in a range of forms, such as histograms, circle graphs, box-and-whisker plots, and stem-and-leaf plots.	**This Section** Students learn how to make scatterplots to display two related sets of data. They then see how the shape of a scatterplot indicates positive or negative correlation, and then go on to practice interpreting scatterplots.	**Future Study** In Algebra I, students derive linear equations of straight lines from graphs.

Pacing Guide – Chapter 6

40- to 50-Minute Class Periods

If your class periods are 40-50 minutes, we recommend allowing **12 days** for teaching Chapter 6.
As well as the **9 days of basic teaching**, you have **3 days** remaining to allocate 3 of the 5 optional activities (to be delivered at any appropriate point during the Chapter).

The table shows the 9 teaching days as well as all of the **optional days** you may choose for Chapter 6, in the order we recommend.

Day	Lesson	Description
Section 6.1 — Analyzing Data		
Optional		*Exploration — Reaction Rates*
1	6.1.1	Median and Range
2	6.1.2	Box-and-Whisker Plots
3	6.1.3	More on Box-and-Whisker Plots
4	6.1.4	Stem-and-Leaf Plots
5	6.1.5	Preparing Data to be Analyzed
6	6.1.6	Analyzing Data
Optional		*Assessment Test — Section 6.1*
Section 6.2 — Scatterplots		
Optional		*Exploration — Age and Height*
7	6.2.1	Making Scatterplots
8	6.2.2	Shapes of Scatterplots
9	6.2.3	Using Scatterplots
Optional		*Assessment Test — Section 6.2*
Chapter Investigation		
Optional		*Investigation — Cricket Chirps and Temperature*

Accelerating and Decelerating

- To **accelerate** Chapter 6, allocate fewer than 3 days to the optional material. This will give you extra days to allocate to other Chapters.
  Note that you may use the remaining optional days at the end of the 160-day course.
- To **decelerate** Chapter 6, consider allocating more than 3 days to the optional Assessment Tests, Section Explorations, or Chapter Investigation, or spend longer teaching some Lessons. Also consider preparing students for difficult Lessons by reviewing previous coverage of math topics on related Skills Review Worksheets.
  Note that decelerating Chapter 6 will result in fewer days being available for teaching other Chapters.

90-Minute Class Periods

If you are following a block schedule with 90-minute class periods, we recommend allowing **6 days** for teaching Chapter 6.
The basic teaching material will take up **4.5 days**, and you can allocate the remaining **1.5 days** to the **optional material**.
To accelerate or decelerate a block schedule, follow the same advice as given above.

Section 6.1

Exploration — Reaction Rates

Purpose of the Exploration

This reaction time Exploration allows students to perform an experiment to produce their own data. They then review how to calculate the mean, median, mode, and range of their results. Ownership of the data makes the analysis more meaningful to the students.

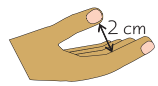
Resources

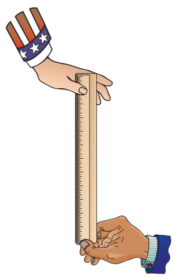
- 30-cm rulers
- calculators

Section 6.1 introduction — an exploration into: Reaction Rates

In this Exploration, you'll test your reaction time by catching a ruler dropped by another student. You'll collect data and calculate its mean, mode, median, and range. From this analysis you'll be able to draw conclusions about your typical reaction time.

The experiment requires one person to **drop** the ruler and another person to **catch it**. The catcher is seated with his or her arm resting on a table. The catcher's hand is off the table, with the distance between his or her thumb and pointer finger at **2 cm**.





The dropper holds a ruler so that **0 cm** is level with the catcher's finger and thumb. The dropper then **releases the ruler without warning**, and the catcher tries to catch it **as soon as possible**.

The dropper **records** the position of the catcher's pointer finger and thumb on the ruler.

Exercises

1. Repeat the experiment 12 times and record the results in a copy of the table below.

Trial	Distance (cm)	Trial	Distance (cm)
1		7	
2		8	
3		9	
4		10	
5		11	
6		12	

Switch jobs after the completion of the experiment. The catcher becomes the dropper and the dropper becomes the catcher.

2. What was the range for your data? The least value subtracted from the greatest. This depends on the student's results.

3. What is the median distance for your data? The average of the 6th and 7th values, when all 12 values are arranged in order. This depends on the student's results.

4. What is the modal distance for your data? The most common value (there may be no mode, or more than one). Depends on the student's results.

5. What is the mean distance for your data? Divide the sum of all the reaction times by 12. Depends on the student's results.

6. Explain which value you think represents your typical reaction time best. This depends on the student's results. It's likely to be the mean or the median.

7. Were there any trials that did not seem to fit in with the rest of the results? If so, suggest possible reasons why. Possible reasons might include a student not paying attention, or some warning being given prior to dropping the ruler.

Round Up

It's always a good idea to repeat experiments lots of times, then find the average of all the trials. This means your result is likely to be more accurate. You've found three types of average for your data from this experiment — one will often represent your data better than the others.

Strategic & EL Learners

Strategic learners will benefit from having the definitions of mean, median, and mode in front of them. This will eliminate confusion about which is which and allow them to focus on analyzing the data.

The terms mean, median, and mode can be confusing to EL learners since they all start with "m." Simplify this by pointing out that they are looking for the number that appears most often, the middle number, and the average.

Universal access

Remind students that an important aspect of an experiment is that it is done the same way every time. It is crucial that they measure the distance between the fingers and drop the ruler without warning on every trial.

Common error

Students may calculate the median without placing the numbers in order. Remind students that they should write their results from least to greatest after they have completed the table.

Math background

Students should feel comfortable finding the mean, median, mode, and range for a set of data. They should know how to find the median when there is an even number of items in a data set.

Lesson 6.1.1

Median and Range

In this Lesson, students review how to find the median and range. They then use the median and the range to make comparisons between data sets. A good understanding of the median is important for the work on box-and-whisker plots in the following Lessons.

Previous Study: In grades 4, 5, and 6, students computed the medians of data sets. In grade 6, students also calculated the ranges of data sets.

Future Study: In the next two Lessons, students will use their understanding of the median to produce box-and-whisker plots.

1 Get started

Resources:
- small balls

Warm-up questions:
- Lesson 6.1.1 sheet

2 Teach

Universal access

Give each group of four students a small ball. When you say "Go," they pass the ball from student to student, and count the number of passes until they drop it, or you say "Stop" (after 15 seconds).

Conduct three trials and find the median number of throws. Conduct a fourth trial and find the new median. Discuss the difference between finding the median of a data set with an odd number of values, and the median of a data set with an even number of values.

Concept question

"In 2006, the median salary for a cashier in the US was $14,790 per year. Explain what this statistic means."

This statistic means that of all the cashiers in the US, half of them had a salary of $14,790 per year or more, and the other half had a salary of $14,790 per year or less.

Guided practice

Level 1: q1
Level 2: q1
Level 3: q1

Lesson 6.1.1

Section 6.1

Median and Range

California Standards:

Statistics, Data Analysis, and Probability 1.3

Understand the meaning of, and be able to compute, the minimum, the lower quartile, **the median,** the upper quartile, **and the maximum of a data set.**

What it means for you:

You'll learn the meaning of the terms minimum, median, maximum, and range, and how to find these values from data sets.

Key words:
- median
- minimum
- maximum
- range

Don't forget:

The average of a set of data is the sum of all the numbers divided by the number of data points. So when you work out the average of two numbers, you add them together, and then divide the result by two.

In grade 6, you learned about three different typical values of data sets — the mode, mean, and median. In this Lesson, you'll review the median in preparation for drawing box-and-whisker plots in the next Lesson.

The Median is the Value in the Middle of a Data Set

If you arrange a data set in order, the **middle value** is the **median**. It gives you an idea of a "typical" value for the data set.

Here's a reminder of the process you go through to find the median:

1. **Order** the data from smallest to largest.
2. **Count** the number of values in the data set.
3. If the number of values is **odd**, take the **middle value** as the median.
4. If the number of values is **even**, take the **average of the two middle values** as the median.

Example 1

Find the median of each data set below.

1. {4, 6, 8, 8, 12, 15, 19}

2. {12, 6, 4, 8, 15, 15, 8, 15}

Solution

1. The data is **already ordered** — and there are 7 values.
This is an odd number, so the median is the **middle value**, which is **8**.

4, 6, 8, 8, 12, 15, 19

2. The data **isn't ordered** — so you have to first order the data.
There are 8 values in the data set — an even number. So the median is the **average** of the two middle values, which are **8 and 12**.

4, 6, 8, 8, 12, 15, 15, 15

The average of these values is: (8 + 12) ÷ 2 = 20 ÷ 2 = 10.
So **the median is 10**.

Since the median is the middle of a data set, you know that **half** of the values in the data set are **below** the median, and **half** are **above** it.

Guided Practice

1. A hospital measures the length of newborn babies on a daily basis. On one day the results in inches were:
19, 22, 20, 21, 22, 20, 24, 20, 17, 21.
What was the median length? **20.5 inches**

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  Have students work in groups of five. Ask them to count and record the number of letters in each person's first name. They should then arrange the names in order, from the least to the most letters, and decide which name (or names) represents the median number of letters. Repeat the activity with an even number of students in each group, and discuss the difference in the approach needed for finding the median.
- **English Language Learners**
  Help develop students' understanding of the term "median" by relating it to its use in everyday life. The median strip on a highway is the stretch of land that runs down the middle of the highway. The median in math is in the middle of an ordered set of numbers.

The Range Tells You About the Spread of the Data

The range of a data set tells you about the **spread** of the data. It tells you whether the data is **close together**, or **spaced out**. To calculate the range you first need to find the **minimum** and **maximum** values:

- The **smallest** value in a set is called the **minimum**
- The **largest** value in a set is called the **maximum**
- The **range** is the **difference between** the maximum and the minimum.

Don't forget:
You find the difference between two numbers by taking the smaller number from the larger number.

Example 2

Belinda had the following test scores on her first five tests:

92, 88, 96, 83, 91.

What is the range of her scores?

Solution

The minimum value is **83**, the maximum value is **96**.

The range is the **difference between** the maximum and the minimum.

The range is 96 – 83 = **13**.

Check it out:
It's often safer to put the data into order first, rather than try to find the minimum and maximum values in a long jumbled set of data.

Use Medians and Ranges to Compare Data Sets

Looking at the **medians** and the **ranges** can give you useful information about data sets.

Example 3

Jewelry Store A sells watches with a median price of **$99** and a range of **$60**.

Jewelry Store B sells watches with a median price of **$99** and a range of **$820**.

Describe what these statistics tell about the prices of the watches in each jewelry store.

Solution

Both stores have the **same median** price. But Store A has a **smaller range**, so the prices are all clustered more closely together.
The **minimum price** a watch could be in Store A is $99 – $60 = $39, and the **maximum price** a watch could be in Store A is $99 + $60 = $159.

Store B's price range is much **larger**, so the price of **at least one** of the watches it sells lies much further from the median than any of the watches in Store A. The **maximum price** a watch could be in Store B is $99 + $820 = $919. But some of the watches may be very cheap — cheaper than the cheapest watch in Store A.

Check it out:
A possible data set that has a range of $60, a median of $99, and a minimum price of $39 is: {$39, $99, $99}.
A possible data set that has the same range and median, and a maximum price of $159 is: {$99, $99, $159}.

2 Teach (cont)

Common error
Students will often calculate the median and range without putting the data in order.

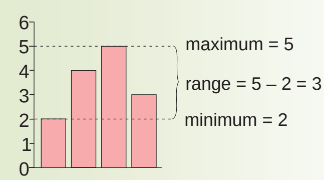
For example, for a data set listed as 91, 88, 96, 83, 93, they may give 96 as the median. 96 is the middle value of this list, but is clearly not the median.

In a similar way, they may calculate the range as 2 — the difference between the first and last values listed, rather than the difference between the greatest and least.

If this is a problem, remind students of the definitions of median and range.

Universal access
Another way to "picture" minimum and maximum values, and the range of a data set is to draw a bar graph.

For example, the data set 2, 4, 5, 3 may be displayed as:



Additional example
City A's buildings have a median age of 150 years. The range of their ages is 320 years.

City B's buildings have a median age of 21 years. The range of their ages is 4 years.

Interpret these statistics.

City A's buildings are generally much older than City B's. None of City B's buildings can be more than 25 years old. City A's buildings have a much wider range of ages — so some are a lot older than others. All of City B's buildings were built around the same time.

- **Advanced Learners**
  Ask students to think about how to find the median value in the general case. Ask them, "If you have an ordered data set of n values and n is odd, which value is the median value?" In this case, the median value is at position $(n + 1) \div 2$.

2 Teach (cont)

Guided practice

Level 1: q2–3
Level 2: q2–4
Level 3: q2–4

Common error

Sometimes students will only list a number once when that number is repeated in the data set.

For instance, when presented with the data set 4, 6, 8, 8, 12, 15, 15, 15, students will write it as 4, 6, 8, 12, 15 and get an incorrect median of 8. Make sure students know that not only are repeats "allowed," but they are needed for correct calculations.

Universal access

A useful technique for finding the median for some students is to cross off a value from each end until they get to the middle, as shown below:

4, 6, 8, 8, 12, 15, 19
~~4,~~ 6, 8, 8, 12, 15, ~~19~~
~~4, 6,~~ 8, 8, 12, ~~15, 19~~
~~4, 6, 8,~~ 8, ~~12, 15, 19~~

The last number (or average of the remaining two numbers) is the median.

Independent practice

Level 1: q1–5, 7–9
Level 2: q1–10
Level 3: q1–15

Additional questions

Level 1: p462 q1–4, 9–11, 15
Level 2: p462 q1–6, 9–13, 15–16
Level 3: p462 q3–8, 11–16

3 Homework

Homework Book — Lesson 6.1.1

Level 1: q1–6
Level 2: q1–7
Level 3: q1–8

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 46 — Median
- Worksheet 48 — Data Sets

Don't forget:
If the values in the data set have units, you need to include units for the range and median.

Guided Practice

A gardener is trying to grow large zucchinis. She has two sets of zucchinis that she treats with different fertilizers. The lengths of the zucchinis in each set are shown below.
Set 1: {11 cm, 15 cm, 16 cm, 19 cm, 23 cm}
Set 2: {24 cm, 13 cm, 61 cm, 55 cm, 41 cm, 22 cm, 55 cm}

2. Find the range of the lengths in each set. **Set 1 = 12 cm, Set 2 = 48 cm**
3. Find the median length for each set. **Set 1 = 16 cm, Set 2 = 41 cm**
4. If you were only told the median length and the range of lengths for Set 1, what could you say about the minimum and maximum values of the set? **minimum possible value = 4 cm, maximum possible value = 28 cm**

Independent Practice

Find the median of the data sets in Exercises 1–4.
1. {11, 15, 16, 19, 23} **16**
2. {8, 8, 9, 13, 15, 15} **11**
3. {28, 11, 43, 21, 41, 53, 55} **41**
4. {11, 13, 9, 12, 12, 19, 18, 17, 16, 5} **12.5**

5. Frank had the following quiz scores: 18, 16, 15, 20, and 16. What was his median score? **16**

6. Alyssa had the following number of rebounds over her last 8 games: 4, 8, 9, 3, 11, 5, 12, 5. What was the median number of rebounds? **6.5**

Find the minimum, maximum, and range of the data sets in Exercises 7–8.
7. {8, 8, 9, 13, 15, 15} **min = 8, max = 15, range = 7**
8. {11, 13, 9, 12, 12, 19, 18, 17, 16, 5} **min = 5, max = 19, range = 14**

9. Store A sells fine pens with a median price of $29 and a range of $20. Store B sells fine pens with a median price of $40 and a range of $30. What could be the minimum and maximum possible prices of each store's pens? **Store A: min. possible value = $9, max. possible value = $49**
Store B: min. possible value = $10, max. possible value = $70

10. Furniture Store A sells chairs with a median price of $110 and a range of $40. What is the lowest possible price for a chair in Furniture Store A? **$70**

Find the median and range of the sets of data in Exercises 11–15.
11. {86, 78, 81, 80, 80, 85, 72, 90} **med = 80.5, range = 18**
12. {34, 35, 31, 32, 30, 35} **med = 33, range = 5**
13. {101, 104, 107, 102, 98, 100} **med = 101.5, range = 9**
14. {98, 97, 97, 97, 96, 95, 98, 96, 95, 98, 98} **med = 97, range = 3**
15. {61, 60, 63, 65, 61, 62} **med = 61.5, range = 5**

Don't forget:
The median shows you a "typical value" for a data set. The range shows you how spread out the data is.

Now try these:
Lesson 6.1.1 additional questions — p462

Round Up

The median and range are useful for comparing two sets of data. They can give you an idea of which set tends to have higher values and which has the most spread-out values. In a few Lessons, you'll see how box-and-whisker plots show this too, but in a more visual way.

Solutions

For worked solutions see the Solution Guide

Lesson 6.1.2

Box-and-Whisker Plots

In this Lesson, students are introduced to box-and-whisker plots and learn how to construct them. This involves calculating the median and the lower and upper quartiles.

Previous Study: Students have previously calculated the medians of data sets. In grade 5, students displayed single-variable data in appropriate graphs such as histograms and circle graphs.

Future Study: In the next Lesson, students will analyze box-and-whisker plots, and use them to compare two sets of data.

Lesson 6.1.2

Box-and-Whisker Plots

California Standards:

Statistics, Data Analysis, and Probability 1.1
Know various forms of display for data sets, including a stem-and-leaf plot or **box-and-whisker plot**; **use the forms to display a single set of data** or to compare two sets of data.

Statistics, Data Analysis, and Probability 1.3
Understand the meaning of, and be able to compute, the minimum, **the lower quartile**, the median, **the upper quartile**, and the maximum **of a data set.**

What it means for you:
You'll learn how to find lower and upper quartiles, and how to make box-and-whisker plots of data sets.

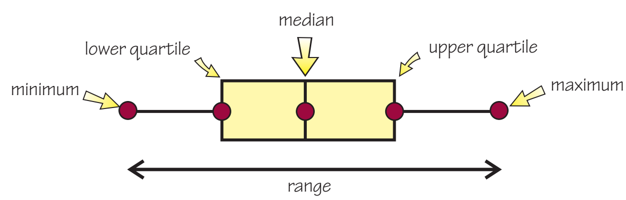
Key words:
- box-and-whisker plot
- upper quartile
- lower quartile
- number line

Box-and-whisker plots are useful because you can use them to directly compare the medians and ranges of data sets. To plot them, you need five key values for the data set — the minimum, maximum, median, and two values you haven't met before — the lower and upper quartiles.

Box-and-Whisker Plots Show Five Values

Box-and-whisker plots are a way of displaying data sets. They have a central box, and two "whiskers" on either side.

There are **five important values** that are shown in a box-and-whisker plot:



Quartiles Split the Data into Four Equal Parts

There are **three quartiles**. One of them is equal to the median, which splits the data into **two halves**. The other two quartiles are the **lower quartile** and the **upper quartile**:

- The **lower quartile** is the median of the **first half** of the data.
- The **upper quartile** is the median of the **second half** of the data.

To find the lower and upper quartiles:
1. First **order the data** and find the **position of the median** of the full set.
2. Next find the **median value of each half of the data**:

If the total number of data points is **odd** —

The lower quartile is the average of these two values.

median

30 31 35 37 40 41 42 45 48

lower quartile — upper quartile

When the number of points is odd, you **don't include the median value** when you work out the lower and upper quartiles...

If the total number of data points is **even** —

median

30 31 35 37 40 | 40 41 42 45 48

lower quartile — upper quartile

...but when you have an even number of points, **you do include the two middle values.**

1 Get started

Resources:
- pictures of animals with whiskers
- rulers
- individual whiteboards
- small slips of paper

Warm-up questions:
- Lesson 6.1.2 sheet

2 Teach

Universal access
An alternative way to approach this Lesson is to have students explore and explain what is meant by the "middle 50%" of the data (this is the portion of data values that make up the box).

Give students a data set and ask them to circle the middle 50% of the values.

For example, the bold values in this data set would be circled:
67, 69, 73, 73, 75, **76, 76, 78, 78, 79, 80, 80, 82, 82, 84,** 88, 89, 92, 95, 95

Some students may have misconceptions of what the "middle 50%" of the data means, and this is a good opportunity to identify and correct any erroneous ideas.

Math background
Quartiles basically divide data arranged in ascending order into four roughly equal parts. However, there are alternative methods of computing quartiles, which can lead to different results.

For example, in the method given in the text for finding the lower and upper quartiles in a data set with an odd number of data points, you don't include the median value in either the lower or upper halves of the data.

In other methods the median value is included in both the lower and upper halves of the data.

- **Strategic Learners**
  Have each student draw a straight line on their whiteboard. Tell them to mark point "A" in the middle of the line, point "B" halfway from the beginning of the line to point "A," and point "C" halfway between "A" and the end of the line: _________B__________A__________C__________ .
  Explain that points A, B, and C divide the line into four equal sections. This is similar to how the quartiles split up a data set.

- **English Language Learners**
  Bring in pictures of animals with whiskers (cats, dogs, seals, fish, tigers, etc.). Point out that whiskers are thin lines that go out symmetrically left and right from the head. Show a box-and-whisker plot, and ask someone to point to the "whiskers."

2 Teach (cont)

Additional example

The following approach to finding the median and the lower and upper quartiles allows students to manipulate the numbers physically.

Give students a set of 11 numbers written on individual slips of paper. Ask them to sort the slips into numerical order, and count the number of data points.

Students should then find the median, and color that slip of paper.
For example:

5 6 8 11 11 13 14 14 18 20 20

Students should be able to clearly "see" the lower and upper halves to the left and right of the median respectively.

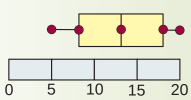
The medians of these halves (the lower and upper quartiles) can then be found in the same way.

Lower half 5 6 8 11 11

Median 13

Upper half 14 14 18 20 20

These values can then be used, together with the minimum and maximum, to make a box-and-whisker plot:



Guided practice

Level 1: q1–2
Level 2: q1–2
Level 3: q1–2

Example 1

Find the lower and upper quartiles of the following data set:

20, 21, 21, 24, 25, 25, 27, 29, 30, 31, 33, 37

Solution

First, find the position of the median of the full data set.

20, 21, 21, 24, 25, 25, 27, 29, 30, 31, 33, 37

median

There's an even number of data points so the median is the average of the two middle values.

There are **six values on each side** of the median, so:

20, 21, 21, 24, 25, 25, 27, 29, 30, 31, 33, 37

Lower quartile = average of 3rd and 4th values = (21 + 24) ÷ 2 = **22.5**

Upper quartile = average of 9th and 10th values = (30 + 31) ÷ 2 = **30.5**

Guided Practice

Find the lower and upper quartiles of the following data sets:

1. Test scores for class A: **56, 57, 57, 59, 62, 64, 64, 68, 69, 70, 72**
l = 57, u = 69

2. Test scores for class B: **45, 52, 53, 53, 55, 57, 61, 61, 65, 68**
l = 53, u = 61

Make a Box-and-Whisker Plot on a Number Line

You draw a box-and-whisker plot on a number line — this gives you a scale to line the numbers up on.

Follow these steps for making a box-and-whisker plot:

1. Find the **five important values for the data set** — the minimum, lower quartile, median, upper quartile, and maximum.

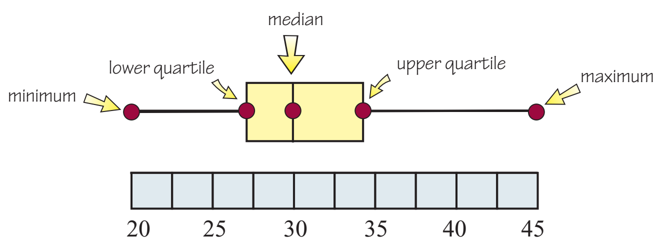
2. Draw out a **number line** that goes from the minimum to the maximum of the data.

3. **Plot** the five values on the number line, and draw a **box** from the lower quartile to the upper quartile. Mark the **median** across the box.

4. Draw **whiskers** to the minimum and maximum values.

Check it out:

The box represents half of the data. A quarter of the data is above the median line, and a quarter of the data is below the median line.
The **larger** the space from the quartile to the median, the **more spaced out** the data in that quarter is. If the quartile is **very close** to the median, it means that the data is very **concentrated**.



Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask advanced learners to collect a set of real data of their choosing. For example, they could measure the hand span of each member of the class. They should then display this data as a box-and-whisker plot.

Example 2

Draw a box-and-whisker plot to illustrate the following data:

{45, 46, 47, 47, 49, 51, 51, 53, 55, 57, 57}

Solution

First find the five key values for the set:

The minimum and maximum are 45 and 57.

There's an odd number of data points, so the median value is 51.

Now find the lower and upper quartiles:

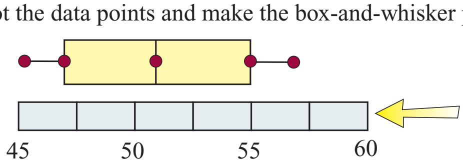
45, 46, 47, 47, 49, 51, 51, 53, 55, 57, 57

lower quartile — median — upper quartile

Lower quartile = 47

Upper quartile = 55

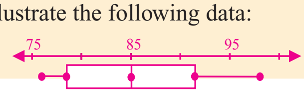
Plot the data points and make the box-and-whisker plot:



Check it out:
This data set has an odd number of values. So the median is the middle value. You don't include this value when you're finding the lower and upper quartiles.

Guided Practice

3. Draw a box-and-whisker plot to illustrate the following data:
98, 76, 79, 85, 85, 81, 78, 94, 89



Independent Practice

1. Mrs. Walker wants to compare the test results of her period 1 and period 4 science classes. Find the maximum, minimum, median, and lower and upper quartiles of the sets of data, and display the data sets on box-and-whisker plots. **see below for box-and-whisker plots**

Period 1: 56, 78, 10, 43, 32, 20, 67, 65, 58, 72, 74, 67, 68, 55, 59, 49
Minimum: 10; maximum: 78; median: 58.5; lower quartile: 46; upper quartile: 67.5
Period 4: 75, 64, 65, 68, 62, 52, 42, 38, 53, 64, 64, 72, 73, 59, 59, 63
Minimum: 38; maximum: 75; median: 63.5; lower quartile: 56; upper quartile: 66.5

Now try these:
Lesson 6.1.2 additional questions — p462

Round Up

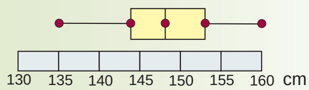
There's a lot of information in this Lesson. You need to remember the five key values for drawing box-and-whisker plots — the minimum, lower quartile, median, upper quartile, and maximum. Remember that the box goes from the lower quartile to the upper quartile, and there are two whiskers — one from the minimum to the lower quartile, and another from the upper quartile to the maximum. Next Lesson you'll see how box-and-whisker plots can be used to analyze and compare sets of data.

2 Teach (cont)

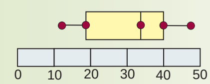
Additional example
Make a box-and-whisker plot to display this data set:

Heights of students in a class (in cm):
135, 141, 142, 143, 143, 145, 145, 146, 147, 148, 148, 148, 150, 151, 151, 153, 153, 155, 156, 157, 160

Minimum: 135; maximum: 160; median: 148; lower quartile: 144; upper quartile: 153



Concept question
"Is it possible that the value 8 is in the data set represented by this box-and-whisker plot? Explain your answer."



No. The left-hand whisker only extends to a minimum value of about 12.

Guided practice
Level 1: q3
Level 2: q3
Level 3: q3

Independent practice
Level 1: q1
Level 2: q1
Level 3: q1

Additional questions
Level 1: p462 q1–2, 6–9
Level 2: p462 q1–12
Level 3: p462 q1–12

3 Homework

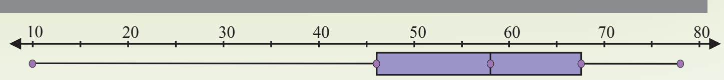
Homework Book — Lesson 6.1.2
Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

4 Skills Review

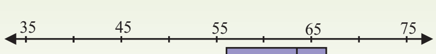
Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 46 — Median
- Worksheet 47 — Data Displays
- Worksheet 48 — Data Sets

Solutions
For worked solutions see the Solution Guide

1. Period 1:



Period 4:



Lesson 6.1.3 More on Box-and-Whisker Plots

In this Lesson, students learn to analyze what box-and-whisker plots show. They then compare two data sets by considering the minimum and maximum values, the medians, and the interquartile ranges.

Previous Study: In grade 6, students described the characteristics of data samples. In the previous Lesson, they learned to draw box-and-whisker plots.

Future Study: Later in this Section, students will organize, display, and analyze larger sets of real-life data. This involves drawing box-and-whisker plots.

1 Get started

Resources:
- Assortment of pebbles or shells, etc.
- Balance or ruler

Warm-up questions:
- Lesson 6.1.3 sheet

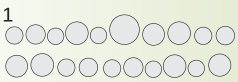
2 Teach

Universal access

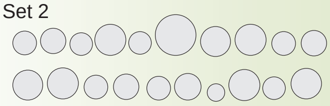
This activity allows students to see the difference between two sets, and relate this to the differences between their box-and-whisker plots.

Give students two sets of about 20 objects, such as pebbles or shells — the objects in one set should tend to be larger than the objects in the other. For example:

Set 1



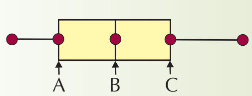
Set 2



Ask students to weigh each object, or measure the diameters, and produce a box-and-whisker plot for each set.

Concept question

"What is each of the following points the median of?"



A is the median of the lower half of the data set. B is the median of the entire data set. C is the median of the upper half of the data set.

Concept question

"In a box-and-whisker plot, the data is divided into four quartiles. Each quartile represents one quarter of the data. Explain why the quartiles don't always look equal."

The values in some quartiles have a bigger range than the values in other quartiles. This makes that section of the box-and-whisker plot wider.

Lesson 6.1.3

More on Box-and-Whisker Plots

California Standards:

Statistics, Data Analysis, and Probability 1.1

Know various forms of display for data sets, including a stem-and-leaf plot or **box-and-whisker plot; use the forms** to display a single set of data or **to compare two sets of data**.

What it means for you:

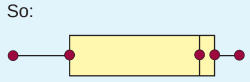
You'll learn how you can compare data sets using box-and-whisker plots.

Key words:
- box
- whisker
- spread
- concentrated

Check it out:

Half the data values are between the minimum value and the median.

So:



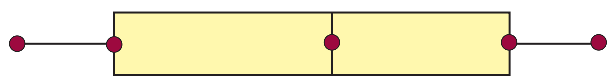
*Last Lesson you learned how to make a **box-and-whisker plot** to display a set of data. In this Lesson you'll use the features of box-and-whisker plots to understand real-life data sets. You'll also see how box-and-whisker plots can be used to **compare two data sets**.*

The Box Shows the Middle 50% of the Data Values

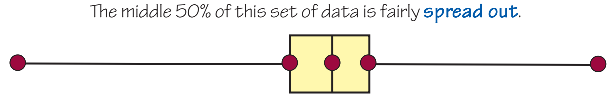
It's useful to be able to **compare** two or more data sets. Drawing two box-and-whisker plots on the **same number line** is a good way of doing this.

Remember these important points:

- The box represents the **middle 50%** of the data.
- The box length shows how **spread out** the middle 50% of the data is.

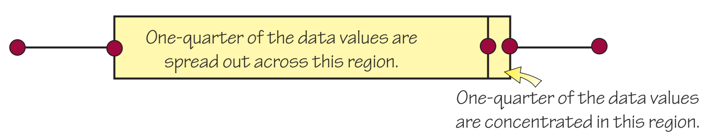


The middle 50% of this set of data is fairly **spread out**.



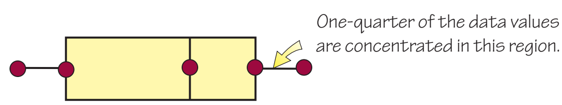
This middle 50% of this set of data is **concentrated** around the median.

- If the median is at the **upper end of the box**, you know that one-quarter of the data values are **concentrated just above the median** value, and that the data is **spread out more below the median**.

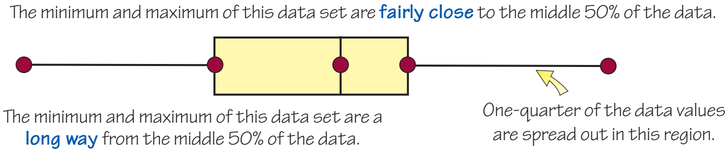


The Whiskers Show the Full Range of the Data Values

The lengths of the whiskers tell you how **far** the very lowest and very highest points are from the middle 50% of the data.



The minimum and maximum of this data set are **fairly close** to the middle 50% of the data.



The minimum and maximum of this data set are a **long way** from the middle 50% of the data.

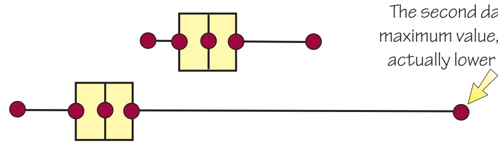
- **Strategic Learners**
  Make it clear to the students that the terms "quartile," "upper quartile," and "lower quartile," each have two meanings. For example, "If your score on a test was in the upper quartile, it means that it was one of the top 25% of all scores on that test." But the upper quartile can also mean a specific number that is the boundary between the top 25% of the data and the lower 75% of the data.

- **English Language Learners**
  Ask students what words they think of when they see the word "quartiles" — quarts, quarters, quadrants, etc. Relate these to the number four. Ask them what there is four of in a box-and-whisker plot. Alternatively, ask students to find three words in the dictionary that begin with "quad" and to write down their meanings. The words and meanings can then be shared with a partner.

There are Different Ways to Compare Data Sets

Comparing two sets of data can be quite complicated. There are many differences that you may need to think about.

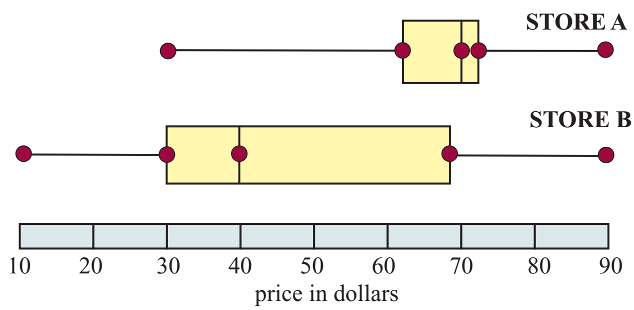
For instance, if you **only** looked at the **minimum and maximum values** of data sets, you wouldn't get a complete picture. One set of data might have one unusually high value, with the rest of the data really low.



The second data set has a much higher maximum value, but most of its values are actually lower than the first data set's.

Example 1

These box-and-whisker plots show the prices of stock in two stores. What do they tell you about the price differences in the two stores?



Solution

Looking at the minimum and maximum values:
Store B has the **lowest priced item** ($10). Store A's lowest price is much higher ($30). Both stores sell their **most expensive item** for the same price ($90). So Store B has a **greater range** of prices.

Looking at the medians:
Store A's stock has a median price of $70, whereas Store B's stock has a median price of $40. So Store B's items are **typically less expensive** than Store A's.

Another way of looking at this is that **half** of Store A's stock is **under $70**, but **half** of Store B's stock is **under $40**.

Looking at the quartiles:
The **middle 50%** of the prices in Store B are much more **spread out** than they are in Store A. They go from $30 to nearly $70.
The middle 50% of the prices in Store A are **concentrated more tightly** around the median value of $70.

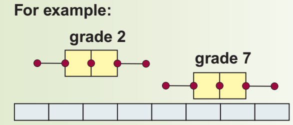
Check it out:
The difference between the lower and upper quartile is called the "interquartile range."

2 Teach (cont)

Concept questions

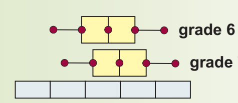
1. "The heights of students in a grade 2 class and a grade 7 class are measured and recorded. Sketch the sort of box-and-whisker plots you might expect to get. You don't need to number the scale."

You'd expect the heights of students in grade 7 to be much greater than those in grade 2. The tallest student in grade 2 might be expected to not be as tall as the smallest student in grade 7.
For example:



2. "The heights of students in a grade 6 class and a grade 7 class are measured and recorded. Sketch the sort of box-and-whisker plots you might expect to get. You don't need to number the scale."

You'd expect the heights of students in grades 6 and 7 to be fairly similar. There'd be a lot of overlap between the two groups. The students in grade 7 are likely to be slightly taller.
For example:



● **Advanced Learners**
Ask students to collect numerical data from two groups of people. For example, they could survey the wrist circumferences, heights or bedtimes of students, or the amount of allowance received by students, in two different grades. Ask them to draw box-and-whisker plots to represent the data sets, and then to use the plots to compare the data sets.

2 Teach (cont)

Math background
Values in a data set that are extremely high or extremely low compared to the rest of the data set are known as "outliers."

They may sometimes be caused by an error in measuring or recording. In any case, these outliers shouldn't usually be given much weight when analyzing data — that's why the middle 50% of the data is emphasized in a box-and-whisker plot.

Guided practice
Level 1: q1
Level 2: q1
Level 3: q1

Independent practice
Level 1: q1
Level 2: q1–2
Level 3: q1–2

Additional questions
Level 1: p462 q1–4
Level 2: p462 q1–5
Level 3: p462 q1–5

3 Homework

Homework Book — Lesson 6.1.3
Level 1: q1–4
Level 2: q1–4
Level 3: q1–5

4 Skills Review

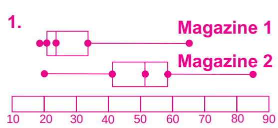
Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 46 — Median
- Worksheet 47 — Data Displays
- Worksheet 48 — Data Sets

Check it out:
If one section of the box is longer than the other, it doesn't mean that they contain different amounts of data. It just means that the data in the larger section is more spread out.

Don't forget:
Always label your box-and-whisker plots so that you know which set of data belongs to which plot.

1. The median age of Saturday's swimmers was much lower than Monday's.
Half of Monday's swimmers were under 58, whereas half of Saturday's swimmers were under 22.
The ages of the middle 50% of the swimmers on Saturday were more spread out than on Monday.
On Mondays, a greater proportion of the people who have free time during the day to swim are likely to be retired people.
On Saturday younger swimmers are more likely to come as they are not in school or at work.

1.

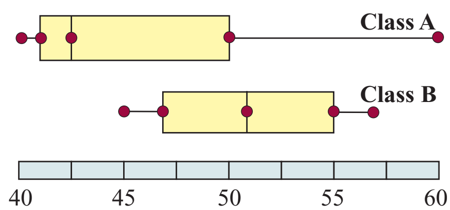


The readers of magazine 1 tend to be much younger than the readers of magazine 2. The median age of the readers of magazine 1 is 23.5, whereas the median age of the readers of magazine 2 is 51.5.

Now try these:
Lesson 6.1.3 additional questions — p462

Example 2

The box-and-whisker plots below show the test scores in two classes. Compare the two sets of scores.



Solution

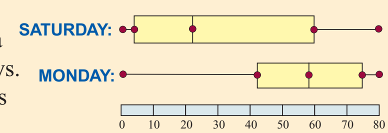
• Class A's test scores had a much **larger range** than Class B's. Both the highest and lowest scores overall were found in Class A.

• Class B's scores were **generally higher** than Class A's. The **median** score for Class B was more than 50, but for Class B it was about 43.

• **More than half** the students in Class B scored **more than 50**, whereas in Class A only **one-quarter** scored more than 50.

Guided Practice

1. These box-and-whisker plots show the ages of people using a public pool on two different days. Compare the differences in ages of the pool-users, and suggest why these differences are seen. **see left**



Independent Practice

1. The data below shows the ages of the people who subscribe to two different magazines. Draw a box-and-whisker plot of each set, and use them to compare the ages of the readers of each magazine.

Magazine 1: **20, 21, 32, 19, 47, 65, 34, 21, 33, 52, 24, 20, 19, 31, 23, 22**

Magazine 2: **45, 67, 20, 72, 54, 37, 51, 54, 50, 52, 44, 39, 85, 29, 57, 60**
see left

2. A group of students took a test in March, then followed a special program for two months before retaking the test in June. Their scores are shown below.
Compare the sets of results by drawing box-and-whisker plots.

March test results: **20, 23, 28, 31, 24, 24, 25, 27, 25, 25, 23, 22, 25**

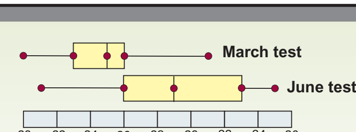
June test results: **33, 26, 35, 28, 21, 30, 31, 35, 23, 26, 33, 29, 26** **see below**

Round Up

There has been a lot covered in the last two Lessons. Make sure you understand everything you've covered — like how to find quartiles, and how to compare data shown on box-and-whisker plots.

Solutions
For worked solutions see the Solution Guide

2. The students' results were generally much higher in the second test than in the first test. In the first test, 75% of the students scored no more than 26. In the second test, 75% of the students scored at least 26.



Lesson 6.1.4

Stem-and-Leaf Plots

In this Lesson, students are introduced to drawing and interpreting stem-and-leaf plots. After displaying single sets of data on stem-and-leaf plots, students use back-to-back stem-and-leaf plots to display and compare two sets of data.

Previous Study: In grade 6, students described the characteristics of data samples. In the previous Lesson, they used box-and-whisker plots to compare data sets.

Future Study: Later in this Section, students will organize, display, and analyze larger sets of real-life data. This involves drawing back-to-back stem-and-leaf plots.

Lesson 6.1.4

Stem-and-Leaf Plots

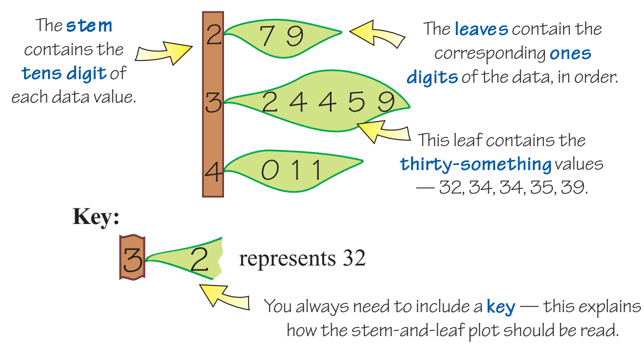
Stem-and-leaf plots are a way of displaying data sets so that you can see their main features more easily. Like box-and-whisker plots, stem-and-leaf plots are also useful for comparing two data sets.

A Stem-and-Leaf Plot Has a Stem and a Leaf

A **stem-and-leaf plot** is a way of displaying data so that you can see clearly **how widely spread** it is and which values are **more common**.

The diagram below displays the data set:

{27, 29, 32, 34, 34, 35, 39, 40, 41, 41}.



Example 1

Make a stem-and-leaf plot to display the following data:

34, 36, 36, 37, 41, 45, 46, 49, 50, 50

Solution

The data contains values in the 30s, 40s, and 50s.
So give the stem **3 rows** — 3 tens, 4 tens, and 5 tens.

Now fill in the leaves. For example, the row with 3 on the stem has a leaf that contains 4, 6, 6, and 7 to represent 34, 36, 36, and 37.

3	4 6 6 7
4	1 5 6 9
5	0 0

Key: 4 | 5 represents 45

Guided Practice

1. Draw a stem-and-leaf plot of the following set of data.

98, 76, 79, 85, 85, 81, 78, 94, 89 see left

California Standards:
Statistics, Data Analysis, and Probability 1.1
Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; **use the forms to display a single set of data or to compare two sets of data.**

What it means for you:
You'll learn how to make a stem-and-leaf plot to display and compare data sets.

Key words:
- stem
- leaf
- tens digit
- ones digit

Check it out:
You don't usually draw the outlines of the stem and leaves — but these have been included here to show you why they're called stem-and-leaf plots.

Check it out:
The stem doesn't always contain tens digits — it depends on the data set. For example, the plot below represents this data set: 1.9, 2.4, 2.7, 3.6

1	9
2	4 7
3	6

Key: 1 | 9 represents 1.9

Other examples of what the stem may contain are hundreds or thousands.

7	6 8 9
8	1 5 5 9
9	4 8

Key: 7 | 6 represents 76

1 Get started

Resources:
- large-square grid paper

Warm-up questions:
- Lesson 6.1.4 sheet

2 Teach

Universal access
Seeing a full stem-and-leaf plot might be confusing for some students. Slow things down by only revealing the stem-and-leaf plot one row at a time.

For instance, initially just uncover

2 7 9

Now work with students to help them understand that there are two values being presented in this row: 27 and 29.

Continue in this way until the whole plot is revealed.

Concept question
"In the leaf below, why does the digit 4 appear twice?"

3 2 4 4 5 9

Because 34 occurs twice in the data set.

Guided practice
Level 1: q1
Level 2: q1
Level 3: q1

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Ask students to write three facts about the data set {48, 49, 52, 54, 55, 55, 57, 59, 60, 60, 61} and to share their facts with a partner. Thinking in terms of stem-and-leaf plots, ask them which numbers would be the stems and which numbers the leaves. They should record this in a table similar to the one shown on the right.

stems	leaves

- **English Language Learners**
  A source of confusion for some English language learners is the word "plot." Explain to students that the word "plot" can be used as a verb — for instance, you plot a coordinate point on a graph. But it's also used as a noun, meaning a diagram or chart displaying data.

2 Teach (cont)

Common error

When making a stem-and-leaf plot, students often make the error of not evenly spacing the digits in the leaves.

A stem-and-leaf plot is similar to a bar graph in shape. If the digits are not correctly spaced, then the shape of the plot is distorted and incorrect conclusions could be drawn.

For some students, writing the data on large-square grid paper is a useful way to keep the spacing even.

Math background

When you're displaying data values of 100 or more, you can follow the same rule and put the numbers of tens on the stem (for example 152 has 15 tens and 2 units). The additional example below illustrates this. Alternatively, if the data has a very large range, you might choose to put the hundred digits on the stem. So, 1 | 52 would represent 152.

Additional example

Construct a stem-and-leaf plot for each of the following data sets. Use your plots to find the median and range of each data set.

Heights of students in Class A (in cm): 135, 141, 142, 143, 143, 145, 145, 146, 147, 148, 148, 148, 150, 151, 151, 153, 153, 155, 156, 157, 160

13	5
14	1 2 3 3 5 5 6 7 8 8 8
15	0 1 1 3 3 5 6 7
16	0

Key: 13 | 5 represents 135

median = 148 cm, range = 25 cm

Heights of students in Class B (in cm): 142, 144, 147, 148, 148, 149, 150, 150, 151, 151, 152, 152, 152, 153, 153, 155, 155, 160, 162

14	2 4 7 8 8 9
15	0 0 1 1 2 2 2 3 3 5 5
16	0 2

Key: 13 | 5 represents 135

median = 151 cm, range = 20 cm

Guided practice

Level 1: q2
Level 2: q2–3
Level 3: q2–3

Find the Median and Range from Stem-and-Leaf Plots

You can find the **median, minimum, maximum, and range** of the data from a stem-and-leaf plot. The example below shows you how.

Check it out:
From the shape of the stem-and-leaf plot you can see that most values are in the 60s and 70s, with just a few in the 50s and 80s.

Example 2

Use the stem-and-leaf plot below to find the:
a) median of the data,
b) minimum, maximum, and range of the data.

This is the minimum value — it's 56.

5	6 9
6	0 3 5 5 6 6 8
7	1 1 3 7 9
8	0

This is the median (the 8th value) — it's 66.

This is the maximum value — it's 80.

Key: 5 | 6 represents 56

Solution

a) You find the median in exactly the same way as usual, except the data points are now spread out over a number of rows.

First count the number of data points, and decide which is the **middle data point**. There are 15 points on this stem-and-leaf plot — this is **odd**, so the median is the middle value.

The middle value is the 8th value, which is **66**.

b) The minimum value is the first number in the top row.
So the minimum value = **56**.

The maximum value is the last number in the bottom row.
So the maximum value = **80**.

The range is the difference between these numbers.
So the range = 80 – 56 = **24**.

Guided Practice

2. The stem-and-leaf plot below shows the number of children who attended an after-school program each week. Find the median number of children and the range.

0	8 9
1	0 2 3 5 7 8 8
2	1 1 1

Key: 1 | 2 represents 12

median = 16, range = 13

3. Find the median and the range of the data shown on the stem-and-leaf plot you made in Guided Practice Exercise 1.

median = 85, range = 22

Solutions
For worked solutions see the Solution Guide

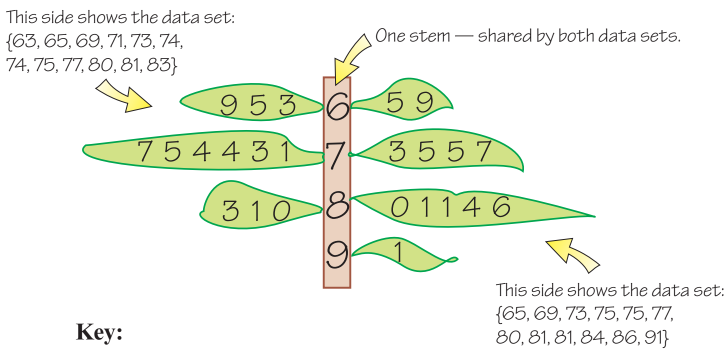
- **Advanced Learners**
  Ask students to refer back to the data they collected in the Advanced Learners activity in the last Lesson. They should display this on a back-to-back stem-and-leaf plot. They can then use this plot to compare the data sets. Ask them whether their conclusions match the conclusions they drew after comparing the box-and-whisker plots.

2 Teach (cont)

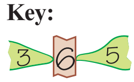
Use Stem-and-Leaf Plots to Display Two Data Sets

A stem-and-leaf plot with leaves on **both sides of the stem** can be used to compare data sets. This is called a **back-to-back** stem-and-leaf plot.

The leaves share the **same stem**, with one data set displayed on one side, and the other on the other side.



Key:



represents 63 from the first data set, and 65 from the second.

Check it out:
The larger values are further away from the stem on each side of a back-to-back stem-and-leaf plot.

Don't forget:
A good first step when drawing stem-and-leaf plots is to order your data from smallest to largest.
This way, you are less likely to make mistakes like leaving numbers out.

Guided Practice

4. Draw a back-to-back stem-and-leaf plot of the following sets of data.
Test scores for class A: {56, 57, 57, 59, 62, 64, 64, 68, 69, 70, 72}
Test scores for class B: {45, 52, 53, 53, 55, 57, 61, 61, 65, 68, 68}

see below

Guided practice
Level 1: q4
Level 2: q4
Level 3: q4

Compare Data Sets Shown on Stem-and-Leaf Plots

To compare the data sets, look at the **shape** of the back-to-back stem-and-leaf plot.

Sometimes there will be a few leaves that are much **longer** than the others — this means the data is more **concentrated** around a certain value. Other sets of data will have **lots of shorter leaves** — which means the data is more **spread out**.

You can also see on the diagram which data set contains the **highest number**, and which data set contains the **lowest number**.

For example:

The data on this side is clustered tightly around the high-30s and 40s.

	1	9 9
	2	0 2 8
9 8 8	3	1 5 7
7 7 5 5 5 3 2	4	0 8

The data on this side is fairly evenly spread between 19 and 48.

Key:
8 | 3 | 1 represents 38 from the first data set, and 31 from the second.

Additional examples

1. Make a back-to-back stem-and-leaf plot to represent the heights of students in Class A and Class B. The data was given in the additional example on the previous page.

Class A heights (cm)		Class B heights (cm)
5	13	
8 8 8 7 6 5 5 3 3 2 1	14	2 4 7 8 8 9
7 6 5 3 3 1 1 0	15	0 0 1 1 2 2 2 3 3 5 5
0	16	0 2

Key: 1 | 14 | 2 represents 141 from Class A and 142 from Class B.

2. Do the students tend to be taller in Class A or Class B? Explain your answer.

The students tend to be taller in Class B. You can tell this from the shape of the back-to-back stem-and-leaf plot. Most students in Class A are under 150 cm tall, whereas most students in Class B are 150 cm or taller.

Solutions
For worked solutions see the Solution Guide

4.

Class A		Class B
	4	5
9 7 7 6	5	2 3 3 5 7
9 8 4 4 2	6	1 1 5 8 8
2 0	7	

Key:
2 | 6 | 1 represents 62 from the first data set, and 61 from the second.

2 Teach (cont)

Concept question

"Look at the back-to-back stem-and-leaf plot shown in Example 3. How many people went on each trip?"

11 people went with Company A.
21 people went with Company B.
You can tell this by the number of data points.

Guided practice

Level 1: q5
Level 2: q5
Level 3: q5

Independent practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1–3

Additional questions

Level 1: p463 q1–11
Level 2: p463 q1–11
Level 3: p463 q1–11

3 Homework

Homework Book — Lesson 6.1.4

Level 1: q1–5
Level 2: q1–6
Level 3: q1–7

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:

- Worksheet 46 — Median
- Worksheet 47 — Data Displays
- Worksheet 48 — Data Sets

Check it out:

The median age for each trip was 66.
The age range on Company A's trip was 14. This was much smaller than Company B's, which was 33.
This supports what the shape of the stem-and-leaf plot shows.

Example 3

Two holiday companies each organized a trip to visit the pyramids in Egypt for people aged over 50. The ages of the passengers on each trip are shown on the stem-and-leaf plot below. Compare the ages of the people who traveled with each company.

Company A		Company B
7 6	5	2 5 5 6 9
9 8 8 7 6 5 4 1	6	0 3 5 5 6 6 8
0	7	1 1 3 7 9
	8	0 2 3 5

Key:
1 | 6 | 0 represents 61 from Company A, and 60 from Company B.

Solution

The ages of the people who traveled with **Company B** were fairly **evenly spread between 52 and 85**. (The leaves are all of a similar length.)

The people who traveled with **Company A** were typically **younger**, and were **closer together** in age — most of them were in their 60s. (This is shown by a very long leaf representing the people in their 60s.)

Guided Practice

5. Look at the back-to-back stem-and-leaf plot of the test scores that you drew in Guided Practice Exercise 4. Compare the two sets of data.
see below

Independent Practice

1. List all the individual data values that are contained in the stem-and-leaf plot below.

25, 25, 26, 27, 29, 31, 31, 31, 34, 34, 38, 39, 39, 41, 41, 42, 42, 42, 44

2	5 5 6 7 9
3	1 1 1 4 4 8 9 9
4	1 1 2 2 2 4

Key:
3 | 1 represents 31

2. In the stem-and-leaf plot in Exercise 1, how many of the values lie between 30 and 35? 5

3. Below is a back-to-back stem-and-leaf plot of golf scores of individuals on two teams. (In golf, the lower the score the better.) Which team has players of a more similar standard? Explain your answer. see below

Team A		Team B
8	7	6 9
7 6 6 5 4 1 1	8	1 3 8 8
0	9	0 1 3

Key: 8 | 7 | 6 represents 78 from Team A, and 76 from Team B.

Now try these:

Lesson 6.1.4 additional questions — p463

Round Up

Stem-and-leaf plots are great for showing the trends in data sets. They show data in a much more visual way than lists of numbers do. And displaying two data sets back to back makes the main differences nice and clear.

Solutions

For worked solutions see the Solution Guide

5. (Gu. Pr.) Class A's test scores were fairly evenly spread between 56 and 69. Class B's test scores were fairly evenly spread between 52 and 68. Class A's scores were higher overall than Class B's.

3. (Ind. Pr.) Team A has players of a more similar standard. Team B's scores are fairly evenly spread between the 70s and the 90s. Team A's scores are mostly in the 80s (they're in one very long leaf), with just a couple of other data values in small leaves.

Lesson 6.1.5

Preparing Data to Be Analyzed

In this Lesson and the next, students answer real-life questions by organizing, displaying, and analyzing data. This Lesson focuses on the organizing and displaying stages of the process.

Previous Study: In grade 5, students organized and displayed single-variable data in appropriate representations, and explained which types of graphs are appropriate for various data sets.

Future Study: In the next Lesson, students will analyze their data displays, and draw conclusions based on them.

Lesson 6.1.5

Preparing Data to Be Analyzed

For data to be meaningful, you usually need to collect quite a lot of it. For instance, if you want to find out the most popular song, then just asking your three closest friends won't give you a very reliable result.

In this Lesson you'll display some larger sets of real-life data, using the methods that you've learned in this Section so far. Then next Lesson you'll think about what the data actually shows.

Real-Life Data is Used to Answer Questions

Real-life data can be collected to try to **answer a question**.

For instance, a company has produced a new medicine designed to **lower cholesterol**. They want to find out if it works, so their question is:

"Does the medicine lower cholesterol when taken daily for two months?"

Before entering into a large study, the company gives the medicine to **25 volunteers**. They record each person's **cholesterol level** before they take the drug, and again after they have been taking it for two months.

The data is shown in the tables below:

Person	Before (units)	After (units)
1	190	185
2	210	210
3	185	185
4	190	170
5	215	210
6	205	205
7	210	165
8	220	205
9	190	185
10	200	205
11	195	200
12	200	180
13	215	210
14	215	185
15	205	165

Person	Before (units)	After (units)
16	215	175
17	220	200
18	190	190
19	195	185
20	210	195
21	190	180
22	215	210
23	210	185
24	200	180
25	185	195

They hope to use this data to answer their question.

California Standards:

Statistics, Data Analysis, and Probability 1.1
Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

What it means for you:
You'll practice preparing a real-life set of data ready to be analyzed.

Key words:
- analyze

Check it out:
From looking at the raw data in the table, you can see that some people's cholesterol did decrease, but other people's stayed the same, while some people's cholesterol actually increased. It's hard to tell what the general trend is though.

1 Get started

Resources:
- internet computers

Warm-up questions:
- Lesson 6.1.5 sheet

2 Teach

Math background

Real medicine studies usually follow a "double blind" procedure. In this cholesterol context, there would be two groups, chosen so that each group had the same make-up of cholesterol levels before any treatment.

One group would be given the medicine, while the other group would be given a placebo. No one in either group would know if they were getting the medicine or the placebo. Also, the doctors administering the medicine would not know if they were dispensing the real medicine or the placebo. Thus the "double blind" term refers to neither the patient nor the health professional knowing if the pills are real medicine or placebos.

This shows whether it's actually the medicine that's improved a person's condition, or just the psychological effects of expecting to get better.

Concept question

"How easy is it to tell from the table of data whether the medicine works?"

It's fairly difficult to tell if the medicine has worked from looking at the data in this form. It's certainly decreased some people's cholesterol levels, but some people's levels haven't changed, and some have even increased. The overall trend is difficult to see.

- **Strategic Learners**
  Some students may find it difficult to manage this many data points. Instruct strategic learners to just use the first 15 points listed. Using just these points, you get the same values for the minimum, lower quartile, median, upper quartile, and maximum. So the resulting box-and-whisker plots are the same as when the whole set of data is used.

- **English Language Learners**
  Provide students with the opportunity to devise their own investigation. For example, they could compare the number of hours of homework assigned for students in different grades. If they have collected their own data about an issue of interest to them, they are more likely to be fully engaged in the related math.

2 Teach (cont)

Common error

Students will often leave out values when putting them in order to calculate the median and quartiles. Provide them with a copy of the data table so that they can cross out the values as they list them in order.

Concept question

"You want to find out if students at High School A tend to travel further to school than the students at High School B. What data would you need to obtain, and how would you collect it?"

You would survey a random selection of students at each school, and ask them how many miles they travel each day. You could select, say, every tenth student from the roll. It might not be a good idea to survey, say, the first 30 students through the gate — they may have all just gotten off the same bus.

Guided practice

Level 1: q1
Level 2: q1
Level 3: q1

Check it out:
It's easy to leave out a value when putting them in order. Check you still have 25 values in each ordered set.

Don't forget:
Finding the minimum, maximum, median, lower quartile, and upper quartile was covered in the last few Lessons. Look back if you can't remember how to find them.

Check it out:
There's an odd number in the full data set — so you don't include the median value when working out the quartiles.

Example 1

The company wants to make a box-and-whisker plot of the cholesterol levels **before** taking the medicine, and a box-and-whisker plot of the cholesterol levels **after** taking the medicine.

Find the minimum and maximum values, the median, and the lower and upper quartiles for each data set.

Solution

The first step is to put the data in order:

"Before" data: 185, 185, 190, 190, 190, 190, 190, 195, 195, 200, 200, 200, 205, 205, 210, 210, 210, 210, 215, 215, 215, 215, 215, 220, 220

"After" data: 165, 165, 170, 175, 180, 180, 180, 185, 185, 185, 185, 185, 185, 190, 195, 195, 200, 200, 205, 205, 205, 210, 210, 210, 210.

The minimum and maximum values:

- Minimum of **"before"** data is **185**, maximum is **220**
- Minimum of **"after"** data is **165**, maximum is **210**

The median:
There are 25 pieces of data in each set, which is odd, so the median is the middle value. This is the 13th data point of each set.

- Median of **"before"** data is **205**
- Median of **"after"** data is **185**

The upper and lower quartiles:
Each half has 12 data points, which is even, so the upper and lower quartiles are the average of the 6th and 7th data points in each half.

- Lower quartile of **"before"** data = $(190 + 190) \div 2 =$ **190**
- Lower quartile of **"after"** data = $(180 + 180) \div 2 =$ **180**
- Upper quartile of **"before"** data = $(215 + 215) \div 2 =$ **215**
- Upper quartile of **"after"** data = $(205 + 205) \div 2 =$ **205**

Guided Practice

1. Marissa is growing sunflowers in her yard. She treats half of the sunflowers with water and a new plant food, and the other half with just water. Marissa wants to answer this question:
"Does the new plant food make sunflowers grow taller?"

The data below shows the heights of the sunflowers at the end of her experiment. Find the values needed to draw a box-and-whisker plot for each set of data.

With food (in cm): 230, 210, 180, 196, 204, 202, 185, 180, 120, 156, 178, 195, 205, 250, 236, 226, 210, 207, 197, 180
Without food (in cm): 205, 230, 210, 196, 186, 198, 204, 176, 134, 156, 202, 185, 182, 178, 208, 165, 174, 182, 110, 162 see below

Solutions

For worked solutions see the Solution Guide

1. "With Food" (in cm): minimum = 120, maximum = 250, median = 199.5, lower quartile = 180, upper quartile = 210.
"Without Food" (in cm): minimum = 110, maximum = 230, median = 183.5, lower quartile = 169.5, upper quartile = 203.

- **Advanced Learners**
The U.S. Census Bureau site, www.census.gov, offers a variety of data sets. Ask students to choose a topic and write a short report, including displays of data. For example, students could compare the populations in a selection of states in different years.

Display Your Data Sets to Show the Trends Clearly

Once you've prepared your data, you can **display it**.

Example 2

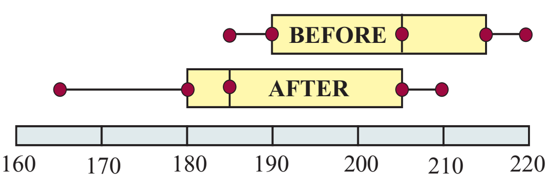
Display the "before" and "after" data in box-and-whisker plots on the same number line.

Solution

The key values for the "before" and "after" data sets were worked out in Example 1:

"Before" data: minimum = **185**, lower quartile = **190**, median = **205**, upper quartile = **215**, maximum = **220**.

"After" data: minimum = **165**, lower quartile = **180**, median = **185**, upper quartile = **205**, maximum = **210**.



Check it out:

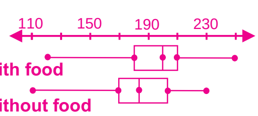
You should choose the type of plot or chart that shows the important features of your data most clearly. Don't forget about the types of charts you've met in previous grades — such as bar graphs.

Check it out:

The number line needs to include the lower minimum (165) and also the higher maximum (220).

Guided Practice

2. Draw box-and-whisker plots to display Marissa's sets of sunflower height data from Exercise 1. see left

2.


Independent Practice

1. A local man is conducting a survey to compare happiness in two nearby towns — Town A and Town B. Inhabitants were asked to rate their happiness on a scale of 1–10. The results are below. Prepare them for analyzing, and display the data in a box-and-whisker plot. see below

Town A: 2, 5, 7, 3, 6, 9, 2, 4, 5, 5, 6, 4, 4, 3, 7, 8, 6, 5, 5, 3, 1, 8, 9, 5

Town B: 5, 7, 7, 8, 8, 6, 6, 5, 4, 4, 6, 9, 9, 10, 3, 7, 8, 8, 9, 6, 7, 4, 8, 9

2. Use a back-to-back stem-and-leaf plot to display the "before" and "after" sets of cholesterol data from p329. see below

Now try these:

Lesson 6.1.5 additional questions — p463

Round Up

Once you've displayed your data, then it's ready for analyzing. This means working out what it means, and seeing whether it answers your question. In the next Lesson, you'll analyze the data sets that you've displayed in this Lesson.

2 Teach (cont)

Math background

Remind students that box-and-whisker plots are not the best way to compare all data sets. For a start, the data must be numerical.

If you wanted, for example, to compare the types of pets owned by two different groups, then you couldn't use a box-and-whisker plot — but a circle or bar graph would be suitable.

Guided practice

Level 1: q2
Level 2: q2
Level 3: q2

Independent practice

Level 1: q1
Level 2: q1–2
Level 3: q1–2

Additional questions

Level 1: p463 q1–10
Level 2: p463 q1–10
Level 3: p463 q1–10

3 Homework

Homework Book — Lesson 6.1.5

Level 1: q1–4
Level 2: q1–4
Level 3: q1–5

4 Skills Review

Skills Review CD-ROM

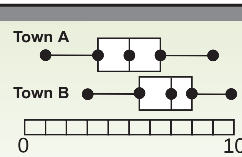
These worksheets may help struggling students:
- Worksheet 46 — Median
- Worksheet 47 — Data Displays
- Worksheet 48 — Data Sets

Solutions

For worked solutions see the Solution Guide

1. Town A: min. = 1, max. = 9, median = 5, lower quartile = 3.5, upper quartile = 6.5

Town B: min. = 3, max. = 10, median = 7, lower quartile = 5.5, upper quartile = 8



2.

before		after
	16	5 5
	17	0 5
5 5	**18**	0 0 0 5 5 5 5 5 5
5 5 0 0 0 0 0	**19**	0 5 5
5 5 0 0 0	**20**	0 0 5 5 5
5 5 5 5 5 0 0 0 0	**21**	0 0 0 0
0 0	**22**	

key: 5 | **18** | 0 represents 185 from the first data set and 180 from the second data set.

Lesson 6.1.6

Analyzing Data

This Lesson follows on closely from the previous one. Students now analyze their box-and-whisker plots and their stem-and-leaf plots, and use them to answer the original questions. Finally, students are encouraged to consider the limitations of statistical investigations.

Previous Study: Students have previously organized data and represented it in a range of forms. In grade 5, students considered the most appropriate ways of displaying data sets.

Future Study: In the next Section, students will display two related sets of data on scatterplots and investigate correlation.

1 Get started

Resources:

- rulers or tape measures
- grid paper

Warm-up questions:

- Lesson 6.1.6 sheet

2 Teach

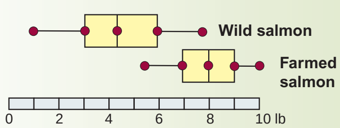
Concept question

"Is this statement true or false? The box-and-whisker plot shows that the medicine decreased the cholesterol levels of everyone who took it."

False. It shows that the cholesterol levels of the group were generally lower after the study. It does not show the effects of the medicine on individuals.

Additional example

Wild salmon and farmed salmon are weighed, and box-and-whisker plots are produced to compare the weights, as shown.



1. Which tends to be heavier, farmed salmon or wild salmon? Suggest why this is.

Farmed salmon. They are given plenty of food and do not exercise as much.

2. Are these statements true or false?

About 50% of the wild salmon are between 3 and 6 lb.

True

About 50% of the farmed salmon weigh more than 9 lb.

False — only about 25% weigh more than 9 lb.

Lesson 6.1.6

Analyzing Data

California Standards:

Statistics, Data Analysis, and Probability 1.1

Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

Mathematical Reasoning 2.6

Express the solution clearly and logically by using the appropriate mathematical notation and terms and **clear language; support solutions with evidence in** both **verbal** and symbolic **work**.

What it means for you:

You'll learn how to analyze data displayed in box-and-whisker plots and stem-and-leaf plots. You'll also consider whether or not the data answers the original question.

Key words:

- compare
- contrast
- conclusion
- limitation

*It's easy to think you're done when you have your data displayed as nice neat plots, but you need to think about what the display is telling you. You also have to **think back** to why you collected the data in the first place, and see if it answers your question.*

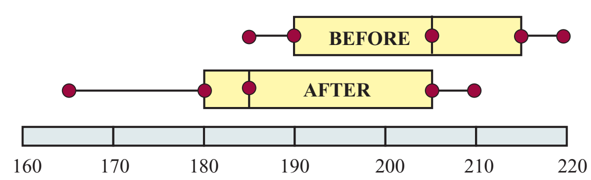
Compare the Similarities; Contrast the Differences

To analyze the results of a study with **two sets of data**, you need to **compare** the displays of each set of data. You have to look at what is **similar** between the data sets and what is **different**.

Example 1

In the previous Lesson, you drew box-and-whisker plots to show the cholesterol levels of a group of people **before and after** they took a certain medicine for two months. These box-and-whisker plots are shown below.

What do they show you about cholesterol levels before and after taking the drug?



Solution

There is a clear **difference** between the two data sets.

The "before" box is much further toward the **higher** end of the scale. This means that cholesterol levels were generally higher **before** the medicine was taken.

The **median** is much lower in the "after" box, which indicates the typical cholesterol level was **reduced** by the medicine.

- **Strategic Learners**
  Suggest some sets of objects for students to measure. For example, flowers in two different areas, or girths of trees in different areas. There should be a clear difference in the sizes of the objects in each set. After students have displayed the results, relate their plots to the differences they observed between the two sets.

- **English Language Learners**
  Provide students with questions to which the answers can easily be predicted. For example, "Which students generally go to bed earlier, first graders or seventh graders?" Give students pairs of box-and-whisker plots (without scales) that could represent relevant data sets. Ask the students to label each box-and-whisker plot (for instance, "first graders," "seventh graders"), and suggest an appropriate scale.

2 Teach (cont)

Guided practice
Level 1: q1
Level 2: q1
Level 3: q1–2

With Food		Without Food
	11	0
0	12	
	13	4
	14	
6	15	6
	16	2 5
8	17	4 6 8
5 0 0 0	18	2 2 5 6
7 6 5	19	6 8
7 5 4 2	20	2 4 5 8
0 0	21	0
6	22	
6 0	23	0
	24	
0	25	

Key: 0 | 21 | 0 represents 210 from the first data set and 210 from the second data set.

Guided Practice

1. Look back at the box-and-whisker plots that you drew last Lesson to illustrate the height of Marissa's sunflowers. Compare the data shown on the plots. see below

2. Plot Marissa's data in a back-to-back stem-and-leaf plot. Which diagram do you think best shows the differences and similarities between the two sets of data? Explain your answer. see left and below

Draw Conclusions After Analyzing Data

Conclusions bring all your analysis together, and relate what you've found out to the **original question**.

Check it out:
You need to be able to back up your conclusions with evidence from the plots of data.

Example 2

The cholesterol data on p329 was collected to try to answer this question:

"Does the medicine lower cholesterol when taken daily for two months?"

Does the data support an answer to this question?

Solution

The data showed that there was a **general reduction** in people's cholesterol levels after taking the medicine.

You can conclude that **yes, the medicine does tend to reduce cholesterol levels when taken daily for two months**.

There were people in the data set for whom the medicine **didn't work**. You can't tell this from the box-and-whisker plots though, because they don't show individual data points. You can just see the **overall trend**.

Concept question
"If the medicine had had no effect, how would you expect the box-and-whisker plots to look?"

The two box-and-whisker plots would look very similar to one another. The median and the quartiles would be very similar in both plots.

Think About Any Limitations of Your Investigation

Your investigation is **unlikely** to give you perfect results.
It's important to understand some reasons for this:

- You might not have used a **big enough sample**. The bigger your sample, the more **accurate and reliable** your results are likely to be.
- Your sample **might not represent the population** well. For instance, if all the people in the cholesterol study were **women**, or **all aged 40**, you couldn't say if the results were likely to be true for everyone.

Don't forget:
The sample is all the people or things you collect data about. It's usually a small part of the whole population. The population is the entire group that you want to know about. You usually use a sample because the whole population is too large for you to be able to collect data on every member.

Concept question
"Suggest what the population and the sample in the cholesterol study might be."

For example, the population might be all adult humans. The sample is all the people whose cholesterol was measured.

Solutions
For worked solutions see the Solution Guide

1. The median of the "without food" data set is slightly lower than the median of the "with food" data set. However, the middle 50% of each set of data is fairly similar.
2. For example, the box-and-whisker plot is more useful here. The stem-and-leaf plot only enables you to see that both data sets follow roughly the same pattern. But on the box-and-whisker plot the differences in the median and the spread of the data between the two sets are more apparent.

- **Advanced Learners**
Ask students to consider other investigations that they have done in this Section. They should examine the methods that they used to collect the data, and identify any sources of bias. Ask them to consider whether their sample would accurately represent the population.

2 Teach (cont)

Guided practice
Level 1: q3
Level 2: q3
Level 3: q3

Universal access
Provide grid paper to help students line up the digits in stem-and-leaf plots, and to help them set up a scale in box-and-whisker plots.

Independent practice
Level 1: q1–4
Level 2: q1–6
Level 3: q1–7

Additional questions
Level 1: p463 q1–8
Level 2: p463 q1–8
Level 3: p463 q1–8

3 Homework

Homework Book — Lesson 6.1.6
Level 1: q1–4
Level 2: q1–4
Level 3: q1–4

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 46 — Median
- Worksheet 47 — Data Displays
- Worksheet 48 — Data Sets

Guided Practice

3. In Exercise 1, you analyzed the results from Marissa's sunflower experiment. Draw some conclusions from your analysis — do you think the plant food has made the sunflowers grow taller? see left

3. The plant food seems to have had a small effect on the heights of the sunflowers — making them grow a little taller. It would be a good idea to repeat the investigation with more sunflowers to be more sure.

Independent Practice

The data below shows the number of vehicles that passed along Road A during each day of August and December.

August: 73, 67, 79, 86, 78, 54, 65, 63, 73, 75, 79, 69, 62, 63, 75, 59, 78, 79, 72, 75, 64, 68, 69, 62, 56, 75, 78, 84, 82, 78, 65

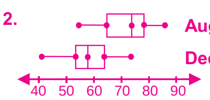
December: 65, 68, 53, 52, 49, 67, 73, 62, 65, 59, 54, 71, 60, 60, 56, 57, 43, 51, 63, 54, 58, 69, 56, 58, 58, 62, 61, 53, 41, 47, 53

Joe, the local town planner, wants to know whether there is a higher demand for the road during the summer months.

1. Find the minimum, lower quartile, median, upper quartile, and maximum of each data set. see left
2. Plot the data on a box-and-whisker plot. see left
3. Plot the data on a back-to-back stem-and-leaf plot. see below
4. Compare the sets of data on each of the plots you have made. Is there a higher demand for the road during the summer months? see left

1. August: minimum = 54, lower quartile = 64, median = 73, upper quartile = 78, maximum = 86

December: minimum = 41, lower quartile = 53, median = 58, upper quartile = 63, maximum = 73

2.


4. Generally, more vehicles pass along Road A each day in August than in December. The data suggests that there is a higher demand for the road in summer, though it's not possible to say for sure as the data only covers August and December.

Moesha wants to compare the heights of the seventh grade boys in her school with the heights of the seventh grade girls.

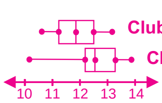
5. Write down a clear question that Moesha might want to know the answer to.
For example, "Are the 7th grade boys in my school generally taller than the 7th grade girls?"
6. What sets of data could Moesha collect?
The heights of all the 7th grade boys and 7th grade girls in her school.

7. Two running clubs both believe that their members are faster at running 100 m than the other club's members. The data below shows the personal best times (in seconds) for the members of each club.

Club A: 12.5, 12.3, 11.3, 11.2, 12.9, 12.7, 12.4, 11.9, 12.0, 11.6, 11.5, 10.7, 10.9, 11.0, 11.2, 12.4, 13.1

Club B: 10.1, 11.9, 13.1, 12.0, 12.2, 12.3, 12.6, 11.9, 12.9, 13.0, 13.5, 13.4, 13.9, 12.6, 12.5, 13.4, 12.2

Display the results clearly and explain whether you think they show which club's members are faster. see left

7.


Club A's times were generally faster than Club B's, although Club B's best time was faster than any of Club A's. More than 50% of Club A's times were below 12 seconds, whereas less than 25% of Club B's were.

Now try these:
Lesson 6.1.6 additional questions — p463

Round Up

You can collect, display, and analyze data to try to answer a question. You've got to be aware that the conclusions you draw have limitations — they can only be definitely true for the sample you used.

Solutions
For worked solutions see the Solution Guide

3. (Independent Practice)

August		December
	4	1 3 7 9
9 6 4	5	1 2 3 3 3 4 4 6 6 7 8 8 8 9
9 9 8 7 5 5 4 3 3 2 2	6	0 0 1 2 2 3 5 5 7 8 9
9 9 9 8 8 8 8 5 5 5 5 3 3 2	7	1 3
6 4 2	8	

key: 4 | 5 | 1 represents 54 from August and 51 from December.

Section 6.2 Exploration — Age and Height

Purpose of the Exploration

The goal of this Exploration is for students to see that there are relationships between variables that are not exact, or strictly linear. The graphing of points provides visual information as to whether a relationship between two variables exists or not.

Resources

- graph paper
- measuring tape

Section 6.2 introduction — an exploration into: Age and Height

The purpose of this Exploration is to determine whether there is a relationship between the age of a person and their height. You can find out if a relationship exists by plotting points on a graph called a scatter plot. This provides a visual check of whether one variable affects the other variable.

A collection of points on a graph sometimes fall in a **diagonal band**. This means the pair of variables represented by the points are **related**.

Example

Plot Variable 1 against Variable 2 on a scatter plot. Say whether these variables are related.

Variable 1	2	3	4	5	5	6	7	8
Variable 2	1	3	2	4	5	6	5	7

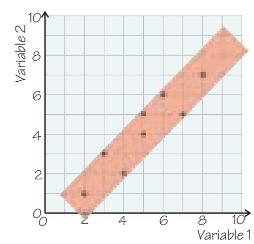
Solution

The following coordinates have been taken from the data set.

(2, 1) (3, 3) (4, 2) (5, 4) (5, 5) (6, 6) (7, 5) (8, 7)

These points can be plotted on a graph.

The points form a diagonal band on the graph — so **the variables are related**. As one variable increases, the other does too.

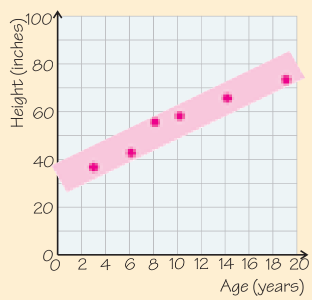


Exercises

1. This table shows the ages and heights of six children in a family.

Family member	Maura	Tobias	George	Spencer	Chloe	Wendy
Age (years)	14	3	8	19	10	6
Height (inches)	63	36	54	72	57	42

Copy these axes onto graph paper. Plot the age and corresponding height of each family member.



2. Does there appear to be a relationship between the age and height of a person? Explain your answer.
See below
3. Plot your age and corresponding height on the graph. Label the point with your name. Depends on the height and age of student.
4. Does your age and height fit with the trend shown in the scatter plot? Depends on the height and age of student.
5. Based on the graph, predict what the height of a 12 year old would be.
Between 53 and 68 inches tall.

Round Up

Some variables are related to each other — the examples on this page are positively related (or correlated). When one increases, so does the other. Other variables are negatively correlated — when one increases, the other decreases. There's more on this later in the Section.

Strategic & EL Learners

Strategic learners will benefit from having their height information rounded, so that they do not have to estimate the location on the y-axis scale.

EL learners may get confused with the term "scatter." A scatter plot is similar to taking a handful of pennies and throwing them in a specific direction. The pennies aren't lined up in a row but a definite direction exists.

Universal access

Start the lesson by asking students whether they believe that there is a relationship between the distance a student lives from school and the time it takes for them to get there. Ask the class how far away they live and the time it takes for them to get to school. Do they think that a relationship exists between the two? After this discussion, introduce the age and height exploration.

Common error

Students will want to connect the points that they have graphed. Remind them that a scatter plot represents a positive or negative trend, rather than allowing exact values to be predicted.

Math background

Students should be familiar with the coordinate plane and the x and y-axis. They should also be able to graph points easily.

Solutions

2. The points fall in a diagonal band, so there appears to be a relationship between a person's age and their height. The graph shows an increase in a person's height as they get older.

Lesson 6.2.1

Making Scatterplots

In this Lesson, students learn how to make scatterplots to display two related sets of data. They are guided in selecting suitable scales to use on the axes.

Previous Study: In the previous Section, and in earlier grades, students learned how to display data in a range of forms, such as histograms, circle graphs, and box-and-whisker plots.

Future Study: In the next two Lessons, students will learn to identify the type of correlation shown by a scatterplot. They will draw lines of best fit through scatterplots and use them to predict other data values.

1 Get started

Resources:
- grid paper
- tape measures and rulers
- internet computers

Warm-up questions:
- Lesson 6.2.1 sheet

2 Teach

Universal access

Provide students with a conjecture such as "the longer a person's forefinger, the wider their span from fingertip to thumb tip will be."

Students should take appropriate measurements and record them in a table. A scatterplot can then be made of this data.

This activity provides students with practice in using measuring devices, and gives them ownership of the data.

Guided practice

Level 1: q1–2
Level 2: q1–2
Level 3: q1–2

Lesson 6.2.1

Section 6.2

Making Scatterplots

California Standards:

Statistics, Data Analysis, and Probability 1.2

Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

What it means for you:

You'll use scatterplots to display sets of data.

Key words:
- conjecture
- scatterplot
- scale
- axes

You'd expect some variables to be related to each other. For example, it might not be a surprise to learn that as grade level increases, so does the average amount of homework that's set. Scatterplots are a way of displaying sets of data to see if and how the variables are related to each other.

Some Things May Be Related to Each Other

Some variables are **related** to other variables. You can make **conjectures**, or educated guesses, about how things might be related.

For example,

- The hotter the day is, the more ice cream will be sold.
- The faster you drive a car, the fewer miles you'll get to the gallon.
- The older a child, the later his or her bedtime.

You Can Collect Data to Test Your Conjecture

To see if your conjecture is correct, you first need to **collect data**.

For example, to see if it's true that ice cream sales increase on hotter days, you need to find the **average temperature** for a number of days, and the **number of ice creams sold** on each of these days. You might end up with a **table** of data that looks like this:

Average temperature of day (°F)	41	63	55	73	70	90	48	66	87	79
Number of ice creams sold that day	16	67	80	101	100	170	36	73	123	114

Example 1

What data could you collect to test the conjecture **"the taller a person, the bigger their feet?"**

Solution

You would need to collect data on the **heights** of a set of people, and on the **size of their feet**.

Check it out:

A conjecture is supposed to be an educated guess, so make sure you have a reason for thinking it might be true.

Guided Practice

1. What data would you need to collect to test the conjecture, "the older a child, the later his or her bedtime"? You would need to collect data on the ages of a set of children and on their bedtimes.

2. Design a table in which to record this data. See below

Solutions
For worked solutions see the Solution Guide

2.

Age of child (years)						
Time child goes to bed						

- **Strategic Learners**
  Some students will find it difficult to set up suitable scales for each axis. If necessary, provide strategic learners with sets of axes on which to draw their scatterplots. Also, review coordinate graphing and give them practice at reading scales on the axes of graphs.

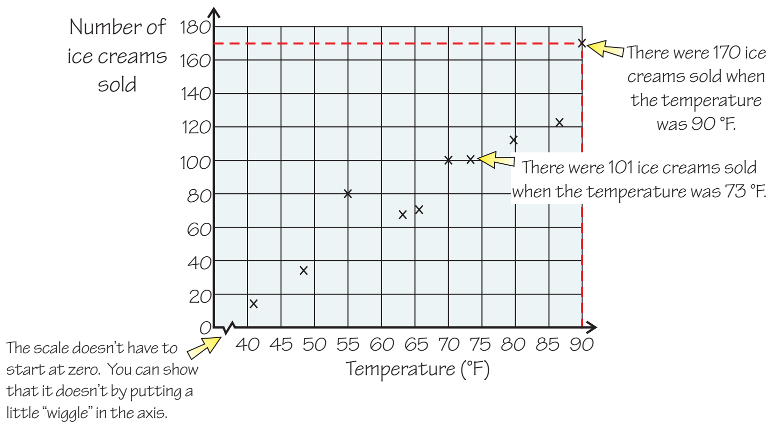
- **English Language Learners**
  Discuss with students things that are likely to be related. For example, the weight and length of a baby, a person's age and the number of gray hairs they have, or the number of hours you study and your test score. Make conjectures for each pair of variables, and ask students to suggest the sets of data that they could collect to test the conjectures.

Mark Data Pairs on a Scatterplot

You can display two sets of data values on a **scatterplot**. The values need to be in **pairs**. A scatterplot is a really good way of seeing if the data sets are related — there's a lot more on this in the next two Lessons.

Below is the scatterplot showing the **number of ice creams sold against the temperature**.

Each cross represents a **pair** of data values — the **number of ice creams** sold on a day of a **certain temperature**.



Check it out:
Each pair of data values is like a coordinate — you plot them in exactly the same way. So 16 ice creams sold at 41 °F can be thought of as (41, 16).
You've got to make sure you get the data in the right order though — temperature is on the horizontal axis (x-axis).

Scatterplots Have Two Axes with Different Scales

You have to think carefully about the **scale** of each axis.
Each axis represents a different thing and is likely to have a different scale.
Here's how to choose a sensible scale for an axis.

1. Look at the **minimum** and **maximum** values of the data set.
You have to choose a **starting point** and an **ending point** for the scale that fits all of the data.

Your scale **doesn't** have to start at zero. If it doesn't, you include a little "wiggle," as above.

The temperatures above were all between 41 °F and 90 °F — so 40 °F and 90 °F were suitable start and end points. It made sense not to start the axis from zero.

2. Choose a sensible **step size**. It must be small enough so that you can show your data clearly, but big enough to fit on your piece of paper.

The temperatures above only had a range of about 50 °F, so 5 °F steps were used.
The number of ice creams sold had a much bigger range, so steps of 20 were used.

Don't forget to **label** each axis clearly. Once you've done all this you can start **plotting your data**.

Check it out:
It doesn't matter which data set goes on which axis.
The scatterplot above could have had "number of ice creams sold" on the horizontal axis and "temperature" on the vertical axis.

Don't forget:
The range is the maximum value minus the minimum value.
So the temperature range is:
90 – 41 = 49 °F

2 Teach (cont)

Math background
Scatterplots are used for examining the relationship between two variables — you use them to find out how one variable changes if the other does.

Each point on the graph must stand for an individual person or object, etc. In this example, each point stands for one day.

Concept question
"Using just the scatterplot, what was the temperature the day 80 ice creams were sold?"

55 °F

Concept question
"Below are the test scores from two Algebra classes presented as a back-to-back stem-and-leaf plot. Explain why this data could not be represented on a scatterplot."

Period 1		Period 6
8 8 7 5 1 1	4	
6 6 4 3 3 2 1 0	5	8
9 8 7 4 2 1 1	6	2 2 6 8 8
	7	3 3 5 6 6 8 9
	8	1 3 3 3

The two sets of data don't come from the same individuals — they aren't paired.

Concept question
"Look back at the cholesterol data used in Lesson 6.1.5. Would a scatterplot have been an appropriate way of displaying this data?"

No. You weren't interested in how the "before" and "after" data is related. You were interested in whether the "after" data was generally lower than the "before" data.

Advanced Learners
Ask students to make a conjecture about how two data sets from the U.S. Census Bureau website, www.census.gov, may be related. For example, students may make the conjecture, “the greater the percentage of high school graduates in a state, the greater the average travel time to work will be.” Then ask them to make a scatterplot of the two sets of data.

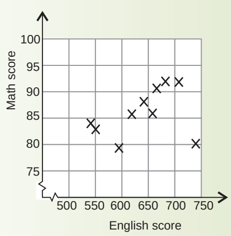
2 Teach (cont)

Additional example

A group of 10 students were tested on English and math skills.
The English test was graded from 0 to 800, while the math test was graded from 0 to 100%. Make a scatterplot from the results below.

Student	English	Math
1	710	92
2	550	83
3	660	86
4	670	91
5	540	84
6	590	79
7	620	86
8	640	88
9	680	92
10	740	80

The English scores range from 540 to 740, so the x-axis can range from 500 to 750. The math scores range from 79 to 92, so the y-axis can range from 75 to 100.



Guided practice
Level 1: q3
Level 2: q3
Level 3: q3

Independent practice
Level 1: q1–2
Level 2: q1–2
Level 3: q1–2

Additional questions
Level 1: p464 q1–5
Level 2: p464 q3–6
Level 3: p464 q3–6

3 Homework

Homework Book — Lesson 6.2.1
Level 1: q1–5
Level 2: q1–6
Level 3: q1–7

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 47 — Data Displays
- Worksheet 48 — Data Sets

Example 2

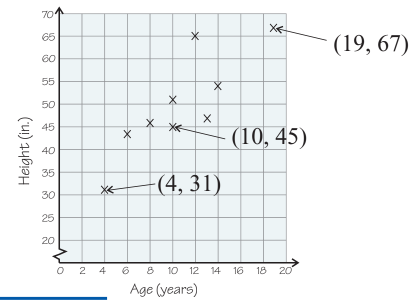
Make a scatterplot of the data below relating people’s ages and heights.

Age (years)	10	19	4	6	10	8	14	12	13
Height (in.)	45	67	31	43	51	46	54	65	47

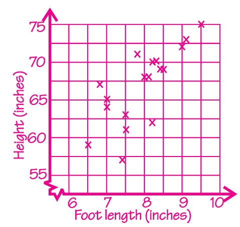
Solution

First you need to decide on a **scale** for each axis.
The ages go from 4 to 19, so a scale might run from 0 to 20, in steps of 2.
The heights go from 31 in. to 67 in. This range is larger, so the scale might go from 20 to 70 in steps of 5 inches.

Then you can **plot the values**. Think of each pair of values as coordinates with the form (*age*, *height*), instead of (x, y). The first three values in the table would be plotted at (10, 45), (19, 67), and (4, 31).



Don't forget:
Take extra care when you’re plotting not to get confused between different scales on your axes. It’s really easy to mark something in the wrong place.



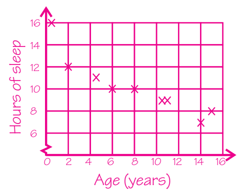
Guided Practice

3. Use the data below to make a scatterplot relating foot length to height.

Foot length (in.)	7	8	8.3	9	8.5	7.4	6.5	7.5	8.1
Height (in.)	64	68	70.1	72	69	57	59	63	68
Foot length (in.)	8.2	9.1	7.5	7	6.8	7.8	8.4	9.5	8.2
Height (in.)	62	73	61	65	67	71	69	75	70

see left

Now try these:
Lesson 6.2.1 additional questions — p464



Independent Practice

1. Miguel makes the conjecture, “the more people there are in a household, the heavier their recycling bins.” What data would you collect to test Miguel’s conjecture? see below

2. The data below was collected to test this conjecture:
“The older a child, the less time he or she will sleep per day.”
Draw a scatterplot of this data. see left

Age (years)	4.5	0.5	8	15	11	14	10.5	2	6
Hours of sleep	11	16	10	8	9	7	9	12	10

Round Up

Now you know what a scatterplot is and how to draw one. The next Lesson shows you how to interpret scatterplots, and how to decide whether, or how closely, the variables are related.

Solutions
For worked solutions see the Solution Guide

1. You would collect data on the number of people in a set of households, and the weight of their recycling bins.

Lesson 6.2.2

Shapes of Scatterplots

In this Lesson, students look at whether the shape of a scatterplot indicates positive or negative correlation, or whether it suggests that there is no correlation between the variables.

Previous Study: In the previous Lesson, students learned to draw scatterplots. In earlier grades and in Section 6.1, they learned to display data in other forms, such as circle graphs and stem-and-leaf plots.

Future Study: In the next Lesson, students will draw lines of best fit through scatterplots. They will then use the lines of best fit to predict other data values.

Lesson 6.2.2

Shapes of Scatterplots

California Standards:
Statistics, Data Analysis, and Probability 1.2
Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

What it means for you:
You'll learn about different types of correlation and what they look like on scatterplots.

Key words:
- slope
- positive correlation
- negative correlation
- strong correlation

Check it out:
If two things are correlated, it doesn't necessarily mean one causes the other.
For instance, shark attacks and ice cream sales may show positive correlation, but one doesn't cause the other. They're both increased by hot weather, which makes people more likely to swim in the sea, and to buy ice cream.

In the last Lesson, you learned how to make scatterplots from sets of data. By looking at the pattern of the points in a scatterplot, you can decide how the variables are related — for example, whether ice cream sales really do increase on hot days.

Positive Slope Means Positive Correlation

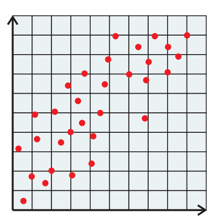
If two things are **correlated**, they are related to each other — if one changes, the other will too.

Two variables are **positively correlated** if one variable **increases** when the other does. For example, children's heights are positively correlated with their ages — because **older** children are typically **taller** than younger ones.

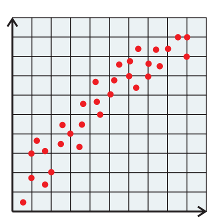
> Variables are **positively correlated** if one variable **increases** as the other does.

If two **positively correlated** variables are plotted on a scatterplot, the points will lie in a band from **bottom left** to **top right**. If you were to draw a line through the points it would have a **positive slope**.

The **thinner the band** of points on the scatterplot, the more **strongly correlated** the data is.



This graph shows **positive correlation**.



This graph shows **strong positive correlation**.

Negative Slope Means Negative Correlation

Negative correlation is when one quantity increases as another decreases. For example, values of cars usually **decrease** as their age **increases**.

> Variables are **negatively correlated** if one variable **increases** as the other **decreases**.

If a scatterplot shows **negative correlation**, the points will lie in a band from **top left** to **bottom right**. They'll follow a line with a **negative slope**.

1 Get started

Resources:
- internet computers
- rulers and tape measures

Warm-up questions:
- Lesson 6.2.2 sheet

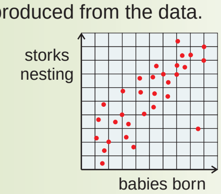
2 Teach

Universal access
Ask students to collect data to test whether "the longer a person's forefinger, the wider their span from fingertip to thumb tip will be."
They should display the data on a scatterplot — they may have already done this part in the previous Lesson.

They should then identify the sort of correlation that is shown.

Then ask the students to repeat this task, comparing a different pair of variables. For example, foot length and head circumference. They can then compare the strengths of the correlations that they find.

Additional example
The number of storks nesting on chimney stacks in towns is recorded, along with the number of babies born in the towns. The scatterplot below is produced from the data.



What does it tell you about the correlation between the number of storks nesting on chimneys in a town and the number of babies born in the town?

They are positively correlated. As one increases, the other generally does too.

Does this mean that:
a) nesting storks cause babies to be born,
b) babies cause storks to nest,
or
c) neither necessarily?

c) neither necessarily. In fact, both variables are likely to depend on the number of homes in a town.

- **Strategic Learners**
  Ask students to come up with pairs of variables that they'd expect to be positively correlated, negatively correlated (this is trickier), and have no correlation. Alternatively, provide students with pairs of variables and ask them to sort them into three sets — "positive correlation," "negative correlation," and "no correlation."

- **English Language Learners**
  Plenty of statistics about sports players can be found on the internet. Ask students to make conjectures about players. For example, they could make the conjecture that taller basketball players are heavier. Encourage the students to predict the shape of the scatterplot they would get. They can then test their conjectures by plotting the statistics on a scatterplot.

2 Teach (cont)

Math background

There is no strict boundary between data that is positively correlated and data that has no correlation — it's a continuum.

In more advanced statistics, there are measures that can be used to analyze exactly how much correlation there is between variables. The most common measure is called a correlation coefficient. It goes from –1 to +1.
A coefficient of –1 implies perfect negative correlation (the points all lie on the same line), a coefficient of +1 implies perfect positive correlation, and a coefficient of 0 implies no correlation.

Additional example

Fifty students participated in a homework study. They were asked how much time they had spent watching television in the past week, and what they had scored on the math test at the end of the week.

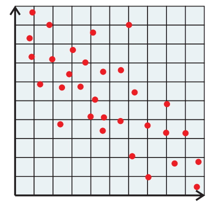
What sort of correlation would you expect? What would a scatterplot of the data look like? Explain your answer.

You'd expect negative correlation, and for the points on the scatterplot to generally go from the upper left of the graph to the lower right.

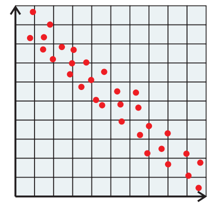
You might expect this because students who watched more TV would be likely to have spent less time studying for the test.

Check it out:
The scatterplot you drew in Independent Practice Exercise 2 in the last Lesson showed negative correlation. The ages of children and the amount of time they sleep are negatively correlated.

The **thinner the band** of points, the more **strongly correlated** the data is.



This graph shows **negative correlation**.

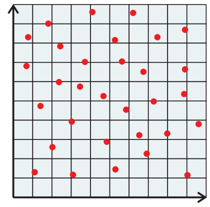


This graph shows **strong negative correlation**.

No Obvious Correlation Means Random Distribution

When points seem to be **spread randomly** all over the scatterplot, then it is said that there is **no obvious correlation**.

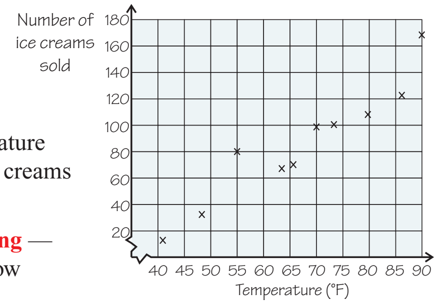
For example, people's heights and their test scores are not correlated — the height of a person has no effect on their expected test score.



This graph shows **no obvious correlation**.

Example 1

Describe the correlation shown in the scatterplot opposite.



Solution

The plot shows **positive correlation**. (As the temperature **increases**, the number of ice creams sold tends to **increase**.)

The correlation is fairly **strong** — the points lie in a fairly narrow band.

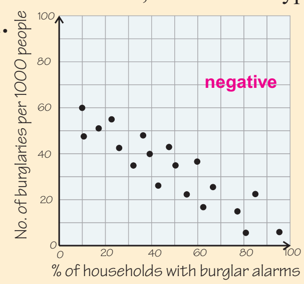
Check it out:
The correlation isn't perfect here. If it was perfect the points would lie in a straight line.

- **Advanced Learners**
  Ask the students to look back at the scatterplots they made using the U.S. Census Bureau data, and consider if they show any correlation. If correlation is shown, ask them to identify which type it is. They should also be encouraged to consider whether a change in one of their variables is likely to cause a change in the other, or whether a third variable may be responsible.

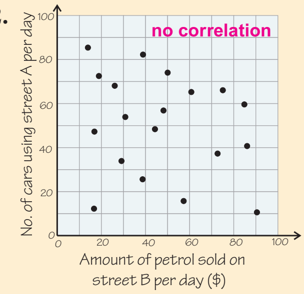
Guided Practice

In Exercises 1–4, describe the type of correlation.

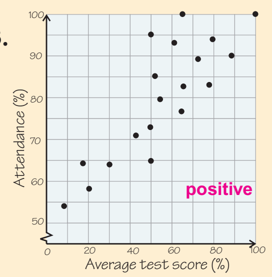
1.



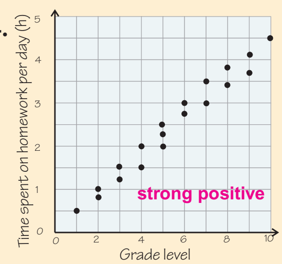
2.



3.



4.



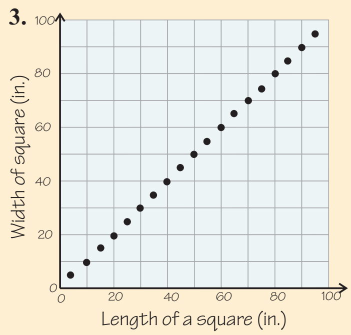
Independent Practice

1. Brandon investigates the relationship between the number of spectators at a football game and the amount of money taken at the concession stand. What kind of correlation would you expect? **positive**

2. If every job you do takes one job off your to-do list, what kind of correlation would you expect between the number of jobs you do and the number of jobs on your to-do list? **negative**

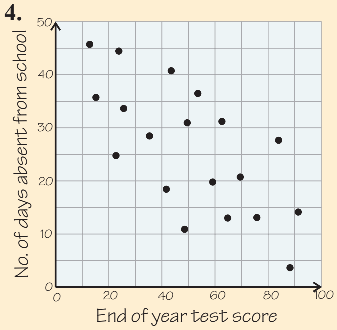
In Exercises 3–4, describe the correlation shown.

3.



perfect positive correlation

4.



weak negative correlation

Now try these:
Lesson 6.2.2 additional questions — p464

Round Up

If the points lie roughly in a diagonal line across a scatterplot, it means the variables are correlated. An "uphill" band means positive correlation, whereas a "downhill" band means negative correlation.

2 Teach (cont)

Guided practice
Level 1: q1–4
Level 2: q1–4
Level 3: q1–4

Concept question
"Independent Practice Exercise 3 shows perfect positive correlation. This is where all the points fall in a perfectly straight line. Give another example of something you'd expect to have perfect positive correlation."

For example, a number, and five times the number. Or, the number of adults attending a movie and the amount spent on adult tickets.

Independent practice
Level 1: q1–4
Level 2: q1–4
Level 3: q1–4

Additional questions
Level 1: p464 q1–6
Level 2: p464 q1–6
Level 3: p464 q1–6

3 Homework

Homework Book — Lesson 6.2.2
Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 47 — Data Displays
- Worksheet 48 — Data Sets

Solutions
For worked solutions see the Solution Guide

Lesson 6.2.3

Using Scatterplots

In this Lesson, students practice interpreting scatterplots. They learn to read the highest and lowest values from them and add lines of best fit, which they then use to predict new values.

Previous Study: In the previous Lessons, students learned to draw scatterplots and identify the types of correlation shown.

Future Study: In Algebra I, students will derive linear equations of straight lines from graphs.

1 Get started

Resources:
- a set of transparencies and dry-wipe pens
- transparent rulers

Warm-up questions:
- Lesson 6.2.3 sheet

2 Teach

Universal access

Refer students back to a particular scatterplot they have made in a previous Lesson.

Pass out a transparency and a dry-wipe pen to each student and have them draw a straight line on the transparency. Then ask them to position the transparency over their scatterplot, so that the line they have drawn acts as a line of best fit. Each student should place the transparency as best they can and then leave it.

Ask the students to compare their line of best fit with those of the students around them. Ask them if the positions are identical, or if there is some variation.

Guided practice

Level 1: q1–2
Level 2: q1–2
Level 3: q1–3

Lesson 6.2.3

Using Scatterplots

California Standards:

Statistics, Data Analysis, and Probability 1.2

Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

Mathematical Reasoning 2.3

Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

What it means for you:

You'll predict data values using scatterplots. You'll also practice finding the highest and lowest values in a data set.

Key words:
- prediction
- scatterplot
- line of best fit

If you have many pairs of values plotted on a scatterplot, and they all fall in a neat band, you know the two variables are correlated. If you plotted some more pairs of values, you'd expect them to lie within the band of points. You can use this idea to predict values. For instance, from the scatterplot of ice cream sales against average temperature, you could predict how many ice creams would be sold when the temperature was 50 °F.

Finding the Highest and Lowest Values

Box-and-whisker plots **don't** show all the raw values — just the maximum and minimum values and the general trends. **Scatterplots** are different — they show the **raw data values**, as well as trends.

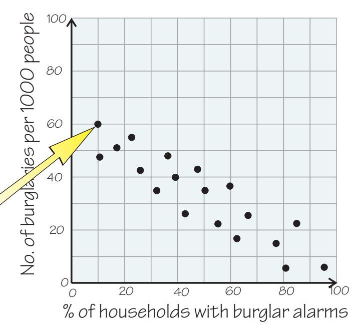
Example 1

The scatterplot below shows the number of burglaries per 1000 people against the percentage of households that have burglar alarms installed. What was the greatest number of burglaries per 1000 people recorded?

Solution

The greatest number of burglaries recorded is the point that lies furthest up the vertical axis on the graph.

The highest number of burglaries recorded per 1000 people is **60**.



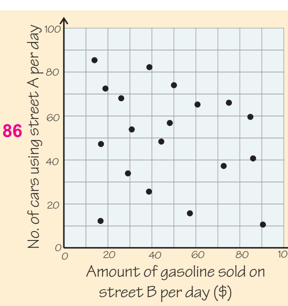
Guided Practice

For Exercises 1–3, refer to the scatterplot on the right.

1. What was the highest number of cars recorded using street A on a single day? 86

2. What was the greatest amount of money spent on gasoline in street B on any day? 91

3. How many cars used street A on the day when the least amount of gasoline was sold on street B? 86



Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  Ask students to use straight lines drawn on transparencies as lines of best fit (see the activity described in the teacher margin of the previous page). When students are ready to mark the lines of best fit onto their plots, provide transparent rulers — these are much easier to work with than opaque rulers.

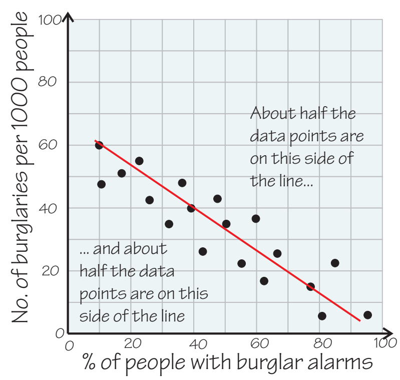
- **English Language Learners**
  Model the positioning of the line of best fit on the overhead projector using a separate transparency. Count the number of points on each side to show that they are approximately equal. Model how to use the "line of best fit" to predict values.

A Line of Best Fit Shows the Trend in the Data

Not many sets of data are perfectly correlated, so a **line of best fit** is used to show the **trend**. If the data was perfectly correlated you'd expect all the points to lie on this line.

Example 2

Draw a line of best fit on the scatterplot below.



The scatterplot shows **negative correlation**, so the line of best fit will have a **negative slope**.

The line of best fit splits the data approximately in **half**. You should have roughly the same number of points on each side of the line.

You can't add a line of best fit to data that has no correlation.

Check it out:

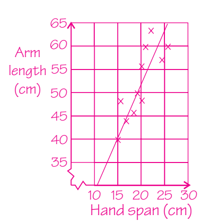
A clear plastic ruler is useful for drawing lines of best fit. You can adjust it until you think the edge is in the right place.

Guided Practice

4. The hand spans of 11 students are measured, together with the lengths of their arms. The measurements are recorded in the table below.

Hand span (cm)	19	18	20	15	21	22	16	17	20	24	26
Arm length (cm)	50	46	56	40	60	63	48	44	48	57	60

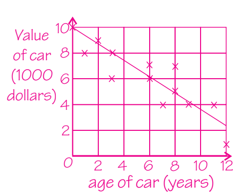
Plot a scatterplot of this data. Add a line of best fit to your scatterplot.
see left



5. The ages and values of a particular type of car are recorded below.

Age of car (years)	0	2	7	12	11	6	8	1	6	3	3	8	9
Value of car (dollars)	10,000	9000	4000	1000	4000	6000	7000	8000	7000	6000	8000	5000	4000

Plot a scatterplot of this data. Add a line of best fit to your scatterplot.
see left



2 Teach (cont)

Universal access

Some students may be uncomfortable with this "eyeball" method of finding a line of best fit as it doesn't seem very mathematical.

There are various ways of finding the position of the line of best fit mathematically. Some students may like to try plotting the median fit line. Below is the procedure for finding this line. It is illustrated with the ice cream sales/temperature data from Lesson 6.2.1.

1. Order the values of each variable.

2. Divide each list into thirds.
If the number of data values isn't divisible by 3, do the best you can making sure that the first and third groups have the same number of values.

3. Find the medians of all six groups.
These are shown below in red.

48 **68** **87**
temp: 41, 48, 55 | 63, 66, 70, 73, | 79, 87, 90

sales: 16, 36, 67 | 73, 80, 100, 101, | 114, 123, 170
36 **90** **123**

4. Plot the three ordered pairs of medians:
A (48, 36), B (68, 90), C (87, 123)

5. Connect the first point (A) to the last (C).

6. Draw a line parallel to AC, but one-third of the way closer to point B. This is the median fit line.

Guided practice

Level 1: q4
Level 2: q4–5
Level 3: q4–5

Solutions

For worked solutions see the Solution Guide

Advanced Learners

Ask advanced learners to try drawing the median fit line. This is described in the teacher margin of the previous page. They should compare this to the line of best fit that they drew "by eye."

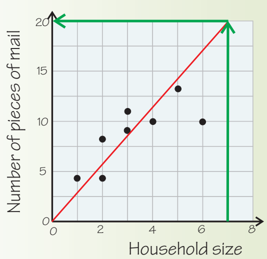
2 Teach (cont)

Additional example

Below is some data relating the number of people in a household and the number of pieces of mail delivered to the household in a given week.

Make a scatterplot of this data and add a line of best fit. Use it to predict the number of pieces of mail that a 7-person household would receive.

Number of people	2	3	3	6	4	2	1	5
Number of pieces of mail received in a week	4	9	11	10	10	8	4	13



About 20 — this depends on the exact positioning of the line of best fit.

Guided practice

Level 1: q6–7
Level 2: q6–9
Level 3: q6–9

Independent practice

Level 1: q1–3
Level 2: q1–3
Level 3: q1–3

Additional questions

Level 1: p464 q1–3
Level 2: p464 q1–3
Level 3: p464 q1–3

3 Homework

Homework Book — Lesson 6.2.3

Level 1: q1–4
Level 2: q1–5
Level 3: q1–5

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:

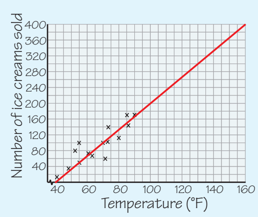
- Worksheet 47 — Data Displays
- Worksheet 48 — Data Sets

Check it out:

You can use scatterplots to predict data points outside of the range you were given. You have to take care though as you don't know whether the data will continue in the same pattern.

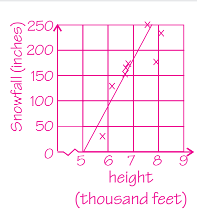
For instance, you could extend the axes and line of best fit in the graph below and predict that 400 ice creams would be sold at a temperature of 160 °F.

This is obviously wrong as people wouldn't survive if it was that hot.



Now try these:

Lesson 6.2.3 additional questions — p464



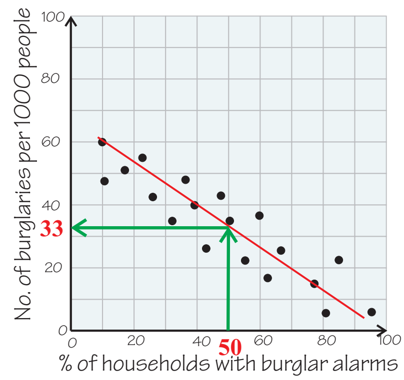
Use Lines of Best Fit to Make Predictions

You can use a line of best fit to predict what other data points might be.

Example 3

Predict the number of burglaries per 1000 people if 50% of households have burglar alarms.

Solution



Start at **50%** on the **horizontal axis**.
Read up to the line of best fit.
Read across from the line of best fit to the **vertical axis**.

When 50% of households have burglar alarms, the number of burgaries per 1000 people is expected to be around 33.

Guided Practice

In Guided Practice Exercise 4, you drew a scatterplot of arm length against hand span. Use your line of best fit to predict:

6. the arm length of a student with a 23 cm hand span. **about 60 cm**
7. the hand span of a student with a 52 cm arm length. **about 20 cm**

In Guided Practice Exercise 5, you drew a scatterplot of values against ages for a certain type of car. Use your line of best fit to predict:

8. the expected value of a 5-year-old car of this type. **about $7000**
9. the age of a car that is valued at $5500. **about 7 years old**

Independent Practice

The table below shows the height (in feet) of mountains with their cumulative snowfall on April 1st (in inches).

1. Create a scatterplot of the data.
2. Draw in a line of best fit for the data.
3. A mountain has a height of 7200 feet. What would you expect its cumulative snowfall to be on April 1st? **about 200 inches**

Height (ft)	6700	7900	7600	6800	6200	5800	8200	6700
Snowfall (in.)	153	174	249	172	128	32	238	162

Round Up

Lines of best fit follow the trend for the data. You can use them to predict values — but remember, chances are your predictions won't be totally accurate. They can give you a good idea though.

Solutions

For worked solutions see the Solution Guide

Investigation — Cricket Chirps and Temperature

Purpose of the Investigation

This Investigation gives students the opportunity to use data to test a real theory — that the frequency of cricket chirps can be used to estimate temperature. Students are asked to display data as a scatterplot and say whether or not the variables are correlated. They then use the data to predict new values, and to arrive at a formula for estimating the temperature from the frequency of cricket chirps.

Chapter 6 Investigation
Cricket Chirps and Temperature

Displaying data in a visual way makes it easier to see whether trends and patterns exist.

For a long time, people have believed that you can estimate the **temperature** from the **number of times a cricket chirps** in a set period of time.

At the same time of evening on 15 consecutive days, a student took the temperature outside and counted the number of times a cricket chirped in **15 seconds**. The results are shown below.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of cricket chirps	15	17	19	13	14	15	17	20	21	21	19	18	16	16	16
Temperature (degrees Fahrenheit)	72	76	83	70	75	80	82	87	93	88	90	85	75	81	72

The data isn't very useful in this form — you can't see any patterns clearly.
You are going to make a **visual display** that makes the data easier to interpret.

Make a **scatterplot** to compare the number of cricket chirps and the temperature.

Explain what, if any, **correlation** exists among the data.
Do you think that you can **estimate** the temperature from the number of cricket chirps?

Extensions

1) Draw a line of best fit on your scatterplot.
Use the line to predict the temperature for each number of cricket chirps from 10 to 25.
Which of these predictions do you have the most confidence in?

2) There are several different formulas for working out temperature from cricket chirps.
One is:
"Count the number of chirps a cricket makes during a 15-second period.
Then, add 45 to the number of chirps. This gives you an estimate of the temperature in degrees Fahrenheit."
Does this formula agree with the data above?

Open-ended Extensions

1) Can you create a formula that fits the line you drew in the first Extension above?

2) There are many formulas for estimating the temperature from cricket chirps.
See how many different ones you can find using reference books, almanacs, or the internet.
Do any of them match the data above?

Round Up

When you've displayed your data in an appropriate way, you'll often immediately be able to see patterns that you couldn't see before. Scatterplots only work when you've got two data sets that are paired. If you haven't, you have to use a different form of display.

Resources
- graph paper for each student
- rulers
- reference books, almanacs, or internet computers

Strategic & EL Learners
Provide strategic learners with a set of suitable axes drawn on graph paper, with an easy to use scale. Check English language learners understand what crickets are, and what chirping is.

Investigation Notes on p345 B-C

Investigation — Cricket Chirps and Temperature

Mathematical Background

Sets of variables that are in pairs can be plotted on a scatterplot. This shows whether the variables are correlated — whether one changes when the other does. If the variables are positively correlated, they'll fall in a diagonal band from the bottom left of the graph to the top right (see the graph below for an example of this). This means that as one variable increases, the other does too.

A line of best fit can be drawn to represent the trend. This line may not include any of the actual values, but you can use it to estimate what other values are likely to be.

This Investigation asks students to investigate whether temperature can be estimated from the rate at which crickets chirp. They are asked to do this by displaying and interpreting provided data.

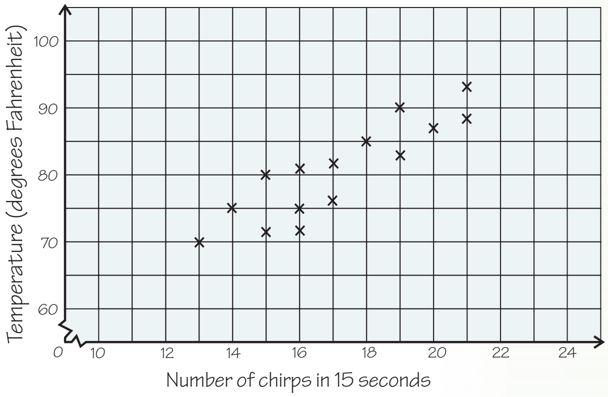
[Crickets make their chirping sound by scraping a ridge on one wing against a series of wrinkles on the other. The rate at which a cricket chirps is dependent on the cricket's species, and on the temperature of the environment.]

Approaching the Investigation

Students first need to devise a suitable scale for their scatterplot. This will depend on the size and scale of the graph paper they are given. It doesn't matter which variable goes on which axis.

The temperatures go from 70 to 93 degrees Fahrenheit, and the number of chirps goes from 13 to 21 (but note that the extension activity requires predictions to be made for 10 to 25 chirps). This means the scale for each axis is likely to be different. Students will get a more useful plot if they do not start either scale at zero.

Below is a scatterplot of the data:



The points fall in a band from the lower left of the graph to the upper right. This indicates that the variables are correlated positively. The correlation is not perfect, but as one variable increases, the other tends to also.

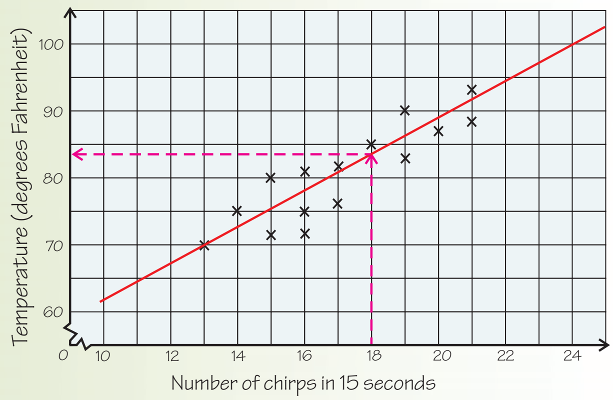
It therefore appears that it is possible to estimate the temperature from the number of cricket chirps in a period of time.

Investigation — Cricket Chirps and Temperature

Extensions

1) In the first extension, students are asked to add the line of best fit to their scatterplot. This line should follow the trend of the points, and divide them roughly in half.

   The lines of best fit the students draw will all differ (meaning the predictions they make will differ too).



   To predict what the temperature will be for a certain number of chirps, you find the number of chirps on the correct axis, and read up (or across if on the vertical axis) to the line of best fit. You then read across (or down) to the corresponding value. The pink line on the graph above indicates how to predict the temperature if there are 18 chirps in 15 seconds.

   Here are the approximate values for each number of chirps from 10 to 25 for the line of best fit drawn above.

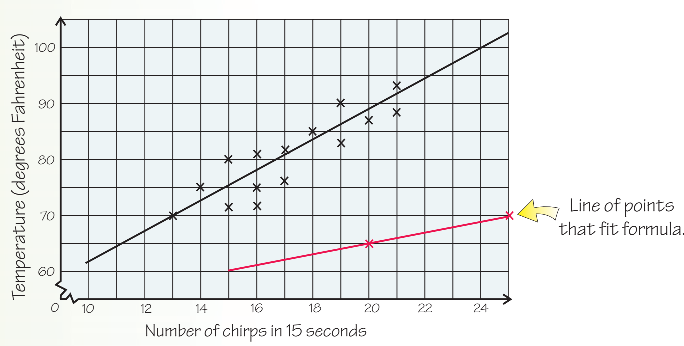
Number of chirps in 15 seconds	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Temperature (degrees Fahrenheit)	62	64	67	70	73	75	78	81	83	86	88	92	94	97	100	103

   It's hard to predict with confidence outside the range for which data has been collected. You don't know what's going to happen at high or low temperatures. The crickets may become too cold or too hot to chirp. So students should be most confident with their temperature predictions for between 13 and 21 chirps.

2) For the formula given, 20 chirps would mean a temperature of about 20 + 45 = 65 degrees Fahrenheit, and 25 chirps would indicate a temperature of about 25 + 45 = 70 degrees Fahrenheit.

   When these points are plotted on the graph, you can see that they are nowhere near the rest of the values, so the formula given doesn't agree with the data.

   The formula may just be wrong, or the data inaccurate. Alternatively, the data may apply only to certain species of cricket, or only to crickets in certain places.



Open-Ended Extensions

1) Students should find that if they multiply the number of chirps in 15 minutes by 3, and add 30, they'll reach approximately the correct temperature in degrees Fahrenheit. Students may figure this out by looking at the corresponding values in the table above, and realising that every time the number of chirps increases by one, the temperature rises by 3 degrees.

2) Most of the formulas to be found have you count the number of cricket chirps in a set time, such as 15 seconds, and then add a number. The most common numbers are 37, 39, and 40. None of these formulas fit the data collected in this Investigation.

Chapter 7

Three-Dimensional Geometry

How Chapter 7 fits into the K-12 curriculum

Section 7.1 — Shapes, Surfaces, and Space		
Section 7.1 covers Measurement and Geometry 2.1, 2.2, 2.3, 3.5, 3.6, Mathematical Reasoning 1.3, 2.2 **Objective: To use nets to calculate surface areas of 3-D figures**		
Previous Study Students met prisms, cylinders, cones and pyramids in previous grades. In grade 6, students calculated the volumes of prisms and cylinders. In grade 5 they used nets of rectangular prisms to calculate surface area.	**This Section** Students review common 3-D figures, and then relate nets to 3-D figures. They then use nets to calculate surface areas of 3-D figures, and finally see how lines and planes can be arranged in 3-D space.	**Future Study** Later in this Chapter, students will investigate the effect of scale factor on surface area, edge lengths, and volume.

Section 7.2 — Volume		
Section 7.2 covers Measurement and Geometry 2.1, Algebra and Functions 3.2, Mathematical Reasoning 2.2, 2.3, 3.2 **Objective: To understand the concept volume and the relationships between dimension and volume of 3-D figures**		
Previous Study At grade 5 students were first introduced to the concept of volume. In grade 6 they learned how to calculate the volumes of rectangular prisms, triangular prisms, and cylinders.	**This Section** Students first develop their understanding of volume, and learn formulas for calculating volumes of regular prisms. They then plot dimension against volume on coordinate axes to investigate the relationship.	**Future Study** In Geometry students will derive and solve problems involving the volume of common solids. In Algebra I students will graph non-linear functions.

Section 7.3 — Scale Factors		
Section 7.3 covers Measurement and Geometry 1.2, 2.1, 2.3, 2.4 **Objective: To identify similar solids, to calculate their areas and volumes, and to convert between area and volume units**		
Previous Study In grade 6, students solved problems involving similar polygons. In Chapter 3, they were introduced to scale factors and explored their effect on the perimeter and area of 2-D figures.	**This Section** Students review the use of scale factors in two dimensions, and then expand this to three-dimensional space. They then calculate surface area and volumes of similar shapes. Finally, they convert between unit systems of area and volume.	**Future Study** In Geometry, students will prove basic theorems involving congruence and similarity.

Pacing Guide – Chapter 7

40- to 50-Minute Class Periods

If your class periods are 40-50 minutes, we recommend allowing **15 days** for teaching Chapter 7.
As well as the **10 days of basic teaching**, you have **5 days** remaining to allocate 5 of the 7 optional activities (to be delivered at any appropriate point during the Chapter).

The table shows the 10 teaching days as well as all of the **optional days** you may choose for Chapter 7, in the order we recommend.

Day	Lesson	Description
Section 7.1 — Shapes, Surfaces, and Space		
Optional		*Exploration — Nets*
1	7.1.1	Three-Dimensional Figures
2	7.1.2	Nets
3	7.1.3	Surface Areas of Cylinders and Prisms
4	7.1.4	Surface Areas & Perimeters of Complex Shapes
5	7.1.5	Lines and Planes in Space
Optional		*Assessment Test — Section 7.1*
Section 1.1 — Sets and Expressions		
Optional		*Exploration — Build the Best Package*
6	7.2.1	Volumes
7	7.2.2	Graphing Volumes
Optional		*Assessment Test — Section 7.2*
Section 1.1 — Sets and Expressions		
Optional		*Exploration — Growing Cubes*
8	7.3.1	Similar Solids
9	7.3.2	Surface Areas & Volumes of Similar Figures
10	7.3.3	Changing Units
Optional		*Assessment Test — Section 7.3*
Section 1.1 — Sets and Expressions		
Optional		*Investigation — Set Design*

Accelerating and Decelerating

- To **accelerate** Chapter 7, allocate fewer than 5 days to the optional material. This will give you extra days to allocate to other Chapters.
  Note that you may use the remaining optional days at the end of the 160-day course.
- To **decelerate** Chapter 7, consider allocating more than 5 days to the optional Assessment Tests, Section Explorations, or Chapter Investigation, or spend longer teaching some Lessons. Also consider preparing students for difficult Lessons by reviewing previous coverage of math topics on related Skills Review Worksheets.
  Note that decelerating Chapter 7 will result in fewer days being available for teaching other Chapters.

90-Minute Class Periods

If you are following a block schedule with 90-minute class periods, we recommend allowing **7.5 days** for teaching Chapter 7.
The basic teaching material will take up **5 days**, and you can allocate the remaining **2.5 days** to the **optional material**.
To accelerate or decelerate a block schedule, follow the same advice as given above.

Section 7.1

Exploration — Nets

Purpose of the Exploration

The goal of the Exploration is to have students make connections between two-dimensional figures and three-dimensional figures. The nets show students how the dimensions of a three-dimensional rectangular prism are produced.
This Exploration is also a good introduction to the concept of surface area.

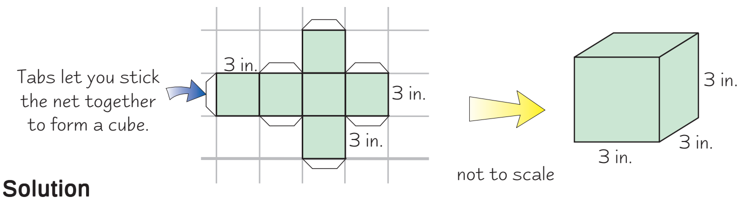
Section 7.1 introduction — an exploration into: Nets

In this Exploration, you will be using two-dimensional figures called nets to make three-dimensional rectangular prisms. You'll construct nets for rectangular prisms with given dimensions.

The example below shows a **net** of a cube. You make the cube by cutting the net out, folding it along the lines and taping it together.

Example

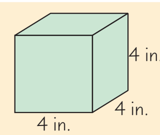
What dimensions will the cube made from the net below have?



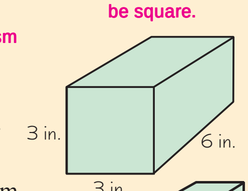
Solution

The cube made by this net has a length, width, and height of 3 inches.

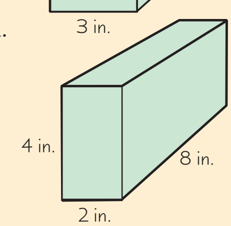
Exercises



1. Create a net for a cube that has a length, width, and height of 4 inches. Then build your cube. see below
2. Find a rectangular prism that isn't a cube and draw around each face. It may be useful to mark each face after you have drawn around it.
   a. How many faces are there? 6 **b.** What shapes are the faces? Depending on the actual prism, 2, 4, or all 6 faces will be rectangles. The other faces will be square.
3. Measure the length and width of each face you drew in Exercise 2. Do any of the faces match each other?
   The faces will always be in matching pairs from opposite sides of the prism (or 4 faces may be identical, depending on the actual prism used).



4. This rectangular prism has twice the length of the cube in the example at the top of the page. Its height and width are the same. What dimensions do the faces of this rectangular prism have?
   Two are 3 in. by 3 in., and four are 6 in. by 3 in.
5. Create a net for the prism shown on the right. Then build the prism. see below



6. The prism shown on the right has the following dimensions:
   Length = 8 in., width = 2 in., height = 4 in.
   What dimensions do the faces of this prism have?
   Two are 4 in. by 2 in., two are 4 in. by 8 in., and two are 2 in. by 8 in.
7. Create a net for a rectangular prism with the dimensions given in Exercise 6. Then build the prism. see below

Round Up

Cubes have six identical square faces — so their nets are made of six identical squares. You have to make sure you join them to each other correctly so that the net folds into a cube though. Rectangular prisms are trickier to draw nets for. They usually have three different pairs of faces.

Resources

- grid paper
- sample rectangular prisms (for example, small cereal boxes)
- rulers
- scissors
- tape
- card

Strategic & EL Learners

Strategic learners may benefit from having examples of the rectangular prisms they are trying to construct in front of them. If these are made from nets in advance, students can then take them apart and study how they are made if necessary.

EL learners may have difficulty with "two-dimensional" and "three-dimensional." Tell these students that two-dimensional means flat and three-dimensional means a solid.

Universal access

If the nets have tabs, they will be easier to tape together.
Tabs should be put on alternate "edges" of the net (you can start at any point on the net).

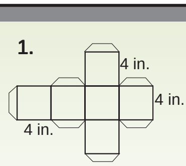
Common error

Students may have difficulty constructing nets for rectangular prisms with given dimensions.
It may help them to draw and cut out rectangles to represent each face of the rectangular prism they are trying to make. They can then manipulate the rectangles before taping them together to make the net.

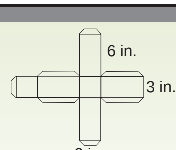
Math background

There are different ways that the nets of prisms and cubes can be drawn. (There are 11 different nets for the cube.) This is an interesting point for discussion with the class.

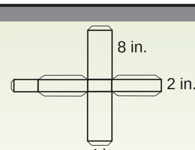
Solutions 1.



5.



7.



Other nets are possible.

Lesson 7.1.1

Three-Dimensional Figures

This Lesson gives mathematical names to common types of three-dimensional figures and explains their properties. The concept of diagonals of a 3D-shape is also introduced and explained.

Previous Study: Students have met prisms, cylinders, cones, and pyramids in previous grades. In grade 6, students calculated the volumes of prisms and cylinders.

Future Study: Later in this Section, students will calculate the volumes and surface areas of three-dimensional figures.

1 Get started

Resources:
- geoblocks / everyday 3-D objects

Warm-up questions:
- Lesson 7.1.1 sheet

2 Teach

Universal access

Another way to approach this Lesson is to have students derive the definitions on their own, before giving them the text.

Place a selection of prisms out for display, and agree with the students that they are all prisms. Ask the students to try to write a definition in pairs. Then have them all present their definitions. Discuss them among the class to see if people agree with the definition or not.

You could make a "game" of trying to create an example that fits their definition, but is not a prism. For instance, if students omit that the bases have to be congruent, draw a figure with different sized bases, to demonstrate the importance of this part of the definition.

This activity can be repeated with pyramids, cylinders, and cones.

Universal access

If available, geoblocks are a great way to reinforce the definitions of prisms, pyramids, cylinders, and cones. Have students sort the geoblocks into these 4 categories (with a fifth category for others).

Alternatively, have students look around the classroom and try to find examples of prisms, pyramids, cylinders, or cones.

Lesson 7.1.1

Section 7.1

Three-Dimensional Figures

California Standards:

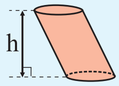
Measurement and Geometry 2.1
Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of **basic three-dimensional figures, including** rectangles, parallelograms, trapezoids, squares, triangles, circles, **prisms, and cylinders**.

Measurement and Geometry 3.6
Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

What it means for you:
You'll practice recognizing some important 3-D shapes, such as prisms, cylinders, cones and pyramids.

Key words:
- prism
- cylinder
- cone
- pyramid
- diagonal
- right
- oblique

Check it out:
The height of a three-dimensional solid is always the perpendicular distance from the top to the bottom. So the height of cylinder C is the distance straight down from the circle at the top to the circle at the bottom. It doesn't matter that there isn't a side that goes straight down.

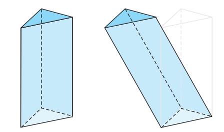


Real life is full of three-dimensional figures. This Lesson's about some of the special figures that have mathematical names.

Prisms and Cylinders Have Two Identical Ends

A **prism** is a 3-D shape formed by joining two **congruent polygon faces** that are **parallel** to each other. The polygon faces are called the **bases** of the prism.

- If the edges joining the bases are at right angles to the bases, it is called a **right prism**.
- If the edges joining the bases are not at right angles to the bases, it's an **oblique prism**. Oblique prisms appear to lean to one side.

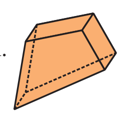
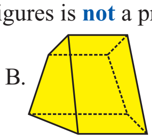
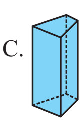
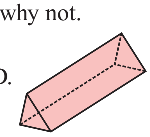


right prism oblique prism

Prisms are often named according to their bases — if the bases are rectangles, it's a **rectangular prism**. If the bases are triangles, it's a **triangular prism**.

Example 1

Which of these figures is **not** a prism? Explain why not.

A.  B.  C.  D. 

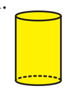
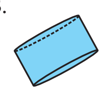
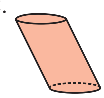
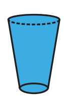
Solution

B is not a prism. The shape at one end is not the same as the shape at the other. **A, C, and D are prisms**. D is a triangular prism.

A **cylinder** is just like a prism, except that the bases have curved, rather than straight, edges. All the cylinders in this book will be **circular cylinders** with circle bases, though it is possible to have cylinders with ellipse bases. As with prisms, cylinders can be **right** or **oblique**.

Example 2

Which of these figures is not a cylinder? Explain why not.

A.  B.  C.  D. 

Solution

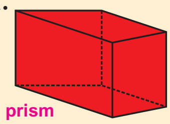
D is not a cylinder because the circle at the bottom is not the same size as the circle at the top. **A, B, and C are cylinders** because they have congruent circular faces that are parallel to each other. A and B are right cylinders, and C is an oblique cylinder.

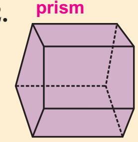
- **Strategic Learners**
  Have students identify the shapes of faces on three-dimensional figures. They should count how many faces of each shape the figure has, and identify the number of vertices and edges. The sorting activity suggested in the Universal access section at the bottom of the previous page is also useful for strategic learners.

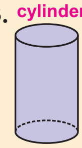
- **English Language Learners**
  Put a variety of wooden or plastic 3-dimensional shapes into a paper bag. Ask pairs of students to take turns to reach inside the bag, and identify a shape without looking at it. They should then check that their partner agrees that they are right.

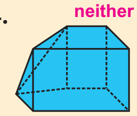
Guided Practice

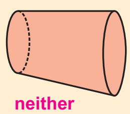
In Exercises 1–10, identify each shape as either a prism, a cylinder, or neither.

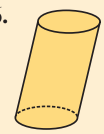
1.  prism

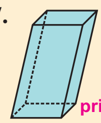
2. prism 

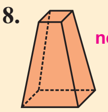
3. cylinder 

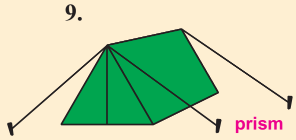
4. neither 

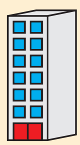
5.  neither

6.  cylinder

7.  prism

8.  neither

9.  prism

10.  prism

Pyramids and Cones Have Points

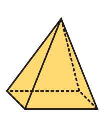
Don't forget:
A polygon is any shape that is made from straight lines that have been joined, end-to-end into a closed shape.
See Section 3.1 for a reminder on polygons.

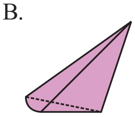
A **pyramid** is a three-dimensional shape that has a **polygon** for its base, and all the other faces come to a **point**. The point **doesn't** have to be over the base.

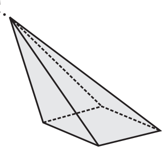
A pyramid with a **rectangular base** is known as a **rectangular pyramid**. Similarly, a pyramid with a **triangular base** is a **triangular pyramid**.

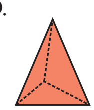
Example 3

Which of these figures is not a pyramid? Explain why not.

A. 

B. 

C. 

D. 

Solution

B is not a pyramid. Its base has a curved edge, so it isn't a polygon. The base of a pyramid is always a polygon.

A, C and D are all pyramids. It doesn't matter that C leans, as the sides all still come to a point.

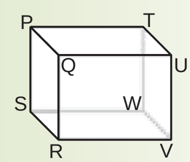
2 Teach (cont)

Guided practice

Level 1: q1–8
Level 2: q1–10
Level 3: q1–10

Additional example

Below is a three-dimensional figure



a. How many vertices are there? Name them.
There are 8 vertices in the figure above. They are P, Q, R, S, T, U, V, and W.

b. How many edges are there? Name them.
There are 12 edges in the figure above. They are PQ, QR, RS, PS, TU, UV, VW, TW, PT, QU, RV, and SW.

c. How many faces are there? Name them.
There are 6 faces in the figure above. They are PQRS, TUVW, PTUQ, RSWV, PTWS, and QUVR.

Solutions

For worked solutions see the Solution Guide

● **Advanced Learners**
Ask students to play a questioning game in pairs to identify shapes. One student thinks of a three-dimensional shape, and the other asks questions to try to identify it. The questions can only be answered with either "Yes" or "No." For example, a student might ask "Is one of the faces a circle?" Students must not ask questions mentioning the names of particular three-dimensional figures (such as, "Is it a cone?").

2 Teach (cont)

Math Background
The definitions of the solids covered in this Lesson will vary depending on the source. For example, in this book, cylinders and cones are restricted to those with curved-edge bases. However, cylinders and cones can also be defined more generally, by allowing the base to be any flat shape. With these more general definitions, prisms become special cases of cylinders, and pyramids become special cases of cones.

Guided practice
Level 1: q11–15
Level 2: q11–16
Level 3: q11–17

Universal access
Some students might be interested in trying to sketch the shapes.

Draw their attention to the fact that the top face of a shape appears to be "squashed" — a rectangle becomes a parallelogram, a circle becomes an ellipse, and so on. Get them to look at common 3-D shapes from different angles, and see how the "2-D" figures which make them up change shape.

Concept question
"Does a pyramid have any diagonals? Explain your answer."

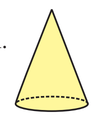
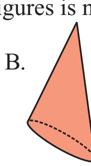
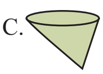
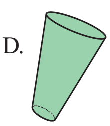
No. A line drawn between any two vertices in a pyramid will lie along a face.

A **cone** is like a pyramid, but instead of having a polygon for a base, the base has a curved edge. All of the cones in this book will be **circular cones** with bases that are **circles**.

Check it out:
Pyramids and cones which have their point directly above the center of their base are called "right pyramids" or "right cones."

Example 4

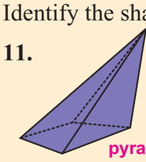
Which of these figures is not a cone? Explain why not.

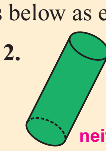
A. 
B. 
C. 
D. 

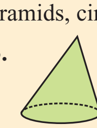
Solution
D is not a cone because it doesn't make a point at the end.
A, B, and C are all cones. It doesn't make any difference that B and C are leaning to the side.

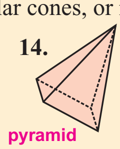
Guided Practice

Identify the shapes below as either pyramids, circular cones, or neither.

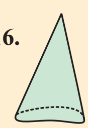
11. 
pyramid

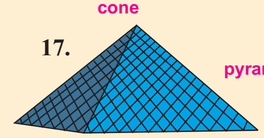
12. 
neither

13. 
circular cone

14. 
pyramid

15. 
neither

16. 
neither

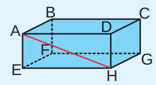
17. 
pyramid

Check it out:
Pyramids and prisms are polyhedrons. The sides of polyhedrons are all polygons, and are known as **faces**. The line where two faces meet is called an **edge**. A point where edges meet is called a **vertex**.

Diagonals Go Through the Insides of Solids

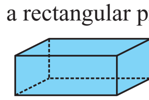
Diagonals are a type of line segment. In a 3-D shape, they connect two **vertices** that **aren't on the same face**.

Check it out:
Diagonals must pass through the inside of the shape. So the line from A to H **isn't** a diagonal.

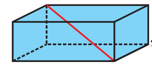
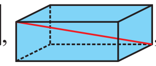
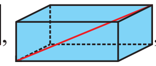
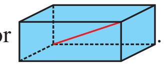


Example 5

The diagram below shows a rectangular prism or cuboid. Mark a diagonal on it.



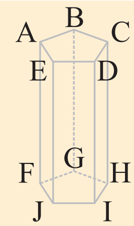
Solution
There are four diagonals that you could mark:

, , , or .

There are no other possible diagonals.

Solutions
For worked solutions see the Solution Guide

Guided Practice



18. Crystal says that any line that goes through a prism is a diagonal of the prism. Is she correct?
No — the lines must go through vertices of the prism

Exercises 19–20 are about the prism shown on the right.

19. How many diagonals does this shape have? **10**

20. Name all the diagonals by giving their starting vertex and ending vertex.
AH, AI, BI, BJ, CJ, CF, DF, DG, EG, EH

Check it out:

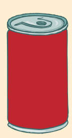
For Exercises 19–20 — start with vertex A and see how many diagonals you can draw from it. The same number of diagonals will start from vertices B, C, D, and E.

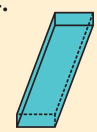
Independent Practice

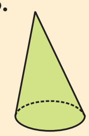
1. What is similar about a cylinder and a prism? **See below**

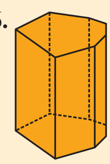
2. What is different about a cylinder and a prism? **See below**

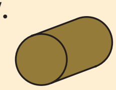
In Exercises 3–13, identify each shape as a prism, cylinder, pyramid, cone, or as none of those.

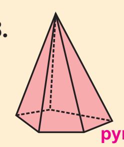
3.

cylinder

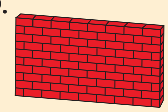
4.

prism

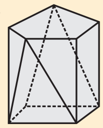
5.

cone

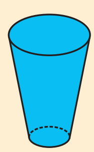
6.

prism

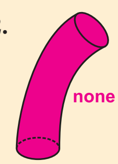
7.

cylinder

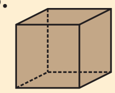
8.

pyramid

9.

prism

10.

none

11.

none

12.

none

13.

prism

In Exercises 14–17, say whether each statement is true or false. If any are false, explain why.

14. A cylinder is a type of prism. **False — the base is not a polygon**

15. All cubes are prisms. **True**

16. Pyramids have no diagonals. **True**

17. A cylinder is any shape with two circles for bases.
False — the circles must be parallel and the same size

Now try these:

Lesson 7.1.1 additional questions — p465

Round Up

You need to be able to identify the different kinds of three-dimensional figures. The hard part is remembering precisely when you can use each name and when you can't. Once you've mastered that, you can try nets — which are like flat "patterns" that can be folded to make 3-D figures.

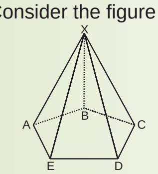
2 Teach (cont)

Guided practice

Level 1: q18
Level 2: q18–20
Level 3: q18–20

Additional examples

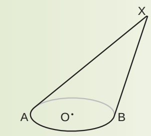
1. Consider the figure below.



a. What type of figure is it? Be as specific as you can.
A right pentagonal pyramid

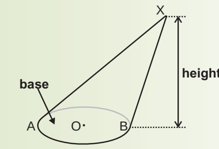
b. Name the base.
ABCDE

2. Consider the figure below.



a. Is this a right cone? Explain your answer?
No. The point is not directly above the center of the base, O.

b. Label the base and the height.



Independent practice

Level 1: q1–13
Level 2: q1–17
Level 3: q1–17

Additional questions

Level 1: p465 q1–5
Level 2: p465 q1–7
Level 3: p465 q1–8

3 Homework

Homework Book — Lesson 7.1.1

Level 1: q1–5
Level 2: q1–6
Level 3: q1–7

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
• Worksheet 43 — 3-D Figures

Solutions
For worked solutions see the Solution Guide

1. Both have two congruent parallel bases.
2. The bases of a cylinder are circles or ellipses, but the bases of a prism are polygons.

Lesson 7.1.2

Nets

In this Lesson, the concept of nets is introduced, and the nets of specific shapes are shown and explained. Students are taught to calculate the fraction of a circle required for the net of a cone, and how to use π to calculate lengths in cylinder nets.

Previous Study: In grade 5, students made a rectangular box from two dimensional patterns, which they used to compute surface area. Students have already met the 3-D shapes covered in this Lesson.

Future Study: Later in this Section, students will calculate the surface areas of shapes from their nets.

1 Get started

Resources:

- food cans with paper labels
- empty cereal boxes and cardboard containers of other shapes
- paper/scissors/sticky tape
- geoblocks
- graph paper/dot paper
- compasses and protractors

Teacher Resources CD-ROM

- Nets

Warm-up questions:

- Lesson 7.1.2

2 Teach

Universal access

Show students the nets from the **Teacher Resources CD-ROM**, and ask them what 3-D shape they think they will fold into.

Then have the students find out by cutting the nets out and constructing the shapes from them.

Afterwards, it's a good idea to hand out a second copy of the net, and have the students compare the net to the finished shape. Ask them which faces and edges ended up in which positions.

Concept question

"In a net of a rectangular prism, how many differently sized faces can there be?"

There can be faces of three different sizes at most. The faces will be in three pairs.

Lesson 7.1.2

Nets

California Standards:

Measurement and Geometry 3.5

Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

What it means for you:

You'll draw two-dimensional patterns that can be folded up to make three-dimensional shapes, such as prisms. You'll use the formula you learned for finding the circumference of a circle to draw the pattern for a cylinder.

Key words:

- net
- cone
- cylinder
- pyramid
- two-dimensional
- three-dimensional
- rectangular pyramid

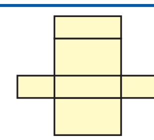
Sometimes you might want to make a model 3-D shape out of card. To do this, you need to figure out which two-dimensional shapes you need for the faces, how big they have to be, and how they should be joined together.

2-D Nets Can Be Folded Into 3-D Figures

A two-dimensional shape pattern that can be **folded** into a three-dimensional figure is called a **net**. The lines on the net mark the **fold lines** — these are the **edges** of the faces.

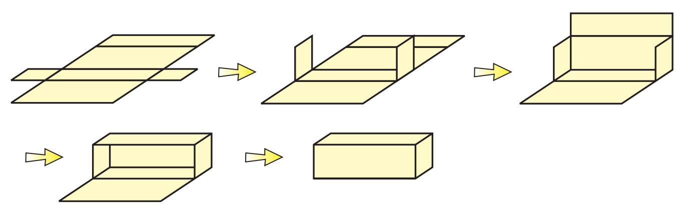
Example 1

What three-dimensional shape is this the net of?



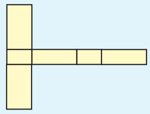
Solution

If you fold along all of the marked lines then you get a prism with a rectangular base. So **this is the net of a rectangular prism or cuboid**.

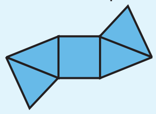


Check it out:

There's often more than one net for a shape.
Here's a different net for the cuboid in Example 1:

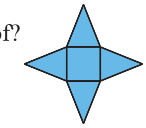


And here's one for the pyramid in Example 2:



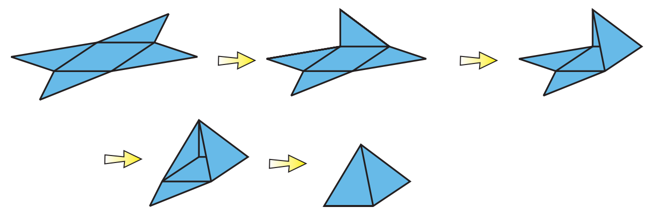
Example 2

What shape is this the net of?



Solution

This is the net of a square-based pyramid (a special type of rectangular pyramid), which is a pyramid with a square base.

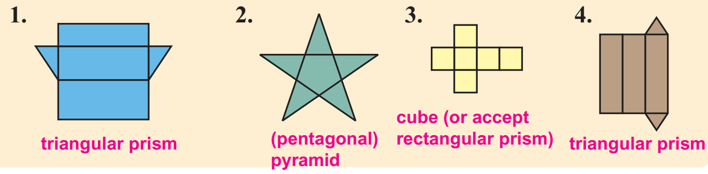


- **Strategic Learners**
Have students bring in an empty cereal box. Ask them to flatten it out and see if any faces match each other, and what shapes they are. Then they should rebuild their cereal box. The activity can be repeated using empty cardboard containers of other shapes.

- **English Language Learners**
Make vocabulary cards for 3-D shapes with the name of the shape on one side and a sketch on the other. Ask students to use two sets of cards to play matching games with a partner.

Guided Practice

In Exercises 1–4, say what shape is made by each net.



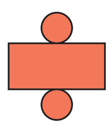
The Net of a Cylinder Has Two Circles

The net of a circular cylinder has **two circles** — one for the top of the cylinder and one for the bottom.

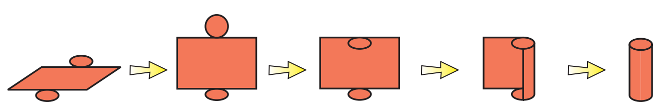
Example 3

Sketch the net of a right circular cylinder.

Solution



The net of a circular cylinder looks like a rectangle with a circle on top and a circle on the bottom. If you were to fold this shape up it would make a cylinder.

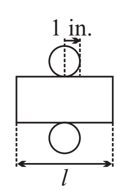


Don't forget:
A **right cylinder** is one in which the two bases are directly above each other. A right cylinder doesn't lean to one side.

The rectangle needs to be wide enough to be **wrapped around the outside** of the circles. So its length needs to be equal to the circumference of the circles.

Example 4

Work out the missing length, l, to the nearest hundredth.



Solution

The rectangle needs to wrap around the circle. So it has to have the **same length as the circumference of the circle**.

$$C = 2\pi r$$
$$l = 2 \times \pi \times r$$
$$= 2 \times \pi \times 1$$
$$= 6.28 \text{ in. (to the nearest hundredth)}$$

Don't forget:
The circumference of a circle is given by the formula $C = 2\pi r$, (where r = radius). That's the same as $C = \pi d$, (where d = diameter).

2 Teach (cont)

Guided practice
Level 1: q1–4
Level 2: q1–4
Level 3: q1–4

Universal access
Cut the paper label off a can to demonstrate that the label is a rectangle.

Another idea is to measure the height and diameter of a can and make a label of the correct size to wrap it. This demonstrates a practical application of using π to work out the circumference.

Universal access
Have students try to create accurate nets for particular cylinders — such as those from a geoblock set, or everyday cylindrical objects, such as food cans.

Students can check their work by wrapping their net around the cylinder.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to make a model using different three-dimensional figures. For example, they could make a car or a house. Careful planning and measuring will be needed so that the shapes fit together accurately. For instance, the base of the triangular prism used for the house roof must exactly match the top of the rectangular prism used for the body of the house. The models students construct will be used in the Advanced Learner activity in Lesson 7.1.4.

2 Teach (cont)

Guided practice

Level 1: q5–6
Level 2: q5–7
Level 3: q5–8

Universal access

It is fun and informative for students to cut out partial circles and wrap them into cones. It is much more memorable to physically see the curve of part of a circumference transform into a circle of its own, rather than simply be told that's what happens.

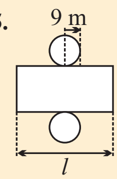
Have students cut out different sized sectors of a circle and tape them together. Then lead them to making the connection between the fraction of the circle and the radius of the base of the cone.

Common error

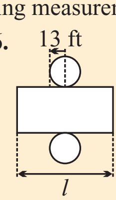
There is often a purely mechanical problem in making accurate circles for the bases for nets of both cylinders and cones. Make sure that students are proficient in using a compass to draw circles, and that their compass is of sufficient quality to allow them to do so.

Guided Practice

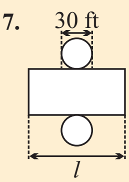
Work out the missing measurements in Exercises 5–8. Use π = 3.14.

5.


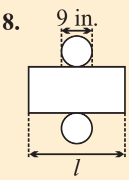
56.52 m

6.


81.64 ft

7.


94.2 ft

8.


28.26 in.

Cutting a Cone Makes Part of a Circle

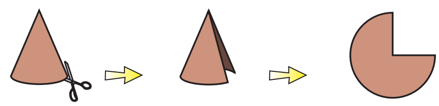
The net of a **circular cone** includes part of a **circle**.

Example 5

Sketch the net of a circular cone.

Solution

Imagine cutting up the side of a cone with no base and laying it flat.

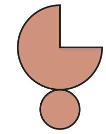


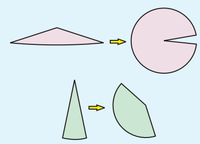
You get **a circle with part missing**.

To make a full cone, you need a base as well — **the base is a circle**.

The base circle can't be as large as the one with a sector cut out, or there'd be no way to roll it up and still have the base fit.

So the net of a cone is a circle with part missing and a smaller circle underneath.

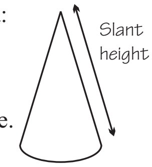


> **Check it out:**
> In Example 5, cutting along the cone makes a circle with exactly $\frac{1}{4}$ missing. But the amount of the circle that is missing could be anything. For very flat cones only a small amount will be missing. For very tall cones a very large amount will be missing.
>
> 

The sketched net above has a part-circle that is $\frac{3}{4}$ of a full circle.

If you're making a cone with a certain **base-radius** and a certain **slant height**, you can work out what **fraction** of a circle you need:

Fraction of circle = radius of base ÷ slant height



You use the **slant height** itself as the radius of the part-circle.

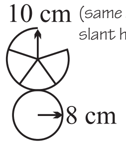
> **Check it out:**
> The circumference of the base circle must be the same as the length of the curved edge of the part-circle.

Example 6

Draw the net of a circular cone with slant height 10 cm and base radius 8 cm.

Solution

If slant height is 10 cm and base radius is 8 cm then the top circle should be $\frac{8}{10}$ of a complete circle. That's the same as $\frac{4}{5}$. **So draw the top circle with $\frac{1}{5}$ missing.**



Solutions

For worked solutions see the Solution Guide

Guided Practice

In Exercises 9–11, say whether each statement is true or false.

9. The net of a cone is two circles. False

10. You can draw the net of a cone without knowing the vertical height of the cone. True

11. The part-circle in the net of a cone is smaller than the full circle. False

Work out what fraction of a circle is needed for the part-circle in the net of each circular cone described in Exercises 12–14.

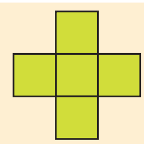
12. A cone with slant height 20 cm and base radius 16 cm. $\frac{4}{5}$

13. A cone with slant height 15 inches and base radius 3 inches. $\frac{1}{5}$

14. A cone with slant height 2 feet and base radius 1 foot. $\frac{1}{2}$

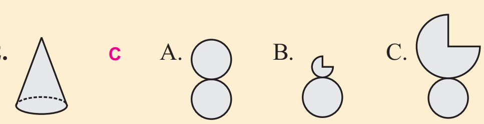
Independent Practice

1. Explain why the net shown on the right doesn't make a cube. It doesn't have enough faces — a cube has 6 but the net only has 5

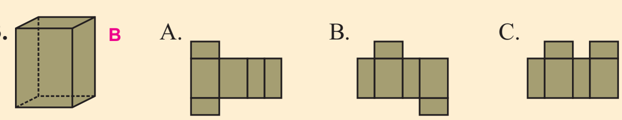


In Exercises 2–4, say which net could make each three-dimensional figure.

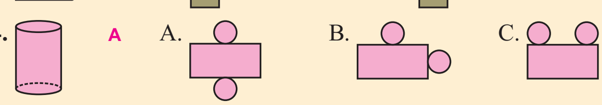
2. C



3. B

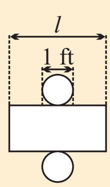


4. A



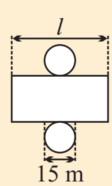
Fill in the missing measurements in Exercises 5–7. Use π = 3.14.

5.



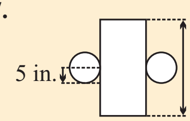
3.14 ft

6.



47.1 m

7.



31.4 in.

Work out what fraction of a circle is needed for the part-circle in the net of each circular cone described in Exercises 8–9.

8. A cone with slant height 3 inches and base radius 1 inch. $\frac{1}{3}$

9. A cone with slant height 11 inches and base radius 3 inches. $\frac{3}{11}$

Now try these:

Lesson 7.1.2 additional questions — p465

Round Up

Nets sound complicated, and some of them even look complicated. The key is to imagine folding along each of the lines, and think about what shape you would get.

2 Teach (cont)

Guided practice

Level 1: q9–12
Level 2: q9–13
Level 3: q9–14

Independent practice

Level 1: q1–7
Level 2: q1–8
Level 3: q1–9

Additional questions

Level 1: p465 q1–2, 4–6
Level 2: p465 q1–8
Level 3: p465 q1–8

3 Homework

Homework Book — Lesson 7.1.2

Level 1: q1–5
Level 2: q1–6
Level 3: q1–8

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:

- Worksheet 43 — 3-D Figures
- Worksheet 44 — Surface Area

Solutions

For worked solutions see the Solution Guide

Lesson 7.1.3

Surface Areas of Cylinders and Prisms

In this Lesson, students find the surface areas of cylinders and prisms by applying their knowledge of the nets of these shapes. The Lesson rounds up with a general formula for calculating the surface areas of prisms and cylinders.

Previous Study: In grade 5, students used nets of rectangular prisms to calculate surface area. In the previous Lesson, nets were introduced and explained.

Future Study: Later in this Chapter, students investigate how the lengths, areas, and volumes of three-dimensional shapes are affected by changes in scale.

1 Get started

Resources:
- paper/scissors/sticky tape
- models of 3-D shapes
- colored pencils
- paint coverage information

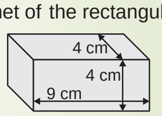
Teacher Resources CD-ROM
- Nets

Warm-up questions:
- Lesson 7.1.3

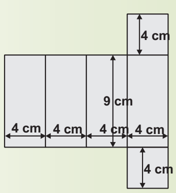
2 Teach

Additional examples

1. Draw the net of the rectangular prism below.



Note there are a number of other ways in which this net could be drawn correctly.



2. Determine the surface area of this prism.

(2 × 4 × 4) + (4 × 4 × 9) = 32 + 144 = 176 cm²

Lesson 7.1.3

Surface Areas of Cylinders and Prisms

California Standards:

Measurement and Geometry 2.1
Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and **the surface area** and volume **of basic three-dimensional figures, including** rectangles, parallelograms, trapezoids, squares, triangles, circles, **prisms, and cylinders**.

Measurement and Geometry 3.5
Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

Mathematical Reasoning 1.3
Determine when and how to break a problem into simpler parts.

What it means for you:

You'll see how to work out the surface area of 3-D shapes like cylinders and prisms.

Key words

- net
- surface area
- cylinder
- prism

Don't forget:

The area of a rectangle is given by $A = lw$.

The area of a triangle is given by $A = \frac{1}{2}bh$.

*Nets are very useful for finding the **surface area** of 3-D shapes. They change a 3-D problem into a 2-D problem.*

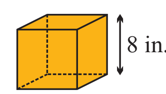
Draw a Net to Work Out the Surface Area

The **surface area** of a **three-dimensional solid** is the total area of all its faces — it's the area you'd paint if you were painting the shape.

The **net** of a **three-dimensional solid** can be folded to make a hollow shape that looks exactly like the solid. So one way to work out the surface area of the solid is to work out **the surface area of the net**.

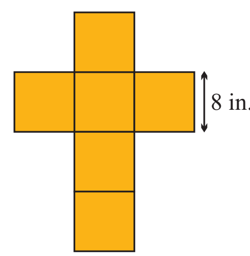
Example 1

What is the surface area of this cube?



Solution

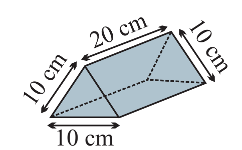
The net of the cube is six squares. So the surface area of the cube is equal to the area of six squares.



The area of each square is $8 \times 8 = 64$ in². So the surface area of the entire cube is $6 \times 64 = 384$ in².

Example 2

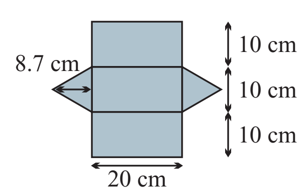
What is the surface area of this prism?



Vertical height = 8.7 cm

Solution

The net of this prism has three identical rectangles. The area of each rectangle is $10 \times 20 = 200$ cm².
So the total surface area of the three rectangles is $3 \times 200 = 600$ cm².



There are also two identical triangles. Each has a base of 10 cm and a height of 8.7 cm. The area of each triangle is $\frac{1}{2} \times 10 \times 8.7 = 43.5$ cm².
So the surface area of both the triangles together is $2 \times 43.5 = 87$ cm².

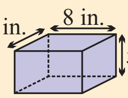
So the total surface area of the prism is $600 + 87 = 687$ cm².

- **Strategic Learners**
The Universal access activity on the previous page is particularly suitable for strategic learners. You may also need to review how to calculate the areas of rectangles, triangles, and circles with them. The English Language Learner activity described below is useful for this.

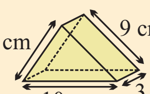
- **English Language Learners**
Ask students to work with a partner, and using paper models of 3-D shapes, count and name the faces that make up the surface of each shape. For example, for a triangular prism they might list "2 triangles, 3 rectangles." They should write the formula needed to calculate the area of each shape, calculate it, and then find the total for the whole 3-D shape. They can then open the model out, and repeat their area calculation.

Guided Practice

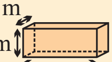
Work out the surface area of the shapes shown in Exercises 1–3.

1. 

236 in²

2. 

Height = 8 cm
164 cm²

3. 

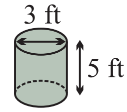
568 m²

Finding the Surface Area of Cylinders

The net of a circular cylinder has a **rectangle** and **two circles**. So you need to use the **formula for the area of a circle** to find its surface area.

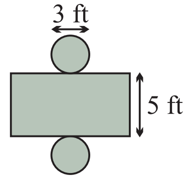
Example 3

What is the surface area of this cylinder? Use π = 3.14.



Solution

The net of the cylinder has one rectangle and two identical circles.



To work out the area of the rectangle, you need to know its length. It's the same as the circumference of the circles, so it is **3 × π = 9.42 ft**.

So the area of the rectangle is **9.42 × 5 = 47.1 ft²**.

The circles have a diameter of 3 feet. So they have a radius of 1.5 feet. The area of each circle is **π × 1.5² = π × 2.25 = 7.065 ft²**. Together the two circles have a surface area of **2 × 7.065 = 14.13 ft²**.

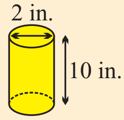
So the total surface area of the cylinder is **47.1 + 14.13 = 61.23 ft²**.

Don't forget:
Use the formula for the circumference of a circle, $C = \pi d$, (where d = diameter) to work out the length of the rectangle in the net of a cylinder.
The rectangle has to be long enough to reach all the way around the circle.

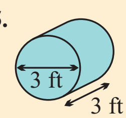
Don't forget:
The area of a circle is given by $A = \pi r^2$ (r = radius).

Guided Practice

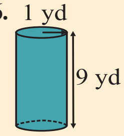
Find the surface areas of the cylinders in Exercises 4–6. Use π = 3.14.

4. 

69.08 in²

5. 

42.39 ft²

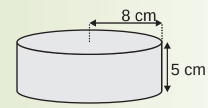
6. 

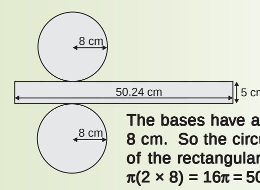
62.8 yd²

2 Teach (cont)

Guided practice
Level 1: q1–3
Level 2: q1–3
Level 3: q1–3

Additional examples
1. Draw the net of the cylinder below and label its dimensions. Use π = 3.14.





The bases have a radius of 8 cm. So the circumference of the rectangular part is π(2 × 8) = 16π = 50.24 cm.

2. Determine the surface area of the cylinder. Use π = 3.14.
Surface area **= (2 × π × 8²) + (5 × 50.24)**
= 653.1 cm²

Guided practice
Level 1: q4–6
Level 2: q4–6
Level 3: q4–6

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Provide students with paint coverage information, perhaps from an empty can. Ask them to calculate how many milliliters of paint they would need to paint particular three-dimensional shapes.

2 Teach (cont)

Common error

Students sometimes try to apply this formula to other shapes, such as pyramids and cones. They need to understand how this formula works in order to use it correctly.

The first thing to be sure of is that they understand where the component parts of the formula come from, "2 × base" comes from the fact there are two bases of equal area in a prism or cylinder.

The "lateral area" is made up of all the faces which join one base to the other. The number of lateral faces varies depending upon the type of 3-D shape. A triangular prism has three, a rectangular prism has four, and a cylinder has just one curved face that wraps itself around the entire base.

The second thing is to be sure that the students are accounting for the entire lateral area. When looking at a 3-D object, it can be easy to omit one face. A good way to avoid this problem is by sketching a net for each surface area problem they complete. If the student decides to work out the lateral area in sections, they should shade each section once its area has been found. This way any omissions are clear.

Independent practice

Level 1: q1–4
Level 2: q1–5
Level 3: q1–7

Additional questions

Level 1: p466 q1–6
Level 2: p466 q1–6
Level 3: p466 q1–9

3 Homework

Homework Book — Lesson 7.1.3

Level 1: q1–6
Level 2: q1–7
Level 3: q1–9

4 Skills Review

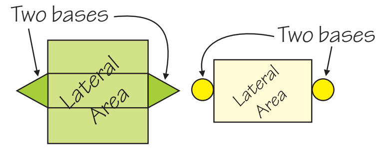
Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 43 — 3-D Figures
- Worksheet 44 — Surface Area

Use Formulas For Prism and Cylinder Surface Areas

The way you work out the surface area of a cylinder, and the way you work out the surface area of a prism are similar.
The surface area of either is **twice** the **area of the base** plus the area of the **part between the bases** of the net.

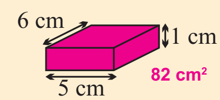
The part between the bases is sometimes called the **lateral area**.



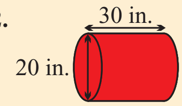
Area = (2 × base) + lateral area

Independent Practice

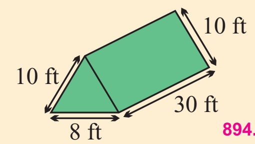
Work out the surface areas of the shapes shown in Exercises 1–4.
Use $\pi = 3.14$.

1.


82 cm²

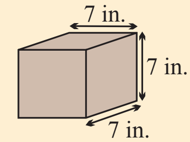
2.


2512 in²

3.


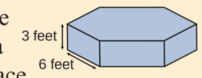
894.4 ft²

Vertical height = 6.8 feet

4.


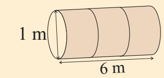
294 in²

5. A statue is to be placed on a marble stand, in the shape of a regular-hexagonal prism. Find the area of the stand's base, given that the stand has a surface area of 201.5 square feet and dimensions as shown.



46.75 ft²

The inside of a large tunnel in a children's play area is to be painted. The tunnel is 6 meters long and 1 meter tall. It is open at each end.



6. What is the area to be painted? 18.84 m²
7. Cans of paint each cover 5 m². How many cans do they need to buy? 4

Now try these:
Lesson 7.1.3 additional questions — p466

Round Up

Working out the surface area of a 3-D shape means adding together the area of every part of the outside. One way to do that is to add together the areas of different parts of the net.
Just make sure you can remember the triangle, rectangle, and circle area formulas.

Solutions
For worked solutions see the Solution Guide

Lesson 7.1.4

Surface Areas & Edge Lengths of Complex Shapes

This Lesson covers finding the total edge length and the surface area of complex shapes. Students need to analyze each complex shape, and carefully apply what they have learned previously about finding surface areas.

Previous Study: Students calculated the surface areas of simple three-dimensional shapes in the previous Lesson. This built on work in earlier grades involving finding areas of two-dimensional shapes.

Future Study: The effect of scale factor on surface area, edge lengths, and volume will be investigated later in this Chapter.

Lesson 7.1.4

Surface Areas & Edge Lengths of Complex Shapes

California Standards:

Measurement and Geometry 2.1
Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and **the surface area** and volume **of basic three-dimensional figures, including** rectangles, parallelograms, trapezoids, squares, triangles, circles, **prisms, and cylinders**.

Measurement and Geometry 2.2
Estimate and compute the area of more complex or irregular two-and **three-dimensional figures by breaking the figures down into more basic geometric objects.**

Measurement and Geometry 2.3
Compute the length of the perimeter, the surface area of the faces, and the volume **of a three-dimensional object built from rectangular solids.** Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.

Mathematical Reasoning 1.3
Determine when and how to break a problem into simpler parts.

What it means for you:

You'll work out the surface area and edge lengths of complex figures.

Key words:

- net
- surface area
- edge
- prism

Prisms and cylinders can be stuck together to make complex shapes. For example, a house might be made up of a rectangular prism with a triangular prism on top for the roof. You can use lots of the skills you've already learned to find the total edge length and surface area of a complex shape — but there are some important things to watch for.

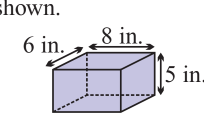
Finding the Total Edge Length

An **edge** on a solid shape is a line where **two faces meet**.

The tricky thing about finding the **total edge length** of a solid, is making sure that you include **each edge length only once**.

Example 1

Find the total edge length of the rectangular prism shown.



Solution

There are four edges around the top face:
6 + 6 + 8 + 8 = **28 in**.

The bottom is identical to the top, so this also has an edge length of **28 in**.

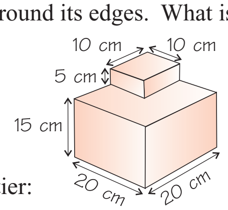
There are four vertical edges joining the top and bottom: 4 × 5 = **20 in**.

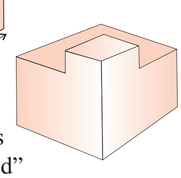
So total edge length = 28 + 28 + 20 = **76 in.**

When complex shapes are formed from simple shapes, some of the edges of the simple shapes might "disappear." These need to be subtracted.

Example 2

A wedding cake has two tiers. The back and front views are shown below. The cake is to have ribbon laid around its edges. What is the total length of ribbon needed?



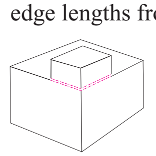


Solution

Total edge length of the top tier:
(10 × 8) + (5 × 4) = **100 cm**

Total edge length of the bottom tier:
(20 × 8) + (15 × 4) = **220 cm**

But, two of the 10 cm edges on the top tier aren't edges on the finished cake. You have to subtract these "shared" edge lengths from both the top and the bottom.



Total edge length = top tier edge lengths
+ bottom tier edge lengths – (2 × shared edge lengths)
= 100 + 220 – (2 × 10 × 2) = 280 cm

So 280 cm of ribbon is needed.

1 Get started

Resources:
- blank cards (or paper and scissors)
- tape
- grid paper

Warm-up questions:
- Lesson 7.1.4 sheet

2 Teach

Universal access

If students struggle with finding the "top face edge lengths" of a prism, multiplying them by two (to find the "bottom face edge lengths") and then adding the verticals, get them to write each group out separately in order to make the link for themselves.
Example 1 for instance, would become:
Top Face = 6 + 8 + 6 + 8
Bottom Face = 6 + 8 + 6 + 8
Vertical Lengths = 5 + 5 + 5 + 5

Students should be encouraged to put a cross through each edge in the diagram once it's included in the sum so that it isn't counted twice.

Providing models of three-dimensional shapes for students to measure the edge lengths of may also be useful.

- **Strategic Learners**
Make a complex three-dimensional shape by sticking two rectangular prisms together. Ask students to cover the edges with tape, then calculate the total amount of tape that they have used. Then separate out the prisms and have the students find the additional amount of tape needed now to cover all the edges. Discuss why extra tape is needed.

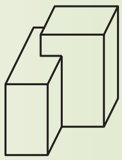
- **English Language Learners**
The Strategic Learners activity described above will also be useful for helping English Language Learners to understand the concept of total edge length. To illustrate how to find the surface area of a complex shape, make flat shapes that match each face of the complex shape. Then compare these to the nets of the separate figures that make up the complex shape.

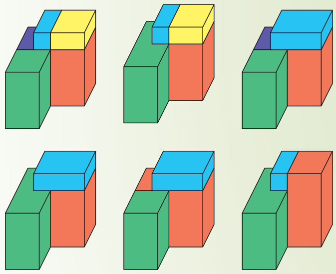
2 Teach (cont)

Guided practice
Level 1: q1–2
Level 2: q1–2
Level 3: q1–2

Universal access
Ask students to cover the surface of complex shapes with grid paper, and then to count the squares to find the surface area. They can compare this to the number of grid squares required to cover the separate shapes that make up the complex figure.

Universal access
Provide students with multiple copies of a complex shape, and ask them to break it up in different ways.
Invite students to come and draw their versions on the board.
Discuss with the class the merits of each one for calculating surface area. Students should see quickly that the fewer blocks formed, the fewer numbers they will have in their calculations.
An example of a shape and its possible breakdowns are shown here.





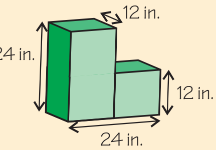
Don't forget:
All the edges on a cube are the same length.

Guided Practice

A display stand is formed from a cube and a rectangular prism.

1. Find the total edge lengths of the cube and rectangular prism before they were joined.
cube = 144 in., rectangular prism = 192 in.

2. Find the total edge length of the display stand.
264 in.



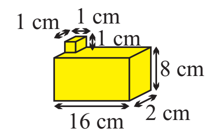
Break Complex Figures Up to Work Out Surface Area

You work out the **surface areas** of complex shapes by breaking them into **simple shapes** and finding the surface area of each part.

The place where two shapes are stuck together **doesn't** form part of the complex shape's surface — so you need to **subtract** the area of it from the areas of **both** simple shapes.

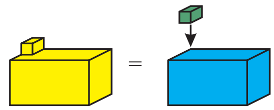
Example 3

What is the surface area of this shape?



Solution

The complex shape is **like two rectangular prisms stuck together.** You can **work out the surface area of each individually, and then add them together.**



But **the bottom of the small prism is covered up, as well as some of the top of the large prism.** So you lose some surface area.

The amount covered up on the big prism must be the same as the amount covered up on the small prism.

So you have to **subtract the area of the bottom face of the small prism twice** — once to take away the face on the small prism, and once to take away the same shape on the big prism.

The surface area of the big prism is **352 cm²**.

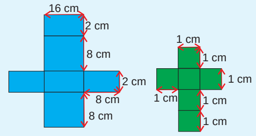
The surface area of the small prism is **6 cm²**.

The surface area of the bottom face of the small prism is **1 cm²**.

Total surface area = surface area of big prism + surface area of small prism – (2 × bottom face of small prism).

So the surface area of the shape is **352 + 6 – (2 × 1) = 356 cm²**.

Don't forget:
To find the surface areas of the prisms, first draw the nets:



Big prism:
Surface area = $2(16 \times 2) + 2(16 \times 8) + 2(8 \times 2) = 352\text{ cm}^2$

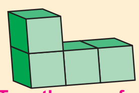
Small prism (actually a cube):
$1 \times 1 \times 6 = 6\text{ cm}^2$

Solutions
For worked solutions see the Solution Guide

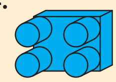
- **Advanced Learners**
  Ask students to calculate the surface area of the model that they constructed in the Advanced Learner activity for Lesson 7.1.2.

Guided Practice

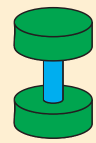
In Exercises 3–5, suggest how the complex figure could be split up into simple figures.

3. 

Two, three, or four rectangular prisms

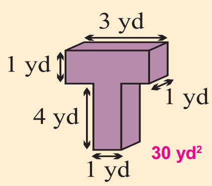
4. 

Four cylinders and a rectangular prism

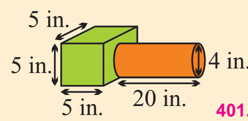
5. 

Three cylinders

In Exercises 6–7, work out the surface area of each shape. Use $\pi = 3.14$.

6. 

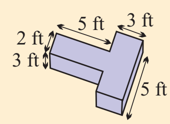
30 yd²

7. 

401.2 in²

Independent Practice

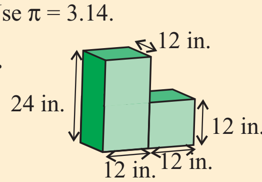
A kitchen work center is made from two rectangular prisms. It is to have a trim around the edge.



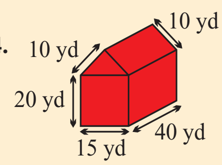
1. Find the total edge length of each of the prisms separately. 40 ft and 44 ft

2. Find the length of trim needed for the work center. 76 ft

Work out the surface areas of the shapes shown in Exercises 3–6. Use $\pi = 3.14$.

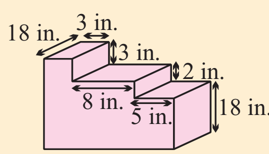
3. 

2016 in²

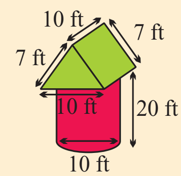
4. 

3699 yd²

Total height of shape = 26.6 yd

5. 

2042 in²

6. 

917 ft²

Total height of shape = 24.9 ft

Don't forget:

The edges that "disappear" when the complex shape is formed are lost from both prisms.

Now try these:

Lesson 7.1.4 additional questions — p466

Round Up

So to find the total edge length of a complex shape, first break the shape up into simple shapes. Then you can find the edge length of each piece separately. But then you have to think about which edges "disappear" when the complex shape is made. The surface area of complex shapes is found the same way. Remember — the edge lengths and surface areas that "disappear" need to be subtracted twice — once from each shape.

2 Teach (cont)

Guided practice
Level 1: q3–5
Level 2: q3–6
Level 3: q3–7

Independent practice
Level 1: q1–5
Level 2: q1–6
Level 3: q1–6

Additional questions
Level 1: p466 q1–4, 8
Level 2: p466 q1–10
Level 3: p466 q1–10

3 Homework

Homework Book — Lesson 7.1.4
Level 1: q1–3, 5–6
Level 2: q1–7
Level 3: q1–8

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 43 — 3-D Figures
- Worksheet 44 — Surface Area

Solutions
For worked solutions see the Solution Guide

Lesson 7.1.5

Lines and Planes in Space

In this Lesson, students look at how lines and planes can be arranged in three-dimensional space.

Previous Study: In grade 4, students identified lines that were parallel or perpendicular.

Future Study: In Geometry, students prove and use theorems involving the properties of parallel lines cut by a third line. They also construct the line parallel to a given line through a point off the line.

1 Get started

Resources:
- toothpicks and clay (or other modeling materials)
- string
- large sheet of paper
- index cards

Warm-up questions:
- Lesson 7.1.5 sheet

2 Teach

Universal access

This topic is best addressed through manipulatives and exploration.

Start by only considering lines in two dimensions. Ask students to consider combinations of two lines on a plane (they'll either be parallel, intersecting, or intersecting perpendicularly).

After this, have students think about the different ways two lines can be arranged in space. They might be surprised that there is only one other possibility (skew lines) to those already found.

This might be difficult for students to accept, so use the suggestion in the Common error section on the next page to show this.

Guided practice

Level 1: q1–6
Level 2: q1–6
Level 3: q1–6

Lesson 7.1.5

Lines and Planes in Space

California Standards:

Measurement and Geometry 3.6

Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and **describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).**

What it means for you:

This Lesson is all about the different ways planes and lines can be arranged in space.

Key words:
- coplanar
- skew lines
- perpendicular
- intersects

Imagine two endless, flat sheets of paper in space. Unless they are parallel to each other, they'll end up meeting sooner or later. Planes are similar to these endless sheets of paper — except they can pass through one another. This Lesson's about all the different ways that lines and planes can meet.

Planes and Lines Can Meet in Different Ways

Planes are **flat 2-D surfaces** in the **3-D world**. They go on forever.

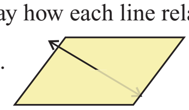
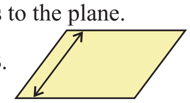
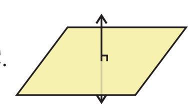
Lines are **1-D shapes in a 2-D plane**. They extend forever in both directions.

There are three ways for a line and a plane to **meet**:

1. The line might **rest on the plane**, so every point of the line is touching the plane.
2. The line might **pass through the plane** — so it **intersects** the plane.
3. The line might **intersect the plane at a right angle to it**. In this case, the line is **perpendicular** to the plane.

Example 1

Say how each line relates to the plane.

A. 
B. 
C. 

Solution

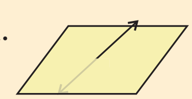
A. **The line intersects the plane.**
Part of it is above the plane and part of it is below the plane.

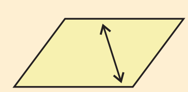
B. **The line rests on the plane.** Every point on the line touches the plane.

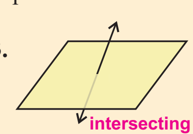
C. **The line intersects the plane at right angles to it.**
So the line is **perpendicular** to the plane.

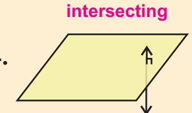
Guided Practice

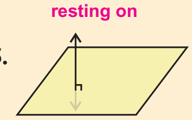
In Exercises 1–6, say how each line relates to the plane.

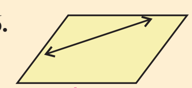
1. 

2. 

3. 

4. 

5. 

6. 

Solutions

For worked solutions see the Solution Guide

- **Strategic Learners**
  Ask students to identify pairs of parallel, intersecting, and perpendicular lines in the classroom. Emphasize that these lines are all coplanar.

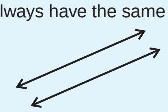
- **English Language Learners**
  Have students use toothpicks (or straws or popsicle sticks) and modeling clay to build three-dimensional figures, e.g. a triangular prism. Use index cards to show the parallel, and intersecting planes different faces lie in. Cut slots in the index cards and fit them together to show intersecting planes.

Lines Can Be Coplanar or Skew

Two lines are **coplanar** if there is a plane that they both lie on — imagine trying to hold a giant piece of paper so that both lines lie on it. Lines that **intersect** or are **parallel** are always coplanar — there's always one plane on which they both lie.

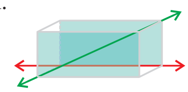
If two lines neither intersect nor are parallel, they're **skew**. This means there's no plane that they both lie on.

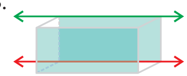
Check it out:
Parallel lines go on forever and never meet — they always have the same slope.

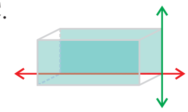


Example 2

Say whether each pair of lines is coplanar or skew. Explain your answers.

A. 

B. 

C. 

Solution

A. **The lines never intersect and aren't parallel**. You can't find a single plane that both rest on. That means they aren't coplanar. **So they are skew lines.**

B. **The lines are parallel. That means they are coplanar.** Both lines rest on the plane at the back of the diagram.

C. **The lines intersect. That means they are coplanar.** They both rest on the plane at the back of the diagram.

Guided Practice

In Exercises 7–10, say whether each statement is true or false. If it is false, explain why.

7. All lines are either coplanar or skew. True

8. Skew lines can be drawn on the coordinate plane. False — all lines on the coordinate plane are coplanar

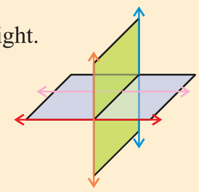
9. Lines that intersect are always coplanar. True

10. Lines that don't intersect are never coplanar. False — parallel lines are coplanar

Exercises 11–12 are about the lines shown on the right.

11. How many lines are skew with the blue line? 2

12. How many lines are coplanar with the red line? 2



Planes Can Meet in Different Ways Too

Two planes are **parallel** if they **never meet**.

If they do meet, then the two planes are **intersecting**. Intersecting planes look like one plane going through another. Planes are **perpendicular** if they make a **right angle** where they meet.

2 Teach (cont)

Common error

Students will not always accept that two parallel lines (or two intersecting lines for that matter) always lie on a single plane. They will often think of a plane as being horizontal.

To demonstrate that lines may lie on a vertical or slanted plane, ask students to hold pieces of string to represent the lines, then use a large sheet of paper to represent a plane. Show how it can be positioned such that both lengths of string lie along it.

Guided practice

Level 1: q7–10
Level 2: q7–12
Level 3: q7–12

Solutions

For worked solutions see the Solution Guide

● **Advanced Learners**
The Universal access activity below is quite challenging and is suitable for advanced learners. It requires visualizing lines in space.

2 Teach (cont)

Universal access

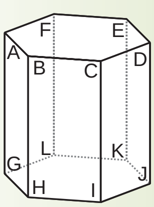
Ask students to try and generate examples of the concepts of the Lesson. Here are some sample questions and possible answers.

What is an example of parallel planes? — Floors in a building.
What is an example of skew lines? — A street lamp and the line down the center of the street.
What is an example of a line intersecting a plane? — A flagpole on a sidewalk.

Obviously there are many possible answers, but the point of the exercise is to have the students start to look at the three-dimensional nature of the world.

Additional example

1. In the prism below, give an example of:
a. segments of skew lines
b. faces that lie on parallel planes
c. faces that lie on intersecting planes.



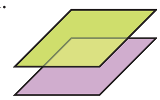
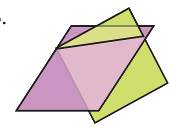
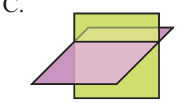
a. There are many answers, such as AB and HI, or FL and DE.
b. There 4 pairs of parallel planes, such as ABCDEF and GHIJKL.
c. There are many different pairs of intersecting planes, such as BCIH and DEKJ.

Guided practice

Level 1: q13–18
Level 2: q13–21
Level 3: q13–21

Example 3

Say whether the planes in each pair are parallel or intersecting.

A.

B.

C.


Solution

A. **The planes are parallel.** They never touch.

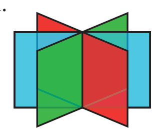
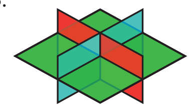
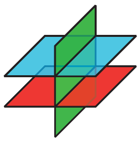
B. **The planes intersect** — one goes through the other.

C. **The planes intersect** — one goes through the other. They make a right angle where they meet, so **they are perpendicular planes**.

When two planes intersect, they always meet along a **line**.
When three planes all intersect at the same place, they can either meet along a **line** or at a **single point**.

Example 4

Say if each of these sets of planes intersect, and if so whether they meet along a line or at a point.

A.

B.

C.


Solution

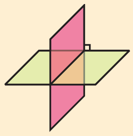
A. **The planes all intersect. They meet along a line.**
The line is shown vertically down the middle of the picture.

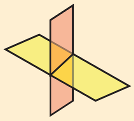
B. **The planes all intersect. They meet at a point.**
The point is at the centre of the picture.

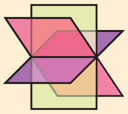
C. **The planes do not all intersect.**
The blue plane never intersects with the red plane (they're parallel).

Guided Practice

In Exercises 13–18, say whether the planes meet, and if so describe how.

13.

planes intersect — perpendicularly

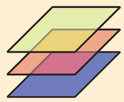
14.

planes intersect — not perpendicularly

15.

Three intersecting planes, meet along a line

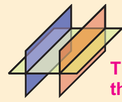
Solutions

For worked solutions see the Solution Guide

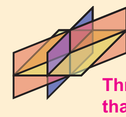
2 Teach (cont)

16. 

Parallel planes — no intersection

17. 

Three planes that do not all meet

18. 

Three planes that meet at a point

Fill in the missing words in Exercises 19–21.

19. Planes that never touch are parallel.

20. Two intersecting planes always meet along a line.

21. Three intersecting planes meet either along a line or at a point.

Independent Practice

In Exercises 1–5, say whether each statement is true or false. If any are false, explain why.

1. When a line goes through a plane, it is said to be perpendicular to the plane. False — perpendicular means the line and plane meet at a right angle

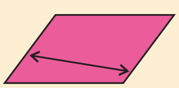
2. Skew lines never lie on the same plane. True

3. Two lines are coplanar if they both intersect the same plane. See below

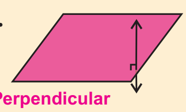
4. If two planes aren't parallel, then they meet along a line. True

5. Three planes always all meet each other at a line or a point unless they're all parallel to each other. False — sometimes they intersect each other but don't all meet at the same place

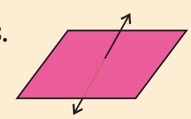
In Exercises 6–8, say how each line relates to the plane.

6. 

Resting on

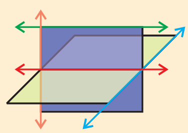
7. 

Perpendicular

8. 

Intersecting

Exercises 9–11 are about the lines shown on the right.

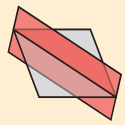


9. How many lines are coplanar with the red line? 3

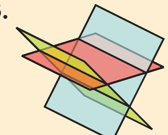
10. How many lines are coplanar with the green line? 2

11. How many lines are skew to the blue line? 2

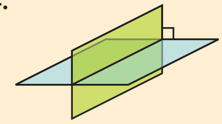
In Exercises 12–14, identify whether the planes meet and if so say how.

12. 

Yes — intersect along line

13. 

The planes do not all intersect

14. 

Yes — intersect perpendicularly

Now try these:
Lesson 7.1.5 additional questions — p467

Round Up

It's hard to imagine the 2-D drawings in 3-D with lines and planes that go on forever. You've got to try to visualize the planes and lines intersecting in space — then you'll be most of the way there.

Independent practice
Level 1: q1–8
Level 2: q1–11
Level 3: q1–14

Additional questions
Level 1: p467 q1–3
Level 2: p467 q1–9
Level 3: p467 q1–9

3 Homework

Homework Book — Lesson 7.1.5
Level 1: q1–7
Level 2: q1–9
Level 3: q1–10

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 42 — Parallel and Perpendicular Lines

Solutions
For worked solutions see the Solution Guide

3. False — two lines are only coplanar if they rest on the same plane.

Section 7.2

Exploration — Build the Best Package

Purpose of the Exploration

The goal of the Exploration is for students to see how the dimensions of a prism affect both the surface area and volume. Students will see that it is possible to construct a rectangular prism with a given volume by using different amounts of material. The activity culminates in a challenge where students are asked to design and build the most efficient package.

Resources

- grid paper
- rulers
- card, scissors and tape

Strategic & EL Learners

Strategic learners will benefit from having the possible integer dimensions for a package with a volume of 24 cubic centimeters given to them. This will eliminate having to determine all of the possible combinations and they can focus solely on the task.

EL learners may be unfamiliar with the terms “minimize” and “maximize.” Inform students that minimize means to make as small as possible, and maximize means to make as big as possible.

Universal access

Start the Exploration by reminding students on how to calculate the surface area and volume of a shape.

Stop students before they try the more challenging final Exercise. Ask them to look at the three packages that had a volume of 64 cubic centimeters and decide what type of design used the least amount of material.

Common error

Students may have difficulty finding the surface area of the figures. Have students mark each face on the net as they find its area.

Math background

Students should be able to calculate the volume and surface area of basic three-dimensional figures. They should be comfortable enough with these concepts to apply them in a problem solving capacity.

Section 7.2 introduction — an exploration into:

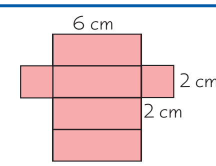
Build the Best Package

When companies design packaging, they often aim to maximize volume and minimize the amount of material used, so that they spend as little money on it as possible. In this Exploration you’ll be asked to design the best package when given specific volume requirements.

To find the **amount of material** needed to make a box, you calculate its **surface area**.

Example

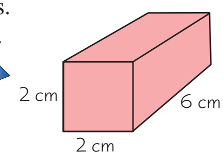
The net on the right can be folded to make a rectangular prism. Calculate the **surface area** and **volume** of the prism.



Solution

The **surface area** is the total area of all the faces.
There are four 6 cm by 2 cm faces, and two 2 cm by 2 cm faces.
So, **surface area** $= 4 \times (6 \times 2) + 2 \times (2 \times 2) = 48 + 8 =$ **56 cm²**.

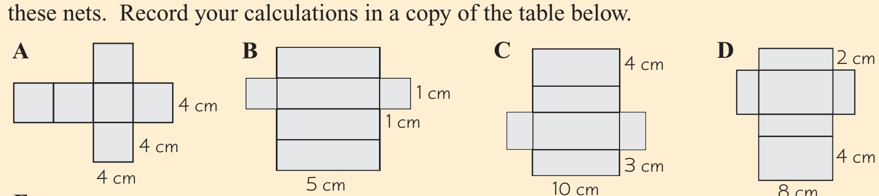
The net folds up to make the rectangular prism on the right.

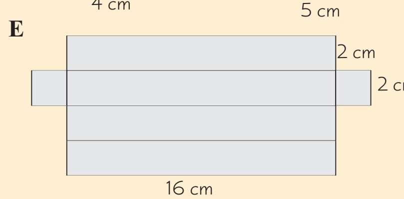


Volume = length × width × height
$= 6 \times 2 \times 2 =$ **24 cm³**

Exercises

1. Calculate the volumes and surface areas of the rectangular prisms that can be made from these nets. Record your calculations in a copy of the table below.





Rectangular Prism	Surface Area	Volume
A	96 cm²	64 cm³
B	22 cm²	5 cm³
C	164 cm²	120 cm³
D	112 cm²	64 cm³
E	136 cm²	64 cm³

2. Is it possible to build packages with the same volume using different amounts of material? Explain your answer.

Yes. A, D and E have the same volume, but different surface areas. Making the prism more “cube-like” gives a bigger volume for the same surface area.

3. Design and construct a box that has a volume of 24 cm³ and uses the least material possible. The length, width, and height of the package must be whole numbers.

A package with dimensions 2×3×4 uses the least material.

Round Up

Volumes and surface areas of rectangular prisms depend on the dimensions — they don’t always increase together. You can change volume without changing surface area, and vice-versa.

Lesson 7.2.1

Volumes

This Lesson develops students' understanding of the concept of volume, and covers the principles behind its calculation. Students learn the formulas for calculating the volumes of rectangular prisms, triangular prisms, and cylinders, and practice using them.

Previous Study: In grade 5, students were first introduced to the concept of volume. In grade 6, they learned how to calculate the volumes of rectangular prisms, triangular prisms, and cylinders.

Future Study: In Geometry, students will learn how to find the volumes of prisms, pyramids, cylinders, cones, and spheres. They will be asked to memorize the volume formulas for prisms, pyramids, and cylinders.

Lesson 7.2.1

California Standards:

Measurement and Geometry 2.1
Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and **volume of basic three-dimensional figures, including** rectangles, parallelograms, trapezoids, squares, triangles, circles, **prisms, and cylinders**.

Mathematical Reasoning 2.2
Apply strategies and results from simpler problems to more complex problems.

Mathematical Reasoning 3.2
Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

What it means for you:

You'll be reminded what volume is, and then see how you can work out the volume of prisms and cylinders.

Key words:

- volume
- cubic units
- prism
- cylinder

Don't forget:

Cubic units can be written as unit^3 — so cubic yards can be written as yd^3.
You multiply three length measurements, such as length, width, and height, to get a volume, so that's why volume is measured in unit^3.

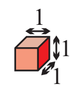
Section 7.2
Volumes

The volume of a 3-D object, like a box, a swimming pool, or a can, is a measure of the amount of space that's contained inside it. Volume is measured in units like cubic feet (ft^3) or cubic centimeters (cm^3). This Lesson, you'll learn how to find the volume of prisms and cylinders.

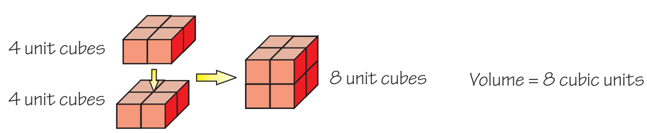
Volume Measures Space Inside a Figure

The amount of **space inside** a 3-D figure is called the **volume**.

Volume is measured in **cubic units**. One cubic unit is the volume of a **unit cube** — a cube with a side length of 1 unit.

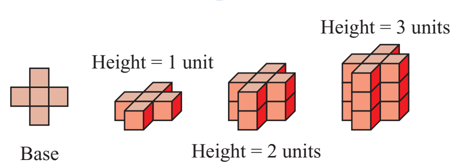


The **number of unit cubes** that could fit inside a solid shape and fill it completely is the volume in cubic units.



The Volume of a Prism is a Multiple of its Base Area

You can work out the volume of a **prism** from the **area of its base**.



The base is made of **5 unit squares**. So it has an **area** of 5 square units.

When the prism's height is 1 unit, it has a **volume** of 5 cubic units because it would take 5 unit cubes to make it.

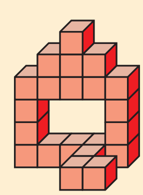
When the prism's height is 2 units, it has a **volume** of 10 cubic units because it would take 10 unit cubes to make it.

When the prism's height is 3 units, it has a volume of 15 cubic units.

Every time you **increase the height** by 1 unit you **add** an extra 5 unit cubes.

Guided Practice

1. The figure on the right is constructed from unit cubes. What is its volume? **21 unit^3**



A prism is one yard high. It has volume 4 yd^3.
2. What is the area of the prism's base? **4 yd^2**
3. A prism with an identical base has a volume of 16 yd^3. How tall is this prism? **4 yards**

1 Get started

Resources:
- 1 cm^3 blocks
- hollow rectangular prisms
- measuring cylinders
- water
- empty boxes
- rulers

Warm-up questions:
- Lesson 7.2.1 sheet

2 Teach

Math background
To find the volume of a prism, students will need to be able to find the area of its base.

Before beginning the topic of volume, briefly review the area formulas for the shapes listed below.

Square:	Area = (side length)^2
Rectangle:	Area = length × width
Triangle:	Area = $\frac{1}{2}$ × base × height
Circle:	Area = πr^2

Guided practice
Level 1: q1–3
Level 2: q1–3
Level 3: q1–3

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Put students into pairs. Give each pair 50 1 cm^3 cubes. Have them build the following: a shape with a volume of 15 cm^3; a shape with a volume of 48 cm^3; a prism with a base of 8 cm^2 and a volume of 40 cm^2. Their shapes do not have to be regular. As they build, compare the different structures, and discuss what properties all of their shapes have in common (the number of cubes, and so their volumes).

- **English Language Learners**
  Put students into pairs. Give each pair an empty box that is a rectangular prism, such as a cereal box, and a ruler. Ask each pair to find the volume of their box, writing down how they did it as they go. Ask them to think about whether it matters which edge is defined as length, width, or height, and which face they call the base. The length is usually taken to be the longest side, but it doesn't have to be.

2 Teach (cont)

Additional example

I have a triangular prism with a base area of 15 inches², and a height of 20 inches. I cut it in half along a line parallel to its base to make two smaller prisms. What is the volume of each smaller prism?

Prism Volume = Base area × Height
= 15 × (20 ÷ 2)
= 15 × 10
= 150 cm^3

Concept question

"I have a prism with a base area of 10 cm^2, and a height of $4x$ cm. What is the volume of the prism?"
$40x$ cm^3

Common error

It can be difficult for students to understand that when the term "height" is used in prism area and volume calculations it does not necessarily refer to vertical height, but to the dimension of the prism running perpendicular to the base.

Make a triangular or rectangular prism out of card. Stand it on its base, and ask students which dimension is the height, and which surface is the base. Label these according to their instructions.

Now lay the prism on its side. Everyone should be able to see that neither the volume nor surface area of the prism have changed, even though the orientation of the height and base have altered.

Area Formulas Help Work Out the Volume of a Prism

When you count the number of unit cubes that make a shape, you find the **number of unit cubes that make up the base layer**, and **multiply it by the height in units**.

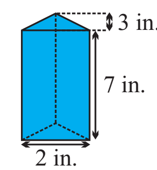
You can't always count the number of unit cubes that are inside a shape, because **not all shapes can fit an exact number of unit cubes inside them**. Instead you can work out the volume of any prism by **multiplying** the **area of the base** by the **height**.

Volume of prism = Base area × Height

Don't forget:
The height you use to work out the area of the base is the height of the base triangle. The height you're using when you work out the volume of the prism is the height of the prism itself.

Example 1

What is the volume of this prism?



Solution

The base of this prism is a triangle. So use the area of a triangle formula to work out its area.

Area of base = $\frac{1}{2}bh = \frac{1}{2} \times 2 \times 3 = 3 \text{ in}^2$.

Then just multiply that area by the height of the prism.

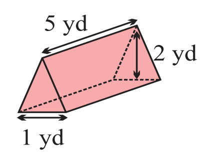
Volume of prism = base area × height = 3 × 7 = 21 in^3.

It doesn't matter if the prism looks like it is **lying down** — the same method of finding volume can still be used.

Check it out:
You've done dimensional analysis before — it's where you check that the units of your answer match the units you should get. When you're figuring out volumes, the answer should always be in units cubed — like cm^3, m^3, ft^3, or yd^3.

Example 2

What is the volume of this prism?



Solution

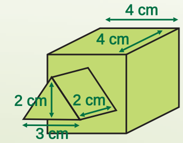
Treat the triangle as the base of the prism, and the length of 5 yards as the height.

The base is always the shape that is the same through the entire prism.

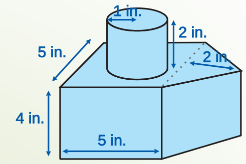
Area of base = $\frac{1}{2}bh = \frac{1}{2} \times 1 \times 2 = 1 \text{ yd}^2$.

So, **volume of prism = base area × height = 1 × 5 = 5 yd^3**.

- **Advanced Learners**
  In Lesson 7.1.4, students saw how to find the areas of complex shapes by splitting them down into smaller regular shapes and adding their areas. The same method can be used to find the volume of complex solids: they can be broken up into smaller 3-D shapes, and their total volumes found by adding the volumes of the smaller 3-D shapes. Give students a copy of the diagrams on the right. Explain the concept to them, and ask them to find the volumes of both solids. If they made a complex 3-D model in the last Section, they could find the volume of it.



Volume = 70 cm³

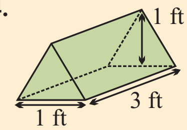


Volume = 126.3 in³

Guided Practice

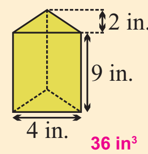
Work out the volumes of the figures in Exercises 4–6.

4.



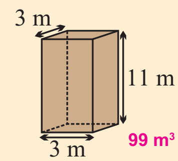
1.5 ft³

5.



36 in³

6.



99 m³

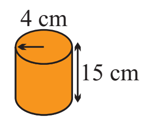
Find the Volume of a Cylinder in the Same Way

Circular cylinders are similar to prisms — the only difference is that the base is a **circle** instead of a **polygon**. So you can work out the volumes of cylinders in the same way as the volumes of prisms — by **multiplying the base area by the height**.

You use the **area of a circle formula** to get the base area of a cylinder.

Example 3

What is the volume of this cylinder? Use $\pi = 3.14$.



Solution

Area of base = $\pi r^2 = \pi \times 4^2 = \pi \times 16 = 50.24$ cm².

Height = 15 cm, so **volume of cylinder = $50.24 \times 15 = 753.6$ cm³**.

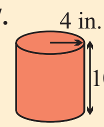
Don't forget:

Make sure you use the radius of the circle in the area formula and not the diameter. If you're given the diameter, you need to halve it to find the radius first of all.

Guided Practice

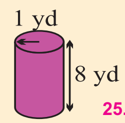
Work out the volumes of the figures in Exercises 7–9. Use $\pi = 3.14$.

7.



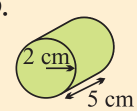
502.4 in³

8.



25.12 yd³

9.

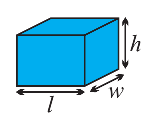


62.8 cm³

Rectangular Prisms and Cubes are Special Cases

The area of the base of a rectangular prism is **length *(l)*** × **width *(w)***.
If you multiply that by height to get the volume then you get
Volume = length *(l)* × width *(w)* × height *(h)*

$$V \text{ (rectangular prism)} = lwh$$

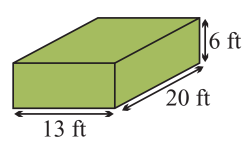


Check it out:

It doesn't matter which dimensions you use as width, height, or length — you can multiply them in any order.

Example 4

What is the volume of this rectangular prism?



Solution

Volume = $lwh = 13 \times 20 \times 6 = 1560$ ft³.

2 Teach (cont)

Guided practice

Level 1: q4–5
Level 2: q4–6
Level 3: q4–6

Universal access

Put students into groups. Give each group a hollow rectangular prism. Empty metal or plastic containers are ideal — they must be waterproof. Also, give each group a ruler.

Ask students to use the ruler to measure the height, width, and length of their prism in cm. Then tell them to use the measurements they have taken to calculate its volume.

Now give each group a measuring cylinder and some water. Have students measure the volume of the prism by filling it with water, and pouring the water out into the measuring cylinder. They may need to know that 1 mL is the same as 1 cm³.

Students should now see that their own calculation of the volume of the prism matches the volume that they measured using the liquid.

This demonstration should help to connect the math formulas they have been learning with everyday measures of volume.

Guided practice

Level 1: q7–9
Level 2: q7–9
Level 3: q7–9

Additional example

What is the volume of a brick that measures 4 cm by 6 cm by 12 cm?

V = 12 × 6 × 4 = 288 cm³.

Solutions

For worked solutions see the Solution Guide

2 Teach (cont)

Concept question

"What is the volume of a cube that has a side length of 1 cm?"

1 cm³

Guided practice

Level 1: q10–12
Level 2: q10–12
Level 3: q10–12

Independent practice

Level 1: q1–5
Level 2: q1–6
Level 3: q1–8

Additional questions

Level 1: p467 q1–6
Level 2: p467 q1–9
Level 3: p467 q1–10

3 Homework

Homework Book — Lesson 7.2.1

Level 1: q1–7
Level 2: q1–8
Level 3: q1–9

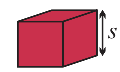
4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:

• Worksheet 45 — Volume

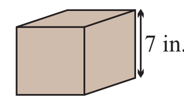
All sides of a cube are **the same length**. For a cube with side length s, the base area is $s \times s = s^2$, and the height is also s, so the volume is $s^2 \times s = s^3$.



$$V\ (cube) = s^3$$ where s is the side length.

Example 5

What is the volume of this cube?

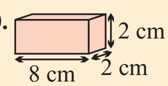


Solution

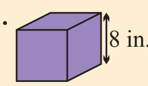
Volume = $s^3 = 7^3 = 343$ in³.

Guided Practice

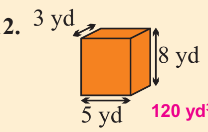
Work out the volumes of the figures in Exercises 10–12.
Figures with only one side length shown are cubes.

10. 

32 cm³

11. 

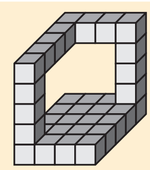
512 in³

12. 

120 yd³

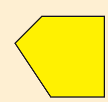
Independent Practice

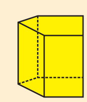
1. The figure on the right is constructed from cubes with a volume of 1 in³. What is its volume? **34 in³**



2. How many unit cubes can you fit inside a figure with dimensions 3 units × 3 units × 5 units? **45**

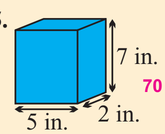
3. What is the volume of the prism shown on the right? **35,000 ft³**


Base Area = 500 ft²

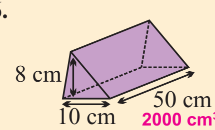

Height = 70 ft

4. A cylinder of volume 32 in³ is cut in half. What is the volume of each half? **16 in³**

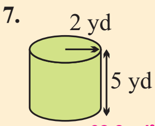
Work out the volumes of the figures shown in Exercises 5–7.
Use π = 3.14.

5. 

70 in³

6. 

2000 cm³

7. 

62.8 yd³

8. What is the volume of a cube with side length 3 yd? **27 yd³**

Now try these:
Lesson 7.2.1 additional questions — p467

Round Up

Volume is the amount of space inside a 3-D figure, and it's measured in cubed units. For cylinders and prisms, you can multiply the base area by the height of the shape to find the volume.

Solutions

For worked solutions see the Solution Guide

Lesson 7.2.2

Graphing Volumes

In this Lesson, students investigate the relationships between the dimensions and volumes of common solids by plotting them against each other on the coordinate plane.

Previous Study: From grade 4 onwards, students have learned how to plot points on the coordinate plane, and how to read and interpret graphs. In grade 6, they learned how to find the volumes of cubes and prisms.

Future Study: In Geometry, students will derive and solve problems involving the volume of common solids. In Algebra I, students will learn how to graph non-linear functions.

Lesson 7.2.2

Graphing Volumes

California Standards:

Algebra and Functions 3.2

Plot the values from the volumes of three-dimensional shapes for various values of the edge lengths (e.g., cubes with varying edge lengths or a triangle prism with a fixed height and an equilateral triangle base of varying lengths).

Mathematical Reasoning 2.3

Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

What it means for you:

You'll plot graphs to show how volume changes as the dimensions of different shapes change. You'll also use graphs to estimate the dimensions of shapes with given volumes.

Key words:

- approximately
- volume

Don't forget:

≈ means "is approximately equal to." You can use it when you know that the answer is near to a number but you don't know exactly what the answer is.

In Section 5.4, you saw the graphs of $y = nx^2$ and $y = nx^3$. When you graph volume against the side length of a cube, or against the radius of a cylinder, you get these types of graphs too.

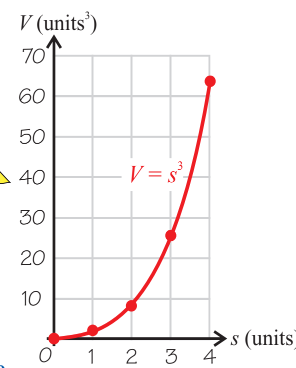
The Volume of a Cube Makes an x^3 Graph

If you **increase the side length** of a cube, the cube's **volume increases**. You can **plot a graph** to see exactly how volume changes with side length.

The volume of a cube with sides of length s is given by $V = s^3$. The first step in plotting a graph is **making a table of values**.

s (units)	V (units^3)
1	1
2	8
3	27
4	64

Plot these points on a graph — put the s-values on the x-axis.
Join the points with a smooth curve.



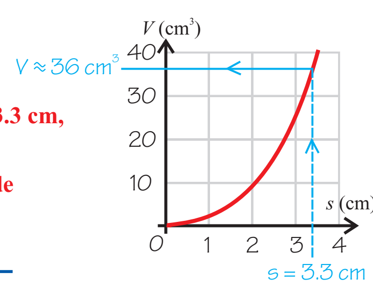
The graph shows that V goes **steeply upward** as s gets bigger — it's the $y = x^3$ graph that you've seen before.

The graph can be used to **find the volume** of other size cubes without doing any multiplication.

Example 1

Using the graph, approximately what is the volume of a cube with side length 3.3 cm?

Solution



Reading off the graph, **when $s = 3.3$ cm, $V \approx 36$ cm^3**.

So **the volume of a cube with side length 3.3 cm is approximately 36 cm^3**.

1 Get started

Resources:
- grid paper
- transparency with axes
- overhead projector
- ruler
- rectangular block

Warm-up questions:
- Lesson 7.2.2 sheet

2 Teach

Universal access

Before starting to teach the content of the Lesson, have the students come up with the cube volume graph themselves.

Put students into groups. Ask group 1 to work out the volume of a 1 cm cube, group 2 to work out the volume of a 2 cm cube, etc.

Put a transparency on the overhead projector with a copy of the axes from the student page, but without the points plotted or the line drawn.

Ask a representative of each group to come and plot the point representing their cube on the axes. Then ask for a volunteer to come and join the points. Discuss briefly with the class what type of graph they have made.

Math background

Students have seen the shape of a cubic graph before in Section 5.4.

The graph that shows how the volume of a cube changes with changing side length is a simple cubic graph, except in one respect — since the volume of a cube cannot possibly be negative, the graph does not extend into Quadrant III.

- **Strategic Learners**
  Give everyone a rectangular block, some grid paper, and a ruler. Have them find the volume of the block, and draw axes of volume against height, plotting a point to represent the block. Now ask them to think how the volume would alter if you stuck 2 blocks together, or if you cut a block in half. In this way students can draw a graph showing what happens to the volume of a prism as you change its length.
- **English Language Learners**
  Ask students to find the volumes of a series of rectangular prisms with a base area of 4 cm^2 and heights ranging from 1 cm to 10 cm. Have them display their data in a table, and use it to plot a graph showing how the volume of the prism changes with its height. Ask them to explain to a partner what their graph shows.

2 Teach (cont)

Math background

To complete this Lesson, students will need to be able to plot a graph from a table of values, and know how to read information from a graph.

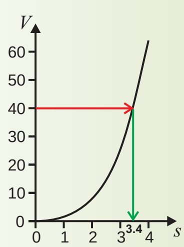
Both of these skills are covered in Section 4.1.

Guided practice

Level 1: q1–2
Level 2: q1–2
Level 3: q1–2

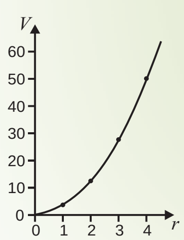
Additional examples

1. A cube has a volume of 40 in^3. Give an estimate for the length of an edge using the graph of $V = s^3$.



On the graph, go along the vertical axis (labeled V) until you get to $V = 40$. Then go across to the graph (red arrow). The point on the graph where $V = 40$ has $s \approx 3.4$ (green arrow). So a cube with a volume of 40 in^3 has an edge of approximately 3.4 in.

2. Graph the volume against the radius of cylinders whose height is 1 inch.
Use the formula $V = \pi r^2 h$. Since $h = 1$, the volume formula now becomes $V = \pi r^2(1) = \pi r^2$. Below is a graph of this.



You can also use the graph to find the side length of a cube if you know the volume.

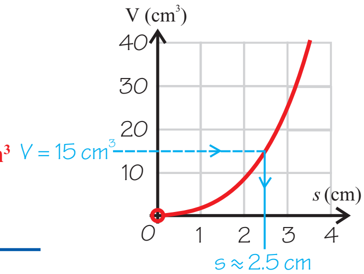
Example 2

What is the side length of a cube with volume 15 cm^3?

Solution

Reading from the graph,
when $V = 15$ cm^3, $s \approx 2.5$ cm.

So, **a cube with volume 15 cm^3 has a side length of approximately 2.5 cm.**



Guided Practice

1. If volume was plotted against side length, with side length along the x-axis, explain how you would find the volume of a cube of side 4 m. See below

2. A cube has a volume of 20 ft^3. Use the graph of volume against side length to find the approximate length of each edge of the cube. 2.7 ft

You Can Graph the Volume of Prisms and Cylinders

You can graph the volumes of **prisms** and **cylinders** as **one of their dimensions**, such as height, **changes**. The rest of the dimensions have to be **kept the same**.

Example 3

Graph the volume against the radius of cylinders of height $\frac{1}{2}$ cm.

Solution

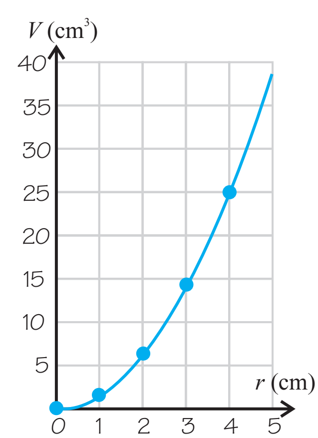
The volume of a cylinder is $V = \pi r^2 h$.

So **cylinders which are $\frac{1}{2}$ cm high have the volume $\frac{1}{2}\pi r^2$.**

Make a table of values to plot:

radius (cm)	volume (cm^3)
1	1.57
2	6.28
3	14.13
4	25.12

Since the radius is squared in the formula for volume, you get a $y = nx^2$ graph — which is a parabola.



Don't forget:
See Lesson 5.4.1 to remind yourself about graphing $y = nx^2$ equations.

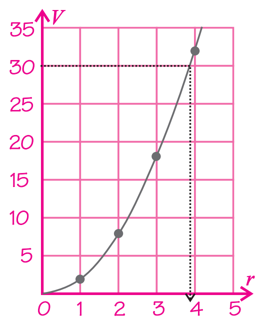
Check it out:
Volume graphs are only drawn for positive values — you can't have negative lengths or volumes.

Solutions
For worked solutions see the Solution Guide

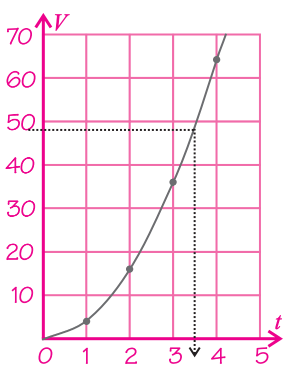
1. You would start at 4 on the x-axis, go straight up until you met the curve of the graph, then go straight across to the y-axis and read off the corresponding volume.

- **Advanced Learners**
  Ask students to independently investigate how the volume of a triangular prism changes as its height changes. Or have them investigate how the volume of a prism changes if you keep the height constant but change the base area. Have them write a paragraph summarizing their findings and any patterns they observed, and share it with a partner (ideally one who did the opposite investigation).

Guided Practice 3 and 4



Guided Practice 5 and 6



You can **use the graph** to find the **volume of a cylinder** of a given height if you know the radius, and you can **find the radius** if you know the volume.

Example 4

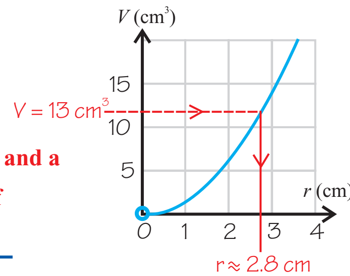
What radius of cylinder with height $\frac{1}{2}$ cm has a volume of 13 cm^3?

Solution

Use the graph from Example 3.

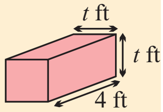
Reading from the graph, **when $V = 13$ cm^3, $r \approx 2.8$ cm.**

So, **a cylinder with height $\frac{1}{2}$ cm and a volume of 13 cm^3 has a radius of approximately 2.8 cm.**



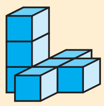
Guided Practice

3. Graph volume against radius for cylinders with height $\frac{2}{\pi}$ units. **see margin**
4. Use your graph from Exercise 3 to estimate the radius of a cylinder with height $\frac{2}{\pi}$ centimeters that has a volume of 30 cubic centimeters. **About 3.9 cm**
5. Graph the volume against t for the figure shown. **see margin**
6. Use your graph from Exercise 5 to estimate the value of t that makes a volume of 48 cubic feet. **About 3.5 feet**



Independent Practice

1. Graph volume against height for prisms with base area 6 units^2. **see margin**
2. Graph volume against radius for cylinders with height 3 units. **see margin**
3. Use your graph from Exercise 1 to find the approximate height of a prism that has a volume of 10.5 cm^3 and a base area of 6 cm^2. **About 1.75 cm**
4. The building on the right is constructed from 7 cubes. Each cube has a side length of s inches. Graph the volume of the building against s. Use the graph to find the side length of cubes needed for a building of volume 36 cubic inches. **About 1.7 inches**



Now try these:
Lesson 7.2.2 additional questions — p468

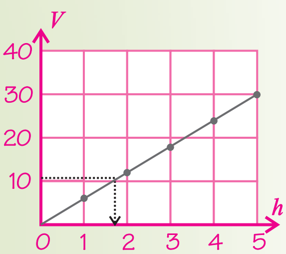
Round Up

*If you **increase the length** of one of the dimensions of a 3-D figure, you **increase its volume**. You can **use a graph** to show the relationship between the **length of a dimension and the volume**.*
*Next Lesson you'll learn about **similar solids** — these are solids of different sizes, which have each of their dimensions in proportion with the **corresponding** dimensions on the other solids.*

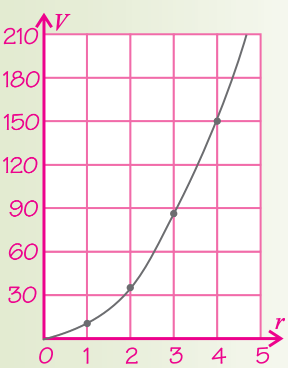
Solutions
For worked solutions see the Solution Guide

2 Teach (cont)

Independent Practice 1 and 3



Independent Practice 2



Guided practice
Level 1: q3–4
Level 2: q3–6
Level 3: q3–6

Independent practice
Level 1: q1–3
Level 2: q1–4
Level 3: q1–4

Additional questions
Level 1: p468 q1–11
Level 2: p468 q1–11
Level 3: p468 q1–11

3 Homework

Homework Book — Lesson 7.2.2
Level 1: q1–4
Level 2: q1–4
Level 3: q1–5

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 28 — Graphing Linear Equations
- Worksheet 45 — Volume

Exploration — Growing Cubes

Purpose of the Exploration

The goal of this Exploration is to lead students to discover the effect that a change in dimensions has on the volume and surface area of a cube. The Exploration challenges students to make predictions based on given information.

Resources

• centimeter cubes

Strategic & EL Learners

Strategic learners may struggle to identify the patterns in their results. Review square and cube numbers and tell them the pattern is connected to these.

EL Learners may not know the meaning of the words "double" and "triple," which are likely to be used during class discussion. Explain that these word mean multiplying by two and by three.

Universal access

Remind students that the sides of a cube are all the same. The surface area of a cube is easily calculated by finding the area of one side and multiplying by six.

Math background

Students should be comfortable finding the volume and surface area of a cube. They should also be able to make predictions from patterns and have knowledge of numbers raised to the powers of 2 and 3.

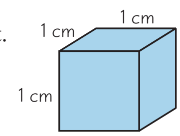
Section 7.3 introduction — an exploration into:
Growing Cubes

Cubes have a length, width, and height that are equal. When you change these dimensions, the volume and surface area also change. The goal of this Exploration is to find out if changing the dimensions by a given amount produces a predictable change in the volume and surface area.

To investigate this, you have to find the **volumes** and **surface areas** of some cubes of **different sizes**.

Example

A cube with a length, width, and height of **1 centimeter** is shown on the right. Calculate its **surface area** and **volume**.



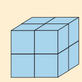
Solution

The **surface area** is the total area of all the faces.
There are six 1 cm by 1 cm faces. So, **surface area** = $6 \times (1 \times 1)$ = **6 cm²**.

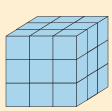
Volume = length × width × height
= $1 \times 1 \times 1$ = **1 cm³**

Exercises

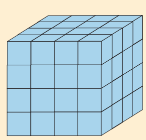
1. Build the following with centimeter cubes. Calculate the surface area and volume of each and record them in a copy of the table shown.



$2 \times 2 \times 2$ cm cube



$3 \times 3 \times 3$ cm cube



$4 \times 4 \times 4$ cm cube

Cube dimensions (cm)	Scale Factor	Volume (cm³)	Surface area (cm²)
original — 1 x 1 x 1	1	1	6
2 x 2 x 2	2	8	24
3 x 3 x 3	3	27	54
4 x 4 x 4	4	64	96

2. What is the volume of the original $1 \times 1 \times 1$ cm cube multiplied by if the cube is enlarged by:
a. a scale factor of 2? 8 **b.** a scale factor of 3? 27 **c.** a scale factor of 4? 64

3. How are the scale factor and the number the original volume is multiplied by connected?
The number the volume is multiplied by is the scale factor cubed (to the power of 3).

4. What is the surface area of the original cube multiplied by if the cube is enlarged by:
a. a scale factor of 2? 4 **b.** a scale factor of 3? 9 **c.** a scale factor of 4? 16

5. What is the connection between the scale factor and the number the original surface area is multiplied by?
The number the surface area is multiplied by is the scale factor squared (to the power of 2).

6. Predict the new volume and surface area if the original cube is enlarged by a scale factor of 5.
Volume = 1×5^3 = 125 cm³, surface area = 6×5^2 = 150 cm²

7. Check your predictions by calculating the volume and surface area of a $5 \times 5 \times 5$ cm cube.
Volume = 125 cm³, surface area = 150 cm²

Round Up

When the dimensions of a cube are increased, the surface area and volume always get bigger. The pattern's more complicated than a linear change though. It involves square and cube numbers.

Lesson 7.3.1

Similar Solids

This Lesson reviews the use of scale factors in 2-D shapes and then expands it to 3-D shapes. Students solve problems involving the lengths in similar 3-D shapes.

Previous Study: In grade 6, students solved problems involving similar polygons. In Chapter 3, students were introduced to scale factors, and explored their effect on perimeter and area of 2-D shapes.

Future Study: In the next Lesson, students investigate the effects of scaling lengths on volume and surface area.

Lesson 7.3.1

California Standards:

Measurement and Geometry 1.2
Construct and read drawings and models made to scale.

What it means for you:
You'll learn about using scale factors to change the size of solid figures.

Key words:
- similar
- congruent
- corresponding
- solid
- scale factor
- image

Don't forget:
For more about scale factors, and similar and congruent figures, look back at Lessons 3.4.3 and 3.4.6.

Section 7.3
Similar Solids

Applying a scale factor makes an image of a shape that is a different size from the original — you saw this with 2-D shapes in Chapter 3. You can also use scale factors with 3-D shapes to produce similar solids of different sizes.

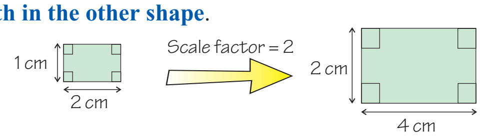
Scale Factors Produce Similar Figures

You've looked at the effect of scale factors on **2-D shapes**. You're going to review this before seeing how scale factors affect **3-D shapes**.

> Two shapes are **similar** if one can be multiplied by a **scale factor** to make a shape that is **congruent** to the other one.

Two shapes are congruent if they're exactly the same — if all the corresponding sides and angles are equal.

Similar shapes have **corresponding angles that are equal**. Also, **any length in one shape is equal to the scale factor times the corresponding length in the other shape**.

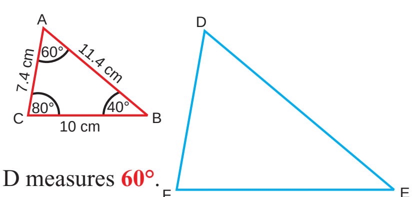


The angles are all 90° in both shapes. The lengths are all twice as big in the larger shape.

Example 1

Triangle ABC is multiplied by a scale factor of 3 to make triangle DEF.

Find the measures of all the sides and angles of triangle DEF. The triangles are not drawn to scale.



Solution

D corresponds to A, so angle D measures **60°**.
E corresponds to B, so angle E measures **40°**.
F corresponds to C, so angle F measures **80°**.

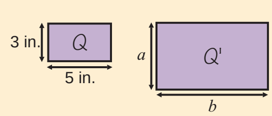
To find the side lengths of DEF, multiply the side lengths of ABC by the **scale factor**.

length of DE = 3 × (length of AB) = 3 × 11.4 cm = **34.2 cm**
length of EF = 3 × (length of BC) = 3 × 10 cm = **30 cm**
length of DF = 3 × (length of AC) = 3 × 7.4 cm = **22.2 cm**

Guided Practice

Rectangle Q is multiplied by a scale factor of 4 to give the image Q'.

1. How long is the side marked a? **12 in.**
2. How long is the side marked b? **20 in.**



1 Get started

Resources:
- a selection of suitable 3-D objects for the Strategic Learners and Advanced Learners activities.
- card, scissors, and rulers

Warm-up questions:
- Lesson 7.3.1 sheet

2 Teach

Universal access

An overhead projector can be used to demonstrate enlargement. Take measurements on the overlay, and compare them to measurements on the projected image.

Lead students toward discovering that the lengths remain in proportion to the original image, and that angles are unchanged.

Ask students what will happen to the scale factor as the projector is moved closer or further away from the wall. "The further away the projector is, the higher the scale factor."

Note that it might be worth testing this activity out before using it with the class, as some overhead setups may distort the image.

Guided practice
Level 1: q1–3
Level 2: q1–4
Level 3: q1–4

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Prepare a selection of 3-D objects, some of which are similar, some of which are not (like the Guided Practice Exercises 11–13 of this Lesson). Give them to students with no measurements, but provide them with rulers and ask them to determine which are similar. Some students may conclude that none are similar — in this case, ask them to try changing the orientation of one of the shapes.

2 Teach (cont)

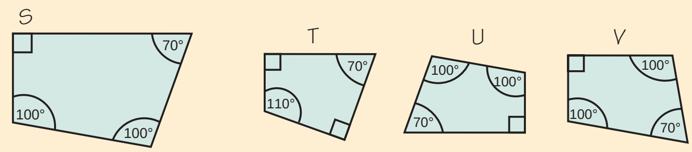
Concept question
"Are congruent figures similar?"
Congruent figures are similar with a scale factor of 1.

Universal access
One way to open the topic is to have students generate some real-world examples of scaling and similarity. Then have students explain these ideas using the vocabulary of the Lesson.

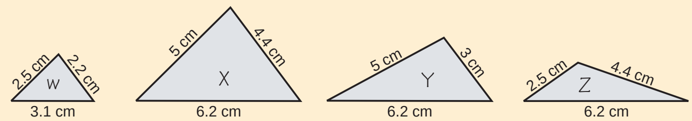
For example, students may suggest a map. In this case, explore the connection between the map's scale and scale factor. What is the scale factor of a map with a scale of 5 miles to the inch? Many students will answer 5. Ask them to imagine the map expanded 5 times — will this be actual size? No, a map with a scale of 5 miles to the inch actually has a scale factor of 316,800. This is calculated by determining how many inches are in 5 miles.

A 3-D example might be a doll house, or a scale model of a car.

3. Which of the quadrilaterals below is similar to S: T, U, or V? U



4. Which triangle below is similar to W: X, Y, or Z? X



3-D Figures Can Be Multiplied by Scale Factors

If you multiply a **3-D figure** by a scale factor you get a **similar figure**.

Just like with 2-D shapes, **corresponding angles** in similar 3-D shapes have **equal measures**. And like 2-D shapes, you multiply any **length** in the original by the **scale factor** to get the corresponding length in the **image**.

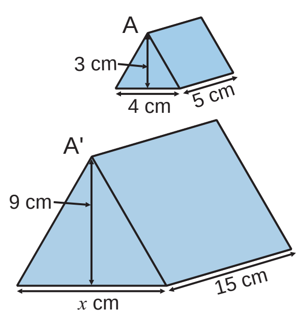
length in image = scale factor × length in original

Don't forget:
When you transform any figure, including changing the size by a scale factor, the new figure is called the image. The image of A is often called A' (read as "A prime").

Check it out:
You could also have used the corresponding lengths 3 cm and 9 cm to find the scale factor.

Example 2

A and A' are similar. Find x.



Solution

First you need to find the **scale factor**.
The lengths in the **original** are multiplied by the **scale factor** to get the lengths in the **image**.

The length of 15 cm in A' corresponds to the length of 5 cm in A.
Rearranging the formula above gives:
Scale factor = length in A' ÷ length in A
= 15 cm ÷ 5 cm
= 3

The length of x cm in A' corresponds to the length of 4 cm in A.

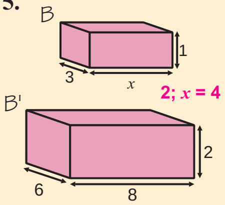
Length in A' = scale factor × length in A
x cm = 3 × 4 cm
$x = 12$

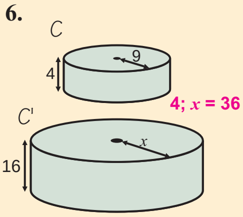
Solutions
For worked solutions see the Solution Guide

- **English Language Learners**
  Illustrate the difference between the mathematical definition of “similar” solids, and the common English language usage of “similar.” Compare a paperback and a hardback version of the same book to see if their measurements are proportional. Repeat this for two differently sized cereal boxes of the same brand, then others, such as frozen juice and oatmeal containers. Conclude that similar looking solids may not be mathematically similar.

Guided Practice

Use the equation to find the scale factor that has been used to produce the image in each of the following pairs of similar solids. Find the missing length, x, in each pair. All lengths are measured in cm.

5.


6.


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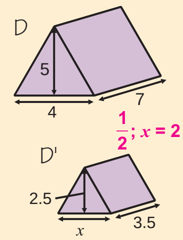
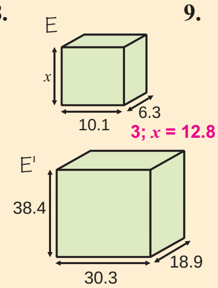
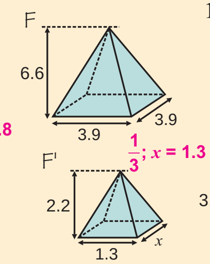
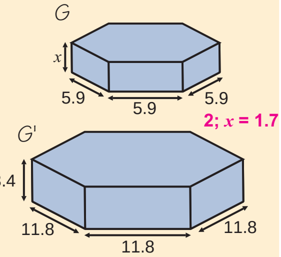


8.


9.


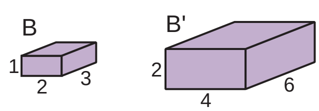
10.


You Can Check Whether Two Solids are Similar

You need to check that all the lengths have been multiplied by the **same scale factor**, and that the **corresponding angles are the same**.

Example 3

Are these rectangular prisms similar?



Solution

If the figures are similar, then all the lengths will have been multiplied by the same scale factor.

scale factor = length in image ÷ length in original

Start with the shortest edge length: $2 \div 1 = 2$
Then the next shortest edge: $4 \div 2 = 2$
And last, the longest edge: $6 \div 3 = 2$

The scale factors are all the same, and all the angles are 90° in each, so **the figures are similar**.

B has been enlarged by **scale factor 2** to make the image B'.

Check it out:

Be careful when you use this formula to check if figures are similar. You must make sure that you are comparing corresponding lengths.

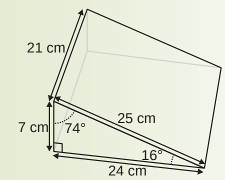
2 Teach (cont)

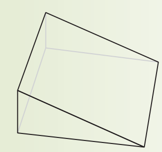
Guided practice

Level 1: q5–8
Level 2: q5–9
Level 3: q5–10

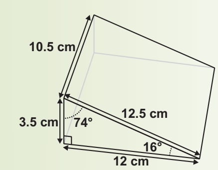
Additional example

The prisms below are similar with a scale factor of $\frac{1}{2}$. Find all the edge lengths of the second prism as well as the angle measures of the base.





Each of these is a triangular prism with a right triangle as the base. The sides of the original base of the prism are 7 cm, 24 cm, and 25 cm, so the sides of the base of the new prism are $\frac{1}{2}$ of this or 3.5 cm, 12 cm, and 12.5 cm. The lateral edges of the original prism are 21 cm, so the lateral edges of the new prism are $\frac{1}{2}$ × 21 = 10.5 cm. The angle measures in the base of the original prism are 16º, 74º, and 90º. So the new angle measures are also 16º, 74º, and 90º.



Solutions

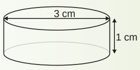
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to construct similar 3-D shapes from card by drawing their nets. You could then ask students to use the similar shapes to investigate the connection between scale factor and volume. They may start by showing that the relation "enlarged volume = scale factor × original volume" is not true, and then start to look for a pattern in the volumes.

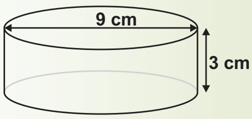
2 Teach (cont)

Guided practice
Level 1: q11
Level 2: q11–13
Level 3: q11–13

Additional example
Draw the image of the cylinder below under an enlargement of scale factor 3.



Since the scale factor is 3, all of the dimensions will be tripled.



Independent practice
Level 1: q1–7
Level 2: q1–10
Level 3: q1–10

Additional questions
Level 1: p468 q1–7
Level 2: p468 q1–12
Level 3: p468 q1–12

3 Homework

Homework Book — Lesson 7.3.1
Level 1: q1–6
Level 2: q1–8
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 39 — Scale and Proportion
- Worksheet 43 — 3-D Figures

Guided Practice

In each set of solids below, say which of the numbered figures is similar to the first figure, and what scale factor created the image. The figures are not drawn to scale. All lengths are measured in cm.

11.

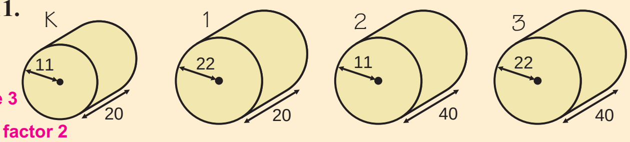


figure 3
scale factor 2

12.

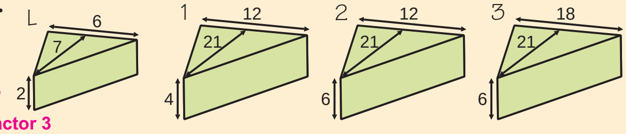


figure 3
scale factor 3

13.

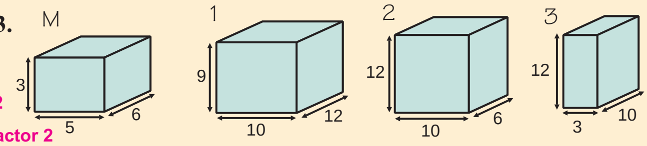


figure 2
scale factor 2

Independent Practice

Copy the following sentences and fill in the missing words:

1. A scale <u>factor</u> of 1 produces an image that is the same <u>size</u>.

2. The <u>image</u> will be <u>smaller</u> than the original if the scale factor is between 0 and 1.

3. If the scale factor is <u>greater than</u> 1, the image is bigger than the <u>original</u>.

Mr. Freeman's history class made a model pyramid, shown below. The class makes a second pyramid, which is twice the size of the first.

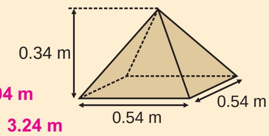
4. What is the height of the second model? **0.68 m**

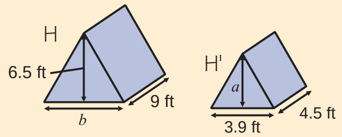
The class builds a third model, which is 3 times the size of the second one.

5. What is the height of the third model? **2.04 m**

6. How long is the base of the third model? **3.24 m**

7. What can you say about the angles in all three models? **They're the same**





8. What scale factor has been used to produce the image H' from prism H? $\frac{1}{2}$

9. What is the length marked *a*? **3.25 ft**

10. What is the length marked *b*? **7.8 ft**

Now try these:
Lesson 7.3.1 additional questions — p468

Round Up

Applying scale factors to solid shapes changes their size — and therefore their surface area and volume. You'll see exactly how surface area and volume are affected by scale factors next Lesson.

Solutions
For worked solutions see the Solution Guide

Lesson 7.3.2 Surface Areas & Volumes of Similar Figures

This Lesson builds on the previous one, which concentrated on length measurements in similar 3-D shapes, and looks at what happens to the surface area and volume of a shape when a scale factor is applied.

Previous Study: In the previous Lesson, students used scale factors to solve problems involving the lengths in similar 3-D shapes.

Future Study: In Geometry, students will prove basic theorems involving congruence and similarity.

Lesson 7.3.2

California Standards:

Measurement and Geometry 1.2
Construct and read drawings and models made to scale.

Measurement and Geometry 2.1
Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and **the surface area and volume of basic three-dimensional figures, including** rectangles, parallelograms, trapezoids, squares, triangles, circles, **prisms, and cylinders.**

Measurement and Geometry 2.3
Compute the length of the perimeter, **the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.**

What it means for you:

You'll find out the effect that applying scale factors has on the surface area and volume of a 3-D figure.

Key words:

- scale factor
- area
- volume
- square
- cube

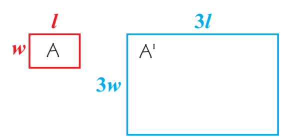
Surface Areas & Volumes of Similar Figures

You've seen already how applying a scale factor affects the perimeter and area of 2-D figures. Well, when you multiply a solid figure by a scale factor, the surface area and the volume are changed. It's not surprising really — bigger shapes have bigger surface areas and volumes. This Lesson looks at exactly how they change.

Area is Multiplied by the Square of the Scale Factor

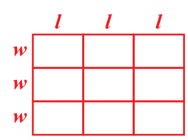
When a **2-D figure** is multiplied by a **scale factor**, the area gets multiplied by the **square of the scale factor**.

Here, rectangle A is enlarged by a scale factor of **3** to give the image A'.



The **area** of A is lw.
The **area** of A' is $3l \times 3w = 9lw$.

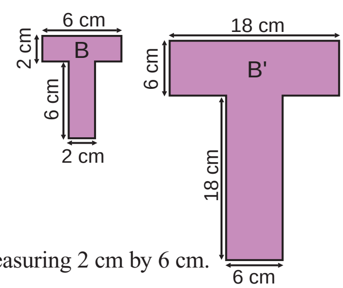
The **side lengths** in the image are **3 times** greater than the original, but the **area** of the image is **3 × 3 = 9 times** greater.



Example 1

Figure B is multiplied by a scale factor of 3 to produce the image B'.

Find the area of figure B.
Use the area of B and the scale factor to find the area of B'.



Solution

B is made up of **2 rectangles**, each measuring 2 cm by 6 cm.

So the area of B is: $2 \times (2 \times 6) = 2 \times 12 = \mathbf{24\ cm^2}$

The area of B' is: area of B × **(scale factor)²**
$= 24\text{ cm}^2 \times 3^2$
$= 24\text{ cm}^2 \times 9 = \mathbf{216\ cm^2}$

Check: Area of B' $= 2 \times (6 \times 18) = 2 \times 108 =$ **216 cm² — Correct**

Guided Practice

Ignacio draws a figure with an area of 5 in². Find the area of the image if Ignacio multiplies his figure by the following scale factors:

1. 2 **20 in²** **2.** 10 **500 in²** **3.** 6 **180 in²** **4.** $\frac{1}{2}$ **1.25 in²**

1 Get started

Resources:
- 1 × 1 × 1 inch cubes (or similar)
- large sheets of poster paper

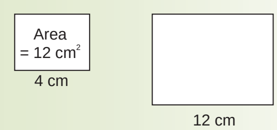
Warm-up questions:
- Lesson 7.3.2 sheet

2 Teach

Universal access

Many students will assume that you find the enlarged area by simply multiplying the original area by the length scale factor. A good way to start the lesson is to show the students that this doesn't work:

Start with two similar rectangles like those below, and find the scale factor of the lengths.

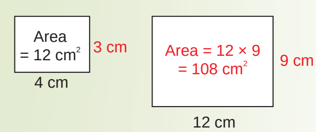


Scale factor = 12 ÷ 4 = 3

Now ask the students how they could use the scale factor to find the area of the enlarged rectangle. Some students will suggest multiplying the area of the smaller rectangle by the scale factor, so write this down.

Enlarged area 12 cm² × 3 = 36 cm²

Now, find the real area by completing the missing dimensions and using "$A = l \times w$". Point out that the actual answer is much larger.



Guided practice
Level 1: q1–3
Level 2: q1–4
Level 3: q1–4

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Give out some 1 × 1 × 1 inch cubes (or similar) to the class. Have the students start with one cube, and ask them to calculate its volume. Ask them what they think the volume would be if the lengths of the cube were all twice as long. Have the students make the new cube, and ask how many of the original cubes fit into the new one. Calculate the volume of the new cube. Try tripling the length measurements. Chart the results on a large poster and discuss. Repeat the activity for surface areas.

2 Teach (cont)

Common error

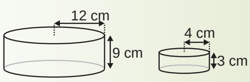
Students may confuse the concepts of volume and surface area.

Some students remember surface area by the use of nets and volume by the use of solid cubes. Another explanation is that surface area is the amount of area that needs to be covered when painting something, and volume is the amount of liquid that would be needed to fill the shape up.

Remind students that surface area is a two-dimensional measure and volume is three-dimensional.

Additional example

The cylinders below are similar with a scale factor of $\frac{1}{3}$.



a) Find the surface area of the first cylinder in terms of π.

SA = $2\pi r^2 + 2\pi rh$
For this cylinder, r = 12 cm and h = 9 cm.
So SA = $2 \times \pi \times (12)^2 + 2 \times \pi \times 12 \times 9$
$= 288\pi + 216\pi$
$= 504\pi$ cm²

b) Determine the surface area of the second cylinder, using the answer to part a) and the scale factor.

Since the scale factor is $\frac{1}{3}$, the surface area of the second figure will be:
SA $= 504\pi \times (\frac{1}{3})^2$
$= 504\pi \times \frac{1}{9}$
$= 56\pi$ cm²

c) Check your work by calculating the surface area of the second cylinder using the given dimensions.

As the scale factor is $\frac{1}{3}$, r = 4 cm and h = 3 cm.
SA = $2\pi r^2 + 2\pi rh$.
SA $= 2 \times \pi \times (4)^2 + 2 \times \pi \times 4 \times 3$
$= 32\pi + 24\pi$
$= 56\pi$ cm²

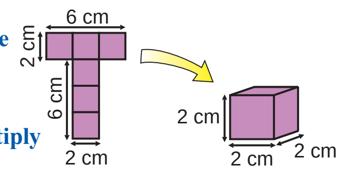
Guided practice

Level 1: q5–7
Level 2: q5–10
Level 3: q5–10

Surface Area is Multiplied by the Scale Factor Squared

Figure B from Example 1 is the **net** of a **2 cm cube**. The **surface area of the cube** is the same as the **area of the net**.

The image B' is the net of a **6 cm cube**. A 6 cm cube is what you get if you **multiply** a 2 cm cube by a **scale factor** of **3**.

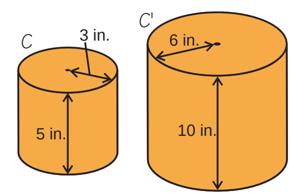


So, if you multiply a cube by a scale factor, the **surface area** is multiplied by the **square** of the **scale factor**. This is true for the surface area of **any solid**.

Example 2

Cylinder C' is an enlargement of cylinder C by a scale factor of 2.

Find the surface area of cylinder C.
Use the surface area of C and the scale factor to find the surface area of C'.
Use $\pi = 3.14$.



Solution

First find the **surface area of C**:

Area = (2 × base area) + lateral area
$= (2 \times \pi r^2) + (2\pi r \times h)$
$= (2 \times 3.14 \times 3^2) + (2 \times 3.14 \times 3 \times 5)$
$= 56.52 + 94.2$
= **150.72 in²**

To find the surface area of C', just multiply the surface area of C by the **square of the scale factor**:

$A = 150.72 \times 2^2 = 150.72 \times 4 =$ **602.88 in²**

You can calculate the surface area of cylinder C' to **check** the answer:

$A = (2 \times \pi r^2) + (2\pi r \times h)$
$= (2 \times 3.14 \times 6^2) + (2 \times 3.14 \times 6 \times 10)$
$= 226.08 + 376.8$ = **602.88 in² — Correct**

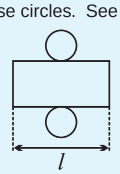
Don't forget:

The formula for surface area of a cylinder in Example 2 uses the formulas for area and circumference of a circle.

They are: $A = \pi r^2$
$C = 2\pi r$

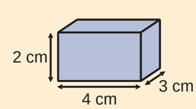
where r is the circle's radius.

Remember — the length of the rectangle (l) that is used to find the "lateral area" is equal to the circumference of the base circles. See Lesson 7.1.3.



Guided Practice

5. What is the surface area of the prism on the left? **52 cm²**



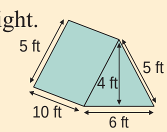
Find the surface area of the image if this figure is multiplied by:

6. Scale factor 3 **468 cm²**
7. Scale factor 4 **832 cm²**

8. Find the surface area of the triangular prism on the right. **184 ft²**

Calculate the surface area of the image if this figure is multiplied by:

9. Scale factor 2 **736 ft²**
10. Scale factor $\frac{1}{2}$ **46 ft²**



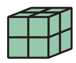
Solutions
For worked solutions see the Solution Guide

- **English Language Learners**
  Use whiteboards to check for understanding of scale factor and corresponding sides. Say, “Rectangular prism A has length 9 inches, width 6 inches, and height 3 inches. Sketch and label A. B is a similar prism. The scale factor of A to B is 3 to 1. Sketch and label B.” Then ask them to calculate and compare the surface areas and volumes of each.

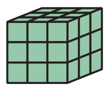
Volume is Multiplied by the Cube of the Scale Factor

D 

Cube D is a **unit cube**. Its volume is **1 cubic unit**.

D' 

D is multiplied by **scale factor 2** to get D'.
Cube D' has a volume of **8 cubic units**.
Multiplying by scale factor 2 increases the volume of D by **2^3 = 8 times**.

D'' 

D is multiplied by **scale factor 3** to get D''.
The volume of D'' is **27 cubic units**.
Multiplying by scale factor 3 increases the volume of D by **3^3 = 27 times**.

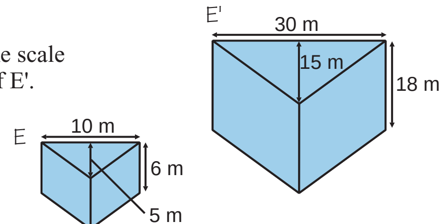
When you multiply any solid figure by a scale factor, the **volume** is multiplied by the **cube of the scale factor**.

This works for **any** solid figure.

Example 3

Prism E' is an enlargement of prism E by a scale factor of 3.

Find the volume of E.
Use the volume of E and the scale factor to find the volume of E'.



Solution

First find the **volume of E**:

$$V = \text{area of base} \times \text{height}$$
$$= \left(\tfrac{1}{2} \times \text{base of triangle} \times \text{height of triangle}\right) \times \text{height of prism}$$
$$= \left(\tfrac{1}{2} \times 10 \times 5\right) \times 6$$
$$= 25 \times 6 = \mathbf{150\ m^3}$$

To find the volume of E', multiply the volume of E by the **cube of the scale factor**:

$$V = 150 \times 3^3 = 150 \times 27 = \mathbf{4050\ m^3}$$

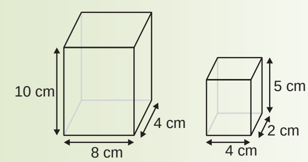
You can calculate the volume of E' to **check** the answer:

$$V = \left(\tfrac{1}{2} \times \text{base of triangle} \times \text{height of triangle}\right) \times \text{height of prism}$$
$$= \left(\tfrac{1}{2} \times 30 \times 15\right) \times 18$$
$$= 225 \times 18 = \mathbf{4050\ m^3} \text{ — Correct}$$

2 Teach (cont)

Additional example

The boxes below are similar with a scale factor of $\frac{1}{2}$.



a) Find the volume of the first box.
$V = lwh$. For this box, l = 8 cm, w = 4 cm, and h = 10 cm.
So $V = lwh$
$= 8 \times 4 \times 10$
$= 320\ \text{cm}^3$

b) Determine the volume of the second box, using the answer in part a) and the scale factor.
Since the scale factor is $\frac{1}{2}$, the volume of the second figure will be:
$V = 320 \times \left(\frac{1}{2}\right)^3$
$= 320 \times \frac{1}{8}$
$= 40\ \text{cm}^3$

c) Check your work by calculating the volume of the second box using the given dimensions.
Since the scale factor is $\frac{1}{2}$, l = 4, w = 2, and h = 5. So
$V = lwh$
$= 4 \times 2 \times 5$
$= 40\ \text{cm}^3$

- **Advanced Learners**
  Have students work out volume scale factors algebraically. Consider the volume formula for a cube: $V = s^3$. Now take a scale factor of 2. Each side of the new cube becomes $2s$, so you now have the volume of the new cube as $V = (2s)^3$ which is also $2^3 \times s^3$. Students should be able to see from this that "the enlarged volume is obtained by multiplying the original volume by the scale factor cubed." Then ask them to try this with a general scale factor, k: $V = (ks)^3 = k^3 \times s^3$. Have students pick a different 3-D shape, such as a cylinder, and do the same analysis.

2 Teach (cont)

Guided practice
Level 1: q11–13
Level 2: q11–16
Level 3: q11–16

Independent practice
Level 1: q1–8
Level 2: q1–10
Level 3: q1–12

Additional questions
Level 1: p469 q1–5, 7–8
Level 2: p469 q1–8
Level 3: p469 q4–9

3 Homework

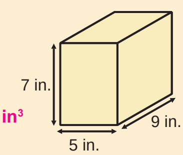
Homework Book — Lesson 7.3.2
Level 1: q1–6
Level 2: q1–7
Level 3: q1–8

4 Skills Review

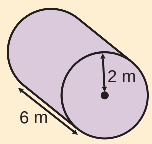
Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 39 — Scale and Proportion
- Worksheet 44 — Surface Area
- Worksheet 45 — Volume

Guided Practice

11. What is the volume of this prism? **315 in³**



Find the volume of the image if this figure is multiplied by:
12. Scale factor 2 **2520 in³** **13.** Scale factor 3 **8505 in³**

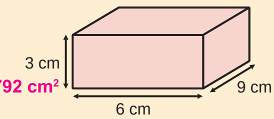


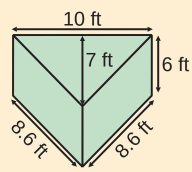
14. Calculate the volume of this cylinder. Use 3.14 for π. **75.36 m³**

Find the volume of the image if this figure is multiplied by:
15. Scale factor 2 **602.88 m³** **16.** Scale factor $\frac{1}{3}$ **2.79 m³**

Independent Practice

Find:
1. The surface area of this prism. **198 cm²**
2. The surface area of the image, if the prism is multiplied by a scale factor of 2. **792 cm²**
3. The volume of this prism. **162 cm³**
4. The volume of the image, if the prism is multiplied by a scale factor of 2. **1296 cm³**





Calculate:
5. The surface area of this prism. **233.2 ft²**
6. The surface area of the image, if the prism is multiplied by a scale factor of 3. **2098.8 ft²**
7. The volume of this prism. **210 ft³**
8. The volume of the image, if the prism is multiplied by a scale factor of 3. **5670 ft³**

9. The surface area of a cube is 480 cm². The cube is enlarged by a scale factor of k. What is the surface area of the new cube? **$480k^2$ cm²**

10. A pyramid has a volume of 24 cm³. If you double all the dimensions of the pyramid, what will be the volume of the new pyramid? **192 cm³**

11. A scale factor enlargement of a cylinder is produced. The surface area of the image is 9 times the surface area of the original. The base of the original cylinder has a radius of 5 in. What is the radius of the base of the image? **15 in.**

12. A scale model of a building has a surface area of 6 ft². If the real building has a surface area of 600 ft², what scale factor has been used to make the model? **$\frac{1}{10}$**

Now try these:
Lesson 7.3.2 additional questions — p469

Round Up

One way to remember how scale factor affects surface area and volume is to think about the units you use to measure them. Surface area is given in square units like m² or in², so the scale factor is squared. Volume is given in cubic units like cm³ or ft³, so the scale factor is cubed.

Solutions
For worked solutions see the Solution Guide

Lesson 7.3.3

Changing Units

This Lesson is about converting area and volume measurements between different units. There are two types of change — changes to account for scale (mm to cm to m to km, and so on), and also changes between measurement systems; for example, cm to inches.

Previous Study: In grade 6, students learned how to convert between units of the customary and metric systems. This was also covered in Chapter 4 of this book.

Future Study: Future math study will require students to switch between units as part of more advanced calculations.

Lesson 7.3.3

Changing Units

California Standards:
Measurement and Geometry 2.4
Relate the changes in measurement with a change of scale to the units used (e.g., square inches, cubic feet) and to conversions between units (1 square foot = 144 square inches or [1 ft²] = [144 in²], 1 cubic inch is approximately 16.38 cubic centimeters or [1 in³] = [16.38 cm³]).

What it means for you:
You'll learn how to convert between different units of area and volume.

Key words:
- area
- volume
- square
- cube

Don't forget:
For a reminder of how to use conversion factors, see Lesson 4.2.3.

Check it out:
Another way to do this conversion is to multiply the area measurement by a conversion fraction equal to 1. This method involves dimensional analysis and was explained in Lesson 4.3.4.

$$500\ \cancel{\text{cm}^2} \times \frac{1\ \text{m}^2}{10{,}000\ \cancel{\text{cm}^2}} = \frac{500}{10{,}000}\ \text{m}^2 = 0.05\ \text{m}^2$$

conversion fraction

You wouldn't measure the area of a country in cm² — km² would be a better unit to use. Similarly, you wouldn't measure the volume of a die in m³. That's why this Lesson's about converting units of area and volume — so you can keep your answers in a sensible range.

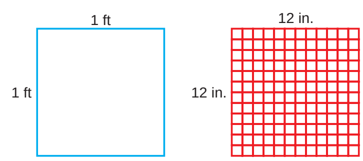
You Can Convert Between Units of Area

You've seen that: **1 foot = 12 inches**

So 1 foot is 1 inch multiplied by a **scale factor of 12**.

If you multiply a **1 inch square** by scale factor 12, you get a **1 foot square**.

So the area **1 ft²**
$= 1\ \text{in}^2 \times 12^2$
$= 1\ \text{in}^2 \times 144$
$= 144\ \text{in}^2$



You can do the same with metric units: **1 meter = 100 centimeters**

So 1 m is 1 cm multiplied by a **scale factor of 100**.

So: $\mathbf{1\ m^2} = 1\ \text{cm}^2 \times 100^2 = 1\ \text{cm}^2 \times 10{,}000\ \mathbf{= 10{,}000\ cm^2}$

$1\ \text{m}^2 = 10{,}000\ \text{cm}^2$ is a **conversion factor**, which you can use to change m² to cm², or cm² to m².

Example 1

A cube has a surface area of 500 cm². What is this surface area in m²?

Solution

$1\ \text{m}^2 = 10{,}000\ \text{cm}^2$, so the ratio of cm² to m² is 10,000 : 1 or $\frac{10{,}000}{1}$.

Write a proportion where there are x m² to 500 cm²: $\frac{10{,}000}{1} = \frac{500}{x}$

Cross multiply and **solve** for x:
$$10{,}000 \times x = 500 \times 1$$
$$10{,}000x = 500$$
$$x = 500 \div 10{,}000 = \mathbf{0.05}$$

The surface area of the cube is **0.05 m²**.

Guided Practice

In Exercises 1–3, convert the areas to cm².

1. 3 m² 30,000 cm² **2.** 0.25 m² 2500 cm² **3.** 1.8 m² 18,000 cm²

In Exercises 4–6, convert the areas to ft².

4. 72 in² 0.5 ft² **5.** 864 in² 6 ft² **6.** 252 in² 1.75 ft²

1 Get started

Resources:
- rulers/yardsticks/metersticks
- base-10 blocks

Warm-up questions:
- Lesson 7.3.3 sheet

2 Teach

Universal access
This is a topic where it is quite helpful to have out rulers, yardsticks, and metersticks. There is nothing better for discussing conversion of units than for students to have hands-on experience with the units.

Universal access
Give the students a handout containing a square, rectangle, and circle. Tell students to measure the sides (and radius) of the figures, and calculate the areas in mm, cm, and m. Discuss which units are the most appropriate. Compare the areas obtained with the different units. For example, "the area in mm is 100 times larger than the area in cm."

Universal access
If you have access to a set of base-10 blocks, this can be a good visual model for the concept of unit conversion. The block that represents 1000 is a 10 × 10 × 10 cube. The edges of the cube can illustrate the conversion from decimeters to centimeters and vice versa. The faces of the cube can illustrate the conversion from square decimeters to square centimeters, and the volume of the cube can illustrate the conversion from cubic decimeters to cubic centimeters.

Guided practice
Level 1: q1–6
Level 2: q1–7
Level 3: q1–8

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  In groups of 4, make 52 "matching" cards using common measures. For example, for 1 yard make four cards: 1 yd, 36 in., 3 ft, and 91.44 cm. Deal 7 cards each and lay down pairs. If you lay down any pairs, make sure you still have 7 cards by picking up from the middle. Take turns to pick a card from the person to your right. If it matches any of yours, lay these down and pick up more cards from the middle. The winner is the person with the most pairs at the end of the game.
- **English Language Learners**
  The Universal access activities on page 383 are particularly suitable for helping English Language Learners to understand the concepts involved in this Lesson. The Strategic Learner activity described above is also useful for reinforcing the names of units and the conversion factors.

2 Teach (cont)

Common error

Many students will at some point make the mistake of multiplying, rather than dividing, or vice versa. For instance, if a square has an area of 4 feet², they may convert it to $\frac{1}{36}$ inch² instead of 576 inch².

Encourage students to consider first whether the answer should be larger or smaller.

Example: "Convert 4 square feet into square inches."

Before starting the calculation, the student should reason along the lines of "square inches are much smaller than square feet, so each square foot will contain many square inches. So 4 square feet will be a much larger number in square inches."

Guided practice

Level 1: q9–15
Level 2: q9–16
Level 3: q9–16

7. Luisa makes a scale drawing of a park. The drawing has an area of 240 in². What is the area of the drawing in ft²? **1.67 ft²**

8. TJ buys 1.2 m² of fabric to make a Halloween costume. What is this area in cm²? **12,000 cm²**

You Can Convert Between Metric and Customary Units

You can convert areas between the metric and customary systems:

1 in = 2.54 cm
So **1 in²** = 2.54 cm × 2.54 cm **= 6.45 cm²**

1 ft = 0.3 m
So **1 ft²** = 0.3 m × 0.3 m **= 0.09 m²**

Example 2

A cube has a surface area of 500 cm². What is this surface area in in²?

Solution

1 in² = 6.45 cm², so the ratio of cm² to in² is 6.45 : 1 or $\frac{6.45}{1}$.

Write a proportion where there are x in² to 500 cm²: $\frac{6.45}{1} = \frac{500}{x}$

Cross multiply and **solve** for x:

$$6.45 \times x = 500 \times 1$$
$$6.45x = 500$$
$$x = 500 \div 6.45 = 77.52$$

The surface area of the cube is **77.52 in²**.

Check the reasonableness: 1 in² is around 6.5 cm², and 77.52 in² ≈ 80 in². 80 × 6.5 = 520, so **the answer is reasonable**.

Don't forget: You should always check the reasonableness of your answers — this just means checking it's about the right size.

Don't forget: There are two ways you could do Exercise 15 — either convert ft² to m² first, then convert m² to cm², or convert ft² to in² then in² to cm². You can do Exercise 16 in two ways also. You'll get slightly different answers depending on which way around you do it — due to rounding errors.

Guided Practice

In Exercises 9–11, convert the following areas to in²:
9. 12.9 cm² **2 in²** **10.** 225 cm² **34.88 in²** **11.** 92 cm² **14.26 in²**

In Exercises 12–14, convert the following areas to m²:
12. 5 ft² **0.45 m²** **13.** 132 ft² **11.88 m²** **14.** 66.7 ft² **6 m²**

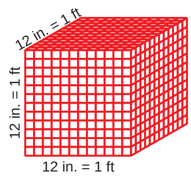
15. What is 1 ft² in cm²? **928.8 cm² or 900 cm²**

16. What is 1 m² in in²? **1550.39 in² or 1600 in²**

Solutions
For worked solutions see the Solution Guide

● **Advanced Learners**
Challenge advanced learners to solve real-life area and volume problems involving unusual units. For example, "A farmer has a square field, with sides that are 200 m long. What is the area of his field in hectares?" (Area of field in square meters = 200 × 200 = 40,000 m^2, 1 hectare = 10,000 m^2, 40,000 m^2 = 4 hectares). Or for volume, "A recipe calls for two tablespoons of lemon juice. If your measuring cylinder is marked in cubic inches, how much should you add?" (2 tablespoons = 30 ml = 30 cm^3, 1 cm^3 = 0.06 in^3, 30 cm^3 = 1.8 in^3). Remember to give them details of any units that may be unfamiliar, such as "1 hectare = 10,000 m^2," and "1 tablespoon = 30 ml."

You Can Also Convert Units of Volume



If you multiply a **1 inch cube** by **scale factor 12**, you get a **1 foot cube**.

So, **1 ft^3** = 1 in^3 × 12^3 = 1 in^3 × 1728 **= 1728 in^3**

In the same way, if you multiply a **1 cm cube** by **scale factor 100**, you get a **1 m cube**.

So, **1 m^3** = 1 cm^3 × 100^3 = 1 cm^3 × 1,000,000 **= 1,000,000 cm^3**

Example 3

A cylinder has a volume of 4 ft^3. What is this volume in in^3?

Solution

1 ft^3 = 1728 in^3, so the ratio of in^3 to ft^3 is 1728 : 1 or $\frac{1728}{1}$.

Write a proportion where there are x in^3 to 4 ft^3: $\frac{1728}{1} = \frac{x}{4}$

Cross multiply and **solve** for x: $1728 \times 4 = x \times 1$
$6912 = x$

The volume of the cylinder is **6912 in^3**.

Check the reasonableness: There are about 1700 in^3 in 1 ft^3. 1700 × 4 = 6800, so **the answer is reasonable**.

You Can Also Convert Volume Units Between Systems

You can convert volumes between the metric and customary systems:

1 in = 2.54 cm
So **1 in^3** = 2.54 cm × 2.54 cm × 2.54 cm **= 16.39 cm^3**

1 ft = 0.3 m
So **1 ft^3** = 0.3 m × 0.3 m × 0.3 m **= 0.027 m^3**

Example 4

A cylinder has a volume of 4 ft^3. What is this volume in m^3?

Solution

1 ft^3 = 0.027 m^3, so the ratio of m^3 to ft^3 is 0.027 : 1 or $\frac{0.027}{1}$.

Write a proportion where there are x m^3 to 4 ft^3: $\frac{0.027}{1} = \frac{x}{4}$

Cross multiply and **solve** for x: $0.027 \times 4 = x \times 1$
$0.108 = x$

The volume of the cylinder is **0.108 m^3**.

Check the reasonableness: There are about 0.03 m^3 in 1 ft^3. 0.03 × 4 = 0.12, so **the answer is reasonable**.

2 Teach (cont)

Additional examples

1. A square has an area of 4 ft^2. What is its area in in^2?

Since there are 12 inches in a foot, and since a foot is larger than an inch, going from feet to inches leads to a conversion factor of 12.

Thus, going from square feet to square inches leads to a conversion factor of 12^2, or 144. So to convert square feet to square inches, multiply by 144:
4 ft^2 = 4 × 144 = 576 in^2.

2. A box has a surface area of 90 cm^2. What is the surface area in m^2?

Since there are 100 cm in a meter, and since a cm is smaller than a meter, going from cm to meters leads to a conversion factor of 1/100.

Thus, going from cm^2 to $meters^2$ leads to a conversion factor of $(1/100)^2$ or 1/10,000. So to convert cm^2 to $meters^2$, multiply by 1/10,000:
90 cm^2 = 90/10,000 = 0.009 m^2.

3. A cone takes up a volume of 24 in^3. What is this volume in ft^3?

Since there are 12 inches in a foot, and since an inch is smaller than a foot, going from inches to feet leads to a conversion factor of 1/12.

Thus, going from cubic inches to cubic feet leads to a conversion factor of $(1/12)^3$ or 1/1728. So to convert in^3 to ft^3, multiply by 1/1728:
24 in^3 = 24/1728 = 1/72 ft^3.

4. A sculpture takes up a volume of 8 m^3. What is this volume in cm^3?

Since there are 100 cm in a meter, and since a meter is larger than a cm, going from meters to cm leads to a conversion factor of 100.

Thus, going from m^3 to cm^3 leads to a conversion factor of $(100)^3$ or 1,000,000. So to convert cubic meters to cm^3, multiply by 1,000,000:
8 m^3 = 8,000,000 cm^3

2 Teach (cont)

Guided practice
Level 1: q17–25
Level 2: q17–27
Level 3: q17–27

Independent practice
Level 1: q1–8
Level 2: q1–10
Level 3: q1–11

Additional questions
Level 1: p469 q1–19
Level 2: p469 q1–19
Level 3: p469 q1–19

3 Homework

Homework Book — Lesson 7.3.3
Level 1: q1–8
Level 2: q1–10
Level 3: q1–11

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 30 — Measuring
- Worksheet 44 — Surface Area
- Worksheet 45 — Volume

Check it out:
There are two ways you could do Exercises 26 and 27. For Exercise 26, either convert from ft^3 to in^3 and then to cm^3, or do ft^3 to m^3 and then to cm^3. You'll get different answers depending on which way around you do it — due to rounding errors.

Guided Practice

In Exercises 17–20, convert the areas to cm^3.

17. 10 m^3 **10,000,000 cm^3** **18.** 21.5 m^3 **21,500,000 cm^3** **19.** 8 in^3 **131.12 cm^3** **20.** 14.3 in^3 **234.38 cm^3**

In Exercises 21–24, convert the areas to ft^3.

21. 2592 in^3 **1.5 ft^3** **22.** 1000 in^3 **0.58 ft^3** **23.** 135 m^3 **5000 ft^3** **24.** 5.8 m^3 **214.8 ft^3**

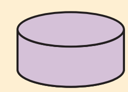
25. A building takes up a space of $20{,}000 \text{ m}^3$. What is this volume in cm^3? **20,000,000,000 cm^3**

26. What is 1 ft^3 in cm^3? **28,321.92 cm^3 or 27,000 cm^3**

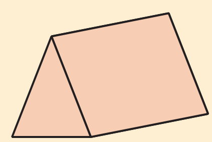
27. What is 1 m^3 in in^3? **61,012.8 in^3 or 64,000 in^3**

Independent Practice

A cylinder has a surface area of $126\pi \text{ cm}^2$.



1. What is this surface area in m^2? **0.0126π m^2**
2. What is this surface area in in^2? **19.53π in^2**

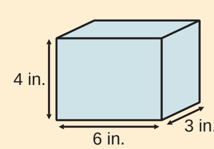


A prism has a volume of 2.85 ft^3.
3. What is this volume in in^3? **4924.8 in^3**
4. What is this volume in m^3? **0.077 m^3**

Karen makes a scale model of her school for a project. The model has a surface area of 376 in^2 and a volume of 480 in^3.
5. What is the surface area of the model in ft^2? **2.61 ft^2**
6. What is the volume of the model in cm^3? **7867.2 cm^3**

7. Julio measures the area of one wall of his bedroom as 35.25 ft^2. What is the area of the wall in square inches? **5076 in^2**

8. The volume of Brandy's suitcase is $184{,}800 \text{ cm}^3$. What is this volume in m^3? **0.1848 m^3**



This rectangular prism is shown with measurements in inches.
9. What is its surface area in square feet? **0.75 ft^2**
10. What is its volume in cubic feet? **0.042 ft^3**

11. An acre is 4840 square yards. What is an acre in square feet? **43,560 ft^2**

Now try these:
Lesson 7.3.3 additional questions — p469

Don't forget:
A yard is 3 feet.

Round Up

Don't forget that when you deal with conversion factors, you should always check that your answer is reasonable. If you convert from small units to large units, the number of units will decrease. If you convert from large units to small units, the number of units will increase.

Solutions
For worked solutions see the Solution Guide

Chapter 7

Investigation — Set Design

Purpose of the Investigation

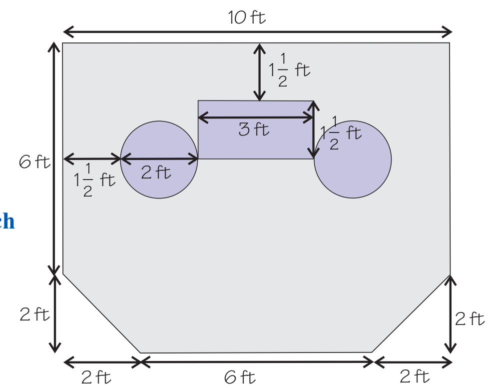
This Investigation gives students a real-life example of scaling and similarity. It relates the math to an area that students are likely to be familiar with — the theater. Students draw scale drawings of theater sets and consider how scale models would be made of the sets.

Chapter 7 Investigation

Set Design

In a play or musical, the actors act on a stage that is often decorated with furniture and scenery. This is called the set. The process of designing a set begins with an idea, and then the set designer creates a scale model of it for the director to see before the real set is built.

Here is a designer's sketch for the layout of a stage set. The stage has a table in the shape of a **rectangular prism** which also serves as a bench and a couch. There are also **two cylinders** that serve as stools or tables.



Part 1:
Make a scale drawing of the set on **¼ inch by ¼ inch** grid paper. Use **1 inch to represent 2 feet** (four small squares are 1 inch).

What is the **scale factor** of the actual objects to the drawing?

Part 2:
The rectangular prism and the cylinders each have a height of 1½ feet.

1) Determine the **volumes** of the rectangular prism and the cylinders. Use $\pi = 3.14$.
2) Determine the **surface areas** of the rectangular prism and the cylinders.
3) Instead of a scale drawing, a set designer would show the director a three-dimensional model. Suppose a model was made of this set using the scale factor in Part 1. What would be the **surface areas** and **volumes** of the model rectangular prism and cylinders?

Extensions

1) Draw this set with a scale of 1 inch to 1 foot. How do your answers to Part 2 change?
2) The entire stage is raised by a platform that exactly fits the stage. The model of it is 3 inches high, when using a scale of 1 inch to 1 foot. What is the volume of the actual platform?

Open-ended Extensions

1) Your school probably has a stage for theatricals. Measure the dimensions of the stage and create a scale drawing of it.
2) Using your measurements of the stage platform, calculate its surface area and volume. Now create a model of the stage using the same scale as you did for your drawing. Calculate your model's surface area and volume, and use these measurements to calculate the volume and surface area of the actual stage. How do your answers compare?

Round Up

Scale models are used in real life to give you a good idea of what something will look like. It's a lot easier and less expensive to build a small model than to build the real thing. So if you don't like what you see, it's less of a problem to change it after only building a scale model.

Resources

- ¼-inch grid paper
- rulers
- compasses
- yard sticks/tape measures
- card, scissors, tape

Strategic & EL Learners

Provide strategic learners with scale models of the rectangular prism and cylinders. This will help them to calculate the surface areas and volumes.

Showing English language learners the school stage, then sketching a plan view of it, may be useful in communicating the key ideas.

Investigation Notes on p387 B-C

Investigation — Set Design

Mathematical Background

First, students are asked to produce a scale drawing from a sketch of a plan-view of a theater set. Students have to convert each real-life measurement to the length it should be in the diagram. They can do this using the scale factor given in the question, and the formula below:

length in drawing = scale factor × length in original

The same idea is used to find lengths in a 3-dimensional model.

Students are also asked to calculate the surface area and volume of a rectangular prism and a cylinder.
The surface areas of prisms and cylinders can be found using the formula: **Surface area = (2 × base area) + lateral area**.
Finding the base area of a cylinder involves using the formula for the area of a circle, which is **area = πr^2**, where r = radius.
The lateral area of a cylinder is a rectangle whose length is the same as the circumference of the base circles.
The circumference of a circle is **circumference = πd**, where d = diameter.

The **volume** of prisms and cylinders is found by multiplying the base areas by the height.

The Investigation also allows students to review the surface areas and volumes of similar figures. When a figure is multiplied by a scale factor of k, then its **surface area** is multiplied by a scale factor of k^2, and its **volume** is multiplied by a scale factor of k^3.

The open-ended extensions require students to make a model of a stage. This tests their skills in creating **nets** for figures with certain dimensions.

Approaching the Investigation

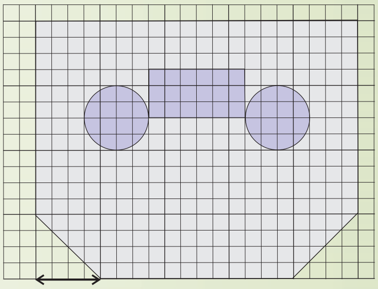
Part 1:

Since 1 inch on the drawing is 24 inches in reality, the scale factor from actual objects to the diagram is $\frac{1}{24}$.
Students can use this scale factor in the formula **length in drawing = scale factor × length in original**.
For example, the real-life length of 10 feet converts to:

length in drawing = scale factor × length in original

$= \frac{1}{24} \times 10 = \frac{5}{12}$ feet = **5 inches** (4 ¼-inch grid squares = 1 inch, so 5 inches = 20 ¼-inch grid squares)

The other dimensions can be converted in the same way, and used to produce the scale drawing below (not to scale here).



1 inch = 4 squares = 2 feet

Part 2:

1) **Volume of actual rectangular prism:**

The rectangular prism has dimensions of $1\frac{1}{2}$ feet by 3 feet by $1\frac{1}{2}$ feet.

Volume = length × width × height = $1\frac{1}{2} \times 3 \times 1\frac{1}{2} = \frac{3}{2} \times 3 \times \frac{3}{2} = \frac{27}{4} = \mathbf{6\frac{3}{4}}$ **feet^3**

Volume of actual cylinder:

Each cylinder has a circular base with a diameter of 2 feet (so a radius of 1 foot), and a height of $1\frac{1}{2}$ feet.

Volume = height × base area = height × $\pi r^2 = 1\frac{1}{2} \times (3.14 \times 1^2) = \frac{3}{2} \times 3.14 =$ **4.71 feet^3**

2) **Surface area of actual rectangular prism:**

The **rectangular prism** has six faces: four of $1\frac{1}{2}$ feet by 3 feet, and two of $1\frac{1}{2}$ feet by $1\frac{1}{2}$ feet.

So the **surface area** = $[4 \times (1\frac{1}{2} \times 3)] + [2 \times (1\frac{1}{2} \times 1\frac{1}{2})] = \mathbf{22\frac{1}{2}}$ **feet^2**

Surface area of actual cylinder:

The **cylinders** have two circular bases, each of area 3.14 feet^2 (calculated for the volume).

The lateral area is a rectangle, which has a length of $1\frac{1}{2}$ feet. Its width is equal to the circumference of the circular bases.
Circumference = $\pi \times d$
$= 3.14 \times 2 = 6.28$ feet
So lateral area = length × width = 6.28 × 1.5 = 9.42 feet^2
Surface area = (2 × base area) + lateral area = (2 × 3.14) + 9.42 = 15.7 feet^2

Investigation — Set Design

3) The scale factor of the models is $\frac{1}{24}$.

Surface area of model rectangular prism = $22\frac{1}{2}\times\left(\frac{1}{24}\right)^2=\frac{45}{2}\times\left(\frac{1}{24}\right)^2=\frac{45}{2}\times\frac{1}{576}=\frac{45}{1152}=\mathbf{\frac{5}{128}}$ **feet² = 0.039 feet²**

To convert to square inches, use the conversion **1 ft² = 144 in²**. The ratio of in² to ft² is 144 : 1 or $\frac{144}{1}$.

Write a proportion: $\frac{144}{1}=\frac{x}{0.039}$. Cross-multiply to solve for x. $144 \times 0.039 = 1 \times x \Rightarrow$ $\boldsymbol{x}$ **= 5.616 in²**

Surface area of model cylinder = $15.7\times\left(\frac{1}{24}\right)^2=15.7\times\left(\frac{1}{24}\right)^2=15.7\times\frac{1}{576}=$ **0.0273 feet²**

$\frac{144}{1}=\frac{x}{0.0273}$. Cross-multiply to solve for x. $144 \times 0.0273 = 1 \times x \Rightarrow$ $\boldsymbol{x}$ **= 3.931 in²**

Volume of model rectangular prism = $\frac{27}{4}\times\left(\frac{1}{24}\right)^3=\frac{27}{4}\times\frac{1}{13{,}824}=\frac{27}{55{,}296}=\mathbf{\frac{1}{2048}}$ **feet³ = 4.88 × 10⁻⁴ feet³**

1 ft³ = 1728 in³, so the ratio of in³ to ft³ is 1728 : 1 or $\frac{1728}{1}$.

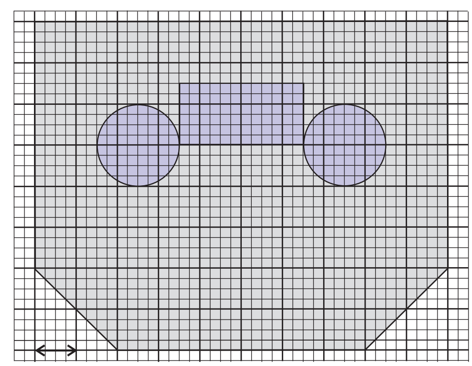
Write a proportion: $\frac{1728}{1}=\frac{x}{4.88\times10^{-4}}$. Cross-multiply to solve for x. $1728 \times 4.88 \times 10^{-4} = 1 \times x \Rightarrow$ $\boldsymbol{x}$ **= 0.843 in³**

Volume of model cylinder = $4.71\times\left(\frac{1}{24}\right)^3=4.71\times\frac{1}{13{,}824}=$ **3.41×10^{-4} feet³**

1 ft³ = 1728 in³, so the ratio of in³ to ft³ is 1728 : 1 or $\frac{1728}{1}$.

Write a proportion: $\frac{1728}{1}=\frac{x}{3.41\times10^{-4}}$. Cross-multiply to solve for x. $1728 \times 3.41 \times 10^{-4} = 1 \times x \Rightarrow$ $\boldsymbol{x}$ **= 0.589 in³**

Extensions

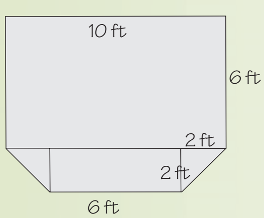


1 inch = 4 squares = 1 foot

1) Since 1 inch on the drawing is now 12 inches in reality, the scale factor from the actual objects to the diagram is $\frac{1}{12}$. Each 4 squares now represent 1 foot.
   The surface areas and volumes of the real rectangular prism and cylinders stay the same, but the dimensions of the models from before will be multiplied by a scale factor of 2. The new surface areas and volumes are summarized below:

	Original model surface area (in²)	New model surface area (in²) (original surface area × 2²)	Original model volume (in³)	New model volume (in³) (original volume × 2³)
Rectangular prism	5.616	22.464	0.843	6.744
Cylinder	3.931	15.724	0.589	4.712

2) The model stage is 3 inches high, using a scale of 1 inch to 1 foot. So the real stage is 3 feet high. To calculate the volume of the stage, split it into simple geometric shapes — two rectangular prisms, and two identical triangular prisms.



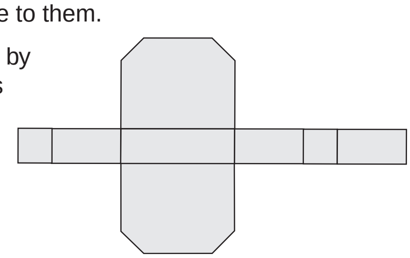
Volume of one triangular prism = base area × height = $\frac{1}{2}(2 \times 2) \times 3 = 6$ ft³

So, volume of stage = (10 × 6 × 3) + (6 × 2 × 3) + (6 × 2) = **228 ft³**

Open-Ended Extensions

1) The scale drawing students produce should match the stage that it is based on. Students should decide on a scale that allows them to conveniently fit the drawing on the grid paper that is available to them.

2) After calculating the volume and surface area of the stage, students should create a model by drawing the appropriate nets. They can either break the stage into simple geometric figures and build them separately before sticking them together, or they can create one net for the whole shape. For example, a model of the stage above could be constructed from this net:



   Students should calculate the volume and surface area of their model stage, and then use the scale factor to determine the volume and surface area of the actual stage, using the same method as above. They should get the same answers this way as they did by calculating the volume and surface area directly from measurements of the stage.

Chapter 8

Percents, Rounding, and Accuracy

How Chapter 8 fits into the K-12 curriculum

Section 8.1 — Percents		
Section 8.1 covers Number Sense 1.3, 1.6 **Objective: To change fractions and decimals to percents and to calculate percent increases and decreases**		
Previous Study In grades 5 and 6, students calculated given percents. In grade 6, they also solved problems involving discounts at sales, interest earned, and tips.	**This Section** Students begin by reviewing percents. They then go on to convert fractions and decimals into percents. Finally, they learn to increase and decrease quantities by given percents.	**Future Study** In Algebra I students will use algebraic techniques to solve more problems involving percents.

Section 8.2 — Using Percents		
Section 8.2 covers Number Sense 1.3, 1.7 **Objective: To use percents to solve real-life problems**		
Previous Study In grade 6 students calculated percents and solved real-life problems involving discounts, tips, and interest.	**This Section** Students apply their knowledge of percents to discounts, tips, tax, profit, and simple and compound interest.	**Future Study** In Algebra I, students will solve rate problems, work problems, and percent mixture problems.

Section 8.3 — Rounding and Accuracy		
Section 8.3 covers Number Sense 1.3, Mathematical Reasoning 2.1, 2.3, 2.7, 2.8, 3.1 **Objective: To round numbers and estimate calculations correctly and appropriately**		
Previous Study In grade 3 students first learned how to round numbers to the nearest ten, hundred, and thousand. In grade 4 they rounded decimal numbers. They have also considered reasonableness of answers since grade 3.	**This Section** Students review rounding, and then go on to see when it is appropriate to round numbers in calculations. They then consider when it is appropriate to use approximate answers, and when to use estimation.	**Future Study** In all further study of math and science, students will need to be adept at rounding answers, justifying the degree of accuracy, and checking that their answers are reasonable.

Pacing Guide – Chapter 8

40- to 50-Minute Class Periods

If your class periods are 40-50 minutes, we recommend allowing **17 days** for teaching Chapter 8.
As well as the **12 days of basic teaching**, you have **5 days** remaining to allocate 5 of the 7 optional activities (to be delivered at any appropriate point during the Chapter).

The table shows the 12 teaching days as well as all of the **optional days** you may choose for Chapter 8, in the order we recommend.

Day	Lesson	Description
Section 8.1 — Percents		
Optional		*Exploration — Photo Enlargements*
1	8.1.1	Percents
2	8.1.2	Changing Fractions and Decimals to Percents
3	8.1.3	Percent Increases and Decreases
Optional		*Assessment Test — Section 8.1*
Section 8.2 — Using Percents		
Optional		*Exploration — What's the Best Deal?*
4	8.2.1	Discounts and Markups
5	8.2.2	Tips, Tax, and Commission
6	8.2.3	Profit
7	8.2.4	Simple Interest
8	8.2.5	Compound Interest
Optional		*Assessment Test — Section 8.2*
Section 8.3 — Rounding and Accuracy		
Optional		*Exploration — Estimating Length*
9	8.3.1	Rounding
10	8.3.2	Rounding Reasonably
11	8.3.3	Exact and Approximate Answers
12	8.3.4	Reasonableness and Estimation
Optional		*Assessment Test — Section 8.3*
Chapter Investigation		
Optional		*Investigation — Nutrition Facts*

Accelerating and Decelerating

- To **accelerate** Chapter 8, allocate fewer than 5 days to the optional material. This will give you extra days to allocate to other Chapters. Note that you may use the remaining optional days at the end of the 160-day course.
- To **decelerate** Chapter 8, consider allocating more than 5 days to the optional Assessment Tests, Section Explorations, or Chapter Investigation, or spend longer teaching some Lessons. Also consider preparing students for difficult Lessons by reviewing previous coverage of math topics on related Skills Review Worksheets.
  Note that decelerating Chapter 8 will result in fewer days being available for teaching other Chapters.

90-Minute Class Periods

If you are following a block schedule with 90-minute class periods, we recommend allowing **8.5 days** for teaching Chapter 8.
The basic teaching material will take up **6 days**, and you can allocate the remaining **2.5 days** to the **optional material**.
To accelerate or decelerate a block schedule, follow the same advice as given above.

Section 8.1

Exploration — Photo Enlargements

Purpose of the Exploration

The purpose of the Exploration is to develop students' understanding of percent change. The activity allows students to see what an enlargement or reduction actually means, and that percent increases and decreases are proportional to the sizes of the originals.

Resources

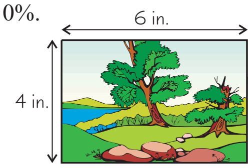
- rulers
- photos/magazine pictures (one per student, brought in from home if possible)
- set of 3 pictures in different sizes

Section 8.1 introduction — an exploration into: Photo Enlargements

You can enlarge or reduce photos — you have to increase or decrease the width and the length by the same percent though, or your image will be stretched. In this Exploration you'll figure out the dimensions a photo would have if it were enlarged or reduced by a given percent.

Example

A photograph that is 4 inches wide and 6 inches long is enlarged by 10%. What are its new dimensions?



Solution

Find **10% of the length** and **10% of the width**, then add these to the original dimensions.

10% of 6 in $= \frac{10}{100} \times 6 = 0.6$ in. 10% of 4 in $= \frac{10}{100} \times 4 = 0.4$ in.

New length = 6 + 0.6 = 6.6 in. **New width = 4 + 0.4 = 4.4 in.**

There's **another way** of doing this:

The original photo is 100%. Increasing it by 10% makes it **110%** of the original size. Find 110% of the length and 110% of the width. These are the new dimensions.

New length = 110% of 6 in $= \frac{110}{100} \times 6 =$ **6.6 in.**

New width = 110% of 4 in $= \frac{110}{100} \times 4 =$ **4.4 in.**

Exercises

Use the photograph that you have brought to class for the following exercises.

1. Measure the length and width of the photograph in inches. Write the dimensions in the first row of a copy of the table below.
2. Calculate the length and width of the photograph when it is enlarged by 20% and 50%, and reduced by 50% and 25%.

   Write the dimensions in your copy of the table.

Sample answers only — students' answers will depend on their photos

	Length	Width
100% — original size	6 inches	4 inches
Enlarged by 20%	7.2 inches	4.8 inches
Enlarged by 50%	9 inches	6 inches
Reduced by 50%	3 inches	2 inches
Reduced by 25%	4.5 inches	3 inches

Round Up

When you enlarge or reduce a photo, you have to keep the dimensions in proportion. So if you increase the length by 20%, you have to increase the width by 20% too. The longer dimension will increase by more inches than the shorter one does. That's what percent change is all about.

Strategic & EL Learners

Strategic learners may have difficulty with the fact that enlarging an image by 10% is different from enlarging it to 10% (which would be a reduction).

EL learners may have difficulty with the words "enlarged" and "reduced." Tell these students that they will be making their pictures bigger and smaller.

Universal access

Start off the activity by showing a picture to the class. Follow this by showing an enlarged and reduced version of the picture. Ask students what happens to the dimensions of the picture under both circumstances. This introduction can be followed by a discussion about proportion and percent change.

Universal access

Advanced students could be challenged to find the percent change needed to convert a photo to a given size.
For example, converting a photo with a length of 6 in. to one with a length of 30 in., requires a percent increase of 400% (30 is 500% of 6).

Common errors

Students will often forget to add the percentage increase to the original length, or forget to subtract the percentage decrease from the original length.

If students are asked to draw the new sizes of their photos, they may encounter difficulty in trying to reconcile decimal answers with the fraction based ruler.

Math background

Students should understand the basic idea of percents and be able to find simple percentages of quantities.

Lesson 8.1.1

Percents

This Lesson introduces and defines percents. It then explains how to calculate percents, and how to find the original number in a percent calculation. It also explains the concept that percents can be greater than 100.

Previous Study: In grades 5 and 6, students calculated given percents. In grade 6, they also solved problems involving sale discounts, interest earned, and tips.

Future Study: In Algebra I, students will learn to apply algebraic techniques to solve problems involving percents.

1 Get started

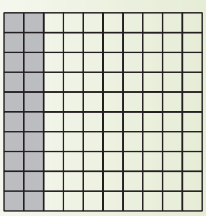
Resources:
- 10 × 10 grid of squares (or squared paper)
- newspapers/magazines/books/website data

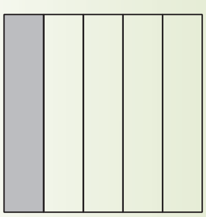
Warm-up questions:
- Lesson 8.1.1 sheet

2 Teach

Universal access

Some students need a visual model for percents. The most common and effective visual model is that of a 10 × 10 grid of squares. On the grid below, 20% is marked by coloring in 20 of the 100 squares. Students may have seen this grid in their work on decimals and fractions, and seeing the same picture used for all these ideas helps students understand that $20\% = \frac{20}{100} = 0.2$.





The grid can also be used to get fractions in their lowest form. On the lower of the above diagrams, the same 20% is marked, but now it is clear that the number of squares colored is $\frac{1}{5}$ of the entire grid.

As well as serving as a picture of percents, fractions, and decimals, this 100 square grid can also be used for money. If each grid is a dollar, then a penny takes up 1 square, a nickel takes up 5 squares, and so on.

Lesson 8.1.1

Section 8.1
Percents

California Standards:

Number Sense 1.3

Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

What it means for you:

You'll see what percents are and how they're related to fractions and decimals.

Key words:
- percent
- fraction
- decimal
- hundredth

Check it out:

On a penny you'll see the words one cent because a penny is one-hundredth of a dollar.

*You hear percents used a lot in everyday life. You might score 83% on a test, or a store might have a 20% off sale. A **percent** is really just a way to write a fraction — it tells you how many **hundredths** of a number you have.*

Percents Tell You How Many Hundredths You Have

A percent is a way to write a **fraction** as a **single number**. It tells you how many **hundredths** of something you have.

The word percent means **out of 100**.

Writing **one percent** or **1%** is the same as writing $\frac{1}{100}$, and writing 10% is the same as writing $\frac{10}{100}$.

Decimals can also be written as percents. The decimal 0.01 means "1 hundredth," so it's the same as 1%. There's more on converting decimals to percents next lesson.

Example 1

In a box of 100 pencils, 26 are blue. What percent of the pencils are blue?

Solution

The fraction of pencils that are blue is $\frac{26}{100}$.

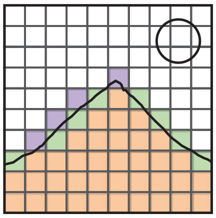
So you can say that **26% of the pencils are blue.**

It's useful to be able to visually estimate a percent.

Example 2

Estimate what percent of the picture on the right is covered by the mountain.

Solution



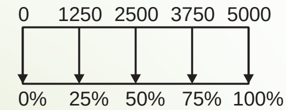
Trace the outline of the picture onto tracing paper. Draw a 10 × 10 grid over the tracing.

Count the number of squares the mountain covers. It covers **37 whole squares**, **8 half squares** and **4 quarter squares**.

$37 + (0.5 \bullet 8) + (0.25 \bullet 4) = 42$ squares.

The grid has 100 squares. So **the mountain covers about 42% of the picture.**

- **Strategic Learners**
  Ask students to draw a "percent box" as shown here. They should label the left lower edge 0%, the middle 50%, the right lower edge 100%, then label 25% and 75%. Ask them to think of a number from 1 to 1,000,000 and put that number above the 100% at the upper right edge. They then fill in appropriate numbers for 0%, 25%, 50%, and 75%. Repeat this activity with several different numbers, including some extending the box beyond the 100% marker.



- **English Language Learners**
  Put students into groups of four. Ask them to finish these sentences using information from the four people in their group: "100% of our group… 75% of our group… 50% of our group… 25% of our group… 0% of our group…" (For example, "75% of our group has brown eyes.") They should compare their results with another group. Repeat the activity for groups of 12 students.

Don't forget:
For a reminder on how to turn fractions into decimals see Section 2.1.

Guided Practice

In Exercises 1–3, write each fraction as a decimal and a percent.

1. $\frac{5}{100}$ 0.05, 5% **2.** $\frac{25}{100}$ 0.25, 25% **3.** $\frac{62}{100}$ 0.62, 62%

In Exercises 4–6, write each percent as a fraction in its simplest form.

4. 1% $\frac{1}{100}$ **5.** 50% $\frac{1}{2}$ **6.** 20% $\frac{1}{5}$

In Exercises 7–9, draw a 10 by 10 square. Shade in the given percent.

7. 8% See below. **8.** 27% See below. **9.** 100% See below.

Percents Can Be Greater Than 100

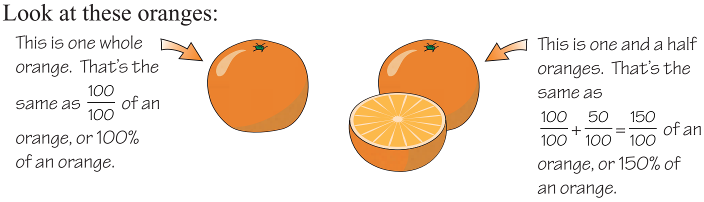
You can also have percents that are **bigger than 100**.

In the same way that $\frac{1}{100}$ is **1%**, $\frac{150}{100}$ is **150%**.

And just as **0.01** is the same as **1%**, **1.5** is the same as **150%**.

Percents bigger than 100 leave you with **more** than the **original number**.

Look at these oranges:



Guided Practice

In Exercises 10–12, write each fraction as a percent.

10. $\frac{120}{100}$ 120% **11.** $\frac{200}{100}$ 200% **12.** $\frac{1200}{100}$ 1200%

In Exercises 13–15, write each decimal as a percent.

13. 1.4 140% **14.** 3.6 360% **15.** 22.0 2200%

To Find a Percent of a Number You Need to Multiply

You already know that to find a fraction of a number, you **multiply** the number by the **fraction**.

Finding a **percent** of a number means finding a fraction out of 100 of the number.

Don't forget:
To multiply a fraction by an integer, just multiply the numerator of the fraction by the integer.

Example 3

What is 25% of 160?

Solution

Write out the percent as a fraction: $25\% = \frac{25}{100}$

$\frac{25}{100} \times 160 = \frac{4000}{100}$ **Multiply the fraction by the number**

$= 40$ **Simplify the answer**

2 Teach (cont)

Guided practice
Level 1: q1–9
Level 2: q1–9
Level 3: q1–9

Universal access
Since percents are so ubiquitous in our world, a nice introductory activity is to have students find two or three examples of percents being used in newspapers, magazines, books, or on websites. You could bring in a stack of materials for this, or have students do the research at home. Then all of these examples can be put out on display and be used as examples throughout the Chapter. For instance, you might examine poll results or business profits or the results of a science experiment. One way to quickly get real sources that use percents is to type in a percent (e.g. "20 percent") into an internet search engine.

Have students prepare displays after they have learned some of the material. In this case, you would want the students to analyze how the story is using the percent. What point is the author making in using the percent? Then you can have the students do the actual calculation. If 30% of the people surveyed favored a certain policy, then what was the actual number of people?

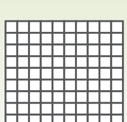
Another way to use these examples is for you to sort through them and tie them to the material being studied. Then when you present a topic, you have a real example found by a student.

Guided practice
Level 1: q10–15
Level 2: q10–15
Level 3: q10–15

Additional example
What is 90% of $8000?

$\frac{90}{100} \times 8000 = \frac{720{,}000}{100} = \$7200$

Solutions
For worked solutions see the Solution Guide

7.


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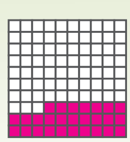

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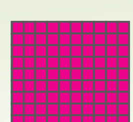


- **Advanced Learners**
  Ask groups of students to choose three newspapers and work out the percentage (in area) of advertising. Then they should sort the advertising into different categories (like cars, groceries, clothing), and work out the area of each category as a percent of all the advertising. Ask them to determine whether the percents vary with different newspapers.

2 Teach (cont)

Additional example

40% of patients trying a new medicine for headaches received relief. If the medicine worked on 360 people, how many people were in the trial altogether?

Call the number of people in the trial n.

Then $\frac{40}{100} \times n = 360$

$40n = 36{,}000$

$n = 900$

So there were 900 people in the trial altogether.

Guided practice

Level 1: q16–22
Level 2: q16–23
Level 3: q16–23

Independent practice

Level 1: q1–11
Level 2: q1–12
Level 3: q1–12

Additional questions

Level 1: p470 q1–10
Level 2: p470 q1–12
Level 3: p470 q1–13

3 Homework

Homework Book — Lesson 8.1.1

Level 1: q1–8
Level 2: q1–9
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
- Worksheet 16 — Percents

Finding the Original Amount — Write an Equation

Sometimes, you'll know how much a certain percentage of a number is and want to find the **original amount**.

Example 4

25% of a number is 40. What is the number?

Solution

Write out the percent as a fraction: $25\% = \frac{25}{100}$

Call the number that you're finding x.

$\frac{25}{100} \times x = 40$ **Multiply both sides by 100.**

$25x = 4000$ **Divide both sides by 25.**

$x = 160$

40 is 25% of 160.

Check it out:
$\frac{25}{100}$ is the same as $\frac{1}{4}$. If you used $\frac{1}{4}$ here, you'd get exactly the same answer.

Guided Practice

Find:

16. 10% of 40 **4** **17**. 60% of 250 **150** **18**. 64% of 800 **512**

In Exercises 19–21, find the value of x.

19. 50% of x is 30 **60** **20**. 4% of x is 7 **175** **21**. 65% of x is 130 **200**

22. Pepe was chosen as president of his class. He got 75% of the votes, and his class has 28 members. How many people voted for Pepe? **21**

23. The school basketball team won 60% of their games this season. If they won 24 games, how many did they play altogether? **40**

Independent Practice

In Exercises 1–4, write the fraction as a percent.

1. $\frac{10}{100}$ **10%** **2.** $\frac{50}{100}$ **50%** **3.** $\frac{23}{100}$ **23%** **4.** $\frac{156}{100}$ **156%**

In Exercises 5–8, write the percent as a fraction in its simplest form.

5. 25% $\frac{1}{4}$ **6.** 17% $\frac{17}{100}$ **7.** 75% $\frac{3}{4}$ **8.** 150% $\frac{3}{2}$

9. Out of 6000 nails made, 2% were faulty. How many were faulty? **120**

10. 150% of the people who were expected turned up at the school fair. If 340 people were expected, how many came? **510**

11. 20% of the students riding a bus are from Town A. If 6 students on the bus are from Town A, how many students ride the bus in total? **30**

12. 80 students auditioned for a play. After the audition, 20% were asked to come to a 2nd audition. 50% of those who came to the 2nd audition were cast. How many were cast? What percent of the original 80 is this? **8, 10%**

Now try these:
Lesson 8.1.1 additional questions — p470

Round Up

Percents say how many hundredths of something you have. You find a percent of a number by converting the percent to a fraction, and then multiplying this fraction by the number.

Solutions
For worked solutions see the Solution Guide

Lesson 8.1.2

Changing Fractions and Decimals to Percents

This Lesson describes how to change both fractions and decimals into percent form. It also includes word problems set in "real life" contexts which make use of these skills.

Previous Study: In grade 5, students found decimal and percent equivalents for common fractions, and explained why they represent the same value.

Future Study: In Algebra 1, students will learn to apply algebraic techniques to solve problems involving percents.

Lesson 8.1.2

Changing Fractions and Decimals to Percents

California Standards:
Number Sense 1.3
Convert fractions to decimals and percents and use these representations in estimations, **computations, and applications.**

What it means for you:
You'll see how to change fractions and decimals into percents.

Key words
- percent
- decimal
- fraction
- hundredth

Changing a fraction or a decimal into a percent is all about working out how many hundredths there are in it. The number of hundredths is always the same as the percent.

Changing Decimals to Percents

1% is the same as the **decimal 0.01** — which is one hundredth.

Each 1% is **one hundredth**. So the **number of hundredths** in the decimal is **the same** as the number of the percent.

0.04 = 4 hundredths = 4%
0.5 = 0.50 = 50 hundredths = 50%
0.31 = 31 hundredths = 31%
0.025 = 2.5 hundredths = 2.5%

The first two digits after the decimal point tell you the number of hundredths.

Any numbers that come after that are parts of 1%.

So to rewrite any **decimal** as a **percent,** you need to **multiply** it by **100** and add a **percent symbol**.

Don't forget:
To multiply a decimal by 100 you just move the decimal point two places to the right.

Example 1

Write 0.25 and 3.12 as percents.

Solution

0.25 • 100 = 25, so 0.25 = **25%**

3.12 • 100 = 312, so 3.12 = **312%**

Multiply decimals by 100 to get the number of the percent

Guided Practice

In Exercises 1–9 write each decimal as a percent.

1. 0.01 1%
2. 0.5 50%
3. 0.75 75%
4. 0.23 23%
5. 0.87 87%
6. 1 100%
7. 0 0%
8. 2.5 250%
9. 11.6 1160%

Don't forget:
1 can be rewritten as 1.00.

Changing Fractions to Percents

You've already seen that **1%** is the same as the fraction $\frac{1}{100}$. To change a **fraction** to a **percent** you need to work out how many **hundredths** are in it — because that's the same as the number of the percent.

1 Get started

Resources:
- tracing paper
- nutritional information from food packaging or the internet

Warm-up questions:
- Lesson 8.1.2 sheet

2 Teach

Universal access

A fun approach to take to this topic is to come up with survey-type questions for the students to answer. Then the results of the surveys can be converted to percents. Suppose you have 28 students in the class and 10 have their favorite color as blue. This means that $\frac{10}{28}$ or 35.7% of the class favors blue. Students are much more keenly interested in topics when they find a personal connection.

Another interesting question is to ask each student what percentage of the class they are. With 28 students, each student is $\frac{1}{28}$ or 3.57% of the class.

Universal access

A more involved way of capturing the students' attention is to investigate percents of fat, sugar, and salt in a selection of popular foods, using food packaging or the internet.

Ask students to calculate how many grams of each substance is provided by a 50 g serving, then work out what this is as a percent of the recommended daily amount. Students could present their findings to the class — are there any surprise findings?

Guided practice

Level 1: q1–5
Level 2: q1–7
Level 3: q1–9

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  This activity uses the percent box from the previous Lesson (see Strategic Learner notes in 8.1.1). Ask students to work in pairs. "Student A" names a simple proper fraction, and "Student B" estimates approximately where the fraction is between 0 and 1. The students then both convert the fraction to a percent and a decimal, and add these labels to the percent box.
- **English Language Learners**
  Have students write pairs of equivalent fractions, decimals, and percents onto cards, and place them all face down in a grid. Ask students to take turns to turn over pairs of cards. Players should keep matching pairs, and the player with the most pairs wins. Encourage the students to make the pairs difficult – for example, it's often thought that 4% is 0.4.

2 Teach (cont)

Universal access

Ask students to work out the percent wastage of carpet required for the classroom if carpet comes in a 12-foot-wide roll and is sold by the foot.

It may be useful to draw a scale drawing of the carpet roll on tracing paper, and a scale drawing of the room on a normal sheet of paper. For an 18-foot-wide room, students will be able to see that a 6-foot-wide section of carpet will go to waste.

Extend the example by including a carpet on a 15-foot roll, or by investigating other rooms in the school.

Common error

In converting a fraction to a percent, the common problems are either dividing the denominator by the numerator or forgetting to multiply by 100. For instance, if the fraction to be converted is $\frac{10}{28}$, some students might divide 28 by 10 and get 2.8% as a final answer. Others will divide and get 0.357 and report 0.357% as a final answer.
As with many common problems, a little estimating and predicting goes a long way. Have students think about $\frac{10}{28}$ — it's less than $\frac{1}{2}$, so that means it must be less than 50%. $\frac{7}{28}$ is 25%, so $\frac{10}{28}$ is between 25 and 50 percent.

Guided practice

Level 1: q10–14
Level 2: q10–16
Level 3: q10–18

Example 2

Write $\frac{3}{4}$ as a percent.

Solution

To write $\frac{3}{4}$ as a percent you need to change it to hundredths.
That means you want the denominator of the fraction to be 100.

So you need to multiply the top and bottom of the fraction by the number that will change the denominator to 100.

$4 \times n = 100$ — ***n* = the number you need to multiply by**

$n = 100 \div 4 = 25$ — **Divide both sides by 4**

Now turn the original fraction into a percent:

$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = \mathbf{75\%}$$

Example 3

Write $\frac{3}{8}$ as a percent.

Solution

This time the denominator of the fraction isn't a factor of 100.
So the fraction won't be a whole number of hundredths.
The most straightforward way of dealing with a fraction like this is to convert it into a decimal first.

$$\frac{3}{8} = 3 \div 8 = 0.375$$

Now change the decimal to a percent by multiplying it by 100:

$0.375 \times 100 = 37.5$, so $0.375 = \mathbf{37.5\%}$

Guided Practice

In Exercises 10–18 write each fraction as a percent.

10. $\frac{1}{100}$ **1%** **11.** $\frac{100}{100}$ **100%** **12.** $\frac{27}{100}$ **27%**

13. $\frac{1}{2}$ **50%** **14.** $\frac{2}{5}$ **40%** **15.** $\frac{3}{2}$ **150%**

16. $\frac{7}{16}$ **43.75%** **17.** $\frac{1}{1000}$ **0.1%** **18.** $\frac{53}{400}$ **13.25%**

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
Have students collect data on a favorite athlete's performance. Ask them to work out what percent of races the athlete has won, and how his or her performance compares with previous seasons' and that of other athletes. Students could adapt this activity for team sports or field events which match their interests more closely.

Check it out:
In real-life questions, it's important to make sure that you get the right numbers in the numerator and the denominator of the fraction.
The phrase "out of" in the question often gives you a clue where the numbers should go — if it's "x out of y," then x is the numerator and y is the denominator. If there isn't an "out of" in the problem, try to reword it a bit so that there is. For example, in Guided practice Ex. 20, you could reword the problem to say, "15 out of 24 seventh graders go to camp."

Using Fractions in Real Life

Percentages are useful for reporting numbers. The meaning of 96% is usually easier for people to understand than the meaning of $\frac{585}{610}$.

Example 4

Tamika is a professional basketball player. She makes 552 out of 625 free throws in a season. What percent of her free throws does she make?

Solution

Tamika made 552 out of 625 free throws. Written as a fraction that's $\frac{552}{625}$.

625 isn't a factor of 100. So first turn the fraction into a decimal.

$$\frac{552}{625} = 552 \div 625 = 0.8832$$

Now change the decimal to a percent by multiplying it by 100:
$0.8832 \times 100 = 88.32$

Tamika made 88.32% of her free throws.
You could round this to **88%**.

Guided Practice

19. In a set of napkins, 18 out of 24 are blue. What percent is this? **75%**

20. In a class of 24 7th graders, 15 go to camp. What percent is this? **62.5%**

21. James buys a ball of string that is 5 m long. Wrapping a parcel he uses a piece that is 0.8 m long. What percent of the string has he used? **16%**

22. A clothing manufacturer makes 3000 T-shirts. If 213 are returned because the color is wrong, what percent are the right color? **92.9%**

Independent Practice

In Exercises 1–4, write each decimal as a percent.

1. 0.05 **5%** **2.** 0.2 **20%** **3.** 3.2 **320%** **4.** 0.235 **23.5%**

5. Puebla and Mark are changing the decimal 0.5 into a percent. Mark says 0.5 = 5%. Puebla says 0.5 = 50%. Who is correct?
Puebla is right 0.5 = 50 hundredths = 50%

In Exercises 6–9, write each fraction as a percent.

6. $\frac{6}{100}$ **6%** **7.** $\frac{0}{100}$ **0%** **8.** $\frac{1}{5}$ **20%** **9.** $\frac{5}{32}$ **15.625%**

10. Out of 50 dogs that are walked every day in the local park, 20 are labradors. What percent is this? **40%**

11. Mandy surveyed 96 seventh graders on their favorite cafeteria meal. 33 students responded "spaghetti bolognese." What percent is this? **34.375%**

12. Tyrone is saving up $80 to buy some hockey skates. He has already saved $47.20. What percent is this of the total that he needs? **59%**

Now try these:
Lesson 8.1.2 additional questions — p470

Round Up

Knowing how to change a decimal or a fraction to a percent will often come in handy. Percents are far easier to compare than fractions and decimals because they're all measured out of one hundred.

2 Teach (cont)

Additional examples

1. Morris is on his school's baseball team. This season he has struck out 37 out of 215 batters he has faced. What percent of batters has he struck out?
Morris has struck out 37 out of 215 batters. So the fraction of strike outs is $\frac{37}{215}$. $\frac{37}{215} \times 100 = \frac{3700}{215} = 17.21$. Morris has struck out 17.21% of the batters he faced.

2. Ms. Smith has received payments from 85 out of 130 students signed up for a field trip. What percent of students have paid Ms. Smith?
85 out of 130 students have paid Ms. Smith. $\frac{85}{130} \times 100 = 65.4$ (to 3 d.p.). 65.4% percent of students have paid Ms. Smith.

3. Amelia's monthly sales target is 300 items. She sells 342 items by the end of the month. What percent of her monthly sales target has Amelia achieved?
She has sold 342 items out of her target of 300. $\frac{342}{300} \times 100 = 114$. Amelia has achieved 114% of her monthly sales target.

Guided practice
Level 1: q19–21
Level 2: q19–22
Level 3: q19–22

Independent practice
Level 1: q1–11
Level 2: q1–12
Level 3: q1–12

Additional questions
Level 1: p470 q1–10
Level 2: p470 q1–11, 13
Level 3: p470 q1–14

3 Homework

Homework Book — Lesson 8.1.2
Level 1: q1–7
Level 2: q2–9
Level 3: q2–9

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 16 — Percents

Solutions
For worked solutions see the Solution Guide

Lesson 8.1.3

Percent Increases and Decreases

In this Lesson, students increase and decrease quantities by given percents. They also describe changes as percents, and see how percents can be used to compare changes fairly. Students apply all of these ideas to real-life contexts.

Previous Study: In grades 5 and 6, students calculated given percents. In grade 6, they also solved problems involving sale discounts, interest earned, and tips.

Future Study: In Algebra I, students will learn to apply algebraic techniques to solve problems involving percents.

1 Get started

Resources:
- a set of about 120 identical cubes/counters/pennies
- 10 × 10 grids
- percent/fraction/decimal cards from the English Language Learners activity in the previous Lesson

Warm-up questions:
- Lesson 8.1.3 sheet

2 Teach

Universal access

Some students will benefit from demonstrations with manipulatives to help them understand the meaning of percent increase and decrease.

Start with 100 identical cubes, counters, or pennies. These are best arranged on a 10 × 10 grid for clarity. Then add 15 to this set. Explain that this is a 15% increase in cubes, because 15 cubes is 15% of the original 100 cubes. Repeat this with other percent increases.

Next move on to starting with 50 cubes and adding 8. 8 is not 8%, but is 16% of the original 50 cubes. So it is a 16% increase.

Guided practice

Level 1: q1–4
Level 2: q1–5
Level 3: q1–5

Lesson 8.1.3

Percent Increases and Decreases

California Standards:

Number Sense 1.3
Convert fractions to decimals and percents and **use these representations in** estimations, **computations, and applications.**

Number Sense 1.6
Calculate the percentage of increases and decreases of a quantity.

What it means for you:
You'll see how to use percents to show how much a quantity has gone up or down by.

Key words:
- percent
- increase
- decrease
- compare

When a number goes up or down, you can use percents to describe how much it has changed by. This can come in useful in real-life situations like comparing price rises or working out sale discounts.

You Can Increase a Number by a Given Percent

You can increase a number by a certain **percent** of itself.
So, say if you want to increase a number by 10%, you have to **work out what 10% is**, then **add this to the original number**.

Example 1

Increase 50 by 20%.

Solution

First work out 20% of 50:

$$20\% \text{ of } 50 = \frac{20}{100} \times 50 = 0.2 \times 50 = 10$$

This is the amount that you need to **increase** 50 by:

$$50 + 10 = 60$$

So, 50 increased by 20% is 60.

Check it out:
"50 increased by 20%" is the same as "120% of 50."
Both are 100% of 50, plus 20% of 50.
So you could do this calculation by multiplying 50 by 1.2 — because 1.2 is the decimal equivalent of 120%.

Example 2

A photograph with a length of 14 cm is enlarged. This increases its length by 8%. What is the final length of the enlarged photograph?

Solution

First work out 8% of 14 cm:

$$8\% \text{ of } 14 \text{ cm} = \frac{8}{100} \times 14 \text{ cm} = 0.08 \times 14 \text{ cm} = 1.12 \text{ cm}$$

This is the amount that you need to **increase** 14 cm by:

$$14 \text{ cm} + 1.12 \text{ cm} = 15.12 \text{ cm}$$

The length of the enlarged photograph is 15.12 cm.

Check it out:
Percents don't have any units. To find a percent you will always be dividing a number by another number that has the same units. So they cancel out.

Guided Practice

In Exercises 1–4, find the total after the increase.

1. 100 is increased by 10% 110 **2.** 20 is increased by 5% 21
3. 165 is increased by 103% 334.95 **4.** 40 is increased by 20.5% 48.2

5. Sarah goes out for lunch. Her bill comes to \$15. She wants to leave an extra 17% as a tip for the server. How much should Sarah leave in total? \$17.55

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Do "error analysis" using common errors in percent problems. Have students work with a partner to find the error and show how to do the problem correctly. For instance, give students the example: "What percent of 80 is 160?" **80 ÷ 160 × 100 = 50%.** They should identify the fact that 160 should have been divided by the original number, 80, to get 200%.
- **English Language Learners**
  The Universal access activity described on the previous page is particularly useful for communicating the concept of percent increase to English language learners.

You Can Describe an Increase as a Percent

When a number goes up, you can give the increase as a percent of the **original number**.

Example 3

A loaf of bread has 24 slices. As a special buy, a larger loaf is sold, which contains 27 slices. What is the percent increase in the number of slices?

Solution

First find the **increase** in the number of slices:
$27 - 24 = 3$

Call x the percent increase and write an equation.
x% of 24 is 3

$$\Rightarrow \frac{x}{100} \times 24 = 3$$

$x \times 24 = 300$ **Multiply both sides by 100.**

$x = 12.5$ **Divide both sides by 24.**

The number of slices has increased by 12.5%.

Guided Practice

6. Reynaldo has 140 marbles. He buys 63 more. By what percent has he increased the size of his marble collection? **45%**

7. A company increases its number of staff from 1665 to 1998. What is this as a percent increase? **20%**

You Can Decrease a Number by a Given Percent Too

You can also **decrease** a number by a **percent** of itself.

Example 4

Decrease 80 by 15%.

Solution

First work out 15% of 80:

$$15\% \text{ of } 80 = \frac{15}{100} \times 80 = 0.15 \times 80 = 12$$

This is the amount you **decrease** 80:

$80 - 12 = 68$

So, 80 decreased by 15% is 68.

Check it out:
You could also do this by multiplying 80 by 0.85. 0.85 is the decimal equivalent of 85% — and finding 85% of a number is the same as decreasing it by 15%.

2 Teach (cont)

Common error
Students often find the change as a percent of the new amount, rather than the original amount.
For instance, in Example 3, they find 3 out of 27 slices as a percent.

If students make this error, try the Universal access activity described on the previous page, so that students gain a fuller understanding of percent change.

Additional example
An airline ticket cost $178 one week and the next week it cost $204. What is the percent increase in the cost of the airline ticket?

14.6%.

Guided practice
Level 1: q6
Level 2: q6–7
Level 3: q6–7

Universal access
Give advanced students problems where they are given the final amount and the percent increase (or decrease). They then have to find the original amount.
For example:

"Profits of a company are $39,960 and up 8% from the previous year. What were the profits last year?"

$39,960 ÷ 1.08 = $37,000

If students find this one-step method difficult to grasp, encourage them to use the following algebraic method, where x is the original amount.

Let last year's profit be x.
8% of x = 0.08x
This year's profits = last year's profits + 8% of last year's profits
$39,960 = x + 0.08x
$39,960 = x(1 + 0.08)
x = $39,960 ÷ 1.08 = $37,000

Solutions
For worked solutions see the Solution Guide

Advanced Learners

Ask students to research the population of a chosen town or county over the past 100 years. They should calculate the percent increase or decrease in the population decade by decade. Then ask them to compare the results with towns which are similar or contrasting (for example, in terms of size or geographical location). They should plot all results on a line graph and write a report comparing them.

2 Teach (cont)

Guided practice

Level 1: q8–11
Level 2: q8–12
Level 3: q8–12

Common error

Students often wrongly conclude that a percent increase is canceled out if it is followed by a percent decrease of the same value.

For example, a _50% increase_ from 100 is: 100 + 50 = _150_.

A _50% reduction_ from 150 is:
150 – 75 = _75_.

The final result is smaller than the value of 100 which was used initially. The reason for this is that the "initial" value is 100 in the first calculation, but 150 in the second.

Additional example

For a clearance sale, a comforter that cost $89.99 was marked down to $59.99. What was the percentage decrease in the cost of the comforter?

33.3%.

Guided practice

Level 1: q13–15
Level 2: q13–15
Level 3: q13–15

Check it out:

Make sure you find the percent of the **original amount**. 12.8 feet is decreased to 9.6 feet, so you find the change as a percent of 12.8 feet, which was the original amount.

Check it out:

There's a formula you can use to work out the percent increase or decrease:

$$\text{Percent change} = \frac{|\text{original amount} - \text{new amount}|}{\text{original amount}} \times 100$$

Absolute value is used so you can use the formula for either a percent increase or decrease.

Guided Practice

In Exercises 8–11, find the total after the decrease.

8. 100 is decreased by 15% **85** **9.** 40 is decreased by 35% **26**
10. 37 is decreased by 8% **34.04** **11.** 10 is decreased by 3.9% **9.61**

12. Tandi has saved $152. She spends 25% of her savings on a shirt. How much does Tandi have left? **$114**

You Can Describe a Decrease as a Percent

When a number goes **down**, you can use a **percent** to describe how much it has changed by. The decrease is described as a percent of the **original number**.

Example 5

A river is 12.8 feet deep on January 1. By September 1, the depth has fallen to 9.6 feet. Find the percent decrease in the river depth.

Solution

First find the **amount** that the depth is **decreased** by:

12.8 feet – 9.6 feet = 3.2 feet

Call x the percent decrease and write an equation.

x% of 12.8 feet is 3.2 feet

$\Rightarrow \frac{x}{100} \times 12.8 \text{ feet} = 3.2 \text{ feet}$

$x \times 12.8 \text{ feet} = 320 \text{ feet}$ **Multiply both sides by 100.**

$x = 25$ **Divide both sides by 12.8 feet.**

The river depth has decreased by 25%.

Guided Practice

Find the percent decreases in Exercises 13–14.

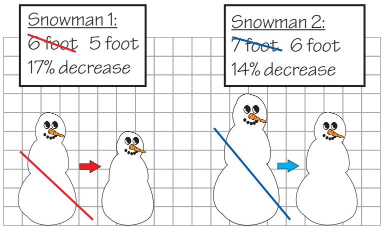
13. 90 is reduced to 81. **10%** **14.** 4 is reduced to 3.5 **12.5%**

15. Jon is selling buttons for a fund-raiser. He starts with 280 buttons and sells all but 21. What percent of his stock has Jon sold? **92.5%**

Use Percents to Compare Changes

You can use **percent increases** and **decreases** to **compare** how much two numbers have changed **relative to each other**. For example:

Snowman 1 and Snowman 2 have both lost the same amount in height as they've melted — 1 ft. But the **percent decrease** is greater for Snowman 1 — 1 ft is a bigger change **relative** to 6 ft than to 7 ft.



Solutions

For worked solutions see the Solution Guide

Example 6

In a store, a bagel is 40¢ and a loaf of bread is \$1.60. The store raises the price of both items by 5¢. Which has the larger percent increase in cost?

Solution

The price of both items is increased by 5¢.

So the percent increase in the cost of a bagel is:

$\frac{x}{100} \times 40¢ = 5¢ \Rightarrow (5¢ \times 100) \div 40¢ = 12.5$, so a <u>12.5%</u> increase.

And the percent increase in the cost of a loaf is:

$\frac{x}{100} \times 160¢ = 5¢ \Rightarrow (5¢ \times 100) \div 160¢ = 3.125$, so a <u>3.125%</u> increase.

You need to have the original value and the increase in the same units. \$1.60 has been converted to 160¢ here.

The bagel shows the larger percent increase in cost.

Check it out:

In terms of actual cost both items increased by the same amount, 5¢. But the bagel has increased by a greater proportion of its original cost than the loaf.

Guided Practice

16. Cindy has 250 baseball cards. Jim has 200 baseball cards. Both buy 50 extra cards. Whose collection increased by the larger percent? Jim

17. Ava and Ian have a contest to see whose sunflower will increase in height by the greatest percent. Ava's starts 10 cm high and grows to 100 cm. Ian's starts 20 cm high and grows to 110 cm. Who won? Ava

Independent Practice

In Exercises 1–6, find the amount after the percent change.

1. Increase 200 by 25% 250
2. Decrease 200 by 75% 50
3. Increase 49 by 7% 52.43
4. Decrease 82 by 56% 36.08
5. Increase 50 by 142.6% 121.3
6. Decrease 80 by 33.2% 53.44

7. At store A, apples used to cost \$1.50 a pound. Then the price rose by 6%. What is the new cost of a pound of apples? \$1.59

8. Kiona's brother Otis is 115.5 cm tall. The last time he was measured, his height was 110 cm. Find the percent increase in his height. 5%

9. Last year, School C had 120 6th grade students. This year they have 5% fewer 6th graders. How many fewer students is this? 6

10. Mr. Hill's house rental costs \$900 a month. He moves to a house with a rental of \$828 a month. Find the percent decrease in his rental. 8%

11. Duena collects comic books. 10 years ago, Comic A was worth \$70 and Comic B was worth \$40. Now Comic A is worth \$84 and Comic B is worth \$49. Which has shown the greater percent increase in value? Comic B

Now try these:

Lesson 8.1.3 additional questions — p470

Round Up

Percent increases and decreases tell you how big a change in a number is when you compare it to the original amount. It's useful to be able to work them out in real-life situations, especially when you're thinking about tips and discounts — and you'll learn more about them in Section 8.2.

2 Teach (cont)

Guided practice
Level 1: q16–17
Level 2: q16–17
Level 3: q16–17

Independent practice
Level 1: q1–9
Level 2: q1–10
Level 3: q1–11

Additional questions
Level 1: p470 q1–8
Level 2: p470 q1–10
Level 3: p470 q1–11

3 Homework

Homework Book — Lesson 8.1.3
Level 1: q1–3, 5–6
Level 2: q2–8
Level 3: q2–9

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 16 — Percents

Solutions
For worked solutions see the Solution Guide

Section 8.2

Exploration — What's the Best Deal?

Purpose of the Exploration

The goal of the Exploration is for students to apply the concepts of percent and discount to a real-life situation. Students will use their knowledge and given information to decide what is the best deal in a comparison of two sales. They'll also develop strategies for calculating percents mentally.

Resources

- advertising materials

Strategic & EL Learners

Strategic learners may benefit from practicing grouping percents into 10s and 5s. For instance, 35% = 10% + 10% + 10% + 5%.

EL Learners may be unfamiliar with the term "original." Tell these students that original simply means "first." The original price is the first price.

Universal access

Start off the Exploration by showing students some advertisements, for example, from newspapers and circulars. Show the students how some stores have sales that take a certain percent off the original price.

Common error

Students sometimes have difficulty remembering that the percentage that they calculate is actually the discount, rather than the new sale price.

Math background

Students need to understand the meaning of fractions, and know simple fraction-percent equivalences, such as $10\% = \frac{1}{10}$, $50\% = \frac{1}{2}$, and $25\% = \frac{1}{4}$.

Section 8.2 introduction — an exploration into: What's the Best Deal?

Discounts on sale items in stores are sometimes advertised as percents off the original prices and sometimes as dollar amounts off the original prices. By working out how much the percent discount is, you can find out which is the better deal among different sales.

You often won't have a calculator or pen and paper to hand when you want to work things like this out. So it's a good idea to develop strategies for calculating percent discounts **in your head**.

The easiest percents to find are **10%** and **50%**. From these you can find pretty much all the percent discounts that are commonly used in sales.

Example

A shirt is originally priced at \$20 and is on sale with 40% off. The shirt is also on sale for \$20 at a different store. You have a store coupon that you could use to save \$7.50 at this store. Which store will the shirt cost less at?

Solution

To answer this question, you can find 40% of \$20 and see if it's a bigger saving than \$7.50.

10% of \$20 = \$20 ÷ 10 = \$2

40% = 4 × 10% = 4 × \$2 = \$8 → This is a bigger saving than \$7.50.

So the shirt costs less at the first store (with 40% off).

Another way of doing this is to find 50% and 10%, then find 40% by subtracting the 10% amount from the 50% amount.

50% of \$20 = £20 ÷ 2 = \$10 and 10% of \$20 = \$20 ÷ 10 = \$2

So 40% = 50% – 10% = \$10 – \$2 = **\$8**

Exercises

For each of the following Exercises explain how you found the percent discount.

1. A pair of sunglasses is originally priced at \$15 and is on sale in a store with 15% off. They are also on sale on an internet site for \$13. Which is the best deal?
   15% = 10% + 5%. Store price = \$12.75. So the best deal is at the store.
2. Two stores are having a sale on the same video game originally priced at \$20. Store A has the game on sale for \$5 off. Store B has the game on sale for 30% off. Which store has the better deal? **30% = 3 × 10%. Store A price = \$15, Store B price = \$14. So the best deal is at Store B.**
3. A \$70 DVD player is on sale in two different stores. One store is selling the DVD player for 40% off. A second store is selling the player for \$35 off the original price. Which is the better deal? **40% = 4 × 10%, or 50% – 10%. First store price = \$42, second store price = \$35. So the best deal is at the second store.**
4. A store has a side table, originally priced at \$55, with \$10 off. The same table can also be found on the internet for \$60 and is on sale for 25% off. Which is the better deal?
   25% = 50% ÷ 2, or 100% ÷ 4. Store price = \$45, internet price = \$45. So they are both the same price.

Round Up

\$20 off might sound like a bigger saving than 5%, and often will be. But not if you're buying something very expensive. So it definitely pays to be able to find percents without a calculator.

Lesson 8.2.1

Discounts and Markups

In this Lesson, students apply what they have learned about calculating percents to the real-life contexts of sale discounts and store markups. They also learn how to calculate the sale price of an article that has had two discounts applied to it.

Previous Study: In grade 6, students calculated percents and solved problems involving sale discounts. In the previous Section, they looked at the meaning of percents greater than 100.

Future Study: Later in this Section, students apply their understanding of percents to other real-life contexts, such as tips, tax, commissions, and simple and compound interest.

Lesson 8.2.1

California Standards:

Number Sense 1.3
Convert fractions to decimals and percents and **use these representations in** estimations, **computations, and applications.**

Number Sense 1.6
Calculate the percentage of increases and decreases of a quantity.

Number Sense 1.7
Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

What it means for you:

You'll see how percents are used in real life to figure out discounts and markups on things that are being sold.

Key words:

- percent
- discount
- markup

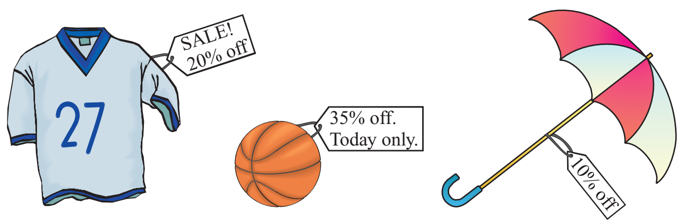
Section 8.2

Discounts and Markups

In the last Section, you learned all about percent increases and decreases. In real life they're used all the time. One thing that they're used for is working out how much items will cost in stores — these price changes are known as discounts and markups. And that's what this Lesson is all about.

A Discount is a Percent Decrease

When you go shopping, you might see items that are **on sale** — they cost less than their **regular price**.



The **difference** between the **regular price** and the **sale price** is called the **discount**. Discounts are often given as **percents** of the **original value** — so they're examples of **percent decrease**.

Example 1

A skirt costing \$28 is on sale at 20% off. What is its sale price?

Solution

Write the percent of the discount as a fraction: $20\% = \frac{20}{100}$

Work out the amount of the discount:

$\frac{20}{100} \times \$28 = 0.2 \times \$28 = \$5.60$

Now subtract the amount of the discount from the original price:
$\$28 - \$5.60 =$ **\$22.40**

OR

The skirt has been discounted by 20%. This means that its sale price is $(100 - 20)\% = 80\%$ of the original price.

Write the percent as a fraction: $80\% = \frac{80}{100}$

Find the reduced price:

$\frac{80}{100} \times \$28 = 0.8 \times \$28 =$ **\$22.40**

Both methods give the **same answer**. You can use whichever one you find easier to remember.

1 Get started

Resources:

- sale flyer from a local store
- internet computers
- catalogs
- school cafeteria menus and wholesale food price lists
- wholesale/store price lists
- grid paper

Warm-up questions:

- Lesson 8.2.1 sheet

2 Teach

Universal access

Display a sale flyer from a grocery or drug store that advertises a certain percent reduction. Using a real flyer from a local store will make the work more meaningful for students.

Ask the students what certain items would cost at the given discount.

It is also useful for students to determine what percent the sale price is of the retail price. This helps students understand the concept that decreasing a price by, say, 25% is exactly the same as finding 75% of it.

Universal access

Ask students to use the internet to investigate prices of goods such as CDs and toys in the US, and in a European country. Use the current exchange rate to convert the European price to a price in dollars. Ask students to work out the percent savings European visitors can make by buying goods in the US. Ask them if they can find any goods that are cheaper in Europe, and if so to calculate the percent saving they could make.

Additional examples

1. A toaster normally costing \$69 is on sale for 25% off. What is the sale price?

$(100 - 25)\% = 75\%$

$\frac{75}{100} \times \$69 = 0.75 \times \$69 = \$51.75$

2. A box of cereal normally costs \$2.70. It's reduced by 20%. What is the new price?

$(100 - 20)\% = 80\%$

$\frac{80}{100} \times \$2.70 = 0.8 \times \$2.70 = \$2.16$

- **Strategic Learners**
  Provide students with catalogs that show prices. Have students work with partners or in small groups to "spend" $100, and complete a chart listing the items they "purchased." Now tell them that the company is having a 10% sale. For each item, they should list the amount of the discount and the sale price. They should then calculate the total amount of money they saved.

- **English Language Learners**
  Review words and phrases that mean "reduced by 10%," such as "decreased by 10%," "discounted 10%," "10% off." Create a word list for students to refer to.

2 Teach (cont)

Guided practice

Level 1: q1–2
Level 2: q1–3
Level 3: q1–4

Additional example

The normal retail price of a CD is $14. It is in a sale with 30% off. You have a student discount card which gives you another 10% off. What will the new price of the CD be?

$\frac{30}{100} \times \$14 = \$4.20$
$\$14 - \$4.20 = \$9.80$
$\frac{10}{100} \times \$9.80 = \$0.98$
$\$9.80 - \$0.98 = \$8.82$

The CD will cost you $8.82.

Guided practice

Level 1: q5–6
Level 2: q5–6
Level 3: q5–7

Universal access

Provide a list of items and their prices at a local wholesaler and at a local store. Ask students to work out the percent savings on store prices when items are bought wholesale.

Ask advanced learners to extend this to calculate the percent savings when different quantities are purchased.

Guided Practice

1. A CD costing $16 goes on sale at 25% off. What is its sale price? **$12**

2. A wheelbarrow has been marked at a discount of 35%. What percent of the original price is it on sale for? **65%**

3. An MP3 player retailing for $90 has been marked down at 15% off. What is the sale price of the MP3 player? **$76.50**

4. A power tool that usually retails at $52 is being sold for $38.74. What is the percent discount on the power tool? **25.5%**

Work Out Two Discounts in a Row Separately

Sometimes the same item might be discounted twice.
You have to work out each discount separately, one after the other.

Check it out:
To find the amount of two discounts in a row you can't add the percents. The 2nd percent is taken after the 1st one has already been applied. So in Example 2 to find the new sale price, you found 10% of 50, and then 15% of 45. This isn't the same as finding 25% of 50.

Example 2

A shirt that usually costs $50 is on sale at 10% off. The store then takes an extra 15% off the discounted price. What is the shirt's new sale price?

Solution

First work out the price after the original discount:

$\frac{10}{100} \times \$50 = \$5$ first discount $\qquad \$50 - \$5 = \$45$

Then work out the price after the second discount:

$\frac{15}{100} \times \$45 = \$6.75$ second discount $\qquad \$45 - \$6.75 = \$38.25$

The new sale price is $38.25.

Guided Practice

5. A pair of sneakers that usually costs $100 is on sale at 50% off. The store takes another 20% off. What is the new sale price? **$40**

6. A computer costing $976 goes on sale at 25% off. The store offers an extra 15% discount for students. What would the student price be? **$622.20**

7. In a store, two sweaters both costing $60 go on sale. Sweater A is put on sale with 20% off, then another 10% is taken off. Sweater B is put on sale with 10% off, and then another 20% is taken off. Which is the least expensive sweater? **They are the same price ($43.20).**

A Markup is a Percent Increase

Stores buy goods at **wholesale prices**. Before selling them, they **increase** the **prices** of the goods in order to cover their expenses and make a profit. The prices that stores sell goods for are called the **retail prices**. The **difference** between the **wholesale** and **retail** price is called the **markup**.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask advanced learners to investigate the price of food served in the school cafeteria. Ask them to calculate the total wholesale price of the ingredients for a dish on the menu. They should then work out the percent markup for the dish. Ask different groups of students to investigate different dishes, so that the markups can be compared.

Example 3

The wholesale price of plain paper is \$3.20 a ream. If the markup is 75%, what is the retail price of a ream of plain paper?

Solution

Write the percent of the markup as a fraction: $75\% = \frac{75}{100}$

Work out the amount of the markup: $\frac{75}{100} \times \$3.20 = 0.75 \times \$3.20 = \$2.40$

Add the markup to the original price: $\$3.20 + \$2.40 = \$5.60$

OR

The markup is 75%. So the retail price is 175% of the wholesale price.

Write the percent as a fraction: $175\% = \frac{175}{100}$

Find the increased price: $\frac{175}{100} \times \$3.20 = 1.75 \times \$3.20 = \$5.60$

Don't forget:
Both methods will give you the same answer, so use whichever one you feel most comfortable with.

Guided Practice

8. The wholesale price of a case of oranges is \$13.50. If a retailer has an 80% markup, what will the retail price of a case of oranges be? \$24.30

9. A \$125 wholesale price chair is marked up 62%. Find its retail price. \$202.50

10. An item is marked up 50% from the wholesale price. What percent of the wholesale price is the retail price? 150%

11. A \$9.20 wholesale price toy retails at \$14.72. Find the percent markup. 60%

Check it out:
The method for finding a change as a percent was covered in the last Lesson.

Independent Practice

1. A hat worth \$70 is on sale at 25% off. What is its sale price? \$52.50

2. A kettle costing \$34 is put on sale at 10% off. The store then offers another 25% off the discounted price. What is the new sale price? \$22.95

3. In a sale you buy a basketball with 20% off a retail price of \$20, sneakers with 40% off a retail price of \$80, and a tennis racket with 20% off a retail price of \$100. What is the total? What percent discount is this on the full amount? \$144, 28%

4. A \$12 wholesale price bag is marked up 40%. Find its retail price. \$16.80

5. The wholesale price of a sweater is \$35. If the markup is 55% what is the retail price of the sweater? \$54.25

6. A shirt with a wholesale price of \$36 is marked up 40%. In store it is put on sale at 20% off its retail price. What is the shirt's sale price? \$40.32

7. A store buys 100 kg of pears for \$1.20/kg. They mark them up 50%. Half sell at retail price and half at 25% off. How much profit does the store make? \$37.50

Now try these:
Lesson 8.2.1 additional questions — p471

Round Up

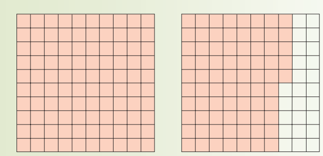
Discounts and markups are real-life examples of percent increase and decrease problems. Whether it's a discount or markup, you need to take care that you find the percent of the original price.

2 Teach (cont)

Common error

Mathematically, there's no difference between how percents of less than 100 are treated and how percents of greater than 100 are treated. Students, however, often freeze up when facing percents greater than 100.

Review the definition of percents covered in Lesson 8.1.1, and show visual examples of percents greater than 100. For example, have them shade in 175% of a 10 × 10 grid and discuss how this is equivalent to 100% + 75%. This can also be related to money — the 10 × 10 grid is worth \$1, and each small square is worth 1¢.



Guided practice

Level 1: q8–9
Level 2: q8–10
Level 3: q8–11

Independent practice

Level 1: q1–2, 4–5
Level 2: q1–6
Level 3: q1–7

Additional questions

Level 1: p471 q1–3, 7–9
Level 2: p471 q1–11
Level 3: p471 q1–11

3 Homework

Homework Book — Lesson 8.2.1

Level 1: q1–4, 6–7
Level 2: q1–9
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 16 — Percents
- Worksheet 19 — Discounts, Tips and Interest

Solutions

For worked solutions see the Solution Guide

Lesson 8.2.2

Tips, Tax, and Commission

In this Lesson, students develop their understanding of how percents are used in the real world. They solve problems in the contexts of adding tips to bills and calculating sales tax. They also look at how percents are involved in earning commission.

Previous Study: In grade 6, students solved problems involving tips. In the previous Section, they reviewed the meaning of "percent," and calculated percentage increases and decreases.

Future Study: Later in this Section, students use percents in the context of profit, and simple and compound interest. In Algebra I, they will apply algebraic techniques to solve percent mixture problems.

1 Get started

Resources:
- examples of sales receipts and restaurant bills
- individual whiteboards

Warm-up questions:
- Lesson 8.2.2 sheet

2 Teach

Universal access

Start the lesson with a general discussion of what students know about tips, tax, and commission.

Some students may think that a restaurant server must be tipped 15% and not know about other options depending on the quality of service, and some students may be surprised at the practice of commissions and will want to know about how prevalent the practice is. Also, students may not know that certain items are exempt from sales tax in California, such as food for consumption at home.

Concept questions

"Explain how you could mentally calculate 35% of an amount."

Find 10% of the amount and multiply it by 3 to get 30%. Then find half of the 10% amount and add this on to get 35%.

"Explain how you could mentally calculate 7.5% of an amount."

Find 10% of the amount and half it to get 5%. Half this new amount to get 2.5%. Now add the 5% and 2.5% amounts together.

Additional example

A customer receives a restaurant bill for $68.

a) If they want to leave a 20% tip, what is the amount of the tip?

$13.60

b) What will they pay in total to the restaurant?

$81.60

Lesson 8.2.2

Tips, Tax, and Commission

California Standards:

Number Sense 1.3
Convert fractions to decimals and percents and **use these representations in** estimations, **computations, and applications.**

Number Sense 1.6
Calculate the percentage of increases and decreases of a quantity.

Number Sense 1.7
Solve problems that involve discounts, markups, **commissions**, and profit and compute simple and compound interest.

What it means for you:
You'll learn about more real-life uses of percent increase.

Key words:
- tip
- tax
- commission
- percent

This lesson is about some more real-life uses of percent increase. You'll come across them in a lot of everyday situations, so they're definitely worth knowing about.

A Tip is Calculated as a Percent of a Bill

When you eat at a restaurant you would usually leave a **tip** for the person who waited on you. The standard amount to leave is **15% of your bill** — though you might **vary** this **percent** depending on the **quality** of the service.

Example 1

Finn's restaurant bill comes to $16. He wants to leave a 15% tip for the server. How much tip should he leave?

Solution

To find how much to leave for a **15%** tip, Finn should multiply his bill by $\frac{15}{100}$ or **0.15**.

$0.15 \times \$16 =$ **$2.40** **So Finn should leave a $2.40 tip.**

You Might Need to Work Out a Tip Mentally

Using **mental math**, 10% is an easier percent to work out than 15%. So find **10%** of the bill and leave that **plus half** as much again.

In Example 1, Finn might first work out that, as his bill is $16, 10% is $1.60.

100%

$1.60	$1.60	$1.60	$1.60	$1.60	$1.60	$1.60	$1.60	$1.60	$1.60

15%

So his tip should be $\$1.60 + (\frac{1}{2} \times \$1.60) = \$1.60 + \$0.80 =$ **$2.40**.

You might sometimes **estimate** a tip, but you should usually **round up** and not down. For example, if your bill was **$54.40** and you wanted to leave a **10%** tip, you could round the bill to **$60**, and leave a **$6 tip**.

Example 2

Raina's taxi fare is $17.61. She wants to give the driver a tip of about 10%. Estimate how much she should give as a tip.

Solution

To estimate the tip needed, **round up** Raina's $17.61 fare to $20.

$0.1 \times \$20 =$ **$2** **So, Raina should give a $2 tip.**

$10\% = \frac{10}{100} = 0.1$

Check it out:
There are other people who you might tip for their service — a taxi driver, a hairstylist, or a parking valet for instance.

Check it out:
If you rounded down to $50 you'd leave a tip of $5 — and that would be less than 10%.

Check it out:
It would also be reasonable here to round to $18 and leave a tip of $1.80.

- **Strategic Learners**
Using whiteboards, check that students can quickly calculate a 15% tip by mentally finding 10% of an amount then adding half of the 10% to get 15%. Ask the students to work out the tip for several different amounts. Have students round up or down the bill total and the cost of the tip where appropriate.

- **English Language Learners**
Check that students understand the key terms. For example, ask them, “Who pays a tip/a tax/a commission?,” “Who gets a tip/a tax/ a commission?” Ask students to work in pairs to list three situations in which a person might pay a tip, a tax, and a commission. Then ask them to list three situations in which a person might get the money from a tip, a tax, and a commission.

Guided Practice

1. Vance’s restaurant bill comes to $40. He leaves a 15% tip. How much is the tip? How much does he leave altogether? $6, $46

In Exercises 2–5, use mental math to find 15% of each amount.

2. $10 $1.50 **3.** $4 $0.60 **4.** $7 $1.05 **5.** $12.60 $1.89

6. Mrs. Clark’s haircut costs $48.59. She wants to leave a 20% tip. Estimate what amount would be sensible for her to leave as a tip. $10

Sales Tax is a Percent Increase on an Item’s Cost

When you buy certain items, you pay a **sales tax** on them — an **extra** amount of money on top of the cost of the item that goes to the government. A sales tax is calculated as a **percent** of the cost of the item.

Tax rates are set by local governments — so they **vary** from place to place.

Check it out:
Like a tip, a tax is an extra amount paid on top of the basic cost of the item.
It’s paid by the buyer.

Example 3

In Fort Bragg, sales tax is 7.75%. Pacho buys a book costing $12 before tax from a bookstore in Fort Bragg. How much sales tax will he pay?

Solution

To find the sales tax Pacho paid, find **7.75%** of the selling price.

$7.75\% = \frac{7.75}{100} = 0.0775$

$0.0775 \times \$12 = \$0.93$

Pacho pays **$0.93** sales tax on his book.

Example 4

On a vacation, you buy a souvenir that was $3.50 before tax. You were charged $3.71. What is the rate of sales tax here?

Solution

The amount of tax paid was $3.71 – $3.50 = $0.21

Let x = the rate of sales tax.

$\frac{x}{100} \times \$3.50 = \$0.21$ **Multiply both sides by 100**

$\$3.50x = \$21$ **Divide both sides by $3.50**

$x = 6$ **So the rate of sales tax is 6%.**

Check it out:
You’re working out what percent $0.21 is of $3.50.
So you could write the fraction $\frac{\$0.21}{\$3.50}$, then convert it to a decimal: $0.21 ÷ $3.50 = 0.06.
This is 6 hundredths, which is 6%.

Guided Practice

7. Hazel bought a calculator costing $29.50 (before tax) in Santa Rosa, where the sales tax is 8%. How much sales tax did she have to pay? $2.36

8. The sales tax in San Francisco is 8.5%, while in Oakland it is 8.75%. What is the price difference in buying a $21,000 (before tax) car in Oakland and San Francisco? $52.50

9. Dale bought a table costing $520 (before tax). He paid $37.70 sales tax on it. What was the rate of sales tax where he bought the table? 7.25%

2 Teach (cont)

Guided practice
Level 1: q1–3
Level 2: q1–3, 6
Level 3: q1–6

Universal access
Provide students with real examples of tipping and of sales tax.

For instance, you could bring in a check from a restaurant and demonstrate how a tip may be calculated.
You could also bring in a credit card receipt to show that there is a space for the customer to fill in the tip.

Sales receipts usually show the sales tax on an item, and some receipts will even have the computation.

Additional example
A customer buys a notebook priced at $12.95 from a store in Oakland, California. The sales tax in Oakland is 8.75%.

a) What is the amount of the sales tax on the notebook?

$\frac{8.75}{100} \times \$12.95 = \$1.13$

b) What will the customer pay in total?

$12.95 + $1.13 = $14.08

Guided practice
Level 1: q7–8
Level 2: q7–8
Level 3: q7–9

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to investigate this problem: A clothing salesperson is offered two jobs. The first pays a salary of $1000 a month plus 5% commission. The second offers $1500 a month with 2% commission. How much would the salesperson need to sell on average per month for the first job to pay more?

  (The equation they need to solve is $1000 + 0.05x = 1500 + 0.02x$; they'd need to sell $16,667 worth of merchandise, to the nearest dollar.) Students may find it useful to plot a line graph of value of merchandise sold against earnings for each job.

2 Teach (cont)

Common error

Most students are familiar with the concepts of tips and taxes, but they may be unfamiliar with commission.

Commission is different from tips and sales tax, because the worker takes the commission from the sales price (usually, the sales price before tax).

Some students will add the commission onto the sales price as is done for tax and tips. Encourage students to read the problems carefully, so they understand in what context they are working.

Guided practice

Level 1: q10–11
Level 2: q10–11
Level 3: q10–12

Independent practice

Level 1: q1–3, 6
Level 2: q1–4, 6
Level 3: q1–7

Additional questions

Level 1: p471 q1–2, 6–9
Level 2: p471 q1–11
Level 3: p471 q1–12

3 Homework

Homework Book — Lesson 8.2.2

Level 1: q1–6, 8
Level 2: q1–8
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 16 — Percents
- Worksheet 19 — Discounts, Tips and Interest

Commission is Paid to a Sales Agent

Commission is sometimes paid to **sales agents** — like realtors, or car salespeople. Realtors may get an amount of money for each property they sell — how much they get is calculated as a **percent** of the **selling price**.

Example 5

Althea is a realtor. She gets 6% commission on the sale price of a house. If a house sells for $210,000 how much commission will she receive?

Solution

To find the commission that Althea gets, find **6%** of the selling price.
$210,000 × 0.06 = **$12,600**
The realtor will receive **$12,600** commission.

Guided Practice

10. A shoe salesman receives a 10% commission on each pair of shoes he sells. What commission will he get on a pair costing $89? **$8.90**

11. A travel agent receives an 8% commission on all cruise sales. If a cruise ticket costs $1689 how much commission will the agent get? **$135.12**

12. An auctioneer takes a commission on all items sold. A lamp sells for $80, and the auctioneer gets $9.60. What percent commission does the auctioneer take? How much does the seller receive? **12%, $70.40**

Independent Practice

1. Shakia wants to leave her hairstylist a 25% tip. If her haircut cost $42, what tip should she leave? **$10.50**

2. In a restaurant, Mr. Baker's bill comes to $76.32. He wants to leave a tip of about 15%. Using mental math, estimate what tip he should leave. **$12**

3. In Santa Clara, the sales tax rate is 8.25%. If Nina buys a radio costing $40 before tax, how much sales tax will she pay? **$3.30**

4. Brad's restaurant bill comes to $25. He leaves a tip of $4. What percent of the bill has he left as a tip? **16%**

5. Leah buys a pair of jeans in Clearlake, where the sales tax rate is 7.75%. If the jeans cost $40 before tax, how much does she pay in total? **$43.10**

6. A salesperson gets 7.5% commission on each car sold. How much commission will the salesperson earn on a car costing $18,600? **$1395**

7. The sales tax rate in Roseland is 8%. Daniel eats in a restaurant in Roseland. The bill is $100 before tax. After the tax has been added he works out 25% of the total to leave as a tip. What tip does he leave? **$27**

Now try these:
Lesson 8.2.2 additional questions — p471

Round Up

Tips, tax, and commission are all just types of percent increases. Don't forget though — tips and tax are paid by the buyer and commission is paid by the seller. So you need to think carefully about what it is that you're being asked to find.

Solutions
For worked solutions see the Solution Guide

Lesson 8.2.3

Profit

In this Lesson, students develop their understanding of profit. They see how calculating profit as a percent of revenue allows businesses to compare their profit fairly each year.

Previous Study: In grade 6, students solved problems involving percents. In the previous Section, they reviewed the meaning of "percent" and calculated percentage increases and decreases.

Future Study: Later in this Section, students use percents in the context of simple and compound interest. In Algebra I, they will apply algebraic techniques to solve percent mixture problems.

Lesson 8.2.3

Profit

California Standards:

Number Sense 1.3
Convert fractions to decimals and percents and **use these representations in** estimations, **computations, and applications.**

Number Sense 1.7
Solve problems that involve discounts, markups, commissions, and **profit** and compute simple and compound interest.

What it means for you:
You'll learn what profit is and how to find profit as a percent of a company's sales.

Key words:
- profit
- revenue
- expenses
- percent
- sales

If you buy something and then sell it for more than the amount that it cost you, the extra money that you get is called profit. Because you end up with more money than you started with, you can think about profit as a percent increase.

Profit is the Amount of Money that a Business Makes

A business has to **spend money** buying stock and paying staff.
The amount of money that a business spends is called its **expenses**.
A business also **has an income** from selling its products or services.
The total amount of money that a business brings in is called its **revenue**.

The **profit** that a business makes is just the **difference between** its **revenue** and its **expenses**.

$$\textit{Profit} = \textit{Revenue} - \textit{Expenses}$$

Example 1

A film had a revenue of $55 million in ticket sales and $35 million in licensing agreements. It had expenses of $4 million in advertising and $48 million in production costs. What profit did the film make?

Solution

The film's total **revenue** = $55,000,000 + $35,000,000 = **$90,000,000**

The film's total **expenses** = $4,000,000 + $48,000,000 = **$52,000,000**

Profit = Revenue – Expenses
= $90,000,000 – $52,000,000 = **$38,000,000**

Check it out:
You know that
Profit = Revenue – Expenses.
Now add "Expenses" to both sides of the equation:
Profit + Expenses = Revenue.

Guided Practice

1. Janet buys a rare baseball card for $15. She later sells it to another collector for $18. What profit has she made? **$3**

2. This year a company had a revenue of $500,000 and $356,000 of expenses. What profit did the company make this year? **$144,000**

3. A school held a fund-raiser. They paid $200 to hire a band, and $400 for food. They took $1000 in ticket sales. How much profit did the event make? **$400**

4. A bookstore's total expenses in one year consisted of $300,000 to buy stock, and $150,000 to pay staff and cover other expenses. Their profit was $40,000. What was their total revenue? **$490,000**

1 Get started

Resources:
- internet computers

Warm-up questions:
- Lesson 8.2.3 sheet

2 Teach

Universal access
Ask students about ways in which they have made money. Some students may have made money babysitting, mowing lawns, shoveling snow, or working for their parents.

Consider the revenue and expenses for the examples students provide. For some ventures, such as babysitting, the revenue will equal the profit as there is no expense.
For other students, they may have to take a bus or other public transportation to work, thus having some expenses.
Or other students might have certain equipment or maintenance cost if they use their own machines.

Guided practice
Level 1: q1–3
Level 2: q1–3
Level 3: q1–4

Solutions
For worked solutions see the Solution Guide

- **Strategic Learners**
  Ask students to imagine that they and a partner are starting a business selling snacks after school. They should list five items they could sell, how much each item would cost to buy, a reasonable sales price, and the profit on each item. Ask them which items would be expected to make the greatest profit and the least profit.

- **English Language Learners**
  To help students understand the meaning of the term "profit", role-play the sale of an item from the manufacturer, to the wholesaler, to the store, and finally to the consumer. Re-price the article at each stage and calculate the profit.

2 Teach (cont)

Universal access

Ask students to use the internet to research recently released, big-budget films. They should find out how much each film cost to make and the box office revenue. From these figures, they can calculate the percent profit.

Remind students that DVD sales and rentals and merchandising, etc., will also boost profits.

Guided practice

Level 1: q5–7
Level 2: q5–7
Level 3: q5–8

Additional examples

1. A company has figured out that they make \$1.45 in profit for every \$19.95 sweatshirt they sell. What percent is their profit on each sweatshirt sale (to the nearest percent)?

$\frac{1.45}{19.95} = 0.073 = 7\%$, **to the nearest %**

2. A company made a profit of \$275,000 on sales of \$1,890,000. What percent of sales was the company's profit (to the nearest percent)?

$\frac{275{,}000}{1{,}890{,}000} \approx 0.15 = 15\%$, **to the nearest %**

Universal access

Students may find it interesting to find out the final selling prices of concert tickets on internet auction sites.

Assuming that the vendors paid the face price of the ticket, ask students to calculate which artists give the greatest percent profit when their tickets are sold.

Profits are Often Given as Percents

You can also work out a **percent profit**. This compares the amount of **profit** to the amount of **sales revenue**.

Example 2

A company makes a profit of \$90,000 on total sales of \$720,000. What is their profit as a percent of sales?

Solution

The company made \$90,000 **profit** on sales of \$720,000.
Write this as a **fraction**, and convert it to a **decimal**.

$$\frac{\$90{,}000}{\$720{,}000} = 0.125$$

Now change the decimal to a **percent** by **multiplying by 100**.
$0.125 \times 100 = 12.5$, so **their profit is 12.5% of their sales.**

Don't forget:
To find x% of a number, just multiply the number by $\frac{x}{100}$.

Guided Practice

5. Sayon's lemonade stand made a \$20 profit. He sold \$80 worth of lemonade. What profit did he make as a percent of sales? **25%**

6. A company made a profit of \$6000 on total sales of \$40,000. What was their profit as a percent of sales? **15%**

7. Sophia buys a set of books for \$75. She later sells the books to a collector for \$90. What percent profit has she made? **20%**

8. Company A made a 12% profit on sales of \$295,000. How much profit did they make? **\$35,400**

You Can Compare Profits Using Percents

Businesses often use **percents** to **compare** the **profits** that they have made in **consecutive years**. This shows how the company is performing over time.

Example 3

This year, Company B increased its profits by 5% over the previous year. If last year's profit was \$43,900, what was this year's profit?

Solution

Write the **percent** of the **increase** as a **fraction**: $5\% = \frac{5}{100}$

Work out the **amount** of the increase:

$$\frac{5}{100} \times \$43{,}900 = 0.05 \times \$43{,}900 = \$2195$$

Now **add** the amount of the increase to the original profit:
$\$43{,}900 + \$2195 =$ **\$46,095**

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to investigate the costs and ticket sales of the school's major fund-raisers. Ask them to find out which fund-raiser made the greatest percent profit. If there is a fund-raiser that is held regularly (say every year perhaps), ask students to investigate how the percent profits have changed over time. They could decide how best to represent this (perhaps as a line graph).

Example 4

Last year, Company C made profits of \$40,000. This year, they made profits of \$28,000. What was the percent decrease in their profits?

Solution

Find the **amount** of the **profit decrease**: \$40,000 – \$28,000 = \$12,000

Now **divide** the amount of the decrease by the first year's profits:

$$\frac{\$12,000}{\$40,000} = 0.3$$

Change the **decimal** to a **percent** by **multiplying by 100**:
0.3 × 100 =30, so they had a **30% decrease in profit**.

Guided Practice

9. This year, Company D increased its profits by 10% over last year. If last year's profits were \$12,000, what was this year's profit? **\$13,200**

10. Company E's profits fell by 4% this year compared to last year. If last year's profits were \$29,500, what were this year's profits? **\$28,320**

11. Last month, Company F made profits of \$1250. This month, they made profits of \$1500. Find the percent increase in their profits. **20%**

12. Last year, Company G made profits of \$200,000. This year, they made profits of \$192,000. Find the percent decrease in their profits. **4%**

Independent Practice

1. In one year, a company has a total revenue of \$185,000 and total expenses of \$155,000. What were the company's profits that year? **\$30,000**

2. A website selling clothes made a profit of \$7890 in a month. In the same month its revenue was \$12,390. Find its expenses for that month. **\$4500**

3. A toy store makes \$12,000 profit on sales of \$300,000. What percent profit has the store made? **4%**

4. A grocer buys \$270 of fruit. He sells it for \$283.50. What is his profit? What is his percent profit? **\$13.50, 5%**

5. This year, Company H's profits fell by 7% compared to the previous year. If last year's profit was \$22,500, what was this year's profit? **\$20,925**

6. Last month, a store made profits of \$4800. This month, they made profits of \$5400. What was the percent increase in their profits? **12.5%**

7. Your class organizes a dance as a fund-raiser. You spend \$100 hiring a DJ, \$180 on food, and \$40 on tickets and fliers. You have 50 tickets — if they all sell, what will you need to price them at to make a 25% profit? **\$8**

Now try these:
Lesson 8.2.3 additional questions — p471

Round Up

Profit is the money that a business is left with when you take away what it spends from what it takes in sales. Percent change in profit is a way of measuring the performance of a business over time.

2 Teach (cont)

Additional example

This year, a company's profits have decreased by 8% from the previous year's.
If the previous year's profit was \$88,500, what was this year's profit?

$\frac{8}{100} \times \$88,500 = \$7080$

$\$88,500 - \$7080 = \$81,420$

This year's profit was \$81,420.

Guided practice
Level 1: q9–10
Level 2: q9–11
Level 3: q9–12

Independent practice
Level 1: q1–4
Level 2: q1–6
Level 3: q1–7

Additional questions
Level 1: p471 q1–3, 5–6, 10
Level 2: p471 q1–3, 5–7, 9–10
Level 3: p471 q1–10

3 Homework

Homework Book — Lesson 8.2.3
Level 1: q1–5, 7
Level 2: q1–9
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM
These worksheets may help struggling students:
- Worksheet 16 — Percents
- Worksheet 19 — Discounts, Tips and Interest

Solutions
For worked solutions see the Solution Guide

Lesson 8.2.4

Simple Interest

In this Lesson, students learn the meaning of simple interest. They use a formula to calculate the amount of interest accrued in real-life situations such as savings accounts and bank loans.

Previous Study: Previously in this Section, students have used percents in a variety of real-life contexts including discounts, tips, tax, and profit.

Future Study: In the next Lesson, students will be introduced to compound interest, and will use a formula to calculate it.

1 Get started

Resources:

- leaflets from banks with information on their accounts and interest rates
- advertisements for loans from newspapers or magazines
- sample bills/bank statements showing interest being calculated
- internet computers

Teacher Resources CD-ROM

- Money Tiles

Warm-up questions:

- Lesson 8.2.4 sheet.

2 Teach

Universal access

If students haven't had experience with interest, it is worthwhile to do a hands-on demonstration.

Have students break into pairs, and using Money Tiles from the **Teacher Resources CD-ROM**, one student "borrows" $12 from the other. The "borrower" pays back $1.05 a month for a year. In other words, this is simple 5% interest.

At the end of 12 months, the partner who "loaned" the money now has $12.60. The 60 cents is interest, the time is 1 year, the principal is $12, and the interest rate is 5%.

Students can then get a feel, in this simplified activity, for what interest is like in the real world. When you take out a loan, you get some money up front and then pay it off slowly, but you have to pay more money than you borrow, which is the interest.

Lesson 8.2.4

Simple Interest

California Standards:

Number Sense 1.3
Convert fractions to decimals and percents and **use these representations in** estimations, **computations, and applications.**

Number Sense 1.6
Calculate the percentage of increases and decreases of a quantity.

Number Sense 1.7
Solve problems that involve discounts, markups, commissions, and profit and **compute simple** and compound **interest.**

What it means for you:
You'll see what interest is and how to work out how much simple interest you could earn over time.

Key words:
- interest
- simple interest
- principal
- interest rate

Check it out:
To invest just means to put money into something.

Interest is an important real-life topic because it's all about saving and borrowing money. If you keep your money in a savings account, the bank will pay you something just for keeping it there. The interest that you gain will be based on how much you put in — and that means it's another use of percent increase.

Interest is a Fee Paid For the Use of Money

When you keep money in a savings account, the bank **pays you interest** for the privilege of using your money.
When you **borrow** money from a bank, the bank **charges you interest** for the privilege of using their money.

Interest is a **fee** that you pay for using someone else's money.

The interest to be paid is worked out as a **percent** of the money invested or loaned. The **percent** that is paid over a given **time** is called the **interest rate**.

Simple Interest is Paid Only on the Principal

The amount of money you **put into** or **borrow from** a bank is called the **principal**. **Interest** that is paid **only** on the principal is called **simple interest**.

With simple interest, the **interest rate** tells you how much money you will get back every year as a **percent** of the **principal**.

For example: think about depositing **$100** in a savings account with a simple interest rate of **5%** per year. For **each year** you leave your money in the account, you will get **5%** of **$100** back.

Number of years	Interest earned	Total
0	$0	$100
1	$5	$105
2	$5	$110
3	$5	$115

Example 1

You deposit $50 in a savings account that pays a simple interest rate of 2% per year. How much interest will you get over 3 years?
How much will be in the account after 3 years?

Solution

First find 2% of $50: $\$50 \times \frac{2}{100} = \$1$. This is the amount of interest you will get each year.

So over 3 years you will earn: $3 \times \$1 = \$3$

After 3 years you will have: $\$50 + (3 \times \$1) = \$53$ in the account.

- **Strategic Learners**
  Ask students to research different savings accounts. What are the rates of interest? Is interest paid monthly or annually? Assuming that the interest is simple, which account will give the best return on $100 after one year? After five years?

- **English Language Learners**
  Use "take notes, make notes" to define the important terms from this page: interest, simple interest, interest rate, loan, fee, charge, principal. As well as a definition, give an example phrase for each term to show its use in a context.
  Students could be split into groups to perform simple role-plays using Money Tiles from the **Teacher Resources CD-ROM**.

Guided Practice

1. If you put money into a savings account which pays simple interest, will the amount of interest you get in the first year be the same as in the second year? Explain your answer. **See below**

2. You borrow $150 from a bank at a simple interest rate of 8% per year. How much interest will you pay in one year? **$12**

3. You deposit $200 in a savings account that pays a simple interest rate of 5% per year. How much interest will you get over 4 years? **$40**

4. You deposit $65 in a savings account that pays a simple interest rate of 4% per year. How much will be in your account after 4 years? **$75.40**

Use the Simple Interest Formula to Calculate Interest

Look back at **Example 1**. To work out **how much interest** you got over 3 years, you worked out the **percent** of the **principal** that you would get **each year** and **multiplied it by 3**.

So the calculation you did was:

$$(50 \times \frac{2}{100}) \times 3 = \$3$$

Don't forget:
You can remove the parentheses from this equation because of the associative property of multiplication — see Lesson 1.1.5.

Now think about what each part of that equation **represents**.

This is the principal.

This is the time that the money is in the account for.

$$50 \times \frac{2}{100} \times 3 = \$3$$

This is the interest rate written as a fraction.

This is the interest earned.

You can use this to figure out a **general formula** for finding **simple interest**. First assign a **variable** to stand for each part of the equation:

- P stands for the **principal**.
- r stands for the **interest rate (in % per year)**, written as a fraction or a decimal.
- t stands for **time (in years)**.
- I stands for the **amount of interest** that has built up.

To find the **amount of interest** that you got, you **multiplied** together the **principal**, the **interest rate**, and the **time** the money was in the account for.

Written as a **formula** this is:

$$I = Prt$$

2 Teach (cont)

Universal access

It can be helpful for students to make a rough estimate before calculating the answer. This will allow them to check that their answer looks reasonable.

For example:
"What is the interest paid on a $3060 loan over 2 years at a simple rate of 6%?"

Build up an estimate from known figures:
1% of $3000 is $30,
so 6% of $3000 is 6 × $30 = $180.
Over 2 years, that would be $360.

Now for the actual answer, the student might have put:
"$3060 × 6 × 2 = $36,720."

Comparing these answers will show the student that an error has clearly been made. In this case, the calculation should have used the decimal form of 6%, which is 0.06, not 6. This would give an answer of $367.20 which is much closer to the estimate.

Additional examples

1. $4200 was put into a savings account for 2 years with a simple interest rate of 6% per year. How much interest was earned?

In this case, P = $4200, r = 0.06 (6% as a decimal), and t = 2 (money was in for 2 years). So now use the formula:

$I = P \times r \times t$
$I = 4200 \times 0.06 \times 2$
$I = 504$

The amount of interest earned is $504

2. Frank received a loan of $6000 for 3 months at an 8% per year simple interest rate. How much interest did he pay?

P = $6000, r = 0.08 (8% as a decimal), and $t = \frac{1}{4}$ (since 3 months is $\frac{1}{4}$ or $\frac{3}{12}$ of a year). So now use the formula:

$I = P \times r \times t$
$I = 6000 \times 0.08 \times \frac{1}{4}$
$I = 120$

Frank paid $120 in interest to get the loan.

Solutions

For worked solutions see the Solution Guide

1. Yes. Simple interest is calculated as a percent of the principal, so you receive the same amount of interest every year.

Advanced Learners

Find out about the Federal Reserve base rate. How does this influence the economy? What factors affect the decision to hold or change the base rate? To extend this activity, research the base rates of central banks around the world, including those of developing countries.

2 Teach (cont)

Common errors

There are several common problems students have when applying the simple interest formula:

1. For an interest rate of 5%, students will often substitute 5 for r instead of 0.05. Remind students of the connections between percents, decimals, and fractions.

For example: $8\% = 0.08 = \frac{8}{100}$

2. Students can confuse the interest earned with the total amount. For example, when a question asks for the total amount of savings, students may correctly use the simple interest formula but not realize that they have to then add this to the original amount.

Guided practice

Level 1: q5–6
Level 2: q5–7
Level 3: q5–8

Independent practice

Level 1: q1–3
Level 2: q1–5
Level 3: q1–6

Additional questions

Level 1: p472 q1–4
Level 2: p472 q1–6
Level 3: p472 q1–7

3 Homework

Homework Book — Lesson 8.2.4

Level 1: q1–7
Level 2: q1–9
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:

- Worksheet 16 — Percents
- Worksheet 19 — Discounts, Tips and Interest

Don't forget:

When you write the interest rate, you can write 6% as either the fraction $\frac{6}{100}$ or the decimal 0.06.

Example 2

You deposit \$276 in a savings account that has a simple interest rate of 6% per year. How much interest will you get over 5 years?

Solution

$I = Prt$

$I = \$276 \times 0.06 \times 5$ **Substitute the values into the formula**

$I = \$82.80$

Over 5 years you'll earn \$82.80 interest.

Guided Practice

5. You borrow \$57 from a bank at a simple interest rate of 9% per year. How much interest will you pay in one year? **\$5.13**

6. You deposit \$354 in a savings account that pays a simple interest rate of 2.5% a year. How much interest will you get over 7 years? **\$61.95**

7. You deposit \$190 in a savings account that pays a simple interest rate of 4% a year. How much will be in your account after 4 years? **\$220.40**

8. You put \$520 in a savings account with a simple interest rate of 6% a year. You take it out after 6 months. How much interest will you get? **\$15.60**

Check it out:

Read each question carefully. Sometimes you'll be asked to just find the amount of interest, other times you'll be asked to find the total amount in the account.

To find the total amount that an account will contain, first calculate the interest. Then add the interest to the principal.

Independent Practice

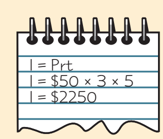
1. You borrow \$75 from a bank at a simple rate of 9% per year. How much interest will you pay over 7 years? **\$47.25**

2. You deposit \$64 in a savings account that pays a simple interest rate of 2.5% a year. How much will be in your account after 17 years? **\$91.20**

3. Ian put \$4000 into a short-term investment for 3 months. The simple interest rate was 5.2% per year. How much interest did Ian earn? **\$52**

4. Luz borrows money from a bank at a simple interest rate of 5% a year. After 4 years she has paid \$50 interest. How much did she borrow? **\$250**

5. Ty puts \$50 in a savings account with a simple interest rate of 3% a year. He works out what interest he will get in 5 years. His calculation is shown on the right. What error has he made? How much interest will he get? **See below.**



6. Anna puts \$50 in a savings account that pays a simple interest rate of 5% a year. After 4 years she takes out all the money, and puts it in a new account that pays a simple interest rate of 6% a year. She leaves it there for 5 years. How much will Anna have in total at the end of this time? **\$78**

Now try these:

Lesson 8.2.4 additional questions — p472

Round Up

Interest is money that is paid as a fee for using someone else's money. Simple interest means that each year you get back a fixed percent of the initial amount you invested. Make sure you understand how simple interest works. You'll use a lot of the same math in the next lesson on compound interest.

Solutions

For worked solutions see the Solution Guide

5. Ty has used 3 instead of 0.03 to represent 3% in his calculation. The correct calculation is: \$50 × 0.03 × 5 = \$7.50

Lesson 8.2.5

Compound Interest

In this Lesson, students learn the meaning of compound interest.
They use a formula to calculate the amount of compound interest in real-life situations.

Previous Study: In the previous Lesson, students learned the meaning of simple interest, and used a formula to calculate it in real-life situations.

Future Study: In Algebra I, students solve rate problems, work problems, and percent mixture problems.

Lesson 8.2.5

Compound Interest

California Standards:

Number Sense 1.3
Convert fractions to decimals and percents and **use these representations in** estimations, **computations, and applications.**

Number Sense 1.6
Calculate the percentage of increases and decreases of a quantity.

Number Sense 1.7
Solve problems that involve discounts, markups, commissions, and profit and **compute** simple and **compound interest.**

What it means for you:
You'll learn about compound interest, and how to work out how much compound interest you could earn over time.

Key words:
- compound interest
- principal
- interest rate
- annually
- quarterly
- monthly

In the last Lesson, you saw what interest was and how to work out simple interest. There's another type of interest that you need to know about called compound interest. And that's what this Lesson is about.

Compound Interest is Paid on an Entire Balance

Simple interest is **only** paid on the **principal**. So although the **balance** of your account **rises**, the **amount** of interest you get is the **same each year**.

Compound interest is paid on the **principal** and on any **interest you've already earned**. Interest is added (or compounded) at regular intervals — and the amount paid is a percent of **everything** in the account.

Think about putting $100 in an account with an interest rate of 5% compounded yearly. **Each year** you leave your money in the account you will get **5%** of **the account's balance** paid into your account.

Number of years	Interest earned	Total
0	$0	$10,000
1	($10,000 × 0.05) = $500	$10,500
2	($10,500 × 0.05) = $525	$11,025
3	($11,025 × 0.05) = $551.25	$11,576.25

Interest can also be worked out **daily**, **monthly**, or **quarterly**. It often isn't an exact number of cents — so the bank **rounds it** to the nearest cent.

Check it out:
Quarterly means every three months — this is a quarter of the year.

Check it out:
0.25 is used here because the interest is compounded quarterly.
If it were compounded yearly, the multiplication factor would be 1.
And if it were monthly, the multiplication factor would be one-twelfth.

Example 1

You put $80 into an account with an interest rate of 5% per year, compounded quarterly. What is the account balance after 6 months?

Solution

After the first 3 months you'll get:
$I = Prt = \$80 \times 0.05 \times 0.25 =$ **$1 interest**.
So you'll have **$81** in the account.

Over the next 3 months, you'll get:
$I = Prt = \$81 \times 0.05 \times 0.25 \approx$ **$1.01 interest**.
So you'll have **$82.01** in the account.

In 6 months you earned $2.01 interest and have **$82.01** in the account.

Guided Practice

1. If you put money into a savings account which pays compound interest, will the amount of interest you get in the first year be the same as in the second year? Explain your answer. **See below**

2. You borrow $100 from a bank at an interest rate of 5% a year compounded annually. How much interest do you pay in 2 years? **$10.25**

1 Get started

Resources:
- leaflets from banks with information on their accounts and interest rates
- advertisements for loans from newspapers or magazines
- sample bills/bank statements showing interest being calculated
- internet computers

Warm-up questions:
- Lesson 8.2.5 sheet.

2 Teach

Universal access
In the student textbook, the distinction between simple and compound interest is illustrated through a worked example at the end of this Lesson. For students who are struggling to understand the differences between the two types of interest, it may be useful to do this illustration at the start of the Lesson. See the Additional example below. This approach may help students to understand the concept before going into the details of how to calculate it.

Additional example
Find the amount after 5 years when $1000 is invested
a) using 3% simple interest
b) 3% compound interest.

a) Principal: $1000

Year 1 Interest	$30
Year 2 Interest	$30
Year 3 Interest	$30
Year 4 Interest	$30
Year 5 Interest	$30

Total: $1150

b) Principal: $1000

Year 1 Interest	$30
Year 2 Interest	$30.90
Year 3 Interest	$31.83
Year 4 Interest	$32.78
Year 5 Interest	$33.77

Total: $1159.27

Guided practice
Level 1: q1–2
Level 2: q1–2
Level 3: q1–2

Solutions
For worked solutions see the Solution Guide

1. No. Compound interest is calculated on the total amount in the account at the end of the year. At the end of the first year, the interest will be based on the principal, but in the next year, the interest will be calculated on the new total in the account, which includes the first year's interest. So in the second year you will get more interest.

- **Strategic Learners**
  Ask the class to think of real-life situations where interest is calculated. Examples may include bank loans, mortgages, and credit cards. Discuss each example — what type of interest is used? How often is it calculated? Over what period is the loan paid off? Explore why it would be better to pay off a loan by making large payments over a short period than small payments over a long period.

- **English Language Learners**
  Review the key terms that were covered in the previous lesson. Use "take notes, make notes" to extend the vocabulary to include more relevant terms, such as: savings account, checking account, deposit, withdrawal, Certificate of Deposit (CD), compound interest, annual, semi-annual, quarterly, monthly, and daily.

2 Teach (cont)

Universal access

An alternative (though less flexible) version of the compound interest formula given on this page is: $A = P(1 + R)^N$, where R is the interest rate per year (or per month) and N = number of years (or months). Some students may find this version of the formula easier to understand.

Universal access

Rather than simply presenting the students with the formula for compound interest, the formula can be derived from a worked example:

"\$200 is invested for 3 years in a savings account that pays 5% per year compound interest, compounded annually."

First, do the problem in stages:
After 1 year: \$200 × 1.05 = \$210
2 years: \$210 × 1.05 = \$220.50
3 years: \$220.50 × 1.05 = \$231.53

Explain that the method above involved multiplying by 1.05 three times and rewrite it as:

\$200 × 1.05 × 1.05 × 1.05 = \$231.53

Now, using the definition of powers:

$\$200 \times (1.05)^3 = \$231.53$
$\$200 \times (1 + 0.05)^3 = \$231.53$

Now this can now be generalized to the simplified version of the compound interest equation:

$A = P(1 + R)^N$

where A = total amount, N = number of years (or months), P = principal and R = yearly interest rate (or monthly).

Guided practice

Level 1: q3–4
Level 2: q3–5
Level 3: q3–6

Calculate Compound Interest Using the Formula

There's a formula for calculating the **amount in an account** (**A**) that has been earning compound interest:

$$A = P(1 + rt)^n$$

- **P is the principal.** This is the amount that is put into the account or loaned in the first place.
- **r is the interest rate**, written as a fraction or a decimal. So an interest rate of 6% could be written as $\frac{6}{100}$ or 0.06.
- **t is the time between each interest payment** in years. In Example 1 this was 0.25 because interest was paid quarterly.
- **n is the number of interest payments made**. In Example 1 this was 2 — in 6 months quarterly interest was paid twice.

Example 2

You put \$80 into an account that pays an interest rate of 6% per year compounded quarterly. Use the compound interest formula to find the account balance after 6 months.

Solution

$A = P(1 + rt)^n$
$A = 80(1 + (0.06 \times 0.25))^2$ **Substitute the values into the formula**
$A = 80 \times 1.015^2$
$A = 82.418$ **Evaluate**

The account balance is **\$82.42** to the nearest cent.

Don't forget:
On bank statements, interest payments are usually rounded to the nearest cent.

Guided Practice

3. You put \$100 into an account with a compound interest rate of 10% per year, compounded annually. What's the account balance after 4 years? \$146.41

4. You put \$150 into an account with a compound interest rate of 4% per year, compounded quarterly. What's the account balance after 6 months? \$153.02

5. You put \$88 into an account with a compound interest rate of 1% per year, compounded quarterly. What's the account balance after 9 months? \$88.66

6. You put \$200 into an account with a compound interest rate of 2% per year compounded monthly. What's the account balance after 7 months? \$202.34

Comparing Simple and Compound Interest

Imagine you have \$10,000 to invest for three years, and you intend to make no transactions during that time. You can choose from two accounts: one pays 5% **simple interest** per year, the other 5% **compound interest** per year, compounded annually.

Solutions

For worked solutions see the Solution Guide

- **Advanced Learners**
  Ask students to research different savings accounts available. Ask them to find out which accounts give the best returns after 1 year, 5 years, and 10 years if $1000, $5000, or $10,000 is invested. Students will need to take into account any bonuses paid. They should compare the returns to those for the same account if interest was only paid on the principal (i.e. simple interest).

This table shows the amount of interest the two accounts would build up:

	SIMPLE INTEREST		COMPOUND INTEREST	
Number of years	Interest earned	Total	Interest earned	Total
0	$0	$10,000	$0	$10,000
1	$500	$10,500	$500	$10,500
2	$500	$11,000	$525	$11,025
3	$500	$11,500	$551.25	$11,576.25

The account with compound interest would earn you **an extra $76.25**.

Comparing two accounts with **the same** annual interest rate:

- If you are **SAVING** a fixed sum of money, the account with **compound interest** will be a better choice because it will earn **MORE** interest.
- If you are **BORROWING** a fixed sum of money, **simple interest** will be a better choice because you'll be charged **LESS** interest overall.

Guided Practice

7. On a loan of $100, Bank A charges simple interest at 6% a year. Bank B charges 6% a year, compounded annually. Neither loan offers repayment in installments. Which bank has the better deal?
Bank A's: the two banks' rates are the same but Bank A's account is simple interest.

8. Myra has $100 to invest for 6 years. She can pick from 2% simple interest a year, or 2% compound interest a year, compounded annually. How much more will be in her account if she picks compound interest? **62¢**

9. Rai has $500 to invest for 3 years. He can pick from 5% simple interest a year or 4% compound interest a year, compounded quarterly. Which will leave him with the greater account balance? **the simple interest account**

Check it out:
To compare simple and compound interest when the interest rates are different you'll have to work out the account balances.

Independent Practice

In Exercises 1–3, work out the account balance using the formula.

1. $100 is invested for 5 years at 5% a year, compounded annually. **$127.63**
2. $50 is invested for 2 years at 3% a year, compounded quarterly. **$53.08**
3. $800 is invested for 8 months at 5% a year, compounded monthly. **$827.06**

4. Ezola borrows $200 at 7% a year compounded quarterly. She makes no repayments in the 1st year. What does she owe at the end of it? **$214.37**

5. Ben puts $2000 in an account that pays 5% a year simple interest. Dia puts $2000 in an account that pays 5% a year compounded annually. What's the difference between their balances after 6 years? **$80.19**

6. Geroy has $1000 to invest for 2 years. He can pick from 3.5% simple interest a year or 3.4% compound interest a year, compounded monthly. Which will leave him with the greater account balance? **the compound interest account**

7. Kim puts $10,000 in an account that pays a rate of 4% interest a year compounded annually. After 2 years the rate goes up to 5% a year compounded quarterly. What is her account balance after 30 months? **$11,088.09**

Now try these:
Lesson 8.2.5 additional questions — p472

Round Up

Compound interest is when you're paid interest on the whole account balance and not just on the money you first put in. It's a great way to save — but not always such a good way to borrow.

2 Teach (cont)

Additional example

$4800 was put into a savings account for 4 years with an interest rate of 6% per year, compounded annually. How much was in the account after the 4 years?

Use the compound interest formula.
$A = P \times (1 + rt)^n$
$P = \$4800$, $r = 0.06$ (6% written as a decimal), $t = 1$, $n = 4$
So:
$A = P \times (1 + r)^t$
$A = 4800 \times 1.06^4 = 6059.8891$
The amount in the account after 4 years is $6059.89.

Guided practice

Level 1: q7–8
Level 2: q7–9
Level 3: q7–9

Common errors

Students can make a variety of errors when trying to calculate compound interest. These include:

1. Using the simple interest formula — students often have problems differentiating between simple and compound interest conceptually.
2. Thinking the compound interest formula gives the total interest rather than the total amount (the simple interest formula gives the interest).
3. Incorrectly evaluating the formula. For instance they may work out $P(1 + rt^n)$ or $P(1 + (rt)^n)$.
4. Using the wrong value of r, for example, using $r = 5$ for 5%, not 0.05.

Independent practice

Level 1: q1–5
Level 2: q1–6
Level 3: q1–7

Additional questions

Level 1: p472 q1–4
Level 2: p472 q1–6
Level 3: p472 q1–6

3 Homework

Homework Book — Lesson 8.2.5

Level 1: q1–4, 7, 8
Level 2: q2–8
Level 3: q2–9

4 Skills Review

Skills Review CD-ROM

These worksheets may help struggling students:
- Worksheet 16 — Percents
- Worksheet 19 — Discounts, Tips and Interest

Solutions

For worked solutions see the Solution Guide

Section 8.3

Exploration — Estimating Length

Purpose of the Exploration

The goal of the Exploration is to have students practice estimating lengths, and then to use their understanding of percents to find their percent error. The Exploration also allows students to practice measuring using the metric system.

Resources

• metric rulers

Strategic & EL Learners

Strategic learners may have difficulty making reasonable estimates for the lengths of objects. For these students it would be helpful to give them certain objects with their actual measurements. This will give students a point of reference for making their estimates.

EL learners may not be familiar with the term "estimate." Give the students practice at estimating things other than length (such as the number of marbles in a jar, or the number of students in the school) so they get a more general understanding of the concept.

Universal access

Students who are not very familiar with the metric system will have difficulty making reasonable estimates. In this situation, provide some examples that will be very close in size to the objects in the Exploration.

Help students make a connection between centimeters and inches by discussing the number of centimeters in one inch. Students will make better estimates if they have the connection between centimeters and inches established.

Common error

Students will often find their error as a percent of their estimate, rather than as a percent of the measured length. Provide them with the formula:

$$\frac{\text{error}}{\text{measured length}} \times 100 = \text{percent error}$$

Section 8.3 introduction — an exploration into:

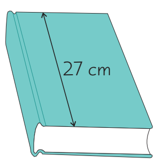
Estimating Length

An estimate is an educated guess about something — such as the size of a measurement. In this Exploration, you'll test your estimation skills by estimating the length of different objects in the classroom. You'll then test your estimates by measuring, and finding your percent error.

If you estimated the **length of a field** and were only 2 inches away from the actual measurement, it would be much more impressive than if you estimated the **length of a pencil** and were 2 inches out. That's why you use **percent error** — 2 inches as a percent of the length of a field, would be **tiny**, whereas 2 inches as a percent of the length of a pencil would be **much bigger**.

Example

A student estimates that the length of a math textbook is 30 centimeters. She measures it, and finds that it's actually only 27 centimeters long. What is her percent error?



Solution

Her error is 30 – 27 = **3 centimeters**.
You have to find 3 centimeters out of 27 centimeters as a percent:

$\frac{3}{27} \times 100 = 11.1$ → **So her percent error was 11.1%.**

Exercises

1. Make a copy of the table below. Complete it by estimating the things listed, measuring them, and then calculating the percent error.

Item	Estimate (cm)	Actual measurement (cm)	Error (cm)	Percent error
Length of student desk				
Width of door				
Diameter of clock				
Width of light switch				

Pick two other items in your classroom. →

2. How did your accuracy change over the course of the Exploration? Did your estimation skills improve? Explain your answer.

Round Up

Percent error tells you how big your error is compared to the size of the thing you are measuring. You normally estimate with "easy" numbers — like whole numbers, or to the nearest 10 or 100. For instance, you'd estimate something as "about a meter" rather than "about 102.3 centimeters."

416 Section 8.3 Exploration — *Estimating Length*

Lesson 8.3.1

Rounding

This Lesson introduces students to the basic principles of rounding numbers to give approximate answers. Students also learn how to give a clear description of how much they have rounded a number by using the decimal place system.

Previous Study: In grade 3, students first learned how to round numbers to the nearest ten, hundred, and thousand. In grade 4, they went on to learn how to round decimal numbers.

Future Study: In all further study of math and science, students will need to be adept at rounding answers, and be able to clearly express how they have rounded an answer.

Lesson 8.3.1

California Standard:

Mathematical Reasoning 2.7

Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

What it means for you:

You'll see how to round numbers, and how to describe how you've rounded them.

Key words:

- rounding
- decimal places
- hundredth
- thousandth

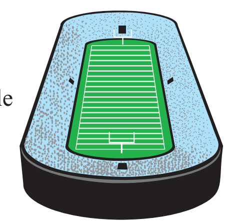
Section 8.3

Rounding

Often, it's fine to give an approximate answer. For instance, if you calculate the length of a yard as 11.583679 meters, then it'd probably be most sensible to say that it's approximately 11.58 meters. Also, rounding numbers that have lots of digits makes them easier to handle. There's a set of rules to follow to help you round any number.

Rounding Makes Numbers Easier to Work With

Sometimes, using **exact numbers** isn't necessary.



For example: the exact number of people who came to a football game might be 65,327. But most people who want to know what the attendance was will be happy with the answer "about 65,000."

Rounding reduces the **number of nonzero digits** in a number while keeping its value similar. Rounded numbers are **less accurate**, but **easier to work with**, than unrounded numbers.

There are Rules to Follow When You Round

Think about rounding 65.3 to the **nearest whole number**.

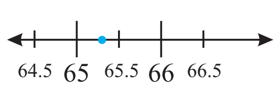
"To the nearest whole number" means that the units column is the last one that you want to keep. So look at the digit to the right of that:

65.3

You're rounding to this place... ...so look at this place.

Because this digit is less than 5, it means that the number is closer to 65 than to 66. So you can round it down to **65**.

It might help to think about where the number is on a number line:



You can see that 65.3 is closer to 65 than to 66.

Rules of rounding:

- Look at the digit to the right of the place you're rounding to.
- If it's 0, 1, 2, 3, or 4, then round the number down.
- If it's 5, 6, 7, 8, or 9, then round the number up.

Check it out:

Rounding is all about figuring out which of two numbers your answer is closer to.

Check it out:

72.5 lies exactly half way between 72 and 73. The rule is to round it up to 73 though.

1 Get started

Resources:

- large decimal number line
- paper dots
- number cards for Strategic Learners activity (see page 418)
- plastic hoops
- copies of bankers' rounding instruction cards (see page 419)

Warm-up questions:

- Lesson 8.3.1 sheet

2 Teach

Math background

In real-life questions, a rounded answer is often all that is needed.

It is sometimes more sensible to give a rounded answer than an exact one. For example, when your answer only makes sense as an integer (such as a number of people), if it came out as a decimal, you would need to round.

This topic is covered in greater depth in Lessons 8.3.2 and 8.3.4.

Universal access

Pin a large number line up on the classroom wall. Have the number line going between 1 and 2 in steps of 0.1.

Write the number 1.4 on the board. Ask a student to come up and stick a paper dot on the number line at 1.4. Now ask what you should round 1.4 to if you were rounding it to the nearest whole number. Ask for a reason along with the answer (that 1.4 is nearer to 1 than 2). Explain the convention for what to do if the number is exactly midway between the two numbers (1.5 in this example) — you round it up.

Repeat with some more examples, such as 1.7, 1.0, and 1.47.

- **Strategic Learners**
  Split students into groups. Give each group 4 plastic rings or hoops, and a set of 14 cards with the following numbers on them: 5.1, 5.3, 5.45, 5.7, 6.01, 6.4, 6.5, 6.8, 7.2, 7.49, 7.6, 7.99, 8.1, and 8.299. Label the hoops with the numbers 5, 6, 7, and 8 using sticky labels. Ask the students to round the numbers on the cards to the nearest whole number by sorting them into the correct hoops.

- **English Language Learners**
  Use individual whiteboards to check that students have understood the terminology of specific place values, including decimal place values. Write a large number like 6,785,429.328 on the overhead. Ask students to round it to the nearest million, write their answer down, and hold it up. Repeat with rounding to the nearest hundred, ten, hundredth, etc. This should highlight any vocabulary problems.

2 Teach (cont)

Concept question
"Round x.8 to the nearest whole number."
$x + 1$

Guided practice
Level 1: q1–3
Level 2: q1–6
Level 3: q1–9

Universal access
Write the number 8 on the board. Tell students you have rounded it to 8 from a number with one decimal place. Have them work with a partner to make a list of all the numbers you might have started with (7.5, 7.6, 7.7, 7.8, 7.9, 8.0, 8.1, 8.2, 8.3, 8.4).

Repeat the exercise with different numbers and different degrees of rounding, such as 40 rounded to the nearest 10 from a whole number, 1.24 rounded to 2 decimal places from a number with 3 decimal places.

Guided practice
Level 1: q10–11
Level 2: q10–13
Level 3: q10–15

Concept question
"Round 3.1847 to 3 decimal places, 2 decimal places, 1 decimal place, and 0 decimal places."
3.185 to 3 decimal places.
3.18 to 2 decimal places.
3.2 to 1 decimal place.
3 to 0 decimal places.

Example 1

Round 57.51 to the nearest whole number.

Solution
You're rounding to the nearest whole number, so look at the units column.

You're rounding to this place...
57.51
...so look at this place.

This digit is 5 — so you can round 57.51 up to **58**.

Guided Practice

In Exercises 1–9, round the number to the nearest whole number.

1. 3.1 **3** **2.** 4.8 **5** **3.** 2.5 **3**
4. 21.6 **22** **5.** 7.01 **7** **6.** 43.19 **43**
7. 0.61 **1** **8.** 127.20 **127** **9.** 1849.271 **1849**

You Need to Say What You're Rounding To

When you round, you need to say in your work what you've rounded your answer to. That might be...

... to the nearest whole number 17.23 → 17
... to the nearest 10 285 → 290
... to the nearest 100 1243 → 1200
... to the nearest one-hundredth 1.379 → 1.38

Check it out:
To help you figure out where to round a number, you can circle the digit you're rounding to.
For example: if you're rounding 1872 to the nearest hundred, ring the digit that represents hundreds.

1872

Now look at the digit to the right of that. In this case, as it is 7, you round up to 1900.

Guided Practice

In Exercises 10–15, round the number to the size given.

10. 726 to the nearest 10 **730** **11.** 1851 to the nearest 100 **1900**
12. 21241 to the nearest 1000 **21000** **13.** 0.15 to the nearest 10th **0.2**
14. 0.2149 to the nearest 100th **0.21** **15.** 0.00827 to the nearest 1000th **0.008**

You Can Round to Decimal Places

Another way of rounding numbers is to round to **decimal places**.

Units
0 decimal places

Thousandths
3 decimal places

1 . 2 3 4

Tenths
1 decimal place

Hundredths
2 decimal places

Check it out:
Rounding to the nearest hundredth is the same as rounding to 2 decimal places. Rounding to the nearest whole number is the same as rounding to 0 decimal places.

Solutions
For worked solutions see the Solution Guide

- **Advanced Learners**
  Introduce the rounding-to-even algorithm, known as bankers' rounding. Give students a copy of the card shown on the right, with instructions for this method, and have them do some simple rounding questions using it. Then give out this list of numbers: 1.5, 2.5, 3.5, 5.5. Have students find the mean of the list (3.25). Ask them to round the numbers to 0 decimal places using round-to-nearest and round-to-even. Have them calculate the mean of both rounded lists (3.75 and 3.5). Ask them to use their averages to suggest why bankers' rounding may be the better method.

> Bankers' Rounding
> 1) Decide which decimal place you are rounding to.
> 2) Round up if the next digit is 6 or more. Leave it the same if the next digit is 4 or less. Example: rounding to 1 decimal place, 4.22 rounds to 4.2, and 4.27 rounds to 4.3.
> 3) If the next digit is a 5, round to the nearest even number. Example: rounding to 1 decimal place, 4.25 rounds to 4.2, and 4.35 rounds to 4.4.

The number of **decimal places** that have been used is just the **number of digits** there are after the **decimal point**.

Example 2

Round 1.48934 to 3 decimal places.

Solution
You're rounding to **3 decimal places**, so look at the number to the right of the third digit after the decimal point.

1.48934
You're rounding to this place...
...so look at this place.

This digit is 3 — so you can round 1.48934 down to **1.489**.

Guided Practice

In Exercises 16–21 round to the number of decimal places given.

16. 0.27 to 1 decimal place 0.3
17. 2.237 to 1 decimal place 2.2
18. 4.118 to 2 decimal places 4.12
19. 1.4619 to 2 decimal places 1.46
20. 0.6249 to 3 decimal places 0.625
21. 0.012419 to 4 decimal places 0.0124

Independent Practice

In Exercises 1–8, round the number to the size given.

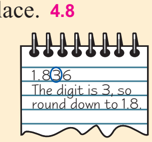
1. 7.8 to the nearest whole number 8
2. 423 to the nearest 10 420
3. 19410 to the nearest 100 19400
4. 1.205 to the nearest 100th 1.21
5. 5.63 to 1 decimal place 5.6
6. 0.74 to 0 decimal places 1
7. 1.118 to 2 decimal places 1.12
8. 7.2462 to 3 decimal places 7.246

9. Duenna's school has 1249 pupils on its roll. How many pupils does it have to the nearest hundred? What about to the nearest 10? 1200, 1250

10. Multiply 1501 by 8. Give your answer to the nearest 1000. 12000

11. Divide 150 by 31. Give your answer to 1 decimal place. 4.8

12. Kelvin is asked to round 1.836 to 2 decimal places. His work is shown on the right. What mistake has he made? What answer should he have gotten? See below



13. The local news reports that, in a survey of 3000 local families, 1000 had 3 or more children below the age of 18. The actual number was 583. Do you think it was sensible to round to the nearest 1000 here? What would you have rounded to? See below

Now try these:
Lesson 8.3.1 additional questions — p472

Round Up

When you don't need to use an exact number you can round. Rounding makes numbers with a lot of digits easier to handle. Use the digit to the right of the one you're rounding to to decide whether you need to round up or down. And don't forget to always say how you've rounded a number — whether it's to the nearest 100, the nearest hundredth, or to a certain number of decimal places.

2 Teach (cont)

Common error
Students sometimes forget that they only need to look at the digit after the one they're rounding to. For example, they will round 1.45 up to 2 on the basis that it ends in a 5.

If a student is inclined to do this, get them into the habit of underlining the digit they are rounding to in blue, and the digit to its right in green, as a reminder that these are the only two digits that they need to consider.

Guided practice
Level 1: q16–19
Level 2: q16–20
Level 3: q16–21

Independent practice
Level 1: q1–5, 9–11
Level 2: q1–7, 9–12
Level 3: q1–13

Additional questions
Level 1: p472 q1–12
Level 2: p472 q1–12
Level 3: p472 q1–12

3 Homework

Homework Book — Lesson 8.3.1
Level 1: q1–8
Level 2: q1–9
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 17 — Rounding Numbers

Solutions
For worked solutions see the Solution Guide

12. He rounded to 1 decimal place — he looked at the wrong digit. The answer is 1.84.
13. It was not sensible to round to the nearest 1000 here because doing so almost doubles the actual answer. It would be more sensible to round to 600 or 580.

Lesson 8.3.2

Rounding Reasonably

In this Lesson, students consider how the rules of rounding apply to real-life situations. They learn to judge whether such situations require them to round an answer up or down on a case-by-case basis.

Previous Study: In grade 4, students learned how to round numbers to specified degrees of accuracy, and looked at how to judge how reasonable a rounded answer is in the context of a question.

Future Study: In all further study of math and science, students will solve real-life math problems, and will need to be able to work out whether it is reasonable to round their answer, and if so, how to round it.

1 Get started

Warm-up questions:
- Lesson 8.3.2 sheet

2 Teach

Math background

This lesson encourages students to think about how their approach to questions and answers needs to change when they apply theoretical math to real-life situations.

Additional example

Round the following numbers to the nearest integer:

1) 3.4 **3**
2) 3.5 **4**
3) 3.6 **4**

Concept questions

"What is 6.19 rounded to the nearest whole number?"

6

"What is 6.19 rounded up to the next integer?"

7

Lesson 8.3.2

Rounding Reasonably

California Standards:

Mathematical Reasoning 2.7

Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Mathematical Reasoning 3.1

Evaluate the reasonableness of the solution in the context of the original situation.

What it means for you:

You'll see how to round numbers in situations where the normal rules of rounding don't apply.

Key words:
- rounding
- decimal places
- round up
- round down

Check it out:

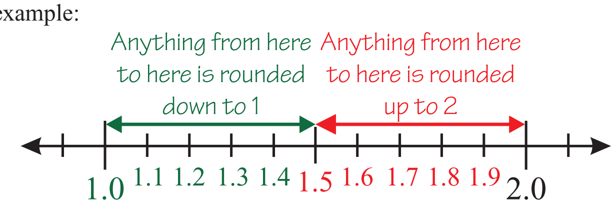
Whenever you're solving a real-life problem, you have to check that your answer is a reasonable one for that particular problem.

If the question asked how many cans Latoria would use, the answer could be 5.2. But she couldn't buy this number of cans.

There are times when the rules about rounding up and down that you learned in the last Lesson don't apply. In some real-life situations it isn't reasonable to round an answer up, and in others it isn't reasonable to round it down. This Lesson is all about being able to spot them.

Ordinary Rounding is Rounding to the Nearest

In the last Lesson, you learned about the ordinary rules of rounding. For example:



- If the digit to the right of the place you're rounding to is 0, 1, 2, 3, or 4 you should round down.
- If the digit to the right of the place you're rounding to is 5, 6, 7, 8, or 9 you should round up.

Rounding with these rules is called "**rounding to the nearest**" because whether you round up or down depends which number the digit is **closest** to.

Sometimes It's Sensible to Round a Number Up

There are real-life situations when it's sensible to round an answer **up** — even though it's actually closer to the lower number.

Example 1

Latoria is decorating. She has to paint a total wall area of 130 m^2. A can of paint covers 25 m^2 of wall. How many cans of paint should Latoria buy?

Solution

To find exactly how many cans of paint Latoria will need, **divide** the total **area of wall** by the area covered by **one paint can**.

$130 \text{ m}^2 \div 25 \text{ m}^2 = \mathbf{5.2}$

But Latoria can only buy a whole number of cans. So you need to round your answer to a whole number.

- Conventional rounding rules would say that the digit to the right of the units column is a **2**. So the answer would round to **5 cans**.
- But if Latoria only buys 5 cans, she **won't have enough** paint to cover the whole wall. So you need to round the answer up to **6 cans**.

- **Strategic Learners**
  Have students try to write their own real-life word problems where the final answer needs to be rounded the opposite way than if it were just a basic math question. When they have written their question they can swap with someone else, and try to answer their question. Finish by having a brief discussion about how they knew from reading the question whether to round the answer up or down.

- **English Language Learners**
  Go through the question from the Additional example section below, as a class. Have students come up and underline any parts that they think are important, saying why. Work out the exact answer (22 ÷ 5 = 4.4). Ask students if they think it needs to be rounded up or down. Have them discuss their answer in pairs. Ask them to make a note of any useful tips they find, and to share them with the class.

Real-life situations where you need to **round up** instead of down include:
- Working out **how many** of something you need for a task — it's better to have a bit left over than not have enough. Example 1 was a good illustration of this.
- Figuring how much to leave for a **tip** — it's fine to leave a little over the percent tip you intended, but you wouldn't want to leave any less.
- Working out how much **money** you need to buy an item — if you give **too much** you get change, but if you don't have enough you can't pay.

Guided Practice

1. A large cake contains 5 eggs. You're baking a small birthday cake that is half the size. How many eggs should you buy? **3 eggs**

2. Reece is laying a path that is 76 m long. Each bag of gravel will cover 3 m of path. How many bags should Reece buy? **26**

3. To get a grade A on a math test, Kate needs to score 80% or higher. If the test has a possible total of 74 points, how many points does Kate need to score an A? **60**

4. Emilio's taxi fare comes to \$17.42. He wants to leave a tip of at least 10%. What is the amount of the smallest tip he can leave? **\$1.75**

5. At Store A, a can of tuna costs \$1.77. Tess is going to the store to buy 3 cans for a recipe. If she only has dollar bills, how many should she take? **6**

Sometimes It's Sensible to Round a Number Down

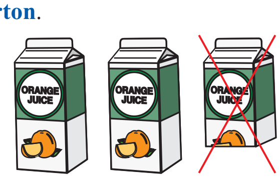
There are real-life situations when it's sensible to round the answer **down** — even though you'd round it up according to the rounding rules.

Example 2

A store charges \$2.50 for a carton of orange juice.
If you have \$7, how many cartons of orange juice can you buy?

Solution
To find exactly how many cartons you can buy, **divide** the money that you have by the price of **one carton**.

\$7 ÷ \$2.50 = **2.8 cartons**



\$2.50 \$2.50 \$2

But you can only buy a whole number of cartons. So you need to round your answer to a whole number.
- Conventional rounding rules would say that because the digit to the right of the units column is an **8**, the answer would round to **3 cartons**.
- But you can't buy 3 cartons because you don't have **enough money** to pay for them. So you need to round the answer down to **2 cartons**.

Don't forget:
There are some things that you can usually buy fractions of. These will mostly be products that are sold by weight or length.
For example, you could buy part of a pound of fruit, or part of a yard of fabric.

2 Teach (cont)

Additional example
Jan wants to make a button for each of the 22 people in her math class. At the craft store, blank buttons come in boxes of 5. How many boxes should Jan buy?
22 ÷ 5 = 4.4
So Jan must buy 5 boxes.

Guided practice
Level 1: q1–2
Level 2: q1–4
Level 3: q1–5

Universal access
Teach students a strategy for working out whether to round the answer to a problem up or down. It involves having them rewrite questions using either the word "minimum" or the word "maximum."

Ask them to rewrite the question in Example 1 using the word "minimum." It could be done like this: "Latoria needs enough paint to cover a minimum area of 130 m^2.
If 1 can of paint covers 25 m^2 of wall, how many cans should Latoria buy?"

Then ask them to rewrite the question in Example 2 using the word "maximum." It could be done like this: "A carton of orange juice costs \$2.50. If you have a maximum of \$7 to spend, how many can you buy?"

Point out that any question can be rewritten in one of these two ways. All questions using the word "minimum" need you to round their answers up — it won't matter if you have more than you need, but it will if you have less. All questions which use the word "maximum" need you to round their answers down — it won't matter if you are under a set limit, but you can't go over it.

Concept question
"What is 6.19 rounded down to the next integer?"
6

Solutions
For worked solutions see the Solution Guide

Advanced Learners
Have students write their own real-life word problems where the final answer needs to be rounded the opposite way than if it were just a basic math question. Ask them to try to answer their question and one written by someone else using inequalities. For example, Exercise 3 in the Independent Practice can be turned into the inequality $4x \geq 57$, which simplifies to $x \geq 14.25$. Since the answer must be an integer, this can be used to answer the question: "If she wants to make 57 gift tags, Lydia must buy at least 15 sheets of card."

2 Teach (cont)

Additional example

Diego is dividing up a pile of baseball cards equally between him and his 7 friends. They have decided to all take exactly the same number, and give any leftover cards to Diego's little sister. If there are 268 cards in the pile, how many will each friend get? How many cards will Diego's little sister get?

268 ÷ 7 = 38.29, or 38 remainder 2

Diego and his friends will each get 33 cards; his little sister will get 4 cards.

Guided practice

Level 1: q6–7
Level 2: q6–8
Level 3: q6–9

Independent practice

Level 1: q1–4
Level 2: q1–6
Level 3: q1–8

Additional questions

Level 1: p473 q1–4
Level 2: p473 q1–6
Level 3: p473 q1–7

3 Homework

Homework Book — Lesson 8.3.2
Level 1: q1–7
Level 2: q1–9
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 17 — Rounding Numbers

Real-life situations where you need to **round down** instead of up include:

- Working out how many **whole items** you can **make** from an amount of material. For instance, if it takes 4 balls of yarn to knit a sweater, and you have 10 balls, you might calculate that you can knit 2.5 sweaters. This isn't a reasonable answer — you can only knit 2.
- Working out **how many items** you can **buy** with a certain amount of money — you can't buy part of an item.

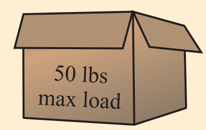
Guided Practice

6. At a local store, pens cost \$2 each. If you go in with \$13.30, how many pens can you buy? **6 pens**

7. You are making up bags of marbles to sell at a fund-raiser. Each bag contains 24 marbles. How many bags can you make from 306 marbles? **12**

8. A room has 4 walls, each with an area of 22 m^2. One can of paint covers 30 m^2. How many whole walls can Trayvon paint with 2 cans? **2 walls**

9. Blanca is packing books into a box that supports a maximum weight of 50 pounds. Each book weighs 2.2 pounds. How many books can Blanca put in the box? **22**



Independent Practice

1. Joel has 26 yards of material. He needs 3 yards to make one cushion. How many cushions can he make? **8**

2. A bread recipe calls for 5 cups of flour. How many loaves can be made from 64 cups of flour? **12 loaves**

3. Lydia is making gift tags. One sheet of card makes 4 tags. How many sheets of card will she need to make 57 tags? **15 sheets**

4. A class is planning to buy their teacher a going-away present. The vase they want to buy costs \$50 and there are 23 people in the class. How much should they each contribute? **\$2.18**

5. Patrick's restaurant bill came to \$22.92. He wants to leave a tip of at least 15%. What is the amount of the smallest tip he can leave? **\$3.44**

6. You have a 1 kg bag of flour. You want to use it to make 7 cakes for a bake sale. How many whole grams of flour will go into each cake? **142 g**

7. You use a payphone to make a call. Calls are charged at \$0.32/minute. If you have \$3 change, how many full minutes can you talk for? **9 minutes**

8. Hannah is saving to buy an MP3 player costing \$80. Each week she gets an allowance of \$6.20, which she saves toward it. How many weeks will she need to save for? **13 weeks**

Now try these:
Lesson 8.3.2 additional questions — p473

Round Up

Usually, when you round a number you use the "rounding to the nearest" method. But in some situations you might need to round a number up or down that you'd usually round the other way. It's all about making sure your answer is reasonable — there's more on that in the next two Lessons.

Solutions
For worked solutions see the Solution Guide

Lesson 8.3.3

Exact and Approximate Answers

In this Lesson, students consider how the context of a question affects how accurate their answer needs to be. They explore the concept of round-off errors, and how it affects the accuracy of a calculation.

Previous Study: From grade 3 onward, students have learned to consider the relative advantages of exact and approximate answers, and to what degree of accuracy it is appropriate to give their answers.

Future Study: In all further study of math and science, students will need to be able to justify the degree of accuracy with which they choose to answer a question, and be aware of the errors that rounding introduces.

Lesson 8.3.3

California Standards:

Mathematical Reasoning 2.7
Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

Mathematical Reasoning 2.8
Make precise calculations and check the validity of the results from the context of the problem.

Mathematical Reasoning 3.1
Evaluate the reasonableness of the solution in the context of the original situation.

What it means for you:
You'll think about how accurate answers to questions need to be, and when it's a good idea to round them.

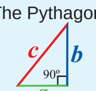
Key words:
- exact
- approximate
- rounding
- round-off error

Check it out:
A wavy equals sign "≈" means "is approximately equal to."

Don't forget:
An irrational number is a decimal that carries on forever without repeating.

Don't forget:
Circumference = π × diameter

Don't forget:
The Pythagorean Theorem:



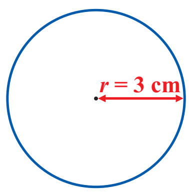
$a^2 + b^2 = c^2$

Exact and Approximate Answers

When you're figuring out the answer to a math question, it's important to think about how precise your answer needs to be. You have to decide if it's sensible to round an answer or not, and how much to round it by. And that's what this Lesson is about.

Leave π and √ In For a Completely Accurate Answer

Sometimes in math you'll need to give very **exact** answers, and sometimes you'll only be able to give an **approximate** answer.



Think about finding the **area** of this **circle**.
The formula is: **Area = πr^2**.

To find the area, you would do the calculation: Area = $\pi \times 3^2$.
But there are different ways that you could write your answer.

- π is an **irrational number**, so the only way to write the answer absolutely **accurately** would be: **Area = 9π cm²**
- If you're asked for an **approximate** answer then round the number off: **Area ≈ 28.3 cm² (to 1 decimal place)**

If the question doesn't tell you **how precise** your answer needs to be then make it **as accurate as possible**. That means leaving **irrational numbers**, like π or square roots, and **non-terminating decimals** in your answer.

Guided Practice

1. What is the area of a circle with a radius of 5 feet? **25π feet²**

2. If the radius of planet Earth at the equator is 6380 km, what is its circumference at the equator? Give your answer to the nearest 100 km. **See below**

3. You are asked to do the calculation $\frac{1}{3} \times 4$. Think of two ways that you could write your answer exactly. $\frac{4}{3}$, $1.\overline{3}$

4. A square has a side length of 10 cm. What is the length of its diagonal? What is the length of its diagonal to 2 decimal places? **See below**

5. 6 people share 10 pears equally. How many pears will each person get to 1 decimal place? How many thirds of a pear will each person get? **See below**

1 Get started

Resources:
- selection of measuring devices, such as, rulers, scales, measuring cylinder
- rulers
- 1 cm grid paper

Warm-up questions:
- Lesson 8.3.3 sheet

2 Teach

Math background
Both π and √2 are irrational numbers. Irrational numbers are non-terminating, non-repeating decimals.

This means that when you do a calculation involving π and √2 it is impossible to give an exact decimal answer.

Concept question
"How many decimal places does π have?"
An infinite number — it goes on forever.

Common error
When writing exact answers that still contain π or a square root symbol, students often forget to include units.

Remind them that things like π and √2 are not just symbols, they are actual numbers, so units must be included if appropriate.

Guided practice
Level 1: q1–3
Level 2: q1–4
Level 3: q1–5

Solutions
For worked solutions see the Solution Guide

2. 40,100 km to the nearest 100 km.
4. The length of its diagonal is √200 cm, which is 14.14 cm to 2 decimal places.
5. 1.7 pears to 1 decimal place, 5 thirds of a pear each.

- **Strategic Learners**
  Give everyone a ruler marked in centimeters and millimeters. Give out 1 cm grid paper. Ask students to draw a right triangle of base 5 cm and height 8 cm. Have them find the hypotenuse length using the Pythagorean theorem. Then ask them to measure it with the ruler. Have a group or class discussion about how different their two answers were, and why.

- **English Language Learners**
  Put students in pairs. Ask each pair to list three everyday situations where using approximate numbers would be fine, and three situations where exact numbers are needed. For example, if you're buying snacks for a party you need to know roughly how many guests are coming to figure out how much food to buy; if you're planning a sit-down wedding dinner you need to know exactly how many guests you will have.

2 Teach (cont)

Universal access
Give students a few multiple choice questions like the ones below.

1) One side of a cube was measured as being 1.056 ft long. What is the volume of the cube?
   a) 1.18
   b) 1.18 ft³ to 2 decimal places
   c) 1.1776 ft³ to 4 decimal places

b — it has the correct units and is to a sensible degree of accuracy.

2) I ran 5.92 km in 1.5 hours. What was my speed?
   a) 3.9 km/hour to 1 decimal place
   b) $3.94\overline{6}$ km/hour
   c) 3.95 km/hour to 2 decimal places

a — it has the correct units and is to a sensible degree of accuracy.

Ask them to decide which answer they think would be the most reasonable in the context of the question. Then have them find a partner, and discuss which answers they picked, and why.

Common error
When working with calculators, students often simply write down the number on the screen without thinking about what it means in practice.

Remind them that the context of the question always matters. The answer should never exceed the accuracy of the least precise data in the question, and be given with the correct units.

In fact, if the question involves multiplying or squaring, any rounding errors will be increased. In these cases it will often be necessary to give the answer to fewer decimal places than the least number given in the question.

Example 3 explores this idea further by demonstrating the actual range of values an answer could have.

Guided practice
Level 1: q6
Level 2: q6–7
Level 3: q6–8

The Data in the Question Decides the Accuracy

In **real-life problems**, approximate answers often make more sense than exact ones. There are two things to think about when deciding **whether to round** your answer, and **how to round it**:

1) The context of the question.
As you saw in the last Lesson, how you round may be affected by what the question is asking you to find.

Example 1

Lupe is making buttons. It cost her \$15 to make 13. What is the lowest price she can sell each one for and make at least as much as she spent?

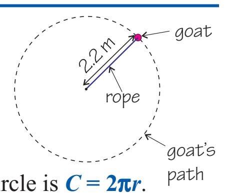
Solution
Each button cost Lupe exactly $\$\frac{15}{13}$ to make. $\frac{15}{13} = 1.\overline{153846}$.
- As she can't charge less than a cent, you should round to 2 decimal places.
- And as she needs to make at least what she spent, round up not down.

So Lupe needs to charge **\$1.16** for each button.

2) The accuracy of the data in the question.
Sometimes data you are given to use in a question will be approximate. If it is, then your answer depends on how precise the data is.

Example 2

A goat is tied to a length of rope, which is measured as 2.2 m long. If the goat walks a complete circle as shown, how far has it walked?



Solution
The formula for finding the **circumference** of a circle is $C = 2\pi r$.
Using $\pi = 3.142$, the goat has walked $2 \times 3.142 \times 2.2 =$ **13.8248 m**.

But the rope's length is **approximate** — it could be a little more or less than 2.2 m. You are told the rope's length **to the nearest 0.1 m**, so it's sensible to give your final answer to the nearest meter.
The goat has walked 14 m to the nearest meter.

Check it out:
Measurements always introduce some error — you can never measure anything completely accurately.
Although the rope in Example 2 is 2.2 m long to the nearest 0.1 m, you don't know the exact length. For example, it could be 2.21 m or 2.18 m.

Check it out:
When you have approximate data in a calculation involving multiplication, any rounding errors are multiplied, making your answer less precise than the data you began with.
In Example 2, the least precise measurement was given to 1 decimal place. This was then multiplied by 2π, giving a bigger rounding error. This makes it unreasonable to give the answer to 1 decimal place — it's more sensible to give the answer to the nearest whole number instead.
The reason for this is demonstrated in more detail in Example 3.

Guided Practice

6. Lee measures the legs of a right triangle as 6.2 in. and 8.3 in., to the nearest tenth of an inch. He calculates the hypotenuse as 10.36 in. Is this an appropriate level of accuracy? Explain your answer. See below

7. La-trice completes a motor race of 190 miles, to the nearest ten miles. She then drives the car a further 0.92 miles back to the pit lane Should the total distance she traveled be given to the nearest 10 miles, to the nearest mile, or to the nearest hundredth of a mile? to the nearest 10 miles

8. Eli wants to make a tablecloth that overhangs by 10 cm for his rectangular table. To what level of accuracy should he measure the length and width of his table? to the nearest cm

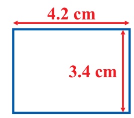
Solutions
For worked solutions see the Solution Guide

6. No — the measurements were only made to the nearest tenth of an inch. The hypotenuse can't be calculated to a higher degree of accuracy than the original measurements.

- **Advanced Learners**
  Ask students to look at Independent Practice Exercise 4 on page 415. It's about compound interest. Ask them to use the compound interest formula to calculate the answer, firstly using a calculation that will give an exact final answer, which they can then round to the nearest cent, and secondly by rounding after each compounding and repeatedly putting the rounded balance back into the formula. Ask them to think about what difference the round-off error made to the account balance, and why you wouldn't want a bank to use the second method if you had invested a large amount of money with them.

Rounding Makes Your Answer Slightly Inaccurate

Rounding numbers creates small inaccuracies called **round-off errors**.



The length of this rectangle's sides have been measured to the nearest tenth of a centimeter. Think about finding its **area**:
Area = length × width = 4.2 × 3.4 = **14.28 cm²** ≈ **14.3 cm²**

But the measurements are **rounded** to the nearest tenth of a centimeter.
So actually: **4.15 cm ≤ length < 4.25 cm** and **3.35 cm ≤ width < 3.45 cm**.

The **minimum area** of the rectangle is found by multiplying the smallest possible length and the smallest possible width, so:
Minimum area = 4.15 × 3.35 = **13.9025 cm²**

And the rectangle's **maximum** area = 4.25 × 3.45 = **14.6625 cm²**

The **actual value** could be anywhere between these two. The **difference** between the true value and your calculated value is a **round-off error**.

Check it out:

The length could be anything from 4.15 cm to just less than 4.25 cm. All the values in this range would round to 4.2 cm.

The width could be anything from 3.35 cm to just less than 3.45 cm. All the values in this range would round to 3.4 cm.

Guided Practice

9. Daisy measures the lengths of 2 planks as 10.2 m and 5.6 m to the nearest 10 cm. She adds them to give a total of 15.8 m. Find the greatest and least possible sums of the lengths. **15.9 m, 15.7 m**

10. Rey measures a triangle's base as 10 mm, and its height as 6 mm to the nearest mm. With round-off error, what is its minimum area? **26.125 mm²**

11. Shantel is finding the product of 1.86 and 0.55. She rounds both numbers to 1 decimal place, multiplies them and gives her answer to 1 decimal place. What round-off error has she introduced? **0.077**

Check it out:

The earlier you round, the bigger your round-off error tends to be. If Shantel had multiplied 1.86 and 0.55, then rounded her solution, her round-off error would have been smaller.

Independent Practice

1. Liam measures the base of a triangle as 2.34 m and its height as 1.69 m. What is the triangle's area to the nearest m²? **2 m²**

2. A square has a side length of one seventh of a meter. What is its exact area? What is its area to 2 decimal places? **$\frac{1}{49}$ m², 0.02 m²**

3. Zoe and Tion both add a third to a seventh and give the answer to 2 decimal places. Their work is below. Which answer is most accurate?

Zoe
$\frac{1}{3}+\frac{1}{7}=\frac{7}{21}+\frac{3}{21}=\frac{10}{21}$
$10 \div 21 = 0.48$ (2 decimal places)

Tion
$\frac{1}{3}\approx 0.33 \quad \frac{1}{7}\approx 0.14$ **See below**
$0.33 + 0.14 = 0.47$ (2 decimal places)

4. Kelly measures the side length of a cube as being 10.1 cm to the nearest mm. With round-off error, what is its minimum volume? **1015.1 cm³**

5. Inez adds the areas of a circle with a 2 cm radius, and a triangle with a base of 6 cm and a height of $\sqrt{2}$ cm. What is her exact answer? **$(4\pi + 3\sqrt{2})$ cm²**

Now try these:

Lesson 8.3.3 additional questions — p473

Round Up

*Sometimes in math you'll be asked to give an **approximate answer**. Always think carefully about **how much** to **round** your answer. And don't forget that rounding always introduces **round-off errors**.*

2 Teach (cont)

Universal access

Bring in a selection of measuring devices, such as a ruler, a set of scales, and a measuring cylinder.

Have students look at them all carefully, and say what degree of accuracy they can be used to measure to (for example "to the nearest mm").

Ask them to think about what difference the accuracy of the measuring device would make if they were using the measurements to do a calculation.

You could ask them to come up with their own example of the inaccuracies it could produce, for example, using the ruler and then doing an area or volume calculation with the measurements.

Guided practice

Level 1: q9
Level 2: q9–10
Level 3: q9–11

Independent practice

Level 1: q1–3
Level 2: q1–4
Level 3: q1–5

Additional questions

Level 1: p473 q1–3
Level 2: p473 q1–5
Level 3: p473 q1–6

3 Homework

Homework Book — Lesson 8.3.3

Level 1: q1–2, 4–5, 7–8
Level 2: q1–9
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM

This worksheet may help struggling students:
- Worksheet 17 — Rounding Numbers

Solutions
For worked solutions see the Solution Guide

3. Zoe: She rounds later, so her answer is more accurate.

Lesson 8.3.4 Reasonableness and Estimation

In this Lesson, students are asked to think about how they can check whether their answers to questions are reasonable. They consider how the context of a question affects the answer, and use estimation to check that their answer is of the right order of magnitude.

Previous Study: From grade 3, students have been asked to consider the reasonableness of answers in the context of questions. From grade 4, they have been taught to use estimation as an answer check.

Future Study: In all further study of math and science, students will need to be able to check that their answers to questions are reasonable in the context of the question and of the right order of magnitude.

1 Get started

Resources:

- set of number and unit cards (see Universal access activity, below)
- local restaurant menu
- meter rulers

Warm-up questions:

- Lesson 8.3.4 sheet

2 Teach

Universal access

Write the following list of numbers and units out onto individual cards several times:

Numbers:	Units:
–2	pencils
3.4	kilometers
3.45	children
6.5	seconds
7.1	miles

Split the students into groups. Give each group a number card and a unit card. Ask the group to look at their cards and think about whether that combination of numbers and units could ever be a sensible answer to a real-life question.

Have each group report back to the class. If they think their combination of unit and number could never be sensible, they must say why. If they think it may be sensible, ask them to give an example of a question it could be an answer to.

Guided practice

Level 1: q1
Level 2: q1–2
Level 3: q1–3

Additional examples

1) Alvar is finding the area of a rectangle with a 10 cm width and a 15 cm length. He does a calculation, and gets an area of 1500 cm^2. Is he likely to be correct?
10 cm by 15 cm is a small rectangle, and 1500 cm^2 is quite a big area. His answer isn't likely to be correct.

2) Hannah is converting the height of her house in meters, which is 9.9 m, into feet. She gets an answer of 27 feet. Is this likely to be correct?
9.9 m is about as big as 5 adults lying head to toe. So is 27 feet. Her answer is likely to be correct.

3) Karl works out how fast he traveled on his 2 mile cycle to school. He does a calculation, and gets the answer 200 mph. Is he likely to be correct?
This is too fast for anyone to cycle, and would mean he did the journey in less than 1 minute. His answer is unlikely to be correct.

Lesson 8.3.4

Reasonableness and Estimation

California Standards:

Mathematical Reasoning 2.1
Use estimation to verify the reasonableness of calculated results.

Mathematical Reasoning 2.3
Estimate unknown quantities graphically **and solve for them by using logical reasoning and arithmetic and algebraic techniques.**

Mathematical Reasoning 3.1
Evaluate the reasonableness of the solution in the context of the original situation.

Number Sense 1.3
Convert fractions to decimals and percents and **use these representations in estimations, computations, and applications.**

What it means for you:

You'll think about ways to check whether the answer to a question is sensible and of roughly the right size.

Key words:

- reasonable
- sensible
- estimate

Check it out:

Think about:
- Whether your answer ought to be a whole number.
- Whether your answer should be negative or positive.
- Whether to round an answer up or down.

When you answer a math question, you need to be sure your answer makes sense and is about the right size. Making an estimate before doing a calculation is a good way to check your answer is sensible — if your estimate is very different from your answer, you'll know there's an error somewhere.

Think About Whether Your Answer is Sensible

The first thing to look at is whether your answer is a **sensible** answer to the **particular question** you've been asked.

Example 1

Mrs. Moore is splitting students into teams. She needs to split 59 students into 4 teams, as equal in size as possible. What would be a reasonable way to split the students up?

Solution

If Mrs. Moore split the class **equally** there would be 59 ÷ 4 = **14.75 people** on a team. This isn't reasonable. You can't put part of a person on a team.

Rounding up doesn't work — 15 people on each of 4 teams needs 60 people. And **rounding down** to 14 means some people are left out.

The most reasonable thing to do would be to split the students into **almost equal teams** of **15, 15, 15, and 14**.

Guided Practice

In Exercises 1–3, say whether the answer given is reasonable. **See below.**

1. A camp has 4 empty tents and 18 new visitors. So the camp supervisor decides to put 4.5 people in each tent.
2. Kea has $7. 1 kg of plums costs $4. Kea says she can buy 1.75 kg.
3. The area of a square is 36 cm^2. Alan finds its side length by taking the square root of 36. He says the side length of the square is ±6 cm.

Look at Whether Your Answer is the Right Size

Another thing to think about is whether your answer is about the **right size**. Sometimes it's quite clear that your answer is the wrong size.

Example 2

Rocio wants to find out how far 2 miles is in meters. She does a calculation and gets the answer 3.2 meters. Is she likely to be correct?

Solution

2 miles is a fairly long walk, but 3.2 meters is only about as big as two adults lying head to toe. Her answer isn't likely to be correct.

Solutions

For worked solutions see the Solution Guide

1. No: you can't put part of a person in a tent.
2. Yes: as the plums are sold by weight, you could buy part of a kg.
3. No: a length cannot be a negative number. The length must be 6 cm.

- **Strategic Learners**
  Bring in a menu from a local restaurant. Put students into pairs, and give each pair a copy of the menu. Have them use estimation to decide what they could order if they had \$10 or \$15. Then ask them to calculate the exact bill (include tax and tip if appropriate) and see how close their first estimate was.

- **English Language Learners**
  Have a class discussion about estimation. Ask students when they use estimation in real life. It could be shopping, to estimate a bill or sales tax, or in a job or hobby. Then go on to discuss using estimation in math. Do students already use estimation to check their answers? Can they remember a specific example where using estimation helped them to solve a problem?

Use an Estimate to Check Your Answer

Sometimes it's not quite so obvious that an answer is the wrong size. So, it's a good idea to **estimate** the answer to a problem before you solve it.

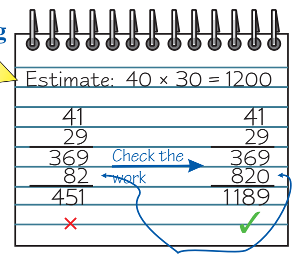
You do this by **rounding** the numbers and using **mental math** to do a simple calculation. If the **estimate** is **about the same** as the **answer**, you'll know it's probably right.

Think about finding the **product** of **41** and **29**. This is what Ralph did:

1) He estimated the answer first by **rounding** both numbers to the nearest 10.

2) Ralph's answer of 451 is **very different** from his estimate of 1200. He thinks there may be an error in his work.

3) He **checks his work** and finds his error. With the error corrected, his worked answer is close to his estimate — so he can be more confident his answer is right.



The error Ralph made was a missing 0 in the tens column.

Whenever you think your answer to a question is **not reasonable**, you should go back and **check your work** to find the **error**.

Estimation can be very useful in **real-life problems** too.

Example 3

Ceria buys a sweater for \$51.99 and jeans for \$39.50.
Sales tax is charged at 8.75%. She pays \$99.50.
Use estimation to check if the total cost is about right.

Solution

First round the costs of the items and add them:

$\$51.99 \approx \$50 \qquad \$39.50 \approx \$40$

$\$50 + \$40 =$ **\$90**

Now round the sales tax rate, and apply it to the total cost:

$8.75\% \approx 10\%$

$\$90 \times 0.1 =$ **\$9**

Add the tax to the item's cost to find the final price: \$90 + \$9 = **\$99**

This is very close to the total cost, so it is probably correct.

Don't forget:
The symbol ≈ means "is approximately equal to."

2 Teach (cont)

Universal access

Put students into pairs. Ask each pair to estimate the area of two objects in the room (for example, in m²), such as their own desktop, or a display board.

Have them compare their estimates with another pair, and revise them if they wish.

Now ask them to measure to check their accuracy, and discuss their results.

Concept question

"If I asked you to find the area of a circle with a radius of 4.1 cm, what estimation could you use to check your answer?"

Area of a circle = πr^2.
Estimation = $3 \times 4^2 = 3 \times 16 = 48$ cm²

Additional example

Write the following multiple choice questions up on the board.
Have students use estimation to pick the correct answer.

1) 22 × 49 = ?
   a) 3418
   b) 298
   c) 1078
   c)

2) 1 kg of bananas costs \$2.20. How much would 5.2 kg cost?
   a) \$11.44
   b) \$21.40
   c) \$5.12
   a)

3) If I drive for 3.5 hours at a speed of 33.3 mph how far will I go?
   a) 116.7 miles
   b) 56.9 miles
   c) 222.74 miles
   a)

4) What's the area of a triangle with a base of 2.6 cm and a height of 15.7 cm?
   a) 4.082 cm²
   b) 20.41 cm²
   c) 40.82 cm²
   c)

● **Advanced Learners**
Ask students to plan, carry out, and write up a measurement project involving estimation. For example, challenge them to work out roughly how much paint would be needed to decorate the gym, or how long it would take to drive to New York. Have them begin by making a list of all the things they would need to know in order to work out the answer. Tell them to find, by measurement or research, rough values for the numbers they need. Then they can use a calculation to estimate the answer. Ask them to include some discussion of how accurate their answer is.

2 Teach (cont)

Guided practice
Level 1: q4–5
Level 2: q4–6
Level 3: q4–7

Math background
Rectangle Length = Area ÷ Width
Circumference = $2\pi r$

Independent practice
Level 1: q1–7
Level 2: q1–10
Level 3: q1–12

Additional questions
Level 1: p473 q1–4
Level 2: p473 q1–5
Level 3: p473 q1–6

3 Homework

Homework Book — Lesson 8.3.4
Level 1: q1–7
Level 2: q1–9
Level 3: q1–9

4 Skills Review

Skills Review CD-ROM
This worksheet may help struggling students:
• Worksheet 18 — Estimating

Guided Practice

4. Find the product of 51 and 68. Check your answer using estimation. **3468 (50 × 70 = 3500)**
5. Karl put $1021 into a savings account paying 6% simple interest per year. Estimate roughly how much interest Karl will earn in a year. **About $60**
6. Ruby's meal cost $39.95. She wants to tip the waiter 15%. She says she should leave about $4. Is she right? Estimate what tip to leave. **No: $4 ≈ 10%. She should leave about $6.**
7. The school council sold 197 tickets to a dance. A ticket entitles you to 2 cartons of juice. If juice cartons come in boxes of 52, estimate how many boxes the school council should buy. **About 8 boxes**

Independent Practice

In Exercises 1–4, say whether the answer given is reasonable.
1. Umar buys lettuce for $2.10, some bananas for $2.05, and a melon for $4. He estimates that his bill will be about $80. **No: the estimate is too big. $8 would be closer.**
2. A shirt selling for $51.30 is discounted by 22%. Clare says this is a reduction of about $10. Is her estimate reasonable? **Yes: $50 × 0.20 = $10**
3. In winter, the temperature outside Iago's house is 23 °F. He converts it to °C, and says it is –5 °C. **Yes: since it is winter the temperature outside may well be below 0 °C.**
4. Lashona measures the legs of a right triangle as 7 in. and 9 in. Using the Pythagorean theorem, she says the hypotenuse is 11.4018 in. long. **No: this is too precise as her measurements are only to the nearest inch.**
5. Rachel's cab fare is $32. She wants to give a 10% tip. Rachel says this is $10. Does this seem reasonable? **No: 10% would be $3–$4.**

6. Jeron and Ann are painting. Jeron paints 1.8 walls/hour, and Ann paints 2 walls/hour. Ann says it will them take less than 2 hours to paint 7 walls. Is this a reasonable thing to say? **Yes: between them they paint 3.8 walls/hour.**
7. Felix finds the length of a rectangle with a 10 mm² area and a 3 mm width. He says its length is 3.333333 mm. Is this a sensible answer? **No: as the measurements are only to the nearest mm, this is too precise.**
8. Tandi is finding the circumference of a circular trampoline. She measures its radius as 3 m, and says its circumference is 19 m. Does this seem reasonable? **Yes: her measurements are to the nearest meter so it is sensible for her answer to be as well. An estimate would be 3 × 2 × 3 = 18 m**
9. Ben is saving up to buy a $98 camera. To earn money he washes cars, charging $12 per car. Estimate the number of cars he will have to wash to earn $98. **9 or 10 cars, depending on what approximation is used (the exact answer is 9 cars).**
10. Xenia uses the Pythagorean theorem to find the hypotenuse of a right triangle. Its legs are 36 cm and 60 cm. She gets 48 cm. Is this sensible? **No: the hypotenuse should be the longest. There must be an error in her work.**
11. Divide 9.88 by 5.2. Check your answer using estimation. **1.9 (10 ÷ 5 = 2)**
12. Evan walked for 1.9 km at 2.8 km/hr, and then for 5.9 km at 3.2 km/hr. Estimate how long his walk took in hours and minutes. **About 2 hours 40 minutes.**

Don't forget:
$°C = \frac{5}{9}(°F - 32)$.

Now try these:
Lesson 8.3.4 additional questions — p473

Round Up

It's always important in math to think about whether your answers are reasonable or not — there's no point in giving an answer that doesn't make sense. Remember that you can always estimate before finding an exact answer. Then you'll have an idea whether your answer is right or not.

Solutions
For worked solutions see the Solution Guide

Chapter 8 Investigation — Nutrition Facts

Purpose of the Investigation

This Investigation shows students a real-life use of percents — in nutrition facts on food labels. It gives students practice in calculating percents, and percent increases and decreases. It also allows students to consider how companies may choose to manipulate figures to their advantage.

Chapter 8 Investigation
Nutrition Facts

Percents are used a lot in real life, so you really need to get confident working with them. In this Investigation, you'll see how percents are used to provide information about foods.

On the right is a **nutrition facts label** from a packet of crackers. The table below shows the recommended daily values you should eat if you need 2000 calories or 2500 calories a day.

	2000 calories per day	2500 calories per day
Total fat	65 g	80 g
Saturated fat	20 g	25 g
Cholesterol	300 mg	300 mg
Sodium	2400 mg	2400 mg
Total carbohydrates	300 g	375 g
Dietary fiber	25 g	30 g

NUTRITION FACTS

Serving size = 30 g   Servings per box = 5

Amount Per Serving	
Calories 120 **Calories from Fat** 40	
	% Daily Value
Total Fat 5.5 g	8%
Saturated Fat 0.6 g	3%
Trans Fat 0 g	
Cholesterol 0 mg	
Sodium 180 mg	7%
Total Carbohydrate 21 g	7%
Dietary Fiber 3 g	12%
Sugars 0 g	
Protein 4 g	

The **Percent Daily Value** figures on the label show what percent of the recommended daily value each serving contains. The number of calories you need depends on things like your gender, and the exercise you do.

1) How many **calories per day** are the **percent daily values** on the nutrition facts label based on?

2) The label reads 7% for the percent daily value of total carbohydrate. What **fraction** was **converted** to report this percent?

3) Suppose there were 8 mg of cholesterol in a serving size. What would you report as the percent daily value for this amount? Explain the calculations and rounding technique you used.

4) The percent daily value of sodium is found by dividing the 180 milligrams of sodium in the crackers by the 2400 milligrams of sodium recommended. What **rounding technique** did the company use to post a percent daily value of 7%? Suggest a reason why this was.

Extension

1) Compute the percent daily values for the crackers above using the 2500 calorie diet.

2) The same company put out a reduced fat version of the same cracker. The nutrition facts label is shown here. Calculate the **percent increase** or **decrease** in the actual amounts in each category (NOT the percent daily values), going from the original to the reduced fat cracker. Round your answers to the nearest whole percent.

NUTRITION FACTS

Serving size = 30 g   Servings per box = 5

Amount Per Serving	
Calories 120 **Calories from Fat** 25	
	% Daily Value
Total Fat 3 g	5%
Saturated Fat 0 g	0%
Trans Fat 0 g	
Cholesterol 0 mg	
Sodium 150 mg	6%
Total Carbohydrate 23 g	8%
Dietary Fiber 3.5 g	14%
Sugars 0 g	
Protein 3 g	

Open-ended Extension

Find food labels for two similar products.
Calculate the percent differences between the products.
Then make a poster comparing the products.

Round Up

Percent daily values make things easier to interpret. It'd be hard to remember how many grams of different things you should have — using percent daily values mean you don't need to.

Resources

- calculators
- labels from two familiar food products, that are similar to each other. For instance, two different brands of chicken soup.

Strategic & EL Learners

Adapt the activity for strategic learners by reducing the number of categories on the labels.
For example, only include the fat and sodium values.
Provide translations of the words used on the label for English Language Learners.

Investigation Notes on p429 B-C

Investigation — Nutrition Facts

Mathematical Background

Whenever you buy packaged food, there is a nutrition facts label on the outside. This label is required by law and contains much information about the product inside. Percent daily values are listed for things such as total fat, sodium and total carbohydrates. These percents are usually based on a 2000 calorie-per-day diet. The number of calories a person actually needs to consume depends on factors such as height, weight, gender, age and activity levels.

In this investigation, students are asked to read food nutrition labels and verify certain percentages. This requires them to recall the **definition of percent** and to **convert fractions into percents**. Students also look at how percents have been rounded, and why a company may choose to round certain percents up or down.

In the Extensions, students use **percent increase** and **decrease** to summarize the differences between two products.

Approaching the Investigation

1) Students firstly have to establish whether the percent daily values are based on a 2000 calorie-per-day diet or a 2500 calorie-per-day diet. To find this out, an amount in grams from the nutrition facts label, should be compared with the corresponding amounts for the 2000 and 2500 calorie-per-day diets.

   For example, total fat in a serving of the crackers = 5.5 g
   As a percent of the total fat needed per day for a 2000 calorie-per-day diet this is: $\frac{5.5}{65} \times 100 = 8.46\% \approx 8\%$

   As a percent of the total fat needed per day for a 2500 calorie-per-day diet this is: $\frac{5.5}{80} \times 100 = 6.88\% \approx 7\%$

   8% is the percent daily value stated on the nutrition facts label, so it must be based on a **2000 calorie-per-day diet**.

2) After establishing that the 2000 calorie-per-day diet is used to determine percent daily value, the Investigation asks students to look at the total carbohydrate. The label says that in a serving, there are 21 grams of carbohydrate. The total carbohydrate for a 2000 calorie-per-day diet is 300 grams.

   So the fraction that is converted to a percent is $\frac{21}{300}$. $\frac{21}{300}$ = 0.07.

   To convert to a percent, multiply by 100 to get 7%.

3) If there were 8 mg of cholesterol in a serving, you would calculate the percent daily value by calculating 8 ÷ 300. This gives 0.026666…, which, to the nearest percent, rounds to 3%.

4) To get the percent daily value of sodium, take 180 ÷ 2400. This gives 0.075. When it's multiplied by 100, you get 7.5%. Now, in rounding to the nearest percent, this would be rounded up to 8%, but it is 7% that is reported on the label. The company has chosen to round 7.5%, which is exactly half way between 7% and 8%, down. As too much sodium is considered unhealthy, the company may have chosen to round down to make the product seem healthier.

Investigation — Nutrition Facts

Extensions

1) Here is a chart that summarizes the percent daily values for a 2500 calorie diet:

	Calculation	2500 calories-per-day Percent Daily Value
Total fat	(5.5 ÷ 80) × 100 = 6.88	7%
Saturated fat	(0.6 ÷ 25) × 100 = 2.4	2%
Cholesterol	unchanged	0%
Sodium	unchanged	7%
Total carbohydrates	(21 ÷ 375) × 100 = 5.6	6%
Dietary fiber	(3 ÷ 30) × 100 = 10	10%

2) Here is a chart that summarizes the percent increases and decreases between the original and the reduced fat crackers:

	Change	Increase or decrease	Percent increase or decrease (to nearest percent)
Total fat	from 5.5 g to 3 g	2.5 g decrease	2.5 ÷ 5.5 × 100 = **45% decrease**
Saturated fat	from 0.6 g to 0 g	0.6 g decrease	**100% decrease**
Cholesterol	no change	no change	no change
Sodium	180 mg to 150 mg	30 mg decrease	30 ÷ 180 × 100 = **17% decrease**
Total carbohydrates	21 g to 23 g	2 g increase	2 ÷ 21 × 100 = **10% increase**
Dietary fiber	3 g to 3.5 g	0.5 g increase	0.5 ÷ 3 × 100 = **17% increase**

Open-Ended Extensions

The results of this open-ended extension depend on which products students investigate. Before they start comparing the products, they should check that the serving sizes are the same. If they are not, they will need to adjust the figures so that they are comparing equal amounts of each product.

Additional Questions

Lesson 1.1.1 — 1.2.1

Level 1: q1–2, 9–10
Level 2: q1–10
Level 3: q1–12

Lesson 1.1.1 — Variables and Expressions

Write the variable expressions in exercises 1 – 4 as word expressions.

1. $10(b + 8)$ 10 times the sum of b and 8
2. $2p + 7$ 7 more than the product of 2 and *p* OR 7 more than 2 times *p*
3. $3r - 4s$ the difference of 3 times *r* and 4 times *s*
4. $5(2x + 6)$ 5 times the sum of 6 and 2 times *x*

Evaluate the expressions in exercises 5 – 8 when $r = 4$ and $s = 7$

5. $2(rs - r^2)$ 24
6. $s^2 - 7r - 3s$ 0
7. $(2rs - 2) \div 6$ 9
8. $5s - 3r + 30$ 53

Mike and Abdul collect model cars. Mike has m cars and Abdul has $10m$ cars.

9. Write a sentence that describes how many cars Abdul has compared to Mike. Abdul has 10 times as many cars as Mike.

10. If Mike has 6 cars in his collection how many does Abdul have? 60

11. A company uses the formula $25 + 13.50h$ to calculate the daily cost in dollars for renting lawn equipment. The variable h represents the number of hours the equipment is rented for. How much would it cost to rent a piece of equipment for 4 hours? $79

12. The formula for the perimeter, P, of a rectangle is given by the formula $P = 2l + 2w$ where l is the length and w is the width. Find the perimeter of a room whose length is 10 feet and width is 14 feet. 48 feet

Level 1: q1–4
Level 2: q1–7
Level 3: q1–10

Lesson 1.1.2 — Simplifying Expressions

Simplify the expressions in exercises 1 – 6 by expanding parentheses and collecting like terms.

1. $3x + 5y + 8x - 3y + 2 + 4$ $11x + 2y + 6$
2. $-5(7c - 8)$ $-35c + 40$
3. $-e(2f - 7)$ $-2ef + 7e$
4. $2x - 5 - 8x + 11$ $-6x + 6$
5. $5(3r + 6) - 23$ $15r + 7$
6. $5(7a + 6) + 4(5 - 3a)$ $23a + 50$

7. Hector is 3 years older than Ami. Kim is twice as old as Hector. Toni is 5 years younger than Kim. If Hector is x years old, write an expression for the combined age of Hector, Ami, Kim and Toni, then simplify your answer as much as possible. $6x - 8$

Lisa earns $8 per hour in her job. She works a fixed 20 hours between Monday and Friday and sometimes works extra hours at the weekend.

8. Write an expression to describe how much money she earns in a week if she works an extra h hours over the weekend. $160 + 8h$

9. Use your expression to find how much Lisa earns in a week if she works 6 hours over the weekend. $208

10. Kendra, Shawn, and Mario are collecting bottles for a recycling project at their school. Kendra collected 5 times as many bottles as Shawn. Mario collected 15 bottles. Let s represent the number of bottles Shawn collected. Write down and simply an expression for the total number of bottles collected. $6s + 15$

Level 1: q1–4
Level 2: q1–6
Level 3: q1–9

Lesson 1.1.3 — Order of Operations

Evaluate.

1. $16 \div 2 \bullet 4 + 5$ 37
2. $24 + 4 \bullet 3 - 8^2$ –28
3. $(2 + 3)^2 \bullet (11 - 8)$ 75

Simplify.

4. $k \bullet (8 + 2) - 12$ $10k - 12$
5. $4 + t^2 \bullet (12 \div 3 + 6)$ $4 + 10t^2$
6. $y + 4^2 - (9 - 2^3) \bullet y + 25$ 41

7. Insert parentheses into the expression $3 + 8 \bullet 4^2 - 10 \div 2$ to make it equal 27. $3 + 8 \bullet (4^2 - 10) \div 2$

The local brake repair shop charges $65 per hour for labor plus cost of parts.

8. Write a calculation to describe the cost of a 3-hour repair if parts cost $54. $(54 + 65 • 3)

9. Evaluate your expression to find the cost of the job. The job would cost $249.

Solutions
For worked solutions see the Solution Guide

Lesson 1.1.4 — The Identity and Inverse Properties

Level 1: q1–5
Level 2: q1–8, 11–12
Level 3: q1–14

1. What is the multiplicative inverse of $\frac{2}{3}$? $\frac{3}{2}$
2. What is the additive inverse of $(x + y)$? $-x - y$ OR $-(x + y)$
3. Does zero have a multiplicative inverse? Explain your answer. **No — no number will multiply zero to give 1.**

Simplify the expressions in Exercises 4 – 10. Justify each step. **See solution guide for justification steps.**

4. $2 + x - x$ **2**
5. $6 - a \bullet 1$ **$6 - a$**
6. $-12a + 12a + 5$ **5**
7. $\frac{1}{2}(4b + 2) + b$ **$3b + 1$**
8. $d \bullet 1 + 2 - d$ **2**
9. $8(p - \frac{1}{8}) + 1$ **$8p$**
10. $2(3y + \frac{1}{2} + 0) + (-8 + 8 - 6y)$ **1**

Determine if the following statements are true or false.

11. Any whole number can be written as a fraction. **True**
12. Any fraction can be written as a whole number. **False**
13. A number multiplied by its reciprocal is always 1. **True**
14. A number divided by itself is zero. **False**

Lesson 1.1.5 — The Associative and Commutative Properties

Level 1: q1–5
Level 2: q1–11
Level 3: q1–14

Identify the property used in Exercises 1 – 3.

1. $2 + x = x + 2$ **Commutative property of addition**
2. $(2 \bullet 3) \bullet 7 = 2 \bullet (3 \bullet 7)$ **Associative property of multiplication**
3. $(2x + 5)8 = 16x + 40$ **Distributive property**

Simplify the expressions in Exercises 4 – 9. Justify your working. **See solution guide for justifications.**

4. $(5 + 2x) + 3x$ **$5 + 5x$**
5. $2v + 7 + 8v$ **$10v + 7$**
6. $12(7g)$ **$84g$**
7. $-3y + (7y + 2 - 3)$ **$4y - 1$**
8. $7r + 6 + 5r + 4$ **$12r + 10$**
9. $5 \bullet w \bullet 7$ **$35w$**

Determine if statements 10 – 14 are true or false.

10. Subtraction is commutative. **False**
11. Division is commutative. **False**
12. Subtraction can be rewritten as addition by adding the opposite. **True**
13. $5 - (a - 3) = (5 - a) - 3$ **False**
14. $a \div b = a \times \frac{1}{b}$ **True**

Lesson 1.2.1 — Writing Expressions

Level 1: q1–4, 7–8
Level 2: q1–10
Level 3: q5–13

Write the variable expressions to describe the word expressions in exercises 1 – 6.

1. the product of 5 and a number, x **$5x$**
2. the quotient of a number, y, and 10 **$y \div 10$**
3. 12 less than a number, c **$c - 12$**
4. 2 increased by twice a number, f **$2 + 2f$**
5. the product of 8 and the sum of a number, r, and 5 **$8(r + 5)$**
6. 15 decreased by twice the quotient of a number, p, and 4 **$15 - (2 \times p \div 4)$**

Are these statements true or false? If false, rewrite the statement so that it is correct.

7. To triple a number means to add 3 to the number. **False — to triple is to multiply by 3.**
8. To quadruple a number means to multiply the number by 4. **True**
9. Twice a number means to raise a number to the second power. **False — twice a number means multiply by 2.**

Write variable expressions for the following. Use x as the variable in each case and say what it represents.

10. The height of a triangle is 8 cm. What is the area of the triangle? **$\frac{1}{2} \times 8 \times x$, x is base of triangle**
11. The width of a rectangle is 9 meters. What is the area of the rectangle? **$9x$, x is length**
12. William has \$25 more than Luke. How much money does William have? **\$$x$ + \$25, x is amount of money Luke has**
13. The video store charges a monthly membership fee of \$15 plus \$2.50 per movie rental. What is the total cost of per month? **\$2.50x + \$15, x is amount of movies rented**

Solutions
For worked solutions see the Solution Guide

Additional Questions

Lesson 1.2.2 — 1.2.7

Level 1: q1–7
Level 2: q1–11
Level 3: q1–14

Lesson 1.2.2 — Variables and Expressions

Prove the equations are true in Exercises 1 – 3. See below

1. $2(10 - 7) + 4 = 55 \div 11 \bullet 2$
2. $8 \bullet 3 \div 6 \bullet 9 = 6^2$
3. $7 - 4 - 2 \bullet \frac{1}{2} + 5 = 5(2^3 + 1) - (3 \bullet 13 - 1)$

Say whether each of the following is an expressions or an equation.

4. $3b$ expression
5. $4x = 12x + 5$ equation
6. $2 + 7 = 32$ equation
7. $2a - 4$ expression

Write an equation to describe each of the sentences in Exercises 8 – 14.

8. Three more than the product of five and b is equal to 40. $5b + 3 = 40$
9. Twenty decreased by the quotient of four and h is equal to 10. $20 - (4 \div h) = 10$
10. Eight increased by the product of three and c is equal to the difference of five and c. $8 + 3c = 5 - c$
11. Three less than the quotient of y and 2 is equal to the sum of 11 and y. $(y \div 2) - 3 = 11 + y$
12. Jessica earns \$8.50 per hour. She earned \$306 for working h hours. \$8.50h = \$306
13. A cell phone company charges a \$10 monthly fee plus \$0.05 a minute for phone calls. Denise's monthly bill for m minutes of phone calls was \$14.75. \$10 + \$0.05m = \$14.75
14. Jose had \$75. He bought 3 movie tickets at \$$d$ a ticket and spent \$25 on food, leaving \$24.50. \$75 – \$3d – \$25 = \$24.50

Level 1: q1–10
Level 2: q1–12
Level 3: q4–12

Lesson 1.2.3 — Solving One-Step Equations

Name the operation that is appropriate for solving each of the equations in Exercises 1 – 3.

1. $5t = 45$ division
2. $b - 7 = 3$ addition
3. $w \div 12 = 4$ multiplication

Find the values of the variables in Exercises 4 – 9.

4. $3k = 39$ $k = 13$
5. $b - 5 = 12$ $b = 17$
6. $-45 = x - 40$ $x = -5$
7. $-18 = -3p$ $p = 6$
8. $h \div -8 = 11$ $h = -88$
9. $a - 24 = -60$ $a = -36$

10. The Spring Hill city council is planning to construct a new courthouse that, at 970 feet, will be twice as tall as the existing courthouse. Use the equation $2x = 970$ to find the height of the existing courthouse. $x = 485$ feet

11. Marcus purchased an outfit for \$54 after receiving a \$15 markdown. This can be described by the equation x – \$15 = \$54. Solve the equation and say what x represents. x = \$69, x is the original price

12. The height of the Washington Monument is 152 meters. The combined height of the Washington Monument and the Statue of Liberty is 245 meters. Write an equation to find the height of the Statue of Liberty and then solve for the height.
$S + 152 = 245$, $S = 93$ meters where S is the height of the Statue of Liberty

Level 1: q1–6
Level 2: q1–10
Level 3: q1–12

1.2.4

1. subtraction, multiplication
2. addition, division
3. multiplication, subtraction
4. subtraction, division

Lesson 1.2.4 — Solving Two-Step Equations

In exercises 1 – 4 say which order you should undo the operations in.

1. $8x - 2 = 22$
2. $w \div 12 + 8 = 12$
3. $3 \bullet (d - 4) = 24$
4. $b \div 2 - 4 = 6$ See margin

Find the values of the variables in Exercises 5 – 10.

5. $3b + 4 = 16$ $b = 4$
6. $30 = 12 + 9x$ $2 = x$
7. $y \div 8 + 10 = 15$ $y = 40$
8. $-45 = x \div 9 - 48$ $27 = x$
9. $18 = -3p + 9$ $-3 = p$
10. $8h - 12 = -76$ $h = -8$

11. A decorator charges \$17 an hour plus a \$25 fee for each job. In one job the decorator made \$620. Write an equation using this information and solve it to find how many hours the job took. \$17h + \$25 = \$620, $h = 35$ hours

12. The video store charges a monthly membership fee of \$15 plus \$2.50 per movie rental. Amanda's monthly bill was \$30. Write an equation and solve it to find the number of movies she rented. \$15 + \$2.50m = 30, $m = 6$ movies

Solutions
For worked solutions see the Solution Guide

1.2.2

1. $2(10-7)+4=55\div 11\cdot 2$
$2(3)+4=5\cdot 2$
$6+4=10$
$10=10$

2. $8\cdot 3\div 6\cdot 9=6^2$
$24\div 6\cdot 9=36$
$4\cdot 9=36$
$36=36$

3. $7-4-2\cdot\frac{1}{2}+5=5(2^3+1)-(3\cdot 13-1)$
$7-4-1+5=5(8+1)-(39-1)$
$3-1+5=5(9)-38$
$2+5=45-38$
$7=7$

Lesson 1.2.5 — More Two-Step Equations

Level 1: q1–3, 7–8
Level 2: q4–12
Level 3: q4–10, 13–14

Find the values of the variables in Exercises 1 – 6.

1. $\frac{1}{3}c = 8$ **$c = 24$** **2.** $6 = \frac{3}{4}a$ **$8 = a$** **3.** $-\frac{4}{3}b = -8$ **$b = 6$**

4. $-\frac{5}{6} \bullet w = 10$ **$w = -12$** **5.** $\frac{3b}{5} = -9$ **$b = -15$** **6.** $\frac{y+7}{4} = 8$ **$y = 25$**

Solve the equations in exercises 7 – 10 and check your solution.

7. $3x - 5 = 7$ **$x = 4$** **8.** $-18 = 2y - 6$ **$-6 = y$** **9.** $n \div 8 + 2 = -7$ **$n = -72$** **10.** $\frac{5}{6}t = -10$ **$t = -12$**

In Exercises 11 – 14 say which order you should undo the operations.

11. $\frac{2}{3}x = 8$ **division, multiplication** **12.** $-\frac{4x}{7} = 3$ **division, multiplication** **13.** $\frac{4c+5}{9} = 3$ **division, addition, multiplication** **14.** $-5 + \frac{3a+2}{8} = -12$ **addition, division, addition, multiplication**

Lesson 1.2.6 — Applications of Equations

Level 1: q1–3
Level 2: q1–5
Level 3: q1–7

In Exercises 1 – 2 use estimation to pick the answer that is reasonable.

1. Joan's annual income in $16,276. Which of the following is her weekly income?
a. $846,352 b. $313 c. $1600 **b**

2. A car travels at a speed of 45 mph. Which of these is the distance it would travel in ½ an hour?
a. 22.5 miles b. 90 miles c. 15 miles **a**

3. Tonya is 10 years younger than her brother. The sum of their ages is 32. How old is Tonya? **11 years old.**

4. A decorating firm needs to order 362 gallons of paint. Paint is sold in 5 gallon containers. Write an equation to describe the number of containers, *n*, they must order. Solve the equation and say if your answer is reasonable in the context of the situation. **$5n = 362$, $n = 72.4$, 72.4 is not reasonable — round up to 73.**

5. Lana got her car repaired at the local garage. She paid $585 for parts and labor. The parts cost $225. She was billed for 8 hours of labor. Solve an equation to find the hourly labor charge, $*d*. **$45 / hour**

6. A parking garage charges $3 for the first 2 hours, then $2.50 for each additional hour. How many hours, *h*, after the first two hours can you keep your car in the garage if you have $15? **4 hours**

7. Joyce spent $205 on supplies for her art work. She paid $175 for the easel and canvas. She also bought two sets of paint brushes which cost $*d* each. How much did each paint brush set cost? **$15.**

Lesson 1.2.7 — Understanding Problems

Level 1: q1–2, 4
Level 2: q1–6
Level 3: q1–8

In Exercises 1 – 3, say what missing piece of information is needed to solve each problem.

1. The length of a rectangle is 5 cm. What is the area? **width of rectangle**
2. Neal has $75. How long is it going to take him to save $200? **rate at which he is saving the money**
3. Greg's car repair bill was $225. Parts cost $85. What was the hourly labor charge? **labor time**

In Exercises 4 – 5, solve the problem and state what information is not relevant.

4. Taylor's cell phone plan charges a monthly fee of $15 plus $0.05 a minute for each call. Taylor's cell phone costs $65. How much was last month's bill if she used 90 minutes? **Bill was $19.50. Cost of phone not needed.**

5. Kenneth makes $12 per hour in his job. Last week he worked 5 days. How many hours did he work if his paycheck was $420? **$h = 35$ hours. Number of days Kenneth worked not needed.**

In Exercises 6 – 7, fill in the blanks to make the statements true.

6. ____ miles ÷ 2 hours = 48 _____ **96, miles/hour**
7. 15 meters/second • _____ = 45 meters **3 seconds**
8. _____ Newtons • 6 _____ = 48 Newton-meters **8, meters**

Solutions
For worked solutions see the Solution Guide

Additional Questions
Lesson 1.3.1 — 2.1.3

Level 1: q1–6
Level 2: q1–11
Level 3: q1–14

Lesson 1.3.1 — Inequalities

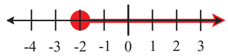
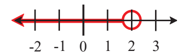
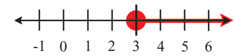
Fill in the blanks in Exercises 1 – 3.

1. If $x > y$ then y __ x. **<**
2. If $m \le n$ then n __ m. **≥**
3. If $a \ge b$ then __ ≤ __. ***b*, *a***

In Exercises 4 – 6 give a number that is part of the solution set of the inequality.

4. $x > -4$ **any number greater than –4**
5. $-5 \ge m$ **any number less than or equal to –5**
6. $x \ge 2$ **any number greater than or equal to 2**

Write the inequality expression for the given number line in Exercises 7 – 9.

7.  **$x \ge 2$**
8.  **$x < 2$**
9.  **$x \ge 3$**

Determine if the statements in Exercises 10 – 13 are true or false.

10. Inequalities have an infinite number of solutions. **True**
11. –4 is in the solution set of $x > -3$. **False**
12. 5 is in the solution set of $y \ge 5$. **True**
13. $t > 2$ means the same thing as $2 < t$. **True**

Level 1: q1–6
Level 2: q1–9
Level 3: q1–11

Lesson 1.3.2 — Writing Inequalities

In Exercises 1 – 3, insert an inequality symbol to make a true statement.

1. 24 inches **<** 3 feet
2. 1 century **>** 6 decades
3. 5 days **>** 50 hours

Write an inequality to describe the sentences in Exercises 4 – 7.

4. A number, p, decreased by four is more than fifteen. **$p - 4 > 15$**
5. Twenty-five times a number, m, is less than or equal to seven. **$25m \le 7$**
6. The quotient of a number, y, and three is less than ten. **$y \div 3 < 10$**
7. Eight increased by a number, c, is at least twelve. **$c + 8 \ge 12$**

Write an inequality to describe the situations in exercises 8 – 11.
You will need to use and define a suitable variable in each case.

8. The maximum weight permitted on the bridge is ten tons. **$w \le 10$ tonnes, w is total weight on bridge**
9. You must be at least twenty-five years old to run for an elective position on the city council. **$a \ge 25$, a is age**
10. The minimum allowed height for entry to a rollercoaster ride is 120 cm. **$h \ge 120$ cm, h is height of person.**
11. Sarah and Miguel have over \$350 in savings combined. Sarah has twice as much as Miguel. **$2m + m > 350$ so $3m > 350$, m is Miguel's savings**

Level 1: q1–4
Level 2: q1–6
Level 3: q1–7

Lesson 1.3.3 — Two-Step Inequalities

Write an inequality to describe the sentences in Exercises 1 – 4.

1. Seven more than the quotient of a number, y, and three is less than twenty. **$y \div 3 + 7 < 20$**
2. Eight increased by the product of a number, w, and five is at least twelve. **$8 + 5w \ge 12$**
3. Nine is less than or equal to the difference of five and the product of two and a number, h. **$9 \le 5 - 2h$**
4. Sixteen less than the quotient of a number, k, and five is at most twenty-four. **$k \div 5 - 16 \le 24$**

5. Danielle ordered a meal that cost under \$20. She had a \$12 main course and a drink and desert each costing \$$d$. Write an inequality to describe how much Danielle spent. **$12 + 2d < 20$**

6. Pam is making a rectangular fenced area in her back yard. She has 100 meters of fencing. The width of the fenced area must be 20 meters. Write an inequality to describe the possible lengths, l. **$2l + 40 \le 100$**

7. Roberto needs an average score of at least 90 from four algebra tests to gain a grade A. His first three test scores are 98, 97, and 82. Write an inequality to describe what Roberto's final test score, x, must be in order to get an A. **$(98 + 97 + 82 + x) \div 4 \ge 90$**

Solutions
For worked solutions see the Solution Guide

Lesson 2.1.1 — Rational Numbers

Level 1: q1–2, 5–6, 11–12
Level 2: q1–7, 9-12
Level 3: q1–14

Show that the numbers in Exercises 1–4 are rational.

1. –6 $-\frac{6}{1}$

2. 4 $\frac{4}{1}$

3. –4.5 $-\frac{9}{2}$

4. 0.75 $\frac{3}{4}$

Convert the fractions in Exercises 5–8 into decimals without using a calculator.

5. $\frac{3}{5}$ **0.6**

6. $\frac{3}{8}$ **0.375**

7. $\frac{84}{5}$ **16.8**

8. $\frac{5}{6}$ **$0.8\overline{3}$**

Say whether the statements in Exercises 9–14 are true or false.

9. All terminating decimals are rational numbers. **True**

10. All decimals can be written as the quotient of two integers. **False**

11. All rational numbers can be written as a decimal. **True**

12. Terminating and repeating decimals are rational numbers. **True**

13. π is a rational number because you can write it as a fraction by putting it over 1. **False**

14. $\frac{1}{6}$ is a terminating decimal because when you do 1 ÷ 6 on a calculator you get 0.166666667. **False**

Lesson 2.1.2 — Converting Terminating Decimals to Fractions

Level 1: q1–2, 4, 7–8
Level 2: q1–5, 7–11
Level 3: q1–11

Convert the decimals in Exercises 1–3 into fractions without using a calculator.

1. 0.37 $\frac{37}{100}$

2. –0.103 $-\frac{103}{1000}$

3. 0.023 $\frac{23}{1000}$

Convert the decimals in Exercises 4–6 into fractions and simplify them if possible.

4. 0.208 $\frac{26}{125}$

5. –12.84 $-\frac{321}{25}$

6. –4.005 $-\frac{801}{200}$

Say whether the statements in Exercises 7–10 are true or false.

7. 0.23 = 0.2300 **True**

8. 2.05 = 2.50 **False**

9. Dividing the numerator and denominator of a fraction by the same number makes another fraction with the same value as the original. **True**

10. Any decimal greater than 1 can be written as a mixed number or an improper fraction. **False**

11. Mario noticed that 8 out of 10 people on a particular team were female. Sarah looked at the same team and said 4 out of 5 people were female. How can they both be right? **$\frac{8}{10}$ and $\frac{4}{5}$ have the same value.**

Lesson 2.1.3 — Converting Repeating Decimals to Fractions

Level 1: q1–3, 7–9
Level 2: q1–6, 10–12
Level 3: q4–12

In Exercises 1–3, use $x = 0.\overline{5}$.

1. Find 10*x*. **$5.\overline{5}$**

2. Use your answer to Exercise 1 to find 9*x*. **5**

3. Write *x* as a fraction in its simplest form. $\frac{5}{9}$

In Exercises 4–6, use $x = 1.\overline{67}$.

4. Find 100*x*. **$167.\overline{67}$**

5. Use your answer to Exercise 4 to find 99*x*. **166**

6. Write *x* as a fraction in its simplest form. $\frac{166}{99}$

Say whether the statements in Exercises 7–9 are true or false.

7. $1.2\overline{35} = 1.235\overline{35}$ **True**

8. $32.3\overline{4} = 323.\overline{4}$ **False**

9. Every rational number is a terminating or repeating decimal. **True**

Find whether each number in Exercises 10–12 is equal to $3.\overline{8}$.

10. $\frac{35}{9}$ **Equal**

11. $3\frac{8}{10}$ **Not equal**

12. $3\frac{8}{9}$ **Equal**

Solutions
For worked solutions see the Solution Guide

Additional Questions
Lesson 2.2.1 — 2.3.4

Level 1: q1–4, 7–8, 10–11, 14
Level 2: q1–8, 12–16
Level 3: q5–6, 9–13, 16–18

Lesson 2.2.1 — Absolute Value

Find the value of the expressions given in Exercises 1–3.

1. $|-2|$ 2 **2.** $|5|$ 5 **3.** $|-3|$ 3

In Exercises 4–6, say which expression has a larger value.

4. $|-8|$ or $|-4|$ $|-8|$ **5.** $|5 - 8|$ or $|24 - 25|$ $|5 - 8|$ **6.** $|6 + 1|$ or $|-3 - 3|$ $|6 + 1|$

Solve the equations given in Exercises 7–9.

7. $|b| = 8$ $b = 8$ or $b = -8$ **8.** $|a| = 0.5$ $a = 0.5$ or $a = -0.5$ **9.** $|h| = 6.5$ $h = 6.5$ or $h = -6.5$

Say whether each statement in Exercises 10–13 is true or false.

10. $|3 - 5| = |3| - |5|$ False **11.** $|3| \times |5| = |3 \times 5|$ True

12. A number and its opposite always have the same absolute value. True

13. Absolute value is the distance of a number from its opposite on the number line. False

Evaluate the expressions given in Exercises 14–17 when $a = -3$, $b = 4$, and $c = 2$.

14. $b - |a|$ 1 **15.** $3 \times |b - c|$ 6

16. $2 \times |a - c| + |b - a|$ 17 **17.** $|b + 4| - |c - 9|$ 1

18. If x and y are not 0, $|x| = |y|$, and $x + y = 0$, then what can we say about x and y? They are opposites

Level 1: q1–8, 10
Level 2: q1–11
Level 3: q1–12

Lesson 2.2.2 — Using Absolute Value

In Exercises 1–6, find the distance between the pairs of numbers given.

1. 8 and 12 4 **2.** 0 and 7 7 **3.** –7 and 7 14

4. –2 and 6 8 **5.** –3 and –8 5 **6.** 2.4 and 8.2 5.8

Say whether each statement in Exercises 7–10 is true or false.

7. $|d - e| = |d| - |e|$ False **8.** $|d - e| = d - e$ False

9. Absolute value can be used to compare numbers. True

10. If $|a - b|$ is small then a and b are far away from each other. False

11. Town A is 500 feet above sea level. Town B is 355 feet below sea level. How much higher is Town A than Town B? 855 feet

12. Which of the statements below can be represented by the inequality $|x - y| < 8$?
a. The length of a rod is 8 cm greater than the length of its bracket.
b. The length of a rod is within 8 cm of the length of its bracket.
b.

Level 1: q1–8, 13
Level 2: q1–13
Level 3: q5–18

Lesson 2.3.1 — Adding and Subtracting Integers and Decimals

Use the number line to work out the calculations in Exercises 1–6.

1. $-2 + 8$ 6 **2.** $9 - 6$ 3 **3.** $4 - (-3)$ 7

4. $-8 - (-2)$ –6 **5.** $5 - 8$ –3 **6.** $-6 + (-4)$ –10

Evaluate the expressions in Exercises 7–12 without using a number line.

7. $346 - 500$ –154 **8.** $500 - 346$ 154 **9.** $-846 + 86$ –760

10. $-36.4 - 45.82$ –82.22 **11.** $8.76 - 27.2$ –18.44 **12.** $14.9 + 5.3$ 20.2

Say whether each statement in Exercises 13–18 is true or false.

13. The sum of two negative numbers is always negative. True

14. $2 - 3 + 5 = 2 - (3 + 5)$ False **15.** $-x - x = -2x$ True

16. $-x + x = 2x$ False **17.** $x - (-y) = x + y$ True

18. $x + (-y) = x - y$ True

Solutions
For worked solutions see the Solution Guide

Lesson 2.3.2 — Multiplying and Dividing Integers

Level 1: q1–11
Level 2: q4–14, q18–19
Level 3: q7–19

Evaluate Exercises 1–4 using the number line.

1. 3×2 **6**
2. -2×4 **–8**
3. $12 \div 4$ **3**
4. $-10 \div 5$ **–2**

Evaluate Exercises 5–6 by drawing a rectangle and breaking the numbers into tens and units.

5. 15×14 **210**
6. 24×18 **432**

Evaluate Exercises 7–8 using long multiplication.

7. 15×14 **210**
8. 24×18 **432**

Evaluate Exercises 9–17 without using a calculator.

9. -12×84 **–1008**
10. -62×-34 **2108**
11. 14×-36 **–504**
12. $816 \div 3$ **272**
13. $448 \div 7$ **64**
14. $736 \div 8$ **92**
15. $1455 \div -5$ **–291**
16. $-3258 \div 9$ **–362**
17. $728 \div 14$ **52**

Say whether each statement in Exercises 18–19 is true or false.

18. The product of two negative numbers is positive. **True**
19. The quotient of a positive number and a negative number is negative. **True**

Lesson 2.3.3 — Multiplying Fractions

Level 1: q1–4, q6–7
Level 2: q2–8, q12
Level 3: q3–12

Use the area model to evaluate the fraction multiplications in Exercises 1–2.

1. $\frac{1}{2} \times \frac{1}{4}$ $\frac{1}{8}$
2. $\frac{3}{4} \times \frac{1}{2}$ $\frac{3}{8}$

Find the product in Exercises 3–5.

3. $\frac{2}{3} \times \frac{5}{7}$ $\frac{10}{21}$
4. $\frac{4}{5} \times \frac{6}{7}$ $\frac{24}{35}$
5. $\frac{3}{8} \times \frac{4}{5}$ $\frac{12}{40}$, or $\frac{3}{10}$

Find the product in Exercises 6–11. Simplify the results as much as possible.

6. $\frac{3}{10} \times \frac{8}{15}$ $\frac{4}{25}$
7. $1\frac{1}{2} \times \frac{2}{3}$ **1**
8. $4\frac{1}{6} \times 3\frac{2}{5}$ $14\frac{1}{6}$
9. $-8 \times \frac{3}{4}$ **–6**
10. $\frac{-6}{25} \times \frac{-5}{12}$ $\frac{1}{10}$
11. $\frac{2}{3} \times \frac{6}{15}$ $\frac{4}{15}$

12. A rectangular patio measures $4\frac{3}{8}$ meters long and $5\frac{1}{4}$ meters wide. What is the area of the patio as a mixed number? $22\frac{31}{32}$ **square feet**

Lesson 2.3.4 — Dividing Fractions

Level 1: q1–6, q13
Level 2: q2–10, q14
Level 3: q4–14

Find the reciprocals of the numbers in Exercises 1–3.

1. $\frac{3}{4}$ $\frac{4}{3}$
2. $\frac{5}{6}$ $\frac{6}{5}$
3. 2 $\frac{1}{2}$

Calculate the divisions in Exercises 4–9. Give your answers as fractions in their simplest form.

4. $\frac{3}{4} \div 3$ $\frac{1}{4}$
5. $\frac{5}{8} \div \frac{3}{7}$ $\frac{35}{24}$
6. $\frac{3}{8} \div \frac{5}{6}$ $\frac{9}{20}$
7. $-\frac{2}{3} \div \frac{4}{9}$ $-\frac{3}{2}$
8. $-\frac{11}{12} \div -\frac{25}{24}$ $\frac{22}{25}$
9. $-5 \div -\frac{5}{4}$ **4**

Evaluate the expressions in Exercises 10–12 and express the solutions as mixed numbers or integers.

10. $-8\frac{1}{2} \div 6$ $-1\frac{5}{12}$
11. $-5\frac{3}{5} \div -\frac{2}{25}$ **70**
12. $1\frac{1}{7} \div \frac{-4}{5}$ $-1\frac{3}{7}$

13. What is the product of a number and its reciprocal? **1**

14. The area of a room is $140\frac{1}{4}$ square feet, and its length is $14\frac{1}{2}$ feet. What is the width of the room? $9\frac{39}{58}$ **feet**

Solutions
For worked solutions see the Solution Guide

Additional Questions
Lesson 2.3.5 — 2.4.3

Level 1: q1–4, 7–9, 13
Level 2: q3–14, 17–18
Level 3: q4–18

Lesson 2.3.5 — Common Denominators

Find the prime factorization of the numbers given in Exercises 1–6.

1. 18 2×3^2
2. 24 $2^3 \times 3$
3. 56 $2^3 \times 7$
4. 11 11
5. 120 $2^3 \times 3 \times 5$
6. 150 $2 \times 3 \times 5^2$

Find the least common multiple of the pairs of numbers in Exercises 7–12.

7. 2 and 3 6
8. 5 and 8 40
9. 4 and 6 12
10. 8 and 12 24
11. 3 and 9 9
12. 12 and 18 36

In Exercises 13–16 put the fractions in each pair over a common denominator to show which is larger.

13. $\frac{3}{4}$ and $\frac{1}{2}$ $\frac{3}{4} > \frac{2}{4} = \frac{1}{2}$
14. $\frac{5}{6}$ and $\frac{3}{8}$ $\frac{5}{6} = \frac{20}{24} > \frac{9}{24} = \frac{3}{8}$
15. $\frac{4}{5}$ and $\frac{7}{8}$ $\frac{7}{8} = \frac{35}{40} > \frac{32}{40} = \frac{4}{5}$
16. $\frac{11}{12}$ and $\frac{9}{10}$ $\frac{11}{12} = \frac{55}{60} > \frac{54}{60} = \frac{9}{10}$

17. Put the fractions $\frac{3}{8}$, $\frac{4}{5}$, $\frac{7}{10}$, $\frac{9}{20}$, and $\frac{3}{4}$ in order. $\frac{3}{8}, \frac{9}{20}, \frac{7}{10}, \frac{3}{4}, \frac{4}{5}$

18. Three-fifths of the senior class voted to have the prom at the same place as last year's prom. One-third voted to change locations. Which option won the vote?
Holding the prom in the same place as last year

Level 1: q1–3, 10–12
Level 2: q1–6, 10–12
Level 3: q4–12

Lesson 2.3.6 — Adding and Subtracting Fractions

Evaluate the expressions in Exercises 1–9. Give your answers as fractions in their simplest form.

1. $\frac{2}{3} - \frac{5}{3}$ -1
2. $\frac{4}{5} - \frac{3}{5}$ $\frac{1}{5}$
3. $\frac{3}{4} + \frac{4}{9}$ $\frac{43}{36}$
4. $-\frac{7}{2} - \frac{11}{12}$ $-\frac{53}{12}$
5. $\frac{7}{10} - \frac{9}{2}$ $-\frac{19}{5}$
6. $\frac{2}{3} + \frac{1}{6}$ $\frac{5}{6}$
7. $-\frac{2}{3} + \frac{1}{2}$ $-\frac{1}{6}$
8. $-\frac{2}{3} - \frac{5}{6}$ $-\frac{3}{2}$
9. $-\frac{11}{15} - \left(\frac{-3}{20}\right)$ $-\frac{7}{12}$

Say whether the statements in Exercises 10–11 are true or false.

10. Subtracting a negative number is the same as adding a positive number. **True**

11. $-\left(\frac{3}{4}\right) = \frac{-3}{-4}$ **False**

12. Jennifer left home and jogged $\frac{5}{8}$ of a mile. She got tired, walked back $\frac{1}{4}$ of a mile, and then took a break. How far away from home was she when she took the break? $\frac{3}{8}$ **of a mile**

Level 1: q1–6
Level 2: q1–9, 13
Level 3: q7–14

Lesson 2.3.7 — Adding and Subtracting Mixed Numbers

Evaluate the expressions in Exercises 1–12. Give your answers in their simplest form.

1. $3\frac{1}{5} + 4\frac{2}{5}$ $7\frac{3}{5}$
2. $8\frac{5}{6} - 3\frac{1}{6}$ $5\frac{2}{3}$
3. $-4\frac{3}{8} - 2\frac{5}{8}$ -7
4. $10\frac{3}{5} - 6\frac{1}{2}$ $4\frac{1}{10}$
5. $5\frac{1}{8} + 4\frac{1}{3}$ $9\frac{11}{24}$
6. $4\frac{2}{9} - 3\frac{1}{6}$ $1\frac{1}{18}$
7. $4\frac{3}{4} - \left(-2\frac{5}{6}\right)$ $7\frac{7}{12}$
8. $-3\frac{5}{6} + 6\frac{1}{4}$ $2\frac{5}{12}$
9. $-3\frac{1}{4} - 1\frac{1}{2}$ $-4\frac{3}{4}$
10. $-5\frac{2}{9} - \left(-4\frac{5}{6}\right)$ $-\frac{7}{18}$
11. $3 - 6\frac{1}{2} + 8\frac{3}{4}$ $5\frac{1}{4}$
12. $4\frac{1}{3} + 7\frac{3}{4} - 7\frac{5}{6}$ $4\frac{1}{4}$

13. The length of a rectangular room is $3\frac{1}{2}$ feet, and its width is $5\frac{3}{4}$ feet. What is the perimeter of the room? $18\frac{1}{2}$ **feet**

14. In the morning, Jacob drank a glass and a half of fruit juice. That evening he drank another three and two-thirds glasses. How much juice did Jacob drink in total? $5\frac{1}{6}$ **glasses**

Solutions
For worked solutions see the Solution Guide

Lesson 2.4.1 — Further Operations With Fractions

Level 1: q1–6
Level 2: q1–9, 13–14
Level 3: q4–14

Do the calculations in Exercises 1–12 and simplify your answers where possible.

1. $\frac{3}{5}+\frac{1}{5}\times\frac{3}{4}$ $\frac{3}{4}$

2. $\frac{3}{8}-\frac{5}{8}\div\frac{7}{16}$ $-1\frac{3}{56}$

3. $2\frac{1}{4}\div\frac{3}{8}\times\frac{1}{5}$ $1\frac{1}{5}$

4. $\left(4-\frac{3}{8}\right)\times\frac{2}{3}$ $2\frac{5}{12}$

5. $3\frac{1}{4}\div 2\frac{1}{2}+4$ $5\frac{3}{10}$

6. $\frac{5}{6}\times 8-2\frac{3}{5}\times\frac{10}{3}$ -2

7. $\left(2\frac{1}{2}-6\frac{3}{4}\right)+5\times\frac{3}{8}$ $-2\frac{3}{8}$

8. $\frac{1}{8}\times\frac{2}{3}+\frac{-7}{12}-\frac{1}{6}$ $-\frac{2}{3}$

9. $\left(-2\frac{3}{8}+\frac{1}{12}\right)\div 4\frac{3}{5}$ $-\frac{275}{552}$

10. $\dfrac{4\frac{2}{3}}{\frac{5}{6}}+\dfrac{\frac{1}{8}}{\frac{3}{4}}$ $5\frac{23}{30}$

11. $\dfrac{\left(\frac{5}{6}-\frac{1}{6}\right)}{\frac{5}{12}}-\frac{3}{4}$ $\frac{17}{20}$

12. $\dfrac{\frac{-2}{3}}{1\frac{3}{4}}+2\times\frac{4}{5}$ $1\frac{23}{105}$

Exercises 13–14 are about a room that is $8\frac{1}{2}$ feet wide and $10\frac{3}{4}$ feet long.

13. What is the area of the room? $91\frac{3}{8}$ square feet

14. What is the perimeter of the room? $38\frac{1}{2}$ feet

Lesson 2.4.2 — Multiplying and Dividing Decimals

Level 1: q1–4, 6–7, 12–13
Level 2: q2–10, 12–14
Level 3: q3–14

Use the area model to solve the multiplications in Exercises 1–2.

1. 0.6×0.1 0.06

2. 0.8×0.1 0.08

Calculate the products in Exercises 3–5 by rewriting the decimals as fractions.

3. 0.6×0.5 $\frac{3}{10}$

4. 0.4×0.05 $\frac{1}{50}$

5. 1.2×2.3 $2\frac{19}{25}$

Calculate the quotients in Exercises 6–8 by rewriting the decimals as fractions.

6. $0.3 \div 0.1$ 3

7. $0.05 \div 0.1$ $\frac{1}{2}$

8. $15.96 \div 4.2$ $\frac{19}{5}$

Find the products in Exercises 9–11.

9. -1.23×-0.006 $\frac{369}{50{,}000}$

10. -2.04×-0.008 $\frac{51}{3125}$

11. 3.5×-0.4 $-\frac{7}{5}$

Say whether each statement given in Exercises 12–13 is true or false.

12. Dividing a number by 100 moves the decimal point two places to the right. False

13. If you multiply together three decimals, each with two decimal places, then the answer can have up to six decimal places. True

14. If $54 \div 18 = 3$, what is $0.54 \div 0.018$? 30

Lesson 2.4.3 — Operations With Fractions and Decimals

Level 1: q1–4
Level 2: q1–6, 10–11
Level 3: q1–11

Calculate the value of each expression given in Exercises 1–9.

1. $\frac{3}{4}\times 0.05$ 0.0375 or $\frac{3}{80}$

2. $-0.28\times\frac{1}{2}$ -0.14 or $-\frac{7}{50}$

3. $\frac{1}{4}\times\left(2.8+\frac{1}{2}\right)$ $\frac{33}{40}$ or 0.825

4. $0.6\times\left(\frac{1}{2}+1.4\right)$ 1.14 or $\frac{57}{50}$

5. $\frac{3}{8}\div(4\times 1.2)$ $\frac{5}{64}$ or 0.078125

6. $\frac{2}{3}-0.5$ $\frac{1}{6}$ or $0.1\overline{6}$

7. $\left(\frac{5}{6}+0.25\right)\times 0.8$ $\frac{13}{15}$ or $0.8\overline{6}$

8. $\dfrac{-2.4+5.6}{\frac{3}{4}}$ $4\frac{4}{15}$ or $4.2\overline{6}$

9. $\dfrac{\frac{4}{5}}{0.5+\frac{3}{4}}$ $\frac{16}{25}$ or 0.64

10. What is the area of a rectangular room that is $16\frac{3}{4}$ feet long and 10.5 feet wide?
175.875 square feet (or $175\frac{7}{8}$ ft²)

11. What is the length of a rectangle with an area of $183\frac{2}{3}$ square feet and a width of 14.5 feet?
$12\frac{2}{3}$ feet (or $12.\overline{6}$ ft)

Solutions
For worked solutions see the Solution Guide

Additional Questions

Lesson 2.4.4 — 2.6.1

Level 1: q1–2, 5–6
Level 2: q1–6
Level 3: q1–7

Lesson 2.4.4 — Problems Involving Fractions and Decimals

1. While cooking, Jose used $6\frac{1}{2}$ cups of flour, $2\frac{3}{4}$ cups of sugar, $5\frac{1}{2}$ cups of water, and $\frac{1}{4}$ cup of chocolate chips. How many cups of ingredients did he use in total? **15 cups**

Exercises 2–4 use the table on the right, which shows what proportion of 850 high school students use various methods of transport to get to school.

Walk	$\frac{4}{17}$
School bus	$\frac{5}{34}$
Public transportation	$\frac{9}{34}$
Car	$\frac{7}{34}$
Other	$\frac{5}{34}$

2. How many students walk or take the school bus to school? **325**
3. How many students use a car to get to school? **175**
4. How many students use public or "Other" transportation to get to school? **350**

A neighborhood has a meeting hall with a floorspace of 206.4 square yards. On Wednesdays half of the floorspace is used by the knitting club.

5. How many square yards do the knitting club use? **103.2 square yards**
6. How much would it cost to carpet the meeting hall if carpet costs $8.25 per square yard? **$1702.80**
7. A shop sells wire in spools and each spool has $9\frac{3}{4}$ yards of wire on it. Jeanie needs nineteen pieces of wire, each 1.5 yards in length. How many spools of wire must Jeanie buy? **4**

Level 1: q1–6, 10–12, 14–17
Level 2: q1–17
Level 3: q1–17

Lesson 2.5.1 — Powers of Integers

Write each of the expressions in Exercises 1–9 as a power in base and exponent form.

1. $4 \cdot 4$ **4^2**
2. $6 \cdot 6 \cdot 6$ **6^3**
3. $9 \cdot 9 \cdot 9 \cdot 9$ **9^4**
4. $-6 \cdot -6$ **$(-6)^2$**
5. $-5 \cdot -5 \cdot -5$ **$(-5)^3$**
6. 7 **7^1**
7. $3 \cdot 3 \cdot 4 \cdot 4 \cdot 5$ **$3^2 \cdot 4^2 \cdot 5^1$**
8. $-4 \cdot -4 \cdot 6 \cdot 6 \cdot 6$ **$(-4)^2 \cdot 6^3$**
9. $-(-4 \cdot -4)$ **$-((-4)^2)$**

Evaluate the expressions in Exercises 10–13.

10. $(-3)^2$ **9**
11. $-(3^2)$ **-9**
12. $(-2)^3$ **-8**
13. $4^2 \cdot 3^1$ **48**

Say whether each of the statements in Exercises 14–17 is true or false.

14. A negative number raised to an even power always gives a positive answer. **True**
15. $(-2)^4 = -(2^4)$ **False**
16. $2 + 2 + 2 + 2 + 2 = 2^5$ **False**
17. $3 \cdot 3 \cdot 3 \cdot 3 = 3^4$ **True**

Level 1: q1–3, 6, 10–11
Level 2: q1–6, 10–12, 13
Level 3: q4–15

Lesson 2.5.2 — Powers of Rational Numbers

Evaluate each of the expressions in Exercises 1–9.

1. $\left(\frac{1}{3}\right)^2$ **$\frac{1}{9}$**
2. $\left(\frac{2}{5}\right)^3$ **$\frac{8}{125}$**
3. $\left(\frac{2}{3}\right)^4$ **$\frac{16}{81}$**
4. $\left(-\frac{5}{7}\right)^2$ **$\frac{25}{49}$**
5. $\left(\frac{-3}{4}\right)^3$ **$-\frac{27}{64}$**
6. $(0.3)^2$ **0.09**
7. $(0.13)^2$ **0.0169**
8. $(-0.4)^3$ **-0.064**
9. $(-0.06)^2$ **0.0036**

Write each of the expressions in Exercises 10–12 in base and exponent form.

10. $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$ **$\left(\frac{1}{3}\right)^4$**
11. $(-0.04)(-0.04)$ **$(-0.04)^2$**
12. $\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{-1}{2} \cdot \frac{-1}{2} \cdot \frac{-1}{2}$ **$\left(\frac{3}{4}\right)^2 \cdot \left(-\frac{1}{2}\right)^3$**

Say whether each of the statements in Exercises 13–15 is true or false.

13. If $\frac{a}{b}$ is between 0 and 1 then $\left(\frac{a}{b}\right)^2 > \frac{a}{b}$. **False**
14. $\left(\frac{-a}{b}\right)^2 = \left(\frac{a}{-b}\right)^2$ **True**
15. Raising a decimal to a power is like repeatedly multiplying the decimal by itself. **True**

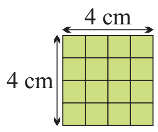
Solutions
For worked solutions see the Solution Guide

Lesson 2.5.3 — Uses of Powers

Level 1: q1–6, 9–10
Level 2: q1–11
Level 3: q1–11

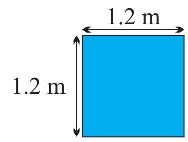
Find the area of the squares in Exercises 1–2.

1.



16 cm^2

2.



1.44 m^2

3. What is the area of a square with a side length of $\frac{1}{2}$ a foot? $\frac{1}{4}$ square foot

4. What is the volume of a cube of side length 2.4 feet? 13.824 $feet^3$

Write the numbers in Exercises 5–8 in scientific notation.

5. 218,534 2.18534×10^5

6. –32,400,000 -3.24×10^7

7. 5,183,000,000 5.183×10^9

8. 500 5×10^2

The numbers in Exercises 9–11 are written in scientific notation. Write them out in full.

9. 7.36×10^4 73,600

10. -8.1×10^6 –8,100,000

11. 6.123929×10^7 –61,239,290

Lesson 2.5.4 — More on the Order of Operations

Level 1: q1–5, 13–14, 16–18
Level 2: q1–10, 13–18
Level 3: q1–18

Evaluate each expression in Exercises 1–12.

1. $12 \div 3 \bullet 4$ 16

2. $10 - 6 + 3$ 7

3. $8 \bullet 6 \div 12$ 4

4. $2^3 \bullet 4 - 20$ 12

5. $(8 - 5)^3 \div 9$ 3

6. $2^4 + 3^2 \bullet (25 - 3^2)$ 160

7. $-5^2 - 8^2$ –89

8. $(-5)^2 - (8)^2$ –39

9. $(3^4 - 2 \bullet 3) \div 5^2$ 3

10. $\left(\frac{3}{4} \div \frac{5}{8}\right) \bullet \frac{10}{3}$ 4

11. $(0.5)^2 \bullet 8(5) + 3^2 \bullet 8$ 82

12. $\frac{4^3 - 6^2}{5 - 3 \bullet 10}$ $-\frac{28}{25}$

Say whether each of the statements in Exercises 13–18 is true or false.

13. $(5 + 3)^2 = 5^2 + 3^2$ False

14. $2 \bullet 3^2 = (2 \bullet 3)^2$ False

15. $(a \bullet b)^3 = a^3 \bullet b^3$ True

16. In the order of operations, division comes before subtraction. True

17. You should always do divisions before multiplications. False

18. It doesn't matter which order you do additions and parentheses in. False

Lesson 2.6.1 — Perfect Squares and Their Roots

Level 1: q1–9
Level 2: q4–15
Level 3: q4–15

Give the perfect square of each of the numbers in Exercises 1–6.

1. 3 9

2. 0 0

3. 5 25

4. –6 36

5. 13 169

6. –11 121

Evaluate the expressions in Exercises 7–12.

7. $\sqrt{225}$ 15

8. $-\sqrt{100}$ –10

9. $9^{\frac{1}{2}}$ 3

10. $121^{\frac{1}{2}}$ 11

11. $-36^{\frac{1}{2}}$ –6

12. $-25^{\frac{1}{2}}$ –5

13. There are 400 people in a marching band. How many people should be in each row if the band want to march in a square formation? 20

A square shaped room has an area of 576 square feet.

14. What is the length of the room? 24 feet

15. As part of a renovation, each wall is being extended by 5 feet. What will be the area of the newly renovated room? 841 $feet^2$

Solutions
For worked solutions see the Solution Guide

Additional Questions

Lesson 2.6.2 — 3.1.3

Level 1: q1–2, 6, 7–10, 16–22
Level 2: q1–12, 16–22
Level 3: q1–22

Lesson 2.6.2 — Irrational Numbers

In Exercises 1–6, prove that each number is rational by writing each one as a fraction in its simplest form.

1. 10 $\frac{10}{1}$ **2.** 0.6 $\frac{3}{5}$ **3.** 0.375 $\frac{3}{8}$

4. –7 $-\frac{7}{1}$ **5.** $\sqrt{25}$ $\frac{5}{1}$ **6.** $3.\overline{2}$ $\frac{29}{9}$

Classify each of the numbers in Exercises 7–15 as rational or irrational.

7. 2π irrational **8.** 2.756 rational **9.** 8 rational

10. $\sqrt{49}$ rational **11.** $\sqrt{6}$ irrational **12.** $-16^{\frac{1}{2}}$ rational

13. $-8^{\frac{1}{2}}$ irrational **14.** $2.24\overline{635}$ rational **15.** –1.2 rational

Say whether each of the statements in Exercises 16–22 is true or false.

16. Irrational numbers can be written as the quotient of two integers. False
17. The square root of a number that is not a perfect square is irrational. False
18. The square root of any integer other than a perfect square is irrational. True
19. Irrational numbers can be displayed in full on calculators. False
20. Terminating and repeating decimals are rational numbers. True
21. All integers are rational numbers. True
22. All rational numbers are integers. False

Level 1: q1–10, 13–14, 17–19
Level 2: q1–19
Level 3: q1–20

Lesson 2.6.3 — Estimating Irrational Roots

Say whether each of the numbers in Exercises 1–4 is rational or irrational.

1. $\sqrt{12}$ irrational **2.** $\sqrt{25}$ rational

3. $-\sqrt{36}$ rational **4.** $-\sqrt{14}$ irrational

Use your calculator to approximate the square roots in Exercises 5–8.
Give your answers to six decimal places.

5. $\sqrt{8}$ 2.828427 **6.** $\sqrt{23}$ 4.795832

7. $\sqrt{129}$ 11.357817 **8.** $\sqrt{520}$ 22.803509

In Exercises 9–12, say which two perfect squares each number lies between.

9. 8 4 and 9 **10.** 17 16 and 25

11. 55 49 and 64 **12.** 2 1 and 4

In Exercises 13–16, find the whole numbers that each root lies between.

13. $\sqrt{18}$ 4 and 5 **14.** $\sqrt{2}$ 1 and 2

15. $\sqrt{33}$ 5 and 6 **16.** $\sqrt{112}$ 10 and 11

In Exercises 17–19, say whether each statement is true or false.

17. $\sqrt{7} = 2.64575131106$ False **18.** $\sqrt{11} \approx 3.31662$ True

19. The square root of a perfect square is irrational. False

20. Will a 13 foot long bookcase fit along the wall of a square room with a floor area of 140 ft^2? Explain your answer.

No. If the room is square then each wall must be $\sqrt{140}$ feet long.
Since $11^2 = 121$ and $12^2 = 144$, $\sqrt{140}$ must be between 11 and 12.
So you could not fit in a 13 foot bookcase.

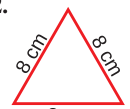
Solutions
For worked solutions see the Solution Guide

Lesson 3.1.1 — Polygons and Perimeter

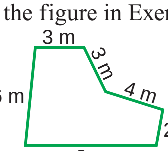
Level 1: q1–6
Level 2: q1–7
Level 3: q1, 4–9

1. Describe how to find the perimeter of a square. Either find the length of each side and add the 4 side lengths, or find the length of one side and multiply that length by 4.

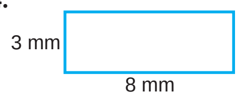
Find the perimeter of the figure in Exercises 2–5.

2.


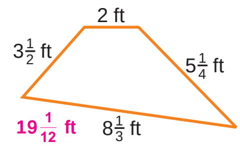
24 cm

3.


26 m

4.


22 mm

5.


$19\frac{1}{12}$ ft

Karl is putting up a fence around an 8 foot square plot.

6. What is the perimeter of the plot? 32 ft

7. The fence is going to be 4 rails high. How many feet of railing will Karl need to complete the fence? 128 ft

Alejandra is walking around the edge of a rectangular field measuring 23 meters by 14 meters.

8. What is the perimeter of the field? 74 m

9. Alejandra walked a total of 407 meters. How many times did she walk around the field? 5.5

Lesson 3.1.2 — Areas of Polygons

Level 1: q1–7
Level 2: q1–7
Level 3: q1–7

Find the area of each figure described in Exercises 1–2.

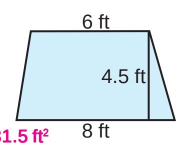
1. A triangle with base of 3 m and height of 4 m. 6 m²

2. A parallelogram with base of $3\frac{2}{3}$ in. and height of $2\frac{1}{2}$ in. $9\frac{1}{6}$ in²

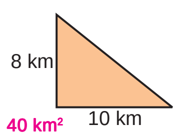
3. Copy the sentence below, and fill in the blank with the term that best completes the statement.
The area of a triangle is __half__ the area of a parallelogram that has the same base and vertical height.

4. Ramon is tiling a square kitchen that is 20 feet on each side.
If each tile is a 1 ft square, how many tiles will he need? 400

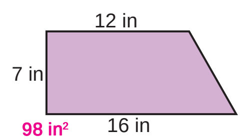
Find the area of each of the shapes in Exercises 5–7.

5.


31.5 ft²

6.


40 km²

7.


98 in²

Lesson 3.1.3 — Circles

Level 1: q1–5, 8–10
Level 2: q3–12
Level 3: q3–12

In Exercises 1–2, determine the missing measure.

1. Radius = __7.5__ cm, diameter = 15 cm.

2. Radius = $18\frac{3}{4}$ in, diameter = __$37\frac{1}{2}$__ in.

3. Explain the difference between the radius and the diameter of a circle. See below

4. Explain how to find the circumference of a circle when given the diameter.
To find the circumference, multiply the diameter by π.

In Exercises 5–12, use 3.14 for π in calculations involving whole numbers and decimals and $\frac{22}{7}$ for those involving fractions.

Find the circumference of each circle described in Exercises 5–7.

5. Radius = 3.3 in. 20.72 in.

6. Radius = 2.52 yds. 15.83 yd

7. Diameter = $6\frac{1}{2}$ mm $20\frac{3}{7}$ mm

Leilani is designing a circular medal with a 3 inch radius. Find:

8. The circumference of the medal. 18.84 in.

9. The area of the circular surface of the medal. 28.26 in²

Find the area of each circle described in Exercises 10–12.

10. Radius = 19 cm 1133.54 cm²

11. Radius = $\frac{1}{8}$ in $\frac{11}{224}$ in²

12. Diameter = $6\frac{2}{3}$ m $34\frac{58}{63}$ m²

Solutions
For worked solutions see the Solution Guide

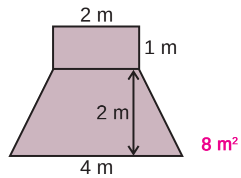
3.1.3 **3.** The radius is the distance from the edge of a circle to its centre.
The diameter is the distance from one side of the circle to the other, through the centre.
The radius is half the diameter.

Additional Questions
Lesson 3.1.4 — 3.3.1

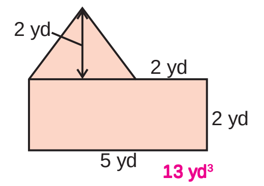
Level 1: q1–2, 4
Level 2: q1–6
Level 3: q1–7

Lesson 3.1.4 — Areas of Complex Shapes

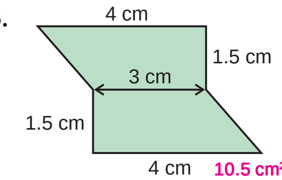
In Exercises 1–3, find the area of each complex shape.

1.


$8\ m^2$

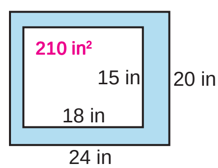
2.


$13\ yd^3$

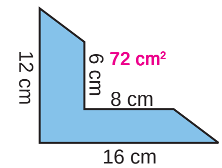
3.


$10.5\ cm^2$

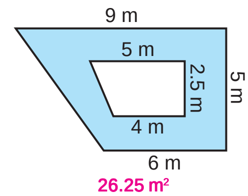
In Exercises 4–6, find the blue area.

4.


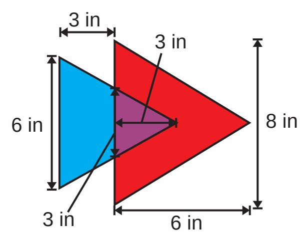
$210\ in^2$

5.


$72\ cm^2$

6.


$26.25\ m^2$

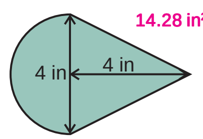


7. The math club created a new logo for their t-shirts. The new logo is shown on the left. What is the area covered by the logo? $37.5\ in^2$

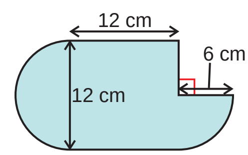
Level 1: q1–4
Level 2: q1–7
Level 3: q1–7

Lesson 3.1.5 — More Complex Shapes

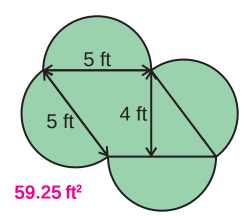
In Exercises 1–3, find the area of each complex shape. Use 3.14 for π.

1.


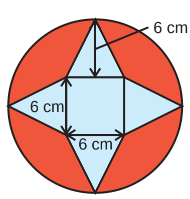
$14.28\ in^2$

2.


$228.78\ cm^2$

3.


$59.25\ ft^2$

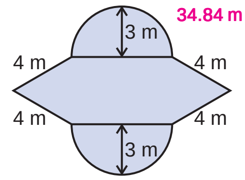


The shape on the left is part of a design for a quilt Regina is making. The four triangles are all the same size.

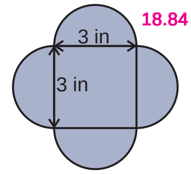
4. What is the area of the blue part of the design? $108\ cm^2$

5. What is the area of the red part of the design? $146.34\ cm^2$

In Exercises 6–7, find the perimeter of each shape.

6.


34.84 m

7.


18.84 in.

Solutions
For worked solutions see the Solution Guide

Lesson 3.2.1 — Plotting Points

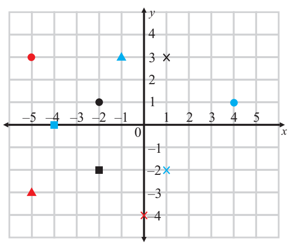
In Exercises 1–3, plot each pair of coordinates on the coordinate plane. Ex. 1–3: See margin

1. (4, –1) 2. (–2, –3) 3. (–3, 2)

4. On the coordinate plane, which axis is the horizontal axis? The x-axis
5. What are the coordinates of the origin on the coordinate plane? (0, 0)

Use the grid below to answer Exercises 6–13.
Identify which shapes are at the following coordinates:

6. (1, 3) Black cross 7. (–5, –3) Red triangle 8. (0, –4) Red cross



Find the coordinates of:

9. The black square (–2, –2) 10. The blue triangle (–1, 3)
11. The red circle (–5, 3)

One of the following pairs of coordinates on this grid does not belong with the others: (4, 1), (–1, 3), (–4, 0), (–5, –3)

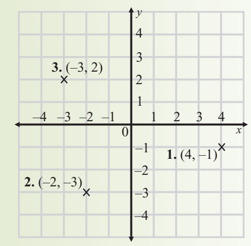
12. Say which pair of coodinates does not belong. Explain your answer.
13. Which pair of coordinates could put instead of your answer to Exercise 15 that would match the others in the set? Explain your answer.

Ex 12–13: See margin

Level 1: q1–8
Level 2: q1–11
Level 3: q4–13

3.2.1

1–3.



12. (–5, –3), because the shape at those coordinates is the red triangle, but the shapes at the other pairs of coordinates are blue.
13. If you replace (–5, –3) with (1, –2), then the shapes at all four pairs of coordinates are blue.

Lesson 3.2.2 — Drawing Shapes in the Coordinate Plane

In Exercises 1–4, plot the points given to find the missing coordinates.

1. Square ABCD: A(?, ?) B(4, 4) C(4, 0) D(0, 0) (0, 4)
2. Rectangle EFGH: E(–2, 4) F(1, 4) G(1, –1) H(?, ?) (–2, –1)
3. Rhombus KLMN: K(–4, 1) L(?, ?) M(0, –1) N(–4, –4) (0, 4)
4. Parallelogram QRST: Q(–3, 4) R(4, 4) S(?, ?) T(–5, 2) (2, 2)

Exercises 5–8 are about the shapes from Exercises 1–4. Ex. 5–6: See below

5. Find the perimeter and area of square ABCD
6. Find the perimeter and area of rectangle EFGH
7. Find the area of rhombus KLMN 20 sq. units
8. Find the area of parallelogram QRST 14 sq. units

9. Alyssa, the yearbook editor, has mapped out the layout of each yearbook page using a grid. If the photo of the volleyball team is placed with the edges at (1,2), (1, 8), (8, 2), and (8, 8), what area will the photo cover? 42 sq. units

Level 1: q1–2, 5–6
Level 2: q1–6, 9
Level 3: q1–9

Lesson 3.3.1 — The Pythagorean Theorem

In Exercises 1–3, copy and complete the following sentences:

1. For any right triangle, $c^2 = a^2 + b^2$, where c is the length of the <u>hypotenuse</u> and a and b are the lengths of the <u>legs</u>.
2. The hypotenuse is always the <u>longest</u> side of a right triangle.
3. In a right triangle, the hypotenuse is always opposite an angle that measures <u>90</u> °.

4. The Pythagorean Theorem is only true for right triangles. If you know the lengths of the three sides of a triangle, how can you use the Pythagorean Theorem to find out if it is a right triangle? See below

In Exercises 6–8, use the Pythagorean Theorem to decide whether a triangle with the given side lengths is a right triangle or not.

5. 4 cm, 9 cm, 12 cm — Not a right triangle
6. 10 ft, 6 ft, 8 ft — Right triangle
7. 5 yd, 7 yd, 5 yd — Not a right triangle

Level 1: q1–3
Level 2: q1–7
Level 3: q1–7

Solutions

For worked solutions see the Solution Guide

3.2.2
5. Perimeter = 4 + 4 + 4 + 4 = 16 units, Area = 4 × 4 = 16 sq. units
6. Perimeter = 3 + 5 + 3 + 5 = 16 units, Area = 3 × 5 = 15 sq. units

3.3.1
4. Call the longest of the three lengths c, and the other two side lengths a and b. Find the squares of a, b and c. If $c^2 = a^2 + b^2$, then the Pythagorean Theorem is true for this triangle, so the triangle is a right triangle.

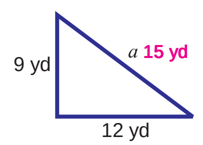
Additional Questions
Lesson 3.3.2 — 3.4.2

Level 1: q1–4, 7
Level 2: q1–8
Level 3: q1–9

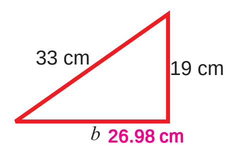
Lesson 3.3.2 — Using the Pythagorean Theorem

In Exercises 1-8, use the Pythagorean Theorem to find the missing length.
Round decimals to the nearest hundredth.

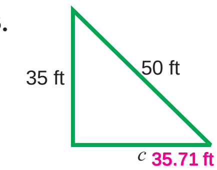
1.



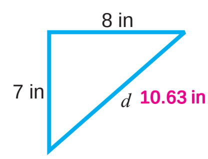
2.



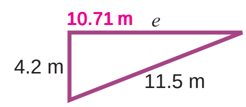
3.



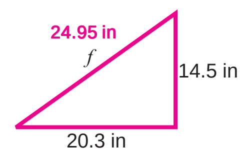
4.



5.



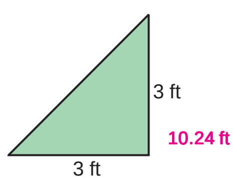
6.



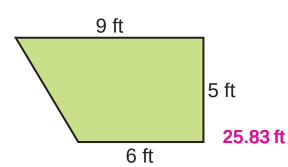
7. Lana has a triangular corner bookshelf. She wants to add rope edging along the hypotenuse. If each of the leg sides is 2.5 feet long, how much rope edging will she need? 3.54 ft

Eduardo and Destiny are planting a vegetable garden. Each plot needs to be fenced in.
In Exercises 8 and 9, determine how much fencing is needed for the plot shown.

8.



9.

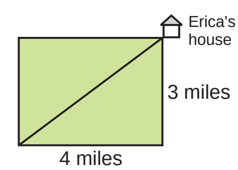


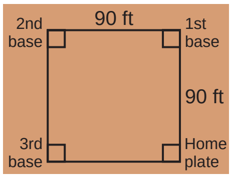
Level 1: q1–3
Level 2: q1–5
Level 3: q1–6

Lesson 3.3.3 — Applications of the Pythagorean Theorem

1. Mrs. Lopez is decorating the class bulletin board. She wants to place decorative trim around the perimeter and along each diagonal. The board is 8 ft. long and 4 ft. wide.
How much trim will Mrs. Lopez need? 41.88 ft

2. Erica usually runs the distance around the park shown on the right each day. When it is raining, she ends the run early by returning home along the diagonal. How much further does Erica run on a dry day compared to a rainy day? 2 miles





3. The school yard has a baseball diamond that is really a 90 foot square as shown on the left. If the catcher throws from home plate to 2nd base, what is the distance thrown? 127.28 ft

A quilt square is stitched along each diagonal to make 4 right triangles. Each diagonal is 12 inches long.
4. What is the perimeter of the square? 33.94 in
5. What is the area of the quilt square? 72 in²
6. How many quilt squares from can be cut from a piece of fabric that is 8 feet long and 2 feet wide? 22

Solutions
For worked solutions see the Solution Guide

Lesson 3.3.4 — Pythagorean Triples and the Converse of the Theorem

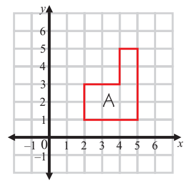
Tell whether the side lengths given in Exercises 1–5 indicate a right, obtuse or acute triangle.

1. 13, 13, 20 obtuse **2.** 8, 9, 11 acute **3.** 45, 60, 75 right
4. 4, 7.5, 8.5 right **5.** 1.2, 1.5, 1.7 acute

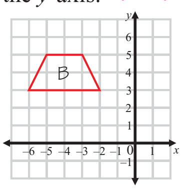
6. A blanket has length of 80 inches, width of 60 inches and a diagonal of 100 inches. Is the blanket a perfect rectangle? Explain your answer. See margin

Exercises 7–9 are about a triangle, XYZ. Side XY is 10.5 cm long. Side YZ is 17.5 cm long.
7. If side XZ was 13 cm long, would XYZ be right, acute or obtuse? obtuse
8. If XYZ was obtuse, and XZ was the longest side, what could you say about the length of XZ? See margin
9. Tammy claims that there is only one possible length for XZ that would make XYZ a right triangle. Is this true? Explain your answer. This is not true. There are two possible ways to make XYZ into a right triangle, because the hypotenuse can be either XZ or YZ.

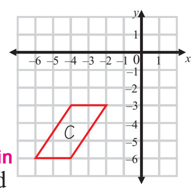
Lesson 3.4.1 — Reflections



Copy shape A on to a set of axes numbered –6 to 6 in both directions.
1. Draw the image A', made by reflecting A over the x-axis. See margin
2. Write the coordinates of the vertices of the image A", made by reflecting A over the y-axis. (–5, 5), (–5, 1), (–2, 1), (–2, 3), (–4, 3), (–4, 5)



Copy shape B on to a set of axes numbered –6 to 6 in both directions.
3. Draw the image B', made by reflecting B over the y-axis. See margin
4. Draw the image B", made by reflecting B over the x-axis. See margin
5. Write the coordinates of the vertices of the image B"', made by reflecting the image B' over the x-axis. (6, –3), (5, –5), (3, –5), (2, –3)

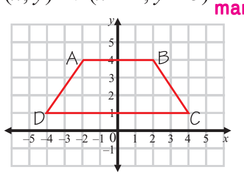


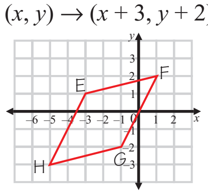
Copy shape C on to a set of axes numbered –6 to 6 in both directions.
6. Draw the image C', made by reflecting C over the x-axis. See margin
7. Write the coordinates of the image C", made by reflecting C over the y-axis. See margin
8. The vertices of the image C"' have the coordinates (–2, –3), (–4, –6), (–6, –6), and (–4, –3). Describe in words the transformation used to create C"' from the image C'. C"' is made by reflecting C' over the x-axis.

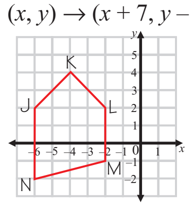
Lesson 3.4.2 — Translations

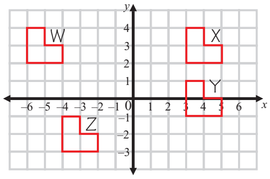
In Exercises 1–3, copy the shapes shown on to grid paper, and graph the indicated translations.

1. $(x, y) \to (x - 2, y - 5)$ See margin **2.** $(x, y) \to (x + 3, y + 2)$ See below **3.** $(x, y) \to (x + 7, y - 2)$ See margin









Use the grid shown on the left to answer Exercises 4–9
In Exercises 4–7, describe the indicated translations in coordinates.
4. X to W $(x, y) \to (x - 9, y)$ **5.** X to Z $(x, y) \to (x - 7, y - 5)$
6. W to Y $(x, y) \to (x + 9, y - 3)$ **7.** Z to Y $(x, y) \to (x + 7, y + 2)$

Exercises 8–9 each describe a translation between two shapes on the grid. Write the translation described in the form "A to B".
8. $(x, y) \to (x, y + 3)$ Y to X **9.** $(x, y) \to (x - 2, y + 5)$ Z to W

Level 1: q1–5
Level 2: q1–7
Level 3: q1–9

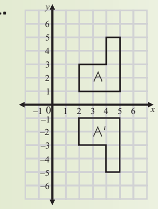
3.3.4

6. $a^2 + b^2 = 60^2 + 80^2 = 3600 + 6400 = 10,000$
$c^2 = 100^2 = 10,000$
$a^2 + b^2 = c^2$, so the triangles made by two sides and the diagonal are right triangles. So the blanket is a perfect rectangle.
8. The length of XZ is more than 20.41 cm.

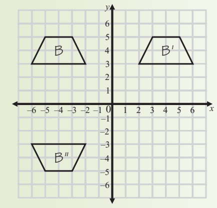
Level 1: q1–4
Level 2: q1–8
Level 3: q1–8

3.4.1

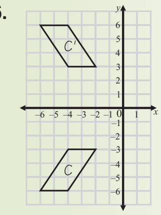
1.



3–4.



6.

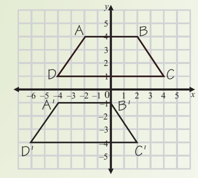


7. (2, –3), (4, –3), (6, –6), (4, –6)

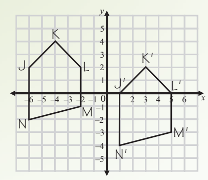
Level 1: q1–5
Level 2: q1–7
Level 3: q1–9

3.4.2

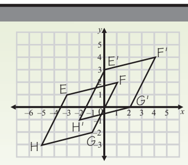
1.



3.



3.4.2 **2.**



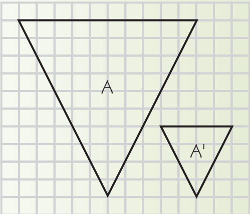
Solutions
For worked solutions see the Solution Guide

Additional Questions
Lesson 3.4.3 — 3.4.6

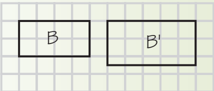
Level 1: q1–7
Level 2: q1–7
Level 3: q1–7

3.4.3

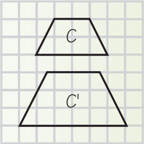
1.



2.



3.

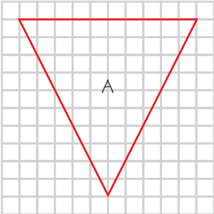


Level 1: q1–10
Level 2: q1–12
Level 3: q1–13

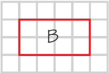
Lesson 3.4.3 — Scale Factor

In Exercises 1–3, draw an image of each figure using the given scale factor. **Ex. 1–3: see margin**

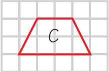
1. Scale factor 0.4



2. Scale Factor 1.25



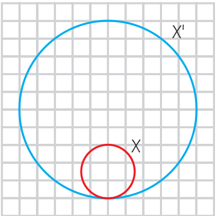
3. Scale Factor 1.5



4. Explain what happens when a scale factor of 1 is applied to a figure. **The image will be exactly the same size as the original.**

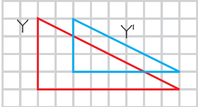
In Exercises 5–7, find the scale factor that produced each transformation.

5.



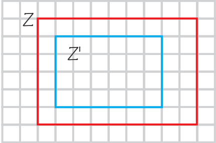
$\frac{10}{3}$

6.



0.75

7.



$\frac{2}{3}$

Lesson 3.4.4 — Scale Drawing

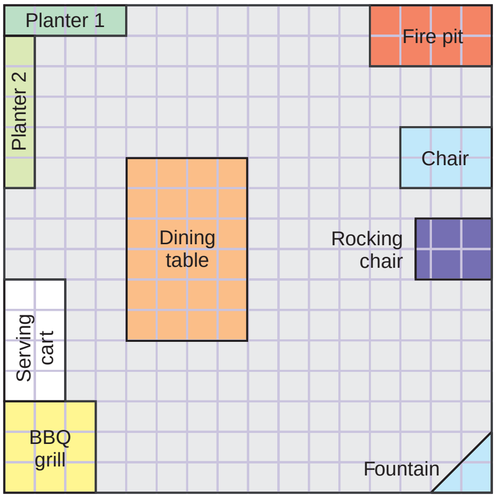
In Exercises 1–5, make the following scale drawings

1. A rectangular 20 × 40 ft. swimming pool, using the scale 1 in = 5 ft **Rectangle 4 in × 5 in**

2. A square play ground with sides of 60 yds using a scale 1 cm = 10 yds **Square 6 cm × 6 cm**

3. A rectangular 28 × 32 ft classroom using a scale 1 cm = 4 ft **Rectangle 7 cm × 8 cm**

4. A circular spa tub with a 9 ft diameter using a scale of 2 in = 3 ft **Circle with diameter 6 in**



This is a scale drawing of Michael's patio, using the scale 1 grid square = 2 ft.
In Exercises 5–12, find the real life dimensions of the following objects.

5. Planter 1 **8 ft by 2 ft**
6. Planter 2 **10 ft by 2 ft**
7. Fire pit **8 ft by 4 ft**
8. Dining Table **12ft by 8 ft**
9. Serving Cart **8ft by 4 ft**
10. BBQ grill **6 ft by 6 ft**
11. Rocking Chair **5 ft by 4 ft**
12. Fountain **4 ft by 4 ft**

13. Ian made a miniature of a portrait he was painting. The miniature was 7 inches long. If the actual portrait is 42 inches long, what is the scale factor he used? $\frac{1}{6}$

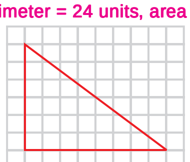
Solutions
For worked solutions see the Solution Guide

Lesson 3.4.5 — Perimeter, Area, and Scale

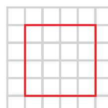
Level 1: q1–2, 6
Level 2: q1–8
Level 3: q1–8

In Exercises 1–5, calculate the perimeter and area if the image if the figure shown is multiplied by the given scale factor.

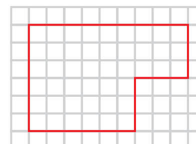
1. Scale factor 1
Perimeter = 24 units, area = 24 sq units



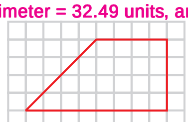
2. Scale factor 5
Perimeter = 80 units
Area = 400 sq units



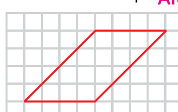
3. Scale factor $\frac{1}{3}$
Perimeter = 10 units
Area = 5 sq units



4. Scale factor 1.5
Perimeter = 32.49 units, area = 54 sq units



5. Scale factor $\frac{1}{4}$
Perimeter = 4.83 units
Area of image = 1 sq unit



In Exercises 6–8, find the scale factor used in each transformation.

6. Perimeter of original = 35 in.; Perimeter of image = 50 in. $1\frac{3}{7}$

7. Area of original = 92 mm; Area of image = 23 mm 0.5

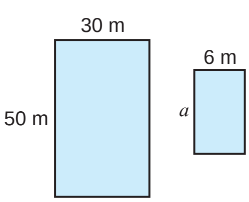
8. Area of original = 12 cm; Area of image = 72 cm 2.45

Lesson 3.4.6 — Congruence and Similarity

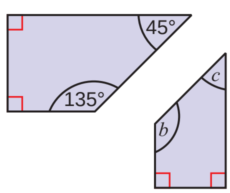
Level 1: q1–3
Level 2: q1–5
Level 3: q1–6

In Exercises 1–6, each pair of figures is similar.

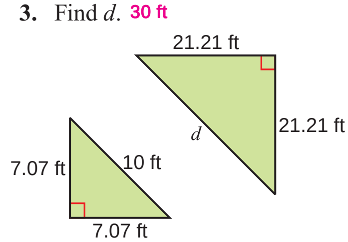
1. Find *a*. 10 m



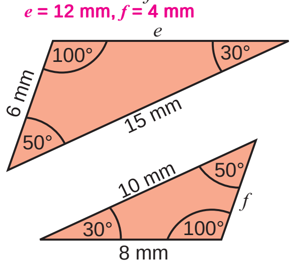
2. Find *b* and *c*. b = 135°, c = 45°



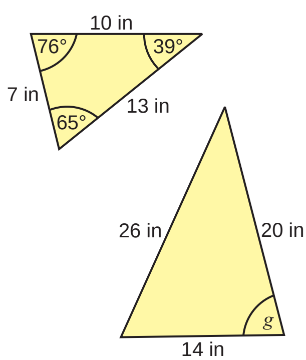
3. Find *d*. 30 ft



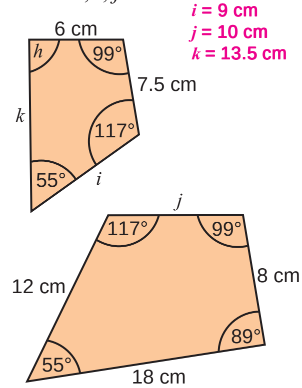
4. Find *e and f*.
e = 12 mm, f = 4 mm



5. Find *g*. g = 76°



6. Find *h*, *i*, *j* and *k*. h = 89°
i = 9 cm
j = 10 cm
k = 13.5 cm



Solutions
For worked solutions see the Solution Guide

Additional Questions
Lesson 3.5.1 — 3.6.2

Level 1: q1–6
Level 2: q1–8
Level 3: q1–8

Lesson 3.5.1 — Constructing Circles

In Exercises 1–6, use a ruler and compass to construct circles with the following features.

1. Circle of radius 2 in, with a chord of length 3 in. and a central angle of 120°.
2. Circle of radius 3 in, with a chord of length 3.5 in. and a central angle of 55°.
3. Circle of diameter 4.5 cm, with a chord of length 3 cm and a central angle of 160°
4. Circle of diameter 5 in, with a chord of length 4 in. and a central angle of 40°
5. Circle of radius 5 cm, with a chord of length 3.5 cm and a central angle of 145°
6. Circle of radius 3.2 cm, with a chord of length 4.8 cm and a central angle of 70°

1–6: Have students check each others' answers by measuring the radius, chord, and central angle of the circles they have drawn.

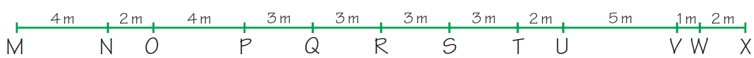
In Exercises 7–8, copy and complete the sentences.

7. The __radius__ is the distance from a point on the circumference of a circle to the center.
A __chord__ is the distance from a point on the circumference to another point on the circumference.
8. A chord of a circle can never be __longer__ than the circle's diameter.

Level 1: q1–9
Level 2: q1–9
Level 3: q1–9

Lesson 3.5.2 — Constructing Perpendicular Bisectors

Use the diagram below to find the midpoints of each segment in Exercises 1–6.



1. Segment NR P
2. Segment PR Q
3. Segment PT R
4. Segment NT Q
5. Segment SV U
6. Segment MX R

In Exercises 7–9, use a compass and straight edge to construct line segments of the following lengths, then construct their perpendicular bisectors.

7. $4\frac{1}{2}$ inches
8. 4.1 cm
9. $3\frac{1}{4}$ inches

7–9. Have students check each others' answers by measuring the line segments they have drawn and checking with a protractor and ruler that the bisector crosses the segment at the midpoint and at a 90° angle.

Level 1: q1–13
Level 2: q1–13
Level 3: q1–13

Lesson 3.5.3 — Perpendiculars, Altitudes, and Angle Bisectors

Draw a line segment, AB, that is 5 inches long. In Exercises 1–8, mark the following points on the line. Draw a perpendicular through each point.

1–10, 12. Have students check each others' answers using protractors and rulers.

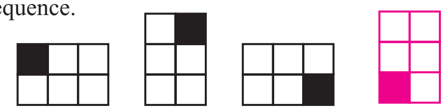
1. Point C, 1 inch away from A.
2. Point D, $2\frac{1}{2}$ inches away from A.
3. Point E, $1\frac{3}{4}$ inches away from B.
4. Point F, $\frac{3}{4}$ inch away from B.
5. Point G, $1\frac{1}{2}$ inches below AB.
6. Point H, $2\frac{1}{2}$ inches below AB.
7. Point I, 1 inch above AB.
8. Point J, $\frac{1}{2}$ inch above AB.
9. Draw a line segment PQ that is 4 cm long. Use a protractor to draw an angle at Q measuring 140°. Mark a point R on the new ray that you have drawn, so that the distance QR is 3 cm.
10. Bisect angle PQR using a compass and straightedge. Mark the point S on the angle bisector that is 5 cm away from Q.
11. What are the measures of the angles PQS and SQR? 70°
12. Use a compass and straightedge to bisect the angle PQS. Mark a point T on the new angle bisector that is 3.5 cm from Q.
13. What are the measures of angles PQT and TQS? 35°

Solutions
For worked solutions see the Solution Guide

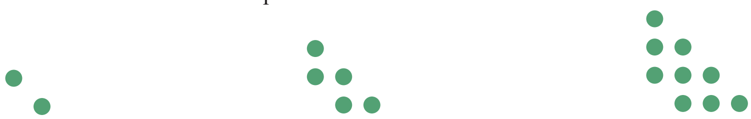
Lesson 3.6.1 — Geometrical Patterns and Conjectures

In Exercises 1–2, draw the next instance in the given sequence.

1. 

2. 

Exercises 3–7 are about the sequence of dots shown below.



Instance 1 Instance 2 Instance 3

Ex. 3–6: see below

3. Make a specific conjecture about instance 4.
4. Make a specific conjecture about instance 5.
5. Make a general conjecture about the pattern.
6. Draw the 6th instance in the sequence.
7. How many dots are in the 10th instance? 65

Are the following conjectures true or false? Explain your answers.

8. There were 365 people inside the school building during the last fire drill.
Conjecture: If John was inside the school at the time, John is a student. Not true. John could have been a teacher, a counselor, a parent or any other non-student.

9. There is a pecan tree in Maria's yard. Yesterday Maria picked up nuts that had fallen in the yard.
Conjecture: The nuts Maria collected must be pecans. Not true. There could have been other kinds of trees in her yard or her neighbors' yards.

10. Marcus made a perfectly square table. One corner is a right angle.
Conjecture: All the corners of the table are right angles. True. A square has 4 sides and 4 right angles.

11. Angela has a round cushion with a diameter of 1 yard.
Conjecture: The radius of the cushion is 18 inches.
True. The radius is half the diameter. 1 yard = 36 inches, 36 ÷ 2 = 18 in.

In Exercises 12–14, find the next three numbers in the series.

12. 6, 17, 28, 39 … 50, 61, 72
13. 3, 7, 12, 18 … 25, 33, 42
14. 1, 1, 2, 3, 5, 8 … 13, 21, 34

Lesson 3.6.2 — Expressions and Generalizations

In Exercises 1–2, find the next term in the following sequences. Explain your answer.

1. January, April, July… October
2. Sunday, Tuesday, Thursday… Saturday

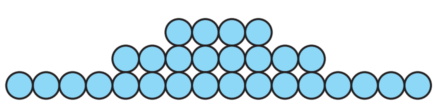
Exercises 3–5 are about the following number sequence: 3, 13, 23, 33…

3. Find the next term in the sequence. 43
4. Write an expression for the nth term in the sequence. $10n - 7$
5. Use your answer to Exercise 4 to find the 17th term in the sequence. 163

Exercises 6–8 are about the following number sequence: –6, 2, 10…

6. Find the next term in the sequence. 18
7. Write an expression for the nth term in the sequence. $8n - 14$
8. Use your answer to Exercise 4 to find the 12th term in the sequence. 82

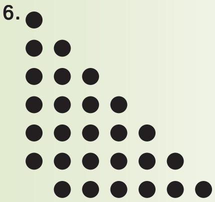
Use the diagram below to answer Exercises 9–12.



9. How many circles are in the 4th row? 32
10. How many circles are in the 5th row? 64
11. How many circles are in the nth row? 2^{n+1}
12. How many circles are in the 9th row? 1024

Level 1: q1–12
Level 2: q1–13
Level 3: q1–14

3.6.1

3. Answers may vary – possible answers include: Instance 4 will have 14 dots; instance 4 will have 5 more dots than instance 3.
4. Answers may vary – possible answers include: Instance 5 will have 20 dots; instance 5 will have 6 more dots than instance 4.
5. Answers may vary – possible answers include: in each instance, the number of dots increases by one more than the number of the instance.
6. 

Level 1: q1–5
Level 2: q1–8
Level 3: q3–12

Solutions
For worked solutions see the Solution Guide

Additional Questions

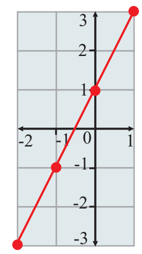
Lesson 4.1.1 — 4.2.3

Lesson 4.1.1 — Graphing Equations

In Exercises 1–4, say whether the equation given is a linear equation or not.

1. $y = 2x + 4$ Linear **2.** $y - 5 = x$ Linear

3. $y^2 + x^2 = 12$ Not Linear **4.** $2y - x = 8$ Linear



The diagram on the right is the graph of the equation $y = 2x + 1$.
Use the graph to explain whether the following are solutions to the equation $y = 2x + 1$.

5. $x = 1, y = 2$ (1, 2) doesn't lie on the line of the graph, so it isn't a solution of the equation.

6. $x = -1, y = -1$ (–1, –1) lies on the line of the graph, so it is a solution of the equation.

7. Show that $x = 2$, $y = 9$ is a solution of the equation $y = 12x - 15$. See below

8. Show that $x = -2$, $y = 1$ is a solution of the equation $y = 3x + 7$. See below

9. Find the solutions to the equation $y = \frac{1}{2}x + 1$ which have x values of –2, –1, 0, 1, and 2. Use the ordered pairs you have found to draw the graph of $y = \frac{1}{2}x + 1$. See left

10. Find the solutions to the equation $y = 2x - 2$ which have x values of 0, 0.5, 1, 1.5, and 2. Use the ordered pairs you have found to draw the graph of $y = 2x - 2$. See left

Lesson 4.1.2 — Systems of Linear Equations

1. How many possible solutions are there to a system of linear equations in two variables? There is only one possible solution.

In Exercises 2–3, write a system of linear equations to represent the statements.

2. Twice a number, y, is equal to a number, x, increased by 9.
Four less than the product of a number, x, and 3 is equal to a number, y. $2y = x + 9$ and $3x - 4 = y$

3. A number, q, increased by the product of a number, p, and 3 is equal to 10.
A number, p, is equal to the quotient of a number, q, and 2. $q + 3p = 10$ and $p = q \div 2$

4. Explain how to find the solution of a system of two linear equations by plotting them both on a graph.
Plot the graphs of both equations on the same axes. The point where the graphs cross is the solution to the system.

5. Check that the point (–1, 1) is the solution to the system of equations $y = 2x + 3$ and $y - x = 2$. See below

Solve the systems of equations in Exercises 6–9 by graphing. For graphs, see left.

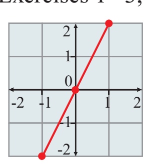
6. $y = x + 1$ and $y = 2x + 1$ (0, 1) **7.** $y = 1 - x$ and $2y = 2x + 2$ (0, 1)

8. $y = 2x$ and $4y = 2x$ (0, 0) **9.** $y = 0.5x + 1$ and $2y = x$ The lines are parallel, so don't cross. The system has no solution.

Lesson 4.1.3 — Slope

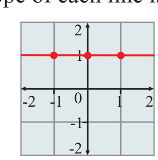
In Exercises 1– 3, say whether the slope of each line is positive, negative, or zero. Then find the slope.

1.



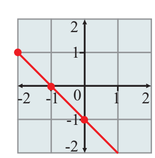
Positive
Slope = 2

2.



Zero
Slope = 0

3.



Negative
Slope = –1

4. Plot the graph of the equation $2y = 1 - x$ and find its slope. Slope = $-\frac{1}{2}$. For graph, see left.

5. Point A with coordinates (–1, 4) lies on a line with a slope of 4. Give the coordinates of any other point that lies on the same line. Any solution to the equation $y = 4x + 8$. For example, (0, 8), (1, 12), (2, 16),or (–2, 0)

In Exercises 6–11, find the slope of the line that passes through the two points given.

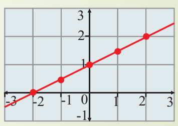
6. (3, 1) and (4, 2) 1 **7.** (1, 1) and (2, 3) 2 **8.** (0, 2) and (2, 0) –1

9. (–2, –3) and (–1, 0) 3 **10.** (–1, 3) and (3, 5) $\frac{1}{2}$ **11.** (2, 1) and (–3, 0) $\frac{1}{5}$

4.1.1

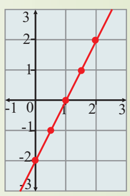
9.

x	y	(x, y)
–2	0	(–2, 0)
–1	0.5	(–1, 0.5)
0	1	(0, 1)
1	1.5	(1, 1.5)
2	2	(2, 2)



10.

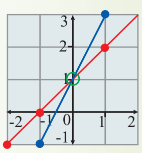
x	y	(x, y)
0	–2	(0, –2)
0.5	–1	(0.5, –1)
1	0	(1, 0)
1.5	1	(1.5, 1)
2	2	(2, 2)



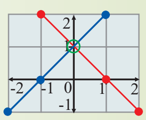
Level 1: q1–5, 7
Level 2: q1–7, 9
Level 3: q1–10

4.1.2

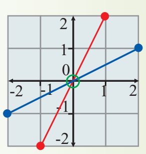
6.



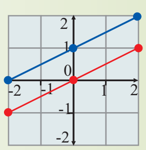
7.



8.



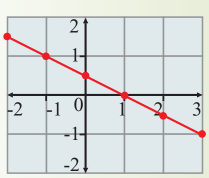
9.



Level 1: q1–5
Level 2: q1–7
Level 3: q1–9

4.1.3

4.



Level 1: q1–7
Level 2: q1–9
Level 3: q1–11

Solutions
For worked solutions see the Solution Guide

4.1.1

7. $y = 12x - 15$
$9 = 12(2) - 15$
$9 = 24 - 15 = 9$
So $x = 2$, $y = 9$ is a solution of the equation.

8. $y = 3x + 7$
$1 = 3(-2) + 7$
$1 = -6 + 7 = 1$
So $x = -2$, $y = 1$ is a solution of the equation.

4.1.2

5. $y = 2x + 3$ $y - x = 2$
$1 = 2(-1) + 3$ $1 - (-1) = 2$
$1 = -2 + 3 = 1$ $2 = 2$
As $x = -1$, $y = 1$ makes both equations true, (–1, 1) is the system's solution.

Lesson 4.2.1 — Ratios and Rates

In Exercises 1–2, express each statement as a ratio in its simplest form.

1. A store sells 2 rulers for every 1 eraser that is bought. **The ratio of rulers to erasers is 2:1, or $\frac{2}{1}$, or 2 to 1.**
2. For every 2 lemons that I have, I also have 8 oranges. **The ratio of lemons to oranges is 1:4, or $\frac{1}{4}$, or 1 to 4.**
3. Clayton is cooking breakfast for his family. He knows that he needs 18 pancakes to feed all 6 people. What is this written as a unit rate? **3 pancakes per person**

In Exercises 4–9 express the quantities as unit rates.

4. 60 apples in 10 pies. **6 apples per pie**
5. $4 for 2 meters of fabric. **$2 per meter**
6. 70 grams of food for 2 gerbils. **35 g per gerbil**
7. 120 miles in 3 hours. **40 miles per hour**
8. 90 books for 15 students. **6 books per student**
9. $2.97 for 3 pens. **$0.99 per pen**

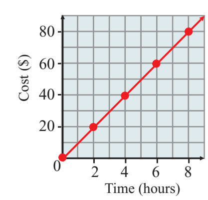
In Exercises 10–13 say which is the better buy. **See below**

10. 1 pen for $2 or 5 pens for $9
11. 2 kg of rice for $6, or 3 kg for $8.40
12. 300 ml of soda for $1.29, or $2.20 for 500 ml.
13. $7 for 100 minutes of calls or $7.20 for 2 hours.
14. A store sells 2kg bags of flour for $3.20, and 500 g bags of flour for $0.90. What is the price per kg for each size? **See below**

Level 1: q1–8, 10–11
Level 2: q1–12
Level 3: q1–14

Lesson 4.2.2 — Graphing Ratios and Rates

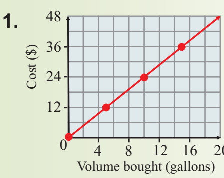
1. At a gas station the price of gas is $2.40 a gallon. Draw a graph to represent the relationship between the cost of the gas and the volume purchased. **See right**
2. A doctor measured a patient's resting pulse rate at 80 beats per minute. Draw a graph to show the relationship between time and the number of times the patient's heart beats. Use it to estimate how many times the patient's heart will beat in 18 minutes. **For graph, see right. 1440 times**



The graph on the right shows the relationship between the cost of hiring a bike and the number of hours you hire it for. Use the graph to answer Exercises 3–5.

3. How much does it cost to hire a bike for 5 hours? **$50**
4. What is the slope of the graph? **$10 per hour**
5. How much does it cost to hire a bike for 1 hour? **$10**
6. Madre works as a waitress. Today she worked an 8 hour shift, and was paid $92. Plot a graph of the amount Madre earns against the time she works for. Then find how much Madre is paid per hour. **For graph, see right. She earns $11.50 per hour.**

4.2.1

1.


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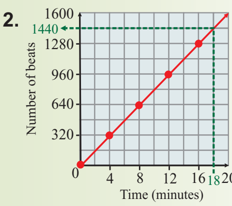

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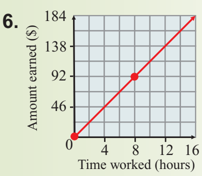


Level 1: q1, 3–5
Level 2: q1–5
Level 3: q1–6

Lesson 4.2.3 — Distance, Speed, and Time

1. Dan walks 12 blocks to school. It takes him 6 minutes. What is his average speed? **2 blocks per minute**
2. If a tortoise is crawling along at a speed of 6 yards per minute, how far will it crawl in 4 minutes? **24 yards**
3. Mr Valdez drove 400 miles in 8 hours. What was his average speed? **50 mph**
4. A hiking club go on a camping expedition. They can cover a distance of 18 km each day. If they plan to go 90 km in total, how long will their expedition take? **5 days**
5. A plane flies 2520 miles in 4 hours and 30 minutes. What is its average speed for the flight? **560 mi/h**
6. A machine can make 5 miles of silk ribbon in an hour. What length of ribbon can the machine make in an average 40 hour working week? **200 miles**
7. Each day Tya cycles to the bus stop to catch the school bus. She rides at an average speed of 10 mph, and the bus goes at an average speed of 40 mph. It takes her half an hour to do the 15 mile trip. Assuming she doesn't have to wait for the bus, find how long she spends riding her bike. **$\frac{1}{6}$ of an hour, or 10 minutes.**

Level 1: q1–5
Level 2: q1–6
Level 3: q1–7

Solutions
For worked solutions see the Solution Guide

4.2.1
10. 5 pens for $9.
11. 3 kg of rice for $8.40.
12. 300 ml of soda for $1.29.
13. $7.20 for 2 hours of calls.
14. Prices per kg: 2 kg bag = $1.60 per kg, 500 g bag = $1.80 per kg.

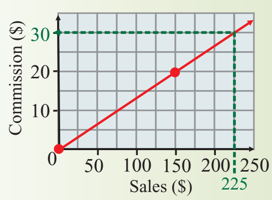
Additional Questions
Lesson 4.2.4 — 4.4.1

Level 1: q1–4, 8–10
Level 2: q1–10
Level 3: q1–12

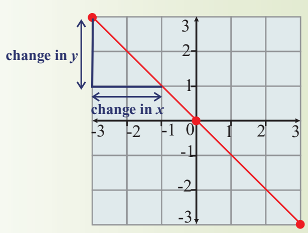
4.2.4

10.

Sales ($)	Commission ($)
0	0
150	20



11.



Lesson 4.2.4 — Direct Variation

The numbers a and b are in direct variation, and $a = 3$ when $b = 4$. In Exercises 1–6, find the value of a when b equals the value given.

1. 8 **6** **2.** 12 **9** **3.** 2 **1.5**
4. 7 **5.25** **5.** –1 **–0.75** **6.** –34 **–25.5**

7. It costs a school \$100 to take 20 students on a trip to the museum. Use direct variation to find how much it would cost to take 55 students. **\$275**

8. What point on the coordinate plane do all graphs that show direct variation pass through? **The origin, (0, 0)**

9. It took Jesse 3 hours to drive the 159 miles to his Grandmother's house. If he was driving at a constant speed, how far would he have driven after 2 hours? **106 miles**

10. A saleswoman receives \$20 in comission for every \$150 worth of goods she sells. Show this relationship on a graph. Use your graph to find how much commission she receives from \$225 of sales. **\$30 commission. For graph see left.**

The numbers x and y are in direct variation, and $x = –1$ when $y = 1$.
11. Write an equation relating x and y, and graph it on the coordinate plane. **$y = –x$. For graph see left.**
12. What is the slope of your graph from Exercise 11? What does the slope represent?
Slope = –1, the slope is the same as k, the constant of proportionality.

Level 1: q1–10
Level 2: q1–11, 15
Level 3: q1–16

Lesson 4.3.1 — Converting Measures

In Exercises 1–6, give the ratio between the units.

1. feet:inches **1:12** **2.** centimeters:millimeters **1:10** **3.** meters:kilometers **1000:1**
4. fluid ounces:cups **8:1** **5.** kilograms:grams **1:1000** **6.** pint:quart **2:1**

7. Complete this equation: 0.1 km = ? m = ? cm = 100,000 mm **0.1 km = 100 m = 10,000 cm = 100,000 mm**

8. Complete this equation: 2 quarts = ? pints = ? cups = 64 fluid ounces **2 quarts = 4 pints = 8 cups = 64 fluid ounces**

In Exercises 9–16, set up and solve a proportion to find the missing value, x, in each case.

9. 480 mm = x cm **48** **10.** 15 kilometers = x meters **15,000** **11.** 64 ounces = x pounds **4**
12. 5.5 pints = x cups **11** **13.** 50 grams = x kilograms **0.05** **14.** 300 pounds = x tons **0.15**

15. Nora is making soup. Her recipe calls for 3 quarts of water. How many one cup servings will it make? **12 cups**

16. Dell needs 70 feet of wallpaper border. If the border comes in 5 yard rolls, how many should he buy? **5 rolls**

Level 1: q1–6, 12
Level 2: q1–8, 11–12
Level 3: q1–12

Lesson 4.3.2 — Converting Between Unit Systems

In Exercises 1–4, give the ratio between the units

1. inches:centimeters **1:2.54** **2.** kilometers:miles **1.6:1**
3. liters:gallons **3.785:1** **4.** kilograms:pounds **1:2.2**

In Exercises 5–10 find the missing value, x, in each case. Give all your answers to 2 decimal places.

5. 18 inches = x cm **45.72** **6.** 49 kg = x pounds **107.80** **7.** 840 yards = x meters **764.40**
8. 31 km = x miles **19.38** **9.** 10 gallons = x liters **37.85** **10.** 14 kg = x ounces **492.80**

11. Salma's house is 3 km from the store. She cycles to the store to buy bread, and then rides on to the library. The library is a further 750 m from the store. How many miles has Salma cycled in total? **2.34 miles (to 2 decimal places)**

12. Bill is working on a science project. His task is to record the daily high temperature outside the school for a week. Bill's table of results is shown below. Fill in the missing temperatures.

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Temperature (°F)	86	**89.6**	**80.6**	86.9	**87.8**	**86**	85.1
Temperature (°C)	**30**	32	27	**30.5**	31	30	**29.5**

Solutions
For worked solutions see the Solution Guide

Lesson 4.3.3 — Dimensional Analysis

1. Match the equations with their missing units.

i) 320 copies × 0.10 $\frac{\text{dollars}}{\text{copy}}$ = 32 ? — A) inches

ii) 100 feet ÷ 5 minutes = 20 ? — B) dollars

iii) 2 feet × 12 $\frac{\text{inches}}{\text{foot}}$ = 24 ? — C) $\frac{\text{feet}}{\text{minute}}$

In Exercises 2–9, find the missing unit

2. 90 miles ÷ 3 hours = 30 miles/hour
3. 4 hours × 20 $\frac{\text{dollars}}{\text{hours}}$ = 80 dollars
4. 2 persons × 3 days = 6 person-days
5. 10 inches × 5 inches = 50 inches²
6. 1095 days × $\frac{\text{1 year}}{\text{365 days}}$ = 3 years
7. 8 miles ÷ 2 miles per hour = 4 hours
8. 15 m/s ÷ 5 s = 3 m/s²
9. 16 m³/person-day ÷ 4 m²/person = 4 m/day

10. The school dance team sell team pins during recess to fund travel to an out of town tournament. They earn \$15 each day. The cost of the trip is \$300. How many days do they need to sell for to cover the trip? **20 days**

11. My go-kart can travel at a maximum speed of 10 miles/hour. How far can it go in 1800 seconds? **5 miles**

Level 1: q1–7
Level 2: q1–10
Level 3: q1–11

Lesson 4.3.4 — Converting Between Units of Speed

In Exercises 1–6 create a conversion factor equal to one for each pair of units.

1. Days and weeks $\frac{\text{7 days}}{\text{1 week}}$
2. Meters and kilometers $\frac{\text{1000 meters}}{\text{1 kilometer}}$
3. Seconds and minutes $\frac{\text{60 seconds}}{\text{1 minute}}$
4. Miles and kilometers $\frac{\text{1 mile}}{\text{1.6 kilometers}}$
5. Days and hours $\frac{\text{1 day}}{\text{24 hours}}$
6. Meters and yards $\frac{\text{0.91 meters}}{\text{1 yard}}$

In Exercises 7–10 perform the conversions to find the missing numbers.

7. 32 miles per hour = w km per hour **w = 51.2**
8. 5 yards per minute = x meters per minute **x = 4.55**
9. 10 cm per second = y cm per hour **y = 36,000**
10. 3 feet per minute = z yards per hour **z = 60**

11. Which is faster, 85 miles per hour, or 120 km per hour? **85 miles per hour**

12. Davina and Juan had a race over a course 1000 m long. Davina's average speed was 9 kilometers per hour. Juan's average speed was 3 meters per second. Who won the race? **Juan**

Level 1: q1–8
Level 2: q1–11
Level 3: q1–12

Lesson 4.4.1 — Linear Inequalities

In Exercises 1–4, write the inequality in words.

1. $p > 5$ **p is more than five.**
2. $q < -13$ **q is less than minus thirteen.**
3. $r \le -2$ **r is less than or equal to minus two.**
4. $s \ge 2.5$ **s is greater than or equal to two point five.**

In Exercises 5–8, plot the inequality on a number line. **See below**

5. $j > 1$
6. $k \le -2$
7. $n \ge -1$
8. $m < 0$

9. A number, x, increased by twelve is at least nineteen. Write this statement as an inequality and solve it. **$x + 12 \ge 19$, $x \ge 7$**

In Exercises 10–13, solve the inequality for the unknown.

10. $a + 2 \le 10$ **$a \le 8$**
11. $b - 4 > 0$ **$b > 4$**
12. $c + (-2) < -1$ **$c < 1$**
13. $d - (-4) \ge 8$ **$d \ge 4$**

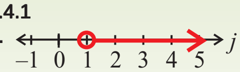
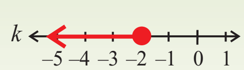
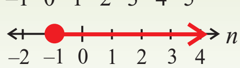
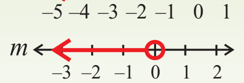
14. Ula is painting a room. She needs at least 5 liters of paint to cover the walls. She already has a 1.5 liter can. Write and solve an inequality to show how many liters of paint, p, Ula needs to buy. **$p + 1.5 \ge 5$, $p \ge 3.5$**

15. Mike is 8 cm taller than Darla. She is less than 160 cm tall. Write an inequality to show Mike's height. **See below**

Level 1: q1–7, 9–12
Level 2: q1–12, 14
Level 3: q1–15

Solutions
For worked solutions see the Solution Guide

4.4.1

5. 
6. 
7. 
8. 

15. Let M = Mike's height (cm)
Darla's height = $M - 8$
$M - 8 < 160$
or $M < 168$

Additional Questions
Lesson 4.4.2 — 5.1.3

Level 1: q1–11, 14–15
Level 2: q1–15
Level 3: q1–17

Lesson 4.4.2 — More On Linear Inequalities

In Exercises 1–6, solve the inequality for the unknown.

1. $2x > 18$ **$x > 9$**
2. $3x < 48$ **$x < 16$**
3. $x \div 8 \geq 8$ **$x \geq 64$**
4. $x \div 12 \leq 11$ **$x \leq 132$**
5. $86x \leq 1032$ **$x \leq 12$**
6. $\frac{1}{2}x \geq 18$ **$x \geq 36$**

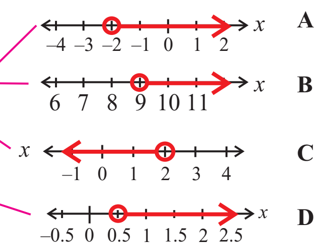
7. Which of the following is the correct solution of the inequality $-2y < 4$? **b) $y > -2$**
a) $y < -2$ b) $y > -2$ c) $y > 2$

In Exercises 8–13, solve the inequality for the unknown.

8. $2x < -4$ **$x < -2$**
9. $x \div 3 > -1$ **$x > -3$**
10. $-4x \geq 8$ **$x \leq -2$**
11. $x \div -10 < 1$ **$x > -10$**
12. $-x < -7$ **$x > 7$**
13. $x \div -5 \leq -6$ **$x \geq 30$**

In Exercises 14–17 say which inequality goes with which solution graphed on the number line.

14. $2x < 4$
15. $\frac{x}{3} > 3$
16. $2x > 1$
17. $-\frac{x}{2} < 1$



Level 1: q1–7, 9–10
Level 2: q1–12
Level 3: q1–15

Lesson 4.4.3 — Solving Two-Step Inequalities

1. Eva is saving money to buy a computer. She needs to save at least \$720. She already has \$200, and thinks that she can save \$40 more each month. Write and solve an inequality to find the number of months, m, that Eva will have to save for to get her computer. **$200 + 40m \geq 720$, $m \geq 13$**

In Exercises 2–7, solve the inequality for the unknown.

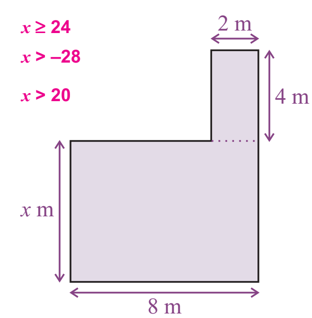
2. $3x + 5 > 8$ **$x > 1$**
3. $2x - 9 < 13$ **$x < 11$**
4. $4x + 5 < -55$ **$x < -15$**
5. $13x - 6 > -58$ **$x > -4$**
6. $-18x + 3 \geq 39$ **$x \leq -2$**
7. $-15x - 3 \leq 72$ **$x \geq -5$**

8. Sean is 6 years older than twice his cousin's age. Given that Sean is over 30, write and solve an inequality to describe his cousin's age, c. **$2c + 6 > 30$, $c > 12$**

In Exercises 9–14, solve the inequality for the unknown.

9. $(x \div 2) + 4 > 7$ **$x > 6$**
10. $(x \div 4) - 5 \geq 1$ **$x \geq 24$**
11. $(x \div 3) + 9 < -2$ **$x < -33$**
12. $(x \div 7) - 4 > -8$ **$x > -28$**
13. $-\frac{x}{4} + 3 \leq 1$ **$x \geq 8$**
14. $-\frac{x}{5} - 7 < -11$ **$x > 20$**

15. The diagram on the right shows the floor of a room in Felicia's house. Felicia has decided to lay a new carpet in the room. She hasn't measured the distance labeled x yet, but she knows that the total area of the floor is less than or equal to 80 m^2. Write and solve an inequality for x. **$8x + 8 \leq 80$, $x \leq 9$**



Solutions
For worked solutions see the Solution Guide

Lesson 5.1.1 — Multiplying With Powers

Level 1: q1–6, 11–13
Level 2: q4–16
Level 3: q4–16

Write the expressions in Exercises 1–3 in base and exponent form.

1. $15 \cdot 15$ 15^2 **2.** $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$ 8^5 **3.** $w \cdot w \cdot w$ w^3

Evaluate the expressions in Exercises 4–9 using the multiplication of powers rule. Give your answers in base and exponent form.

4. $3^2 \cdot 3^3$ 3^5 **5.** $6^1 \cdot 6^{12}$ 6^{13} **6.** $2^5 \cdot 2^7$ 2^{12}

7. $a^5 \cdot a^1$ a^6 **8.** $(-5)^4 \cdot (-5)^9$ $(-5)^{13}$ **9.** $x^8 \cdot x^9$ x^{17}

10. The distance from Anna's middle school to her home is 3^2 times the distance from the school to the library. If the distance from the school to the library is 3^5 yards, how many blocks away is Anna's middle school from her home? 3^7 yards

Evaluate the expressions in Exercises 11–15 using the multiplication of powers rule. Give your answers in base and exponent form.

11. $4 \cdot 8$ 2^5 **12.** $5 \cdot 125$ 5^4 **13.** $3 \cdot 81$ 3^5

14. $8 \cdot 16$ 2^7 **15.** $1000 \cdot 100$ 10^5 **16.** $36 \cdot 7776$ 6^7

16. The 7th grade class has a display at the science fair which is a triangle-shaped board with a base of 3^2 inches and a height of 3^3 inches. What is its area? $\frac{3^5}{2}$ inches^2

Lesson 5.1.2 — Dividing With Powers

Level 1: q1–3, 8–10
Level 2: q1–12, 14
Level 3: q1–14

Evaluate the expressions in Exercises 1–6 using the division of powers rule. Give your answers in base and exponent form.

1. $12^{10} \div 12^3$ 12^7 **2.** $11^9 \div 11^7$ 11^2 **3.** $g^{17} \div g^7$ g^{10}

4. $20^7 \div 20$ 20^6 **5.** $15^{48} \div 15^{19}$ 15^{29} **6.** $(-8)^{15} \div (-8)$ $(-8)^{14}$

7. The area of a rectangular game board is s^6 centimeters^2. The length of the board is s^2 centimeters. What is the width of the board? s^4 centimeters

Evaluate the expressions in Exercises 8–13 using the division of powers rule. Give your answers in base and exponent form.

8. $64 \div 2$ 2^5 **9.** $128 \div 8$ 2^4 **10.** $4096 \div 32$ 2^7

11. $3125 \div 25$ 5^3 **12.** $343 \div 49$ 7^1 **13.** $12167 \div 529$ 23^1

14. Each month, a company purchases 10^7 cell phone minutes and shares them equally among its 10^3 employees. How many minutes does each employee receive? 10^4

Lesson 5.1.3 — Fractions With Powers

Level 1: q1–3, 7
Level 2: q1–8
Level 3: q1–9

Simplify the expressions in Exercises 1–4. Give your answers in base and exponent form.

1. $\left(\frac{3}{8}\right)^2 \cdot \left(\frac{5}{8}\right)^3$ $\frac{3^2 \cdot 5^3}{8^5}$ **2.** $\left(\frac{6}{5}\right)^{11} \cdot \left(\frac{6}{7}\right)^{12}$ $\frac{6^{23}}{5^{11} \cdot 7^{12}}$ **3.** $\left(\frac{2}{7}\right)^5 \cdot \left(\frac{2}{9}\right)^7$ $\frac{2^{12}}{7^5 \cdot 9^7}$

4. $\left(\frac{a}{b}\right)^5 \cdot \left(\frac{a}{3}\right)^{10}$ $\frac{a^{15}}{b^5 \cdot 3^{10}}$ **5.** $\left(\frac{-5}{13}\right)^{12} \div \left(\frac{-1}{10}\right)^9$ $\frac{(-5)^{12} \cdot 10^9}{-13^{12}}$ **6.** $\left(\frac{2}{z}\right)^8 \div \left(\frac{z}{2}\right)^6$ $\frac{2^{14}}{z^{14}}$

7. Can either the multiplication of powers rule or the division of powers rule be used to simplify the expression $\frac{3}{7} \div \frac{1}{14}$? Explain your answer. No — there are no common bases

8. Lisa studied $\left(\frac{3}{2}\right)^2$ hours for her math test and $\frac{3}{4}$ as long for her science test. How long did she study for her science test? Give your answer in base and exponent form. $\frac{3^3}{2^4}$ hours

9. Ashanti runs $\left(\frac{3}{x}\right)^2$ yards every day. Louise runs $\left(\frac{2}{x}\right)^3$ as far as Ashanti. How far does Louise run? Give your answer in base and exponent form. $\frac{3^2 \cdot 2^3}{x^5}$ hours

Solutions
For worked solutions see the Solution Guide

Additional Questions

Lesson 5.2.1 — 5.3.2

Level 1: q1–7, 11–12, 15
Level 2: q1–12, 15
Level 3: q1–15

Lesson 5.2.1 — Negative and Zero Exponents

Evaluate the expressions in Exercises 1–4.

1. 21^0 1
2. $(xy)^0$, $xy \neq 0$ 1
3. $38^0 + 38^0$ 2
4. $(9 - 5)^0$ 1

Rewrite each of the expressions in Exercises 5–10 without a negative exponent. **Exercises 5–10: see below for answers**

5. 3^{-2}
6. 10^{-5}
7. 5^{-13}
8. 41^{-8}
9. 99^{-2}
10. x^{-y}

Rewrite each of the expressions in Exercises 11–14 using a negative exponent.

11. $\frac{1}{10^4}$ 10^{-4}
12. $\frac{1}{t^8}$ t^{-8}
13. $\frac{1}{55 \times 55 \times 55 \times 55 \times 55}$ 55^{-5}
14. $\frac{1}{(-c) \times (-c) \times (-c)}$ $(-c)^{-3}$

15. Lisa's little sister is 1 year old and her big sister is 25. The ages of all three sisters are powers of the same base. How old is Lisa? Give your answer in base and exponent form. 5^1

Level 1: q1–6
Level 2: q1–8
Level 3: q3–11

Lesson 5.2.2 — Using Negative Exponents

Simplify the expressions in Exercises 1–4. Give your answers as powers in base and exponent form.

1. $7^7 \times 7^{-5}$ 7^2
2. $29^8 \div 29^{-2}$ 29^{10}
3. $43^8 \div 43^{-21}$ 43^{29}
4. $12^{-12} \div 12^{-8}$ 12^{-4}

Simplify the expressions in Exercises 5–8 by first converting any negative exponents to positive exponents. Give your answers as powers in base and exponent form.

5. $4^{15} \times 4^{-5}$ 4^{10}
6. $98^{-8} \times 98^{-5}$ 98^{-13}
7. $a^{-56} \times a^{-40}$ a^{-96}
8. $32^{-x} \times 32^{x}$ 32^0
9. $38^2 \div \frac{1}{38^{-9}}$ 38^{-7}
10. $r^{-n} \div \frac{3}{r^{-2}}$ $\frac{r^{(-n-2)}}{3}$

11. $\left(\frac{5}{3}\right)^{-2}$ of the 7th grade class received an A on a recent science test. If there are 25 students in the class, how many students received an A? 9

Level 1: q1–4, 9–10, 13–14
Level 2: q1–6, 9–11, 13–18
Level 3: q3–18

Lesson 5.2.3 — Scientific Notation

Write the numbers in Exercises 1–12 in scientific notation.

1. 420 4.2×10^2
2. 6,000 6.0×10^3
3. 917,000 9.17×10^5
4. –938,700 -9.387×10^5
5. 245,000,000,000 2.45×10^{11}
6. 93,000,000 9.3×10^7
7. 147,396,000,000 1.47396×10^{11}
8. 53,560,000,000,000 5.356×10^{13}
9. 0.00032 3.2×10^{-4}
10. 0.00000000819 8.19×10^{-9}
11. 0.000000064 6.4×10^{-8}
12. 0.000000387 3.87×10^{-7}

Write the numbers in Exercises 13–16 in numerical form.

13. 4.35×10^2 435
14. 8.31×10^6 8,310,000
15. 4.79×10^{-7} 0.000000479
16. 9.101×10^{-12} 0.000000000009101

17. A wealthy businessman is worth 5.28 billion dollars. What is 5.28 billion in scientific notation? 5.28×10^9

18. Pritesh converts the number 16,200 into scientific notation and gets 16.2×10^3. Explain his mistake.
Although $16.2 \times 10^3 = 16{,}200$, the number 16.2 is not between 1 and 10 so 16.2×10^3 is not scientific notation.

Solutions
For worked solutions see the Solution Guide

5.2.1

5. $\frac{1}{3^2}$ **6.** $\frac{1}{10^5}$ **7.** $\frac{1}{5^{13}}$ **8.** $\frac{1}{41^8}$ **9.** $\frac{1}{99^2}$ **10.** $\frac{1}{x^y}$

Lesson 5.2.4 — Comparing Numbers In Scientific Notation

Level 1: q1–4, 7–9
Level 2: q1–4, 7–13
Level 3: q3–14

In Exercises 1–6, say which of the two numbers is greater.

1. 3.27×10^3, 3.27×10^6 **3.27×10^6**
2. 4.9×10^7, 4.9×10^{-7} **4.9×10^7**
3. 7.8×10^9, 7.8×10^{-9} **7.8×10^9**
4. 4.36×10^3, 8.2×10^2 **4.36×10^3**
5. $2(1.5 \times 10^5)$, 3.0×10^{10} **3.0×10^{10}**
6. 9.67×10^{12}, 8.412×10^{13} **8.412×10^{13}**

Order the expressions in Exercises 7–10 from least to greatest.

7. 3.2×10^5, 4.35×10^3, 9.874×10^2, 1.4×10^6, 4.2×10^1 **4.2×10^1, 9.874×10^2, 4.35×10^3, 3.2×10^5, 1.4×10^6**
8. 9.99×10^6, 9.9×10^6, 9.999×10^6, 9.9999×10^6 **9.9×10^6, 9.99×10^6, 9.999×10^6, 9.9999×10^6**
9. 2.7×10^8, 3.965×10^6, 1.982×10^{13}, 8.623×10^8 **3.965×10^6, 2.7×10^8, 8.623×10^8, 1.982×10^{13}**
10. 4.3×10^5, 3.4×10^6, 5.2×10^5, 3.5×10^6 **4.3×10^5, 5.2×10^5, 3.4×10^6, 3.5×10^6**

A space shuttle has two solid rocket boosters that each provide 1.19402×10^6 kg of thrust; 3 main engines that each provide 154,360 kg of thrust, and 2 orbital maneuvering systems engines which each provide 2.452×10^3 kg of thrust.

11. What is the thrust of a single solid rocket booster as a decimal? **1,194,020 kg**
12. Which type of engine has the greatest amount of thrust? **the solid rocket boosters**
13. Which engine has the least amount of thrust? **the orbital maneuvering systems engines**
14. What is the total amount of thrust provided by the engines? Give your answer in scientific notation. **2.856024×10^6 kg**

Lesson 5.3.1 — Multiplying Monomials

Level 1: q1–10, 14
Level 2: q1–16
Level 3: q1–16

State whether or not each expression in Exercises 1–3 is a monomial.

1. $5y^9$ **Is a monomial**
2. $2x + 2y$ **Not a monomial**
3. $\frac{y^8}{2}$ **Is a monomial**

Identify the coefficient in Exercises 4–7.

4. $17a^3$ **17**
5. $31w^4$ **31**
6. $\frac{x}{2}$ **$\frac{1}{2}$**
7. $\frac{51x^6}{9}$ **$\frac{51}{9}$**

Simplify the expressions in Exercises 8–16 by turning them into a single monomial.

8. $17x^3 \times 2x^2$ **$34x^5$**
9. $6ab^2 \times b^3c$ **$6ab^5c$**
10. $18x^2y^2z^2 \times xz^2$ **$18x^3y^2z^4$**
11. $w^3k^2 \times 2wn^6 \times 3k^3n^2$ **$6k^5w^4n^8$**
12. $\frac{40x^3}{9} \times x^2y \times y$ **$\frac{40x^5y^2}{9}$**
13. $\frac{2xz^3}{3} \times \frac{4xy^2}{5} \times \frac{x^6y^2z^2}{3}$ **$\frac{8x^8y^4z^5}{45}$**

Square each monomial in Exercises 14–16.

14. $8y^3$ **$64y^6$**
15. $4t^5uv^3$ **$16t^{10}u^2v^6$**
16. $12d^5e^5f^5$ **$144d^{10}e^{10}f^{10}$**

Lesson 5.3.2 — Dividing Monomials

Level 1: q1–4, 9
Level 2: q1–12
Level 3: q3–14

Evaluate each expression in Exercises 1–8.

1. $6b^7 \div b$ **$6b^6$**
2. $81d^6 \div 9d^4$ **$9d^2$**
3. $18x^2y^2 \div 3xy$ **$6xy$**
4. $27m^{12}n^2 \div 3mn^2$ **$9m^{11}$**
5. $144d^4e^9f^{17} \div 12d^4e^3f^7$ **$12e^6f^{10}$**
6. $20a^{17}b^{14} \div 5a^8b^{12}$ **$4a^9b^2$**
7. $0.6t^3u^2v^3 \div 3xy$ **$0.2t^3u^2v^3x^{-1}y^{-1}$**
8. $\frac{5}{7}x^3y^2z^4 \div \frac{1}{2}xyz^3$ **$\frac{10x^2yz}{7}$**

9. Does $36xy^3 \div 2x^4y$ give a monomial result? **No**

10. If all their chores are done on the weekend, the Anderson children receive $3x^2y^2$ dollars in total, split evenly between the $2x^2$ children. How much does each child receive? **$\frac{3y^2}{2}$ dollars**

Say whether each division gives a monomial result in Exercises 11–14.

11. $5x^2 \div 5x$ **Does give a monomial**
12. $25z^3 \div 5z^4$ **Does not give a monomial**
13. $40x^{34}p^{12} \div 20x^{30}p^{14}$ **Does not give a monomial**
14. $20z^4x^3 \div 5z^4x^3$ **Does give a monomial**

Solutions
For worked solutions see the Solution Guide

Additional Questions
Lesson 5.3.3 — 5.4.3

Level 1: q1–5, 10–12
Level 2: q1–13, 16–17
Level 3: q5–9, 12–15, 16–18

Lesson 5.3.3 — Powers of Monomials

Write the expressions in Exercises 1–9 using a single power.

1. $(3^2)^3$ 3^6
2. $(5^4)^4$ 5^{16}
3. $[(-8)^2]^4$ $(-8)^8$
4. $[(-17)^2]^5$ $(-17)^{10}$
5. $(5^{-2})^{-10}$ 5^{20}
6. $(p^2)^6$ p^{12}
7. $(r^a)^b$ r^{ab}
8. $(s^{-1})^b$ s^{-b}
9. $[(-t)^{-c}]^{-d}$ $(-t)^{cd}$

Simplify the powers of monomials in Exercises 10–15.

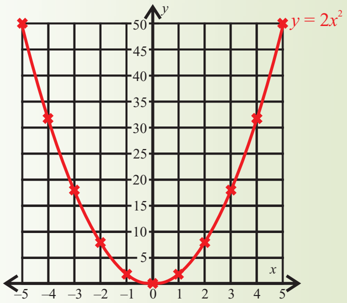
10. $(4x^5)^2$ $16x^{10}$
11. $(2y^7)^3$ $8y^{21}$
12. $(7a^2b^3c)^2$ $49a^4b^6c^2$
13. $(2n^8o^3p^5)^3$ $8n^{24}o^9p^{15}$
14. $(g^{-2}n^3e^2)^4$ $g^{-8}n^{12}e^8$
15. $(-0.3x^{-2}y^mz^4)^3$ $-0.027x^{-6}y^{3m}z^{12}$

16. What is the area of a square room with side lengths of $5x^2y^3z^{10}$ centimeters? $25x^4y^6z^{20}$ centimeters^2

17. What is the volume of a cubic container with side lengths of $2a^5b^3$ feet? $8a^{15}b^9$ feet^3

18. What is the volume of a cylindrical container with a radius of $4x^3y^2$ and a height of $3x^5$? $48\pi x^{11}y^4$ feet^3

5.4.1 Exercise 16 Solution



Level 1: q1–6, 10–12
Level 2: q3–9, 13–19
Level 3: q3–9, 13–19

Lesson 5.3.4 — Square Roots of Monomials

Simplify the expressions in Exercises 1–9.

1. $\sqrt{81}$ 9
2. $\sqrt{36}$ 6
3. $\sqrt{5^2}$ 5
4. $\sqrt{z^2}$ $|z|$
5. $\sqrt{c^{30}}$ $|c^{15}|$
6. $\sqrt{a^2}$ $|a|$
7. $\sqrt{x^4}$ x^2
8. $\sqrt{y^{50}}$ $|y^{25}|$
9. $\sqrt{t^{46}}$ $|t^{23}|$

Find the square roots of each monomial in Exercises 10–15.

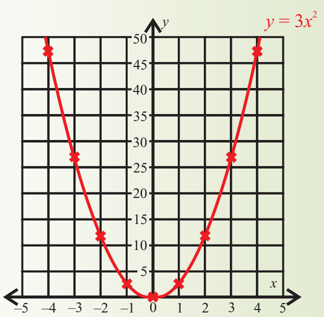
10. $16x^4$ $4x^2$ and $-4x^2$
11. $100x^4z^8$ $10x^2z^4$ and $-10x^2z^4$
12. $a^{12}b^{18}c^{20}$ $a^6b^9c^{10}$ and $-a^6b^9c^{10}$
13. $49k^2h^4j^8$ $7|k| \bullet h^2j^4$ and $-7|k| \bullet h^2j^4$
14. $625r^{12}s^{24}t^{32}$ $25r^6s^{12}t^{16}$ and $-25r^6s^{12}t^{16}$
15. $144a^8b^{10}c^{12}d^{14}e^{16}$ $12a^4c^6e^8 \bullet |b^5||d^7|$ and $-12a^4c^6e^8 \bullet |b^5||d^7|$

16. A square painting has an area of $25x^4y^2z^{18}$ square feet. What is the length of its side? $5x^2 \bullet |y||z^9|$ feet

In Exercises 17–19, determine whether each square root will be a monomial.

17. $\sqrt{x^{15}y^{12}}$ Not a monomial
18. $\sqrt{s^{11}t^8}$ Not a monomial
19. $\sqrt{23x^{16}t^{14}}$ Monomial

5.4.1 Exercise 17 Solution



Level 1: q1–2, 4–5, 16
Level 2: q1–6, 10–12, 16–17
Level 3: q7–18

Lesson 5.4.1 — Graphing $y = nx^2$

In Exercises 16–21, find the y-coordinate of the point on the $y = x^2$ graph for each given value of x.

1. $x = 5$ 25
2. $x = \frac{1}{2}$ $\frac{1}{4}$
3. $x = \frac{3}{4}$ $\frac{9}{16}$

Determine which of the points in Exercises 4–9 lie on the graph of $y = 2x^2$.

4. $(-1, 2)$ On the graph
5. $(2, 8)$ On the graph
6. $(-5, 12)$ Not on the graph
7. $(4, 40)$ Not on the graph
8. $\left(\frac{1}{2}, \frac{1}{2}\right)$ On the graph
9. $(-3, 18)$ On the graph

In Exercises 10–15, calculate the two possible x-coordinates of the points on the graph of $y = x^2$ whose y-coordinate is shown.

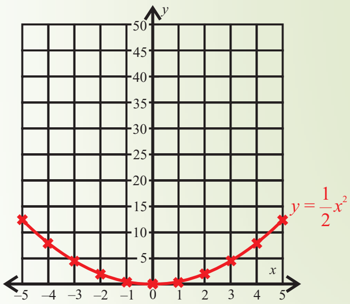
10. 81 9 and –9
11. 144 12 and –12
12. 4 2 and –2
13. 36 6 and –6
14. 14 $\sqrt{14}$ and $-\sqrt{14}$
15. a $\sqrt{a}$ and $-\sqrt{a}$

In Exercises 16–18, draw the graph of each of the given equations.

16. $y = 2x^2$
17. $y = 3x^2$
18. $y = \frac{1}{2}x^2$

Exercises 16–18: see margin

5.4.1 Exercise 18 Solution



Solutions
For worked solutions see the Solution Guide

Lesson 5.4.2 — More Graphs of $y = nx^2$

In Exercises 1–4, plot the graph of the given equation for the values of x between 5 and –5.

1. $y = -x^2$ **2.** $y = -2x^2$ **3.** $-y = x^2$ Exercises 1–3: see margin

4. Using your graph from Exercise 2, what is x if $-2x^2 = -18$? 3

5. Using your graph from Exercise 1, what is x if $-x^2 = -25$? 5

Answer Exercises 6 and 7 without plotting any points.

6. The point (5, 5) lies on the graph of $y = \frac{1}{5}x^2$. What is the y-coordinate of the point on the graph of $y = -\frac{1}{5}x^2$ with x-coordinate 5? –5

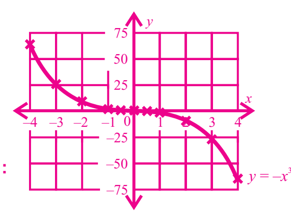
7. The point (–2, 8) lies on the graph of $y = 2x^2$. What is the y-coordinate of the point on the graph of $-y = 2x^2$ with x-coordinate –2? –8

For each point in Exercises 8–13, say which of the equations shown below it would lie on the graph of.

$y = x^2$ $y = -2x^2$ $y = 0.5x^2$
$y = 3x^2$ $y = -x^2$ $y = -4x^2$

8. (–9, 81) $y = x^2$ **9.** (–4, –32) $y = -2x^2$
10. (6, 18) $y = 0.5x^2$ **11.** (8, 32) $y = 0.5x^2$
12. (10, –400) $y = -4x^2$ **13.** (–7, –49) $y = -x^2$

Lesson 5.4.3 Exercise 11:



Lesson 5.4.3 — Graphing $y = nx^3$

1. Make a table of values for $y = 5x^3$ for x between –4 and 4.
2. Use the points in Exercise 1 to plot the graph. see margin

x	–4	–3	–2	–1	0	1	2	3	4
y	–320	–135	–40	–5	0	5	40	135	320

Use your graph of $y = 5x^3$ from Exercise 2 to get approximate solutions to the equations in Exercises 3–8.

3. $5x^3 = 125$ $x = 2.9$ **4.** $5x^3 = 305$ $x = 3.9$ **5.** $5x^3 = -30$ $x = -1.8$
6. $5x^3 = 70$ $x = 2.4$ **7.** $5x^3 = -50$ $x = -2.2$ **8.** $5x^3 = -210$ $x = -3.5$

Graph the equations in Exercises 9–11.

9. $y = 2x^3$ see margin **10.** $y = 3x^3$ see below **11.** $y = -x^3$ see above

Make a table of values with x values from –4 to 4 for each of the equations in Exercises 12–15.

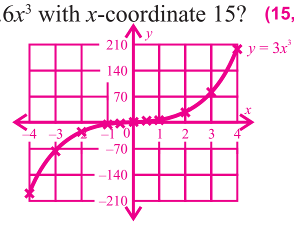
12. $y = -2x^3$ **13.** $y = 3x^3$
14. $y = -3x^3$ **15.** $y = -\frac{2}{3}x^3$ Exercises 12–15: see table below

Answer Exercises 16 and 17 without plotting any points.

16. If the graph of $y = 0.2x^3$ goes through (4, 12.8), what are the coordinates of the point on the graph of $-y = 0.2x^3$ with x-coordinate 4? (4, –12.8)

17. If the graph of $y = 0.8x^3$ goes through (15, 2700), what are the coordinate of the point on the graph of $y = -1.6x^3$ with x-coordinate 15? (15, –5400)

Exercise 10:

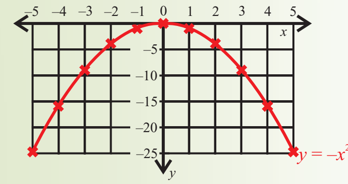


Exercises 12–15:

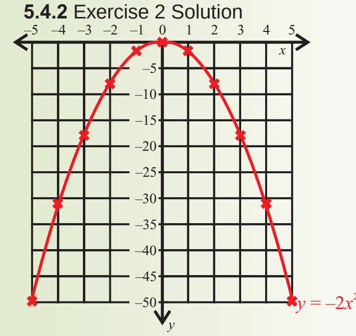
x	–4	–3	–2	–1	0	1	2	3	4
$y = -2x^3$	128	54	16	2	0	–2	–16	–54	–128
$y = 3x^3$	–192	–81	–24	–3	0	3	24	81	192
$y = -3x^3$	192	81	24	3	0	–3	–24	–81	–192
$y = -\frac{2}{3}x^3$	$\frac{128}{3}$	18	$\frac{16}{3}$	$\frac{2}{3}$	0	$-\frac{2}{3}$	$-\frac{16}{3}$	–18	$-\frac{128}{3}$

Level 1: q1–2, 4–5, 6
Level 2: q1–6, 8–13
Level 3: q1–13

5.4.2 Exercises 1 and 3 Solution

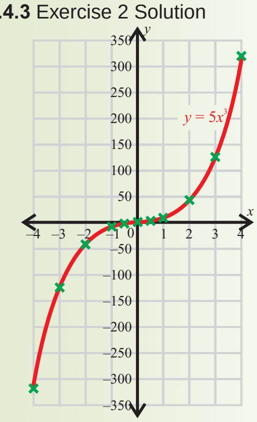


5.4.2 Exercise 2 Solution

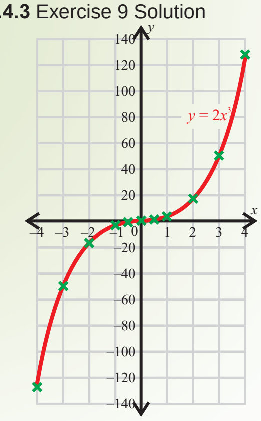


Level 1: q1–4, 9, 12–14
Level 2: q1–10, 12–16
Level 3: q9–17

5.4.3 Exercise 2 Solution



5.4.3 Exercise 9 Solution



Solutions
For worked solutions see the Solution Guide

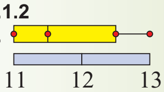
Additional Questions
Lesson 6.1.1 — 6.1.6

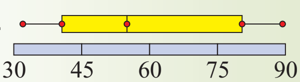
Level 1: q1–4, 9–11, 15
Level 2: q1–6, 9–13, 15–16
Level 3: q3–8, 11–16

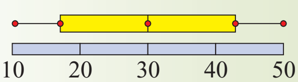
6.1.1
15. Both stores have the same median price of $350, but different ranges. Store X has a range of $50 so all the prices are close to $350 — certainly none are above $400 and none are below $300. Store Y has a range of $250 so the prices are quite spread out and could be as high as $600 or as low as $100.

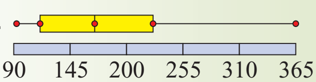
16. The second company has a lower median price, and a lower range of prices, so all of its prices are relatively low and close together. The first company has a higher median but also a higher range, so its prices are more spread out, but it could have a phone that is cheaper than the second company.

Level 1: q1–2, 6–9
Level 2: q1–12
Level 3: q1–12

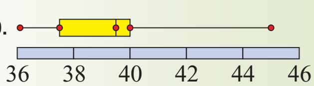
6.1.2
6.


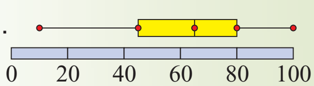
7.


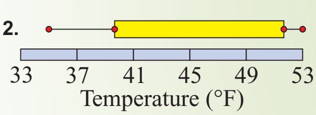
8.


9.


Level 1: q1–4
Level 2: q1–5
Level 3: q1–5

10.


11.


12.


6.1.3
1. Two weeks before the holiday the lowest attendance was 1000 and the highest was 1100 — so there were always at least 1000 students in attendance. One week before the holiday, the data is all much lower and at least 75% of the time attendance is below 1000. So it seems that attendance did reduce before the holiday.

Lesson 6.1.1 — Median and Range

Find the median of each of the data sets in Exercises 1–8.

1. {1, 3, 5, 7, 9, 11, 13} 7
2. {9, 23, 48, 7, 100} 23
3. {10, 4, 2, 8, 6} 6
4. {99, 99, 100, 102, 101, 98, 107, 97} 99.5
5. {30, 33, 30, 33, 33, 33} 33
6. {65, 30, 25, 45, 20, 25} 27.5
7. {18, 15, 13, 6, 9, 12} 12.5
8. {20, 60, 80, 80, 20, 60, 60, 60} 60

Find the range of each of the data sets in Exercises 9–14.

9. {71, 50, 32, 55, 90} 58
10. {11, 11, 11, 11, 12, 12} 1
11. {98, 99, 98, 99, 100} 2
12. {500, 550, 575, 600, 625, 675} 175
13. {365, 90, 90, 200, 250} 275
14. {33.5, 6.5, 200.1, 82.3, 66.4} 193.6

15. Store X sells class rings with a median price of $350 and a range of $50. Store Y sells class rings with a median price of $350 and a range of $250. Interpret these statistics. see margin

16. A cell phone company has packages with a median price of $59.99 per month and a range of $100. Another company has packages with a median price of $49.99 and a range of $50. Interpret these statistics. see margin

Lesson 6.1.2 — Box-and-Whisker Plots

Find the upper and lower quartiles of each data set in Exercises 1–5.

1. {1, 3, 5, 7, 9, 11, 13, 15, 17, 19} 5 and 15
2. {88, 77, 9, 23, 48, 7, 100, 102, 99} 16 and 99.5
3. {99, 99, 100, 99, 99, 100, 102, 101, 98, 107, 97} 99 and 101
4. {20, 22, 24, 25, 30, 33, 30, 33, 33, 33, 50, 53} 24.5 and 33
5. {30, 25, 20, 18, 15, 13, 6, 9, 12} 10.5 and 22.5

Create a box-and-whisker plot to illustrate each of the data sets in Exercises 6–11.

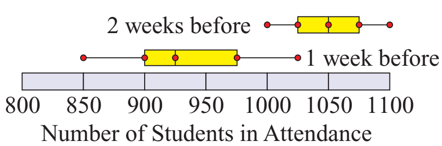
6. {11, 11, 11, 11, 12, 12, 13, 13}
7. {71, 50, 32, 55, 90}
8. {10, 15, 20, 25, 30, 35, 40, 45, 50}
9. {150, 150, 200, 365, 90, 90, 200, 250}
10. {36, 37, 38, 39, 40, 40, 40, 45}
11. {50, 60, 70, 70, 90, 100, 10, 40}

Exercises 6–11: see margin

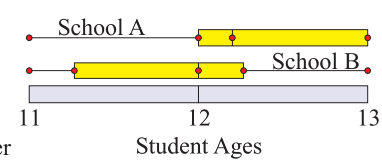
12. Terrell recorded the daily temperature each day for a week and made the data set {45°F, 40°F, 40°F, 53°F, 52°F, 40°F, 33°F}. Draw a box-and-whisker plot to illustrate this data. see margin

Lesson 6.1.3 — More On Box-and-Whisker Plots

1. Principal Garcia is curious to know whether school attendance drops off before a holiday.
The box-and-whisker plots on the right show school attendance 2 weeks before a holiday and 1 week before a holiday. What conclusions can Principal Garcia draw from the plots? see margin



2. The ages of the band members from two middle schools are shown in the box-and-whisker plots on the right. Which school has a larger percentage of younger students? see below



3. What is the difference between the range of a box-and-whisker plot and the interquartile range? The range shows the spread of all the data values. The interquartile range just shows the range of the middle 50%.

4. In a box-and-whisker plot, what is shown by the length of the whiskers? The range of the data.

5. Name the three different areas of a box-and-whisker plot that contain exactly half of the data values. see below

Solutions
For worked solutions see the Solution Guide

2. At School A, 75% of the band members are between 12 and 13 and 25% are between 11 and 12. At School B, 50% are between 12 and 13 and 50% are between 11 and 12. So School B has a larger percentage of young students.
5. The area between the upper quartile and lower quartile, the area from the median to the maximum, and the area from the median to the minimum.

Lesson 6.1.4 — Stem-and-Leaf Plots

Make stem-and-leaf plots to display the data given in each of Exercises 1–7.

1. {11, 13, 24, 29, 33, 35, 37, 39}

2. {13, 14, 15, 17, 18, 18, 24, 34, 42}

3. {34, 35, 39, 40, 47, 50, 54, 56, 57, 60}

4. {2, 3, 6, 6, 10, 12, 14, 19}

5. {2, 12, 13, 14, 22, 23, 27, 33, 35}

6. {82, 82, 83, 83, 83, 84, 84, 85, 85, 89, 92}

7. {3, 4, 4, 4, 12, 15, 23, 25, 27, 33, 34, 34, 34, 36, 42, 44, 54, 57, 60}

Exercises 3–7: see margin

1.
1 | 1 3
2 | 4 9
3 | 3 5 7 9
Key: 2 | 4 represents 24

2.
1 | 3 4 5 7 8 8
2 | 4
3 | 4
4 | 2
Key: 2 | 4 represents 24

Find the median and the range of the stem-and-leaf plots in Exercises 8–9.

8.
4 | 1 3
5 | 4 9
6 | 3 5 7 9
Key: 4 | 3 represents 43

range = 28
median = 61

9.
3 | 4 5 9
4 | 0 7
5 | 0 4 6 7
6 | 0
Key: 4 | 0 represents 40

range = 26
median = 48.5

Draw a back-to-back stem-and-leaf plot for each pair of data sets in Exercises 10–15.

10. {18, 28, 38, 48, 49, 50}
{23, 23, 23, 35, 35, 41, 41, 43, 52}

11. {7, 9, 10, 14, 17, 23, 25, 29}
{13, 13, 14, 14, 21, 22}

Exercises 10–11: see margin

Lesson 6.1.5 — Preparing Data to be Analyzed

40 people participated in the tests for a new health food diet. Half ate a regulated 1500 calorie diet while the other half ate as much as they wanted from a selected group of health foods. Their weight loss in pounds after one month is listed in the chart below. Use this data for Exercises 1–10.

1500 Calorie Group	5	9	10	13	11	15	5	6	8	8	7	3	0	9	7	6	5	5	3	10
Health Food Group	7	10	12	15	14	12	10	9	12	10	10	9	15	13	7	8	9	7	8	15

1. Find the minimum of each data set. 1500 group: 0 health group: 7

2. Find the maximum of each data set. 1500 group: 15 health group: 15

3. Find the median of the 1500 calorie group. 7

4. Find the median of the health food group. 10

5. What is the lower quartile of the 1500 calorie group? 5

6. What is the lower quartile of the health food group? 8.5

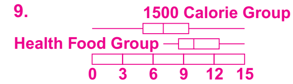
7. What is the upper quartile of the 1500 calorie group? 9.5

8. What is the upper quartile of the health food group? 12.5

9. Create box-and-whisker plots of the two data sets.

10. Create a double stem-and-leaf plot of the two sets.

9.



10. Health Food Group | 1500 Calorie Group
9 9 9 8 8 7 7 7 | 0 | 0 3 3 5 5 6 6 7 7 8 8 9 9
5 5 5 4 3 2 2 2 0 0 0 0 | 1 | 0 0 1 3 5

Key: 3|1|2 represents 13 in the Health Food Group, 12 in the 1500 Calorie Group.

Lesson 6.1.6 — Analyzing Data

A chef was trying to determine which of two daily specials is more popular. He kept track of the number of orders received for each over 22 days. The results are shown below — use this data for Exercises 1–8.

Special 1: {50, 60, 89, 95, 45, 99, 98, 99, 87, 88, 89, 91, 92, 95, 94, 95, 99, 98, 98, 87, 99, 95}

Special 2: {85, 85, 85, 45, 77, 62, 88, 87, 99, 95, 94, 99, 66, 68, 72, 75, 98, 99, 99, 99, 99, 99}

1. Find the minimum and maximum of each data set. Special 1: min = 45, max = 99, Special 2: min = 45, max = 99

2. Find the range of each data set. Special 1: 54, Special 2: 54

3. Find the median of each data set. Special 1: 94.5, Special 2: 87.5

4. Find the lower and upper quartile of each data set. Special 1: lower = 88, upper = 98 Special 2: lower = 75, upper = 99

5. Find the interquartile range of each data set. Special 1: 10, Special 2: 24

6. Draw a box-and-whisker plot of each data set.

7. Draw a back-to-back stem-and-leaf plot of the data sets. Exercises 6–7: see below

8. Compare the popularity of the two specials.

Both data sets have the same range, but special 1 has a higher median and a higher lower quartile, while special 2 has a higher upper quartile. But overall it looks like special A is more consistently popular.

Level 1: q1–11
Level 2: q1–11
Level 3: q1–11

6.1.4

3.
3 | 4 5 9
4 | 0 7
5 | 0 4 6 7
6 | 0
Key: 4 | 0 represents 40

4.
0 | 2 3 6 6
1 | 0 2 4 9
Key: 1 | 4 represents 14

5.
0 | 2
1 | 2 3 4
2 | 2 3 7
3 | 3 5
Key: 1 | 3 represents 13

Level 1: q1–10
Level 2: q1–10
Level 3: q1–10

6.1.4

6.
8 | 2 2 3 3 3 4 4 5 5 9
9 | 2
Key: 9 | 2 represents 92

7.
0 | 3 4 4 4
1 | 2 5
2 | 3 5 7
3 | 3 4 4 4 6
4 | 2 4
5 | 4 7
6 | 0
Key: 3 | 3 represents 33

Level 1: q1–8
Level 2: q1–8
Level 3: q1–8

6.1.4

10.
8 | 1 |
8 | 2 | 3 3 3
8 | 3 | 5 5
9 8 | 4 | 1 1 3
0 | 5 | 2
Key: 8 | 4 | 1 represents 48 in the first data set, 41 in the second data set.

11.
9 7 | 0 |
7 4 0 | 1 | 3 3 4 4
9 5 3 | 2 | 1 2
Key: 4 | 1 | 3 represents 14 in the first data set, 13 in the second data set.

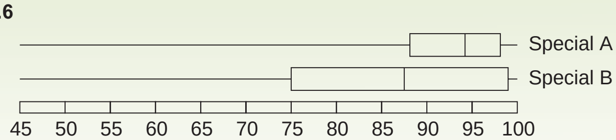
6.1.6

7. Special 1 | | Special 2
5 | 4 | 5
0 | 5 |
0 | 6 | 2 6 8
| 7 | 2 5 7
9 9 8 7 7 | 8 | 5 5 5 7 8
9 9 9 9 8 8 8 5 5 5 5 4 3 1 | 9 | 4 5 8 9 9 9 9 9 9
Key: 7 | 8 | 8 represents 87 for special 1, 88 for special 2.

Solutions
For worked solutions see the Solution Guide

6.1.6

6.



Additional Questions
Lesson 6.2.1 — 7.1.2

Lesson 6.2.1 — Making Scatterplots

1. What data would you need to collect to test the conjecture "the more books read by a student, the higher their grade point average"? You would need to collect data on the number of books read by a student and their grade point average.

2. Design a table in which to record this data. see below

3. What data would you need to collect to test the conjecture "the further you live from school, the fewer after-school clubs you're a member of"? You would need to collect data on the distance each student lives from school and the number of after school clubs they're a member of.

4. Design a table in which to record this data. see below

5. Kayla decides to test the conjecture that "the older the child, the taller they are" and collects the data shown below. Draw a scatterplot of this data.

Age (years)	3	5	4	6	5	6	3
Height (inches)	36	42	40	45	40	46	40

see margin

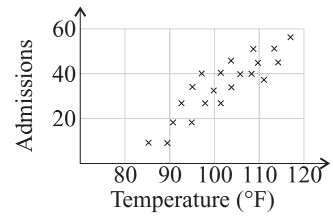
6. The data below was collected to test the conjecture "the more minutes offered by a cell phone contract, the more expensive the contract". Draw a scatterplot of this data.

Number of minutes	300	300	400	400	500	500	500	500	500	600
Monthly Fee	$20	$25	$25	$30	$30	$35	$40	$45	$50	$50

see margin

Lesson 6.2.2 — Shapes of Scatterplots

A hospital in the desert made the scatterplot on the right to determine how outside temperature is related to hospital admissions due to heat stroke. Use it to answer Exercises 1–2.



1. What sort of correlation does the scatterplot show? positive

2. Is the correlation strong or weak? Explain your answer. strong — all the points are in a thin band

3. Draw an example scatterplot with no correlation.

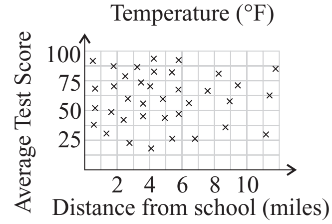
4. Draw an example scatterplot with a weak negative correlation.

5. Draw an example scatterplot with a strong positive correlation.

Exercises 3–5: see margin

6. The scatterplot on the right shows how average test score is related to the distance that students live from school. What sort of correlation does the scatterplot show?

no obvious correlation



Lesson 6.2.3 — Using Scatterplots

Roy decides to test his theory that the older a person in the baseball team is, the more professional baseball games they've seen. He collects the following data:

Age	11	12	11	13	15	14	14	12	11	12	13
Number of games seen	20	15	10	35	30	29	18	10	5	13	20

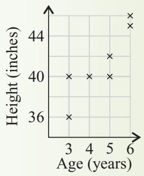
1. Create a scatterplot of the data.

2. Draw a line of best fit. Exercises 1–2: see margin

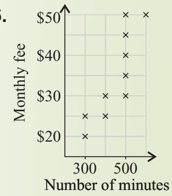
3. Predict how many professional baseball games a 17 year old on the baseball team is likely to have seen. Answers will vary but should be between 30 and 40

Level 1: q1–5
Level 2: q3–6
Level 3: q3–6

5.

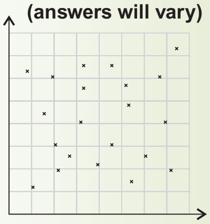


6.



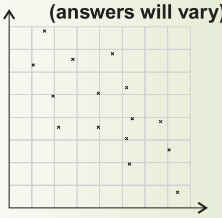
6.2.2

3. (answers will vary)

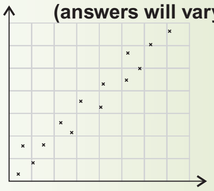


Level 1: q1–6
Level 2: q1–6
Level 3: q1–6

4. (answers will vary)



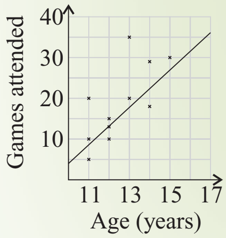
5. (answers will vary)



Level 1: q1–3
Level 2: q1–3
Level 3: q1–3

6.2.3

1. and 2.



Solutions
For worked solutions see the Solution Guide

6.2.1

1.

Number of books read			
Grade point average			

2.

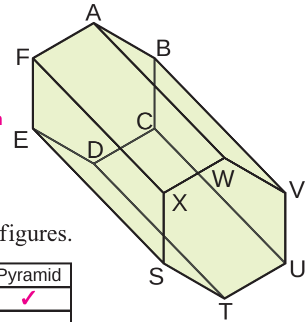
Distance of house from school			
Number of after school clubs			

Lesson 7.1.1 — Three Dimensional Figures

Level 1: q1–5
Level 2: q1–7
Level 3: q1–8

In Exercises 1–4, say whether the statements are true or false.

1. A polygon is any shape that is made from straight lines that are joined end-to-end, in a closed shape. **True**
2. Pyramids and prisms have circles for their base. **False**
3. Prisms must have congruent bases at each end. **True**
4. Pyramids have diagonals. **False**



In Exercises 5–7, refer to the figure at the right.

5. Identify the shape as either a cone, cylinder, prism, or pyramid. **Prism**
6. How many diagonals does this shape have? **18**
7. Name all diagonals by giving the staring and ending vertex.

AS, AT, AU, BS, BT, BX, CS, CX, CW, DV, DX, DW, EU, EV, EW, FT, FU, FV

8. Copy the table below and check all statements that are true about the figures.

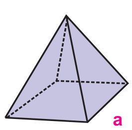
	Cone	Cylinder	Prism	Pyramid
The figure is a polyhedron			✓	✓
The figure has diagonals.			✓	
The base of the figure has a curved edge.	✓	✓		
The base of the figure is a polygon.			✓	✓
The bases are congruent.		✓	✓	
The base of the figure is a polygon, and the other faces meet at a single point.				✓
The base of the figure has a curved edge, and the other end meets at a single point.	✓			

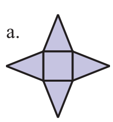
Lesson 7.1.2 — Nets

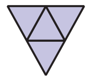
Level 1: q1–2, 4–6
Level 2: q1–8
Level 3: q1–8

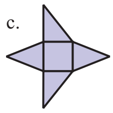
In Exercises 1–3, say which net could make each three-dimensional figure.

1.

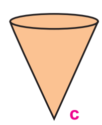

a

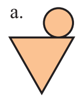
a. 

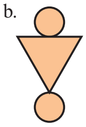
b. 

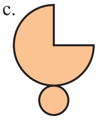
c. 

2.

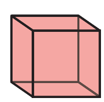

c

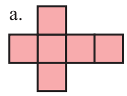
a. 

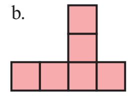
b. 

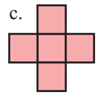
c. 

3.


a

a. 

b. 

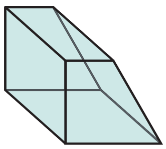
c. 

In Exercises 4–6, say whether the statements are true or false.

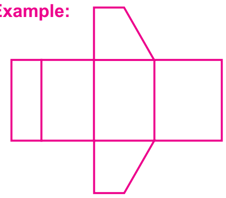
4. Every shape has one unique net. **False**
5. In the net of a cylinder, the length of the rectangle is equal to the circumference of the base circle. **True**
6. The net of a rectangular prism has five rectangles. **False**

In Exercises 7–8, draw a net for the solid.

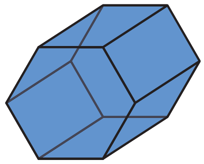
7.



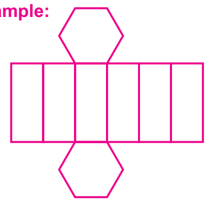
Answers may vary.
Example:



8.



Answers may vary.
Example:



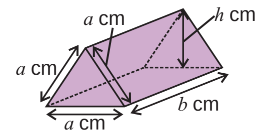
Solutions
For worked solutions see the Solution Guide

Additional Questions
Lesson 7.1.3 — 7.2.1

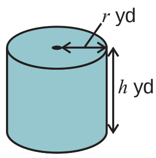
Level 1: q1–6
Level 2: q1–6
Level 3: q1–9

Lesson 7.1.3 — Surface Areas of Cylinders and Prisms

In Exercises 1–3, find the surface area of each triangular prism.



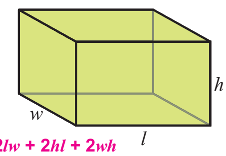
1. $a = 5$, $b = 8$, $h = 4.3$ **141.5 cm²** **2.** $a = 8$, $b = 12$, $h = 6.9$ **343.2 cm²**
3. $a = 20$, $b = 35$, $h = 17.3$ **2446 cm²**



In Exercises 4–6, find the surface area of each cylinder. Use $\pi = 3.14$.
4. $r = 6$ and $h = 11$ **640.56 yd²** **5.** $r = 12$ and $h = 17$ **2185.44 yd²**
6. $r = 30$ and $h = 45$ **14,130 yd²**

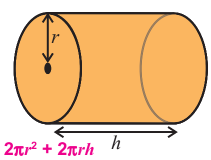
Use each of the diagrams in Exercises 7–9 to write a general formula for finding the surface area of that type of figure.

7.



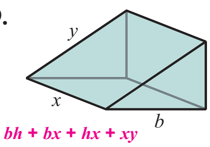
$2lw + 2hl + 2wh$

8.



$2\pi r^2 + 2\pi rh$

9.

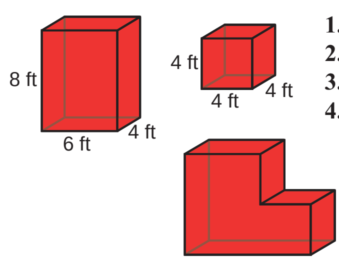


$bh + bx + hx + xy$

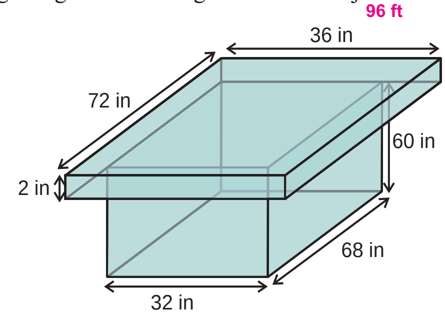
Level 1: q1–4, 8
Level 2: q1–10
Level 3: q1–10

Lesson 7.1.4 — Surface Areas & Perimeters of Complex Shapes

Exercises 1–4 are about the figures shown below:



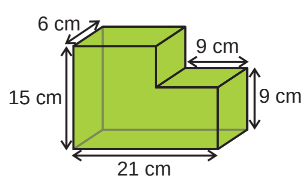
1. Find the edge length of the cube. **48 ft**
2. Find the edge length of the rectangular prism. **72 ft**
3. Find the total edge length of the two figures. **120 ft**
4. Find the total edge length once the figures have been joined. **96 ft**



A work table has the shape shown on the right.
5. What is the total edge length of the tabletop? **440 in**
6. What is the total edge length of the base? **640 in**
7. Find the total edge length of the work table. **980 in**

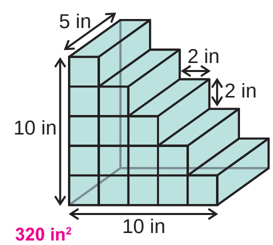
Work out the surface areas of the shapes shown in Exercises 8–10. Use $\pi = 3.14$.

8.



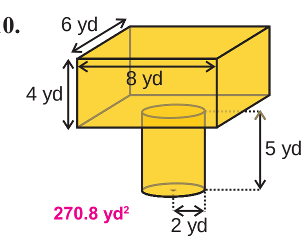
954 cm²

9.



320 in²

10.



270.8 yd²

Solutions
For worked solutions see the Solution Guide

Lesson 7.1.5 — Lines and Planes in Space

Level 1: q1–3
Level 2: q1–9
Level 3: q1–9

In Exercises 1–3, copy the sentences and fill in the missing words.

1. When two planes intersect they can meet along a __line__ or at a single __point__.
2. There are __three__ ways for a line and a plane to meet.
3. Coplanar lines are on the same __plane__.

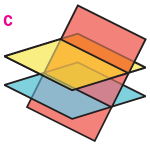
In Exercises 4–6, say whether each statement is true or false. If any are false, explain why.

4. Lines that do not intersect are always parallel. False – coplanar lines that do not intersect are parallel. Lines that rest on parallel planes are skew, not parallel.
5. Perpendicular planes meet at a point. False – perpendicular planes (like all intersecting planes) meet along a line.
6. Skew lines are not coplanar. True

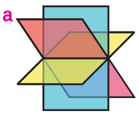
In Exercises 7–9, match each figure to one of the following descriptions:

a. The intersecting planes meet along a line.
b. The intersecting lines meet at a point.
c. One plane intersects two parallel planes.

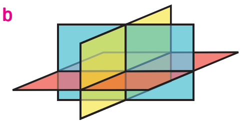
7. c



8. a



9. b



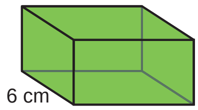
Lesson 7.2.1 — Volumes

Level 1: q1–6
Level 2: q1–9
Level 3: q1–10

1. A prism is 3 meters high. It has volume 12 m^3. What is the area of the prism's base? 4m^2

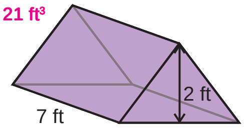
In Exercises 2–6, work out the volume of the figures. Use $\pi = 3.14$.

2.

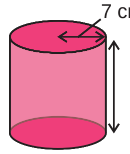


48 cm^3

3. 21 ft^3

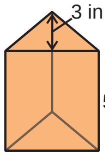


4.



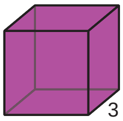
1846.32 cm^3

5.



30 in^3

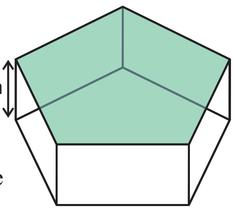
6.



27 m^3

In Exercises 7–8, consider a prism of volume 72 ft^3.

7. What is the height of the prism if the base area is 12 ft^2? 6 feet
8. If the prism is cut in half, what is its new volume? 36 ft^3
9. Find the volume of the figure shown on the right if the area of the shaded base is 26 in^2. 78 in^3



10. The contents of a full baking pan with dimensions 8 in. by 8 in. by 2 in. are poured into a cylindrical container with a diameter of 5 in. and a height of 8 in. Will the cylindrical container hold all the contents of the pan? Explain your answer.
Volume of baking pan = 128 in^3, volume of cylindrical container = 157 in^3
So the cylindrical container will hold the contents of the cake pan.

Additional Questions 467

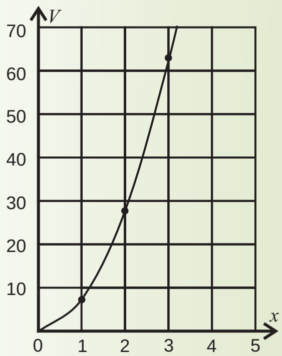
Solutions
For worked solutions see the Solution Guide

Additional Questions
Lesson 7.2.2 — 7.3.3

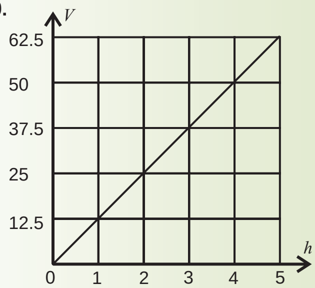
Level 1: q1–11
Level 2: q1–11
Level 3: q1–11

7.2.2

7.

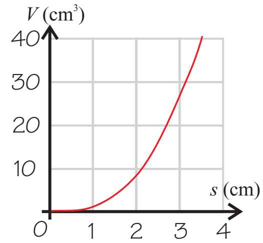


10.



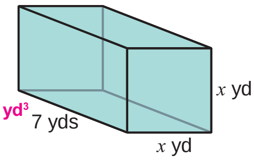
Level 1: q1–7
Level 2: q1–12
Level 3: q1–12

Lesson 7.2.2 — Graphing Volumes

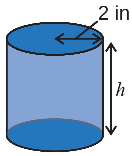


Use the graph of the volume of a cube with side length s, shown on the left, to answer Exercises 1–4.

1. Estimate the volume of a cube with side length 2.1m. **9–10 m³**
2. Estimate the side length of a cube with volume 25 m³. **2.9–3 m**
3. Estimate the volume of a cube with side length 1.5 m. **3–4 m³**
4. Estimate the side length of a cube with volume 33 m³. **3.2–3.3 m**



Exercises 5–8 refer to the figure at the right.
5. Write an expression for the volume of the prism. **$V = 7x^2$**
6. Use your expression to find the volume of the prism when $x = 5$. **$7 \times 5^2 = 175$ yd³**
7. Graph the volume of the figure against the value of x. **See margin**
8. Use your graph to estimate the side length x that makes a volume of 28 yd³. **2 yd**



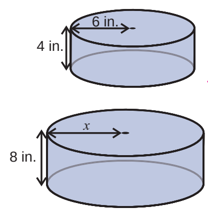
Exercises 9–11 refer to the figure on the left.
9. Find an expression that gives the volume of the figure. Use $\pi = 3.14$. **$V = 12.56h$**
10. Graph the volume of the figure. **See margin**
11. Use your graph to estimate the value of h that makes the volume 55 in³. **4.4 in**

Lesson 7.3.1 — Similar Solids

In Exercises 1–5, say whether the statements are true or false.
1. When multiplying by a scale factor, the corresponding angles of the image are multiplied by the scale factor. **False**
2. If you multiply a three-dimensional figure by a scale factor you get a similar figure. **True**
3. A scale factor of one produces an image the same size as the original figure. **True**
4. Two figures are similar if one can be multiplied by a scale factor to make a shape that is congruent to the other one. **True**
5. The image will be larger than the original if the scale factor is between 0 and 1. **False**

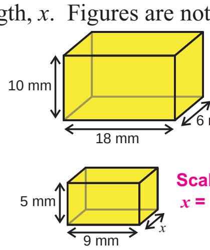
In Exercises 6–8, find the scale factor that has been used to create the image in the following pairs of similar solids and find the missing length, x. Figures are not drawn to scale.

6.



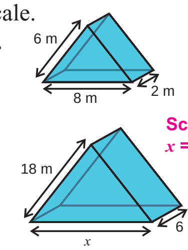
Scale factor = 2
x = 12 in

7.



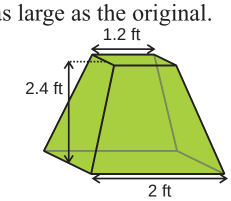
Scale factor = 0.5
x = 3 mm

8.



Scale factor = 3
x = 24 m

A class constructed a scale model of the figure below. The model was twice as large as the original.
9. What is the height of the students' model? **4.8 feet**
10. What is the length of the bottom face of the model? **4 feet**
11. What is the length of the top face of the model? **2.4 feet**
12. The class decided to construct another model one-quarter the size of the original that will be easier to transport.
What is the height of the new model? **0.6 feet**



Solutions
For worked solutions see the Solution Guide

Lesson 7.3.2 — Surface Areas & Volumes of Similar Figures

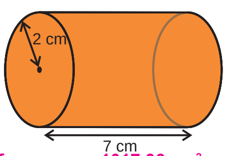
Level 1: q1–5, 7–8
Level 2: q1–8
Level 3: q4–9

Luis draws a figure with area 16 cm². Find the area of the image if Luis multiplies his figure by the following scale factors.

1. 3 **144 cm²** **2.** 20 **6400 cm²** **3.** 0.3 **1.44 cm²**

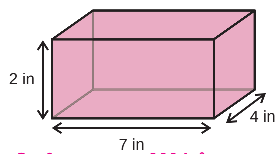
In Exercises 4–6, find the surface area and volume of the image if the figure shown is multiplied by a scale factor of 3. Figures are not drawn to scale. Use $\pi = 3.14$.

4.



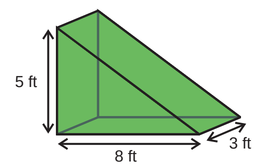
Surface area = 1017.36 cm²
Volume = 2373.84 cm³

5.



Surface area = 900 in²
Volume = 1512 in³

6.

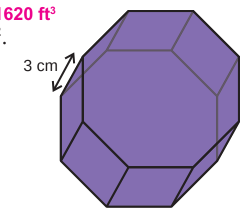


Surface area = 965.7 ft²
Volume = 1620 ft³

Exercises 7–8 refer to figure A at the right. The base area of figure A is 24 cm².

7. Find the volume of figure A. **72 cm²**

8. Find the volume of the image if A was enlarged by a scale factor of 4. **4608 cm³**



9. A scale model of a building has a surface area of 37.5 ft².
If the actual building has a surface area of 15,000 ft², what scale factor was used to make the model? **20**

Lesson 7.3.3 — Changing Units

Level 1: q1–19
Level 2: q1–19
Level 3: q1–19

In Exercises1–3, convert the following areas to cm².

1. 21 m² **210,000 cm²** **2.** 0.085 m² **850 cm²** **3.** 4.5 m² **45,000 cm²**

In Exercises 4–6, convert the following areas to square feet.

4. 864 in² **6 ft²** **5.** 48 in² **$0.\overline{3}$ ft²** **6.** 288 in² **2 ft²**

A cube has a surface area of 240 cm²

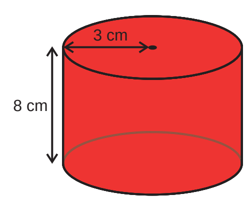
7. What is the surface area in m²? **0.024 m²** **8.** What is the surface area in in²? **37.2 in²**

Exercises 9–11 refer to the cylinder on the right. Use $\pi = 3.14$

9. What is the surface area of the cylinder? **207.24 cm²**

10. What is the surface area in m²? **0.020724 m²**

11. What is the surface area in in²? **32.1 in²**



In Exercises 12–14, convert the volumes to cubic inches.

12. 3 ft³ **5184 in³** **13.** 0.864 ft³ **1492.992 in³**

14. 5.1 ft³ **8812.8 in³**

In Exercises 15–17, convert the volumes to cubic meters. Use the conversion factor 1 m = 0.91 yd. Round to the nearest hundredth where appropriate.

15. 5 ft³ **0.135 m³** **16.** 25 ft³ **0.675 m³** **17.** 24.56 yd³ **18.51 m³**

18. Juan's house has a floor area of 4500 square feet. What is this area in square meters? **405 m²**

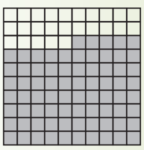
19. An acre is 4840 square yards. What is an acre in square meters? **4008 m³**

Solutions
For worked solutions
see the Solution Guide

Additional Questions
Lesson 8.1.1 — 8.2.3

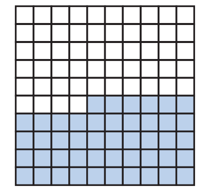
Level 1: q1–10
Level 2: q1–12
Level 3: q1–13

8.1.1

1. 

Lesson 8.1.1 — Percents

1. What percent of the grid shown on the right is shaded? Draw your own 10 by 10 grid and shade 75% of it. **46% is shaded; for grid, see margin.**



In Exercises 2–5 write the fraction as a percent.

2. $\frac{75}{100}$ **75%** **3.** $\frac{36}{100}$ **36%**

4. $\frac{100}{100}$ **100%** **5.** $\frac{124}{100}$ **124%**

In Exercises 6–9 write the percent as a fraction in its simplest form.

6. 1% $\frac{1}{100}$ **7.** 10% $\frac{1}{10}$

8. 40% $\frac{2}{5}$ **9.** 26% $\frac{13}{50}$

10. I have 100 DVDs. 12 of them are comedy films. What percent of my DVDs are comedy films? **12%**

11. If 15% of x is 36, what is x? $x = 240$

12. What is 25% of 64? What is 125% of 64? **16, 80**

13. In a 40 game season, a school soccer team won 70% of their matches. How many games did they win? **28**

Level 1: q1–10
Level 2: q1–11, 13
Level 3: q1–14

Lesson 8.1.2 — Changing Fractions and Decimals to Percents

In Exercises 1–6 write each decimal as a percent.

1. 0.03 **3%** **2.** 0 **0%** **3.** 0.11 **11%**

4. 0.44 **44%** **5.** 0.9 **90%** **6.** 3.6 **360%**

In Exercises 7–12 write each fraction as a percent.

7. $\frac{1}{4}$ **25%** **8.** $\frac{57}{100}$ **57%** **9.** $\frac{140}{100}$ **140%**

10. $\frac{3}{10}$ **30%** **11.** $\frac{7}{1000}$ **0.7%** **12.** $\frac{5}{8}$ **62.5%**

13. A factory has 3000 employees on 2 shifts. The night shift has 600 workers. What is this as a percent? **20%**

14. Shemika grew 48 bean plants for a science project. 33 were grown in soil, and the rest in water. What fraction were grown in soil? What percentage were grown in water? $\frac{11}{16}$, **31.25%**

Level 1: q1–8
Level 2: q1–10
Level 3: q1–11

Lesson 8.1.3 — Percent Increases and Decreases

In Exercises 1–4 find the total after the increase.

1. 80 is increased by 5% **84** **2.** 65 is increased by 20% **78**

3. 100 is increased by 130% **230** **4.** 81 is increased by 80% **145.8**

5. Tulio's curtains are 60 inches long. He lengthens them to put in a larger window. Their new length is 72 inches. By what percent has Tulio lengthened the curtains? **20%**

In Exercises 6–9 find the total after the decrease.

6. 50 is decreased by 10% **45** **7.** 180 is decreased by 30% **126**

8. 25 is decreased by 24% **19** **9.** 600 is decreased by 4.2% **574.8**

10. Nicole has a collection of 110 old coins. She gives 44 of them away to a friend who wants to start their own collection. What is the percent decrease in the size of Nicole's collection? **40%**

11. In 1980 Fremont had a population of 132,000 and Hesperia had a population of 20,600. By 2000, Fremont's population was 203,400 and Hesperia's was 62,600. Over the 20 years, which city's population increased by the largest number of people, and which city saw the biggest percent increase in population? **See below**

Solutions
For worked solutions see the Solution Guide

8.1.3

11. Fremont: Increase = 71,400 people = 54.1% (to 1 decimal place)
Hesperia: Increase = 42,000 people = 203.9% (to 1 decimal place)
Fremont's population increased by the largest number of people.
Hesperia saw the greatest percent increase in population.

Level 1: q1–3, 7–9
Level 2: q1–11
Level 3: q1–11

Lesson 8.2.1 — Discounts and Markups

A stationery store is having a sale. They offer the discounts shown in the table below. Use this information to calculate how much the items in Exercises 1–6 would cost from the sale.

Item	Notebooks	Pencils	Pens	Erasers	Ring Binders	Adhesive Tape
Original Price	\$5.99	\$0.59	\$1.99	\$2.99	\$4.99	\$2.99
Discount	20%	5%	15%	25%	30%	10%

1. 2 ring binders \$6.99
2. 3 erasers \$6.73
3. 3 rolls of adhesive tape \$8.07
4. 10 notebooks \$47.92
5. 1 notebook and 1 pen \$6.48
6. 6 pencils and 5 erasers \$14.58

Find the retail price of each of the items in Exercises 7–11.

7. Cherry vanity unit — wholesale price \$150, markup 45%. \$217.50
8. Cherry dresser — wholesale price \$850, markup 60%. \$1360
9. Florentine mirror — wholesale price \$95, markup 100%. \$190
10. Sleigh bed — wholesale price \$530, markup 85%. \$980.50
11. Mattress set — wholesale price \$999, markup 50.5%. \$1503.50

Level 1: q1–2, 6–9
Level 2: q1–11
Level 3: q1–12

Lesson 8.2.2 — Tips, Tax, and Commission

In Exercises 1–4 use mental math to calculate each tip.

1. 10% tip on a \$15 taxi ride. \$1.50
2. 20% tip on a \$30 hair style. \$6
3. 15% tip on a \$5 salad. \$0.75
4. 15% tip on a \$34.26 family meal. \$5.14

5. A \$25 concert ticket had a 15% entertainment tax added to the price. What was the total cost of the ticket? \$28.75

Work out the total cost after tax of the items in Exercises 6–11.

6. 5% added to a \$39.60 purchase. \$41.58
7. 8% tax added to a \$15,000 car. \$16,200
8. 7% tax added to a \$549 laptop. \$587.43
9. 9% tax added to a \$2.99 notebook. \$3.26
10. 6% tax added to a \$0.49 fruit juice. \$0.52
11. 11% tax added to a \$10.34 fruit basket. \$11.48

12. Jarrod bought a bike priced at \$109.98. After taxes he paid \$119.33. What was the rate of tax? 8.5%

Level 1: q1–3, 5–6, 10
Level 2: q1–3, 5–7, 9–10
Level 3: q1–1-0

Lesson 8.2.3 — Profit

Find the profit made in Exercises 1–4.

1. Expenses: \$800
   Revenue: \$3000 \$2200
2. Expenses: \$954
   Revenue: \$3975.01 \$3021.01
3. Expenses: \$777.77; \$5.89
   Revenue: \$1,258.34 \$474.68
4. Expenses: \$800,034; \$957.45; \$999,381.45
   Revenue: \$12,456,901 \$10,656,528.10

In Exercises 5–8, find the percent profit. Round each answer to the nearest whole percent.

5. Profit: \$245
   Total Sales: \$4,875 5%
6. Profit: \$300
   Total Sales: \$879.20 34%
7. Profit: \$9,000,000
   Total Sales: \$21,000,000 43%
8. Profit: \$4,432,567
   Total Sales: \$6,289,437 70%

9. The math club sold 500 pencils at 10 cents each as a fundraiser. They paid the supplier 2 cents for each pencil. Emmitt says the profit is \$40 but Lonnie says it is \$50. Who is correct? Emmitt

10. A company increased its profits by 33% over the previous year. If the previous year's profits were \$5,000,000, what are the profits this year? £6,650,000

Solutions
For worked solutions see the Solution Guide

Additional Questions

Lesson 8.2.4 — 8.3.4

Level 1: q1–4
Level 2: q1–6
Level 3: q1–7

Lesson 8.2.4 — Simple Interest

1. Jacob put \$3000 in an account with a simple interest rate of 5% per year. How much was in his account after 1 year? \$3150

2. Juan invested \$10,000 and received 6.25% simple interest per year. How much did she have after 5 years? \$13,125

3. Sara borrowed \$500 at a simple interest rate of 5.5% per year. How much did she owe after 5 years? \$637.50

4. A savings account advertises a simple interest rate of 3.5% per year. If Jan puts \$5000 in the account, how much interest will she have earned after 1 year? \$5175

5. A bank advertises a saving scheme which gives 12% simple interest per year if you keep your money in the bank for 8 years with the slogan "double your money". Is their slogan accurate? No

6. Lisa asks her mom to borrow \$20 for new jeans. Her mom says she will charge 3% simple interest per month. If Lisa intends to repay her mom in half a month, how much interest will she owe? \$0.30

7. Janice has decided to buy a new sofa set on credit. One furniture store offers credit with no interest for 6 months and then 18.5% simple interest per year. Another offers 4 months with no interest and then 18% simple interest per year. Which is the better deal for a \$3000 sofa set which Janice has budgeted to pay off in 2 years? The 6 months/18.5% credit deal

Level 1: q1–4
Level 2: q1–6
Level 3: q1–6

Lesson 8.2.5 — Compound Interest

1. If you're saving money, is it usually better to receive simple or compound interest? compound

2. If you're borrowing money, is it usually better to pay simple or compound interest? simple

3. You put \$250 into an account with a compound interest rate of 6%, compounded annually. What is the account balance after 5 years? \$334.56

4. Daniel puts \$1500 in a savings account with 6% interest for 8 years compounded quarterly. What is the account balance in 8 years? \$2415.49

5. Cynthia has \$1700 to invest for 8 years. She can choose between either an investment with a simple interest rate of 15% or one with a compound interest rate of 8%, compounded annually. Which investment would leave her better off? The simple interest

6. Luis puts \$600 into an account with a compound interest rate of 10%, compounded annually. Destiny puts \$600 into an account with a simple interest rate of 12%. What's the difference between the investments after 5 years? After 5 years, Luis has \$6.31 more than Destiny

Level 1: q1–12
Level 2: q1–12
Level 3: q1–12

Lesson 8.3.1 — Rounding

In Exercises 1–3, round each number to the nearest percent.

1. 87.5% 88% **2.** 45.3% 45% **3.** $71\frac{3}{4}\%$ 72%

In Exercises 4–9, round each number to the nearest tenth.

4. 103.785 103.8 **5.** 3.265 3.3 **6.** 40.23 40.2

7. 75.432 75.4 **8.** 7.9854 8.0 **9.** 41.98 42.0

10. Don's favourite group sold 549,873 copies of their last song. How many did they sell to the nearest hundred thousand? 550,000

11. Ana and Ava were asked to round 7199.99 to the nearest hundred. Ana said the answer was 7100, Ava said 7200. Who was correct? Ava

12. 63,495 fans attended a recent football game, which the newspaper rounded to the nearest 10,000. How many did the newspaper report had attended? 60,000

472 Additional Questions

Solutions
For worked solutions see the Solution Guide

Lesson 8.3.2 — Rounding Reasonably

Level 1: q1–4
Level 2: q1–6
Level 3: q1–7

1. Nadia needs 233 yards of ribbon to decorate the hall for the prom. If the ribbon comes on spools 10 yards long how many spools does Nadia need to buy? 24 spools

2. Raul is buying a window shade for a window that measures 95.2 cm wide. The store has shades of 93 cm, 95 cm and 97 cm wide. What width of shade should Raul buy? 97 cm

3. The drama club is using its funds to sponsor students acting at the Shakespeare festival by paying their \$25 entrance fee. If they have \$282, how many full entry fees can they pay? 11 students

4. Latoyah is baking bread. She needs 0.4 kg of flour for each loaf she makes. If she has 1.8 kg of flour altogether, how many loaves can she bake? 4 loaves

5. Michael's restaurant bill came to \$47.10. He wants to leave a 20% tip, but he only has dollar bills with him. What tip should Michael leave? \$10

6. Anjali is saving \$20 per week towards a new computer. If the computer she wants costs \$728.50, how many weeks will she need to save for? 37 weeks

7. Mr Scott is buying new books for the school library. The publishers will sell him each book for \$12. If he has a budget of \$500, what is the maximum number of books he can buy? 41 books

Lesson 8.3.3 — Exact and Approximate Answers

Level 1: q1–3
Level 2: q1–5
Level 3: q1–6

1. Estimate the area of a circle with a radius of 7 cm, using 3.14 as an approximation of π. What is the exact area of the circle? 153.86 cm², 49π

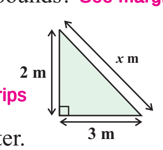
2. Kiana is buying presents for her family. She has \$85, and wants to spend the same amount on each person. If there are 6 people in her family, what is the most that she can spend on each one? \$14.16

3. Rico's kitchen scales measure weights in kilograms to two decimal places. He uses the scales to weigh out 1.45 kg of rice. Given that 1.0 kg ≈ 2.2 pounds, what weight of rice does Rico have in pounds? See margin

8.3.3

3. 3.2 pounds (to 1 decimal place).

4. Find the perimeter of the triangle in the diagram on the right. $(5 + \sqrt{13})$ m



5. Emma has a lawn with the same dimensions as the triangle in the diagram. She wants to put edging round it. If edging comes in 0.4 m strips, how many should Emma buy? 22 strips

6. I measured the side of a square as being 5.6 cm long to the nearest tenth of a centimeter. With round-off error, what are the maximum and minimum possible areas of the square?
Maximum = 31.9225 cm², Minimum = 30.8025 cm²

Lesson 8.3.4 — Reasonableness and Estimation

Level 1: q1–4
Level 2: q1–5
Level 3: q1–6

1. Tion is shopping at the grocery store, and only has \$35 with him. He puts items costing \$5.99, \$3.98, \$10.57, and \$12.99 in his basket. Use estimation to check whether he has enough money with him to pay. See below

2. Rachel puts \$102 in a savings account with a yearly rate of 5.2% simple interest. She says that she will earn \$53 in interest each year. Perform your own estimate, and say whether she is likely to be correct. See below

3. Find the product of 73 and 39. Check your product using estimation. 2847, $70 \times 40 = 2800$

4. Ms Harris is organising a field trip. 144 students are going on the trip, and the bus hire company can provide up to 5 buses that seat 50 students each. How many buses should Ms Harris hire? 3 buses

5. James measures the temperature in his kitchen as being 70°F. He converts this to degrees Celsius, and says that the temperature in his kitchen is $21.\overline{1}$ °C. Is this a reasonable answer? See below

6. Diega is baking scones. Her recipe calls for 0.25 liters of milk. She knows that 1 liter ≈ 4.2 cups. Diega says that she needs to add about 1 cup of milk to the mixture. Is this a reasonable answer? See below

Solutions
For worked solutions see the Solution Guide

8.3.4

1. \$6 + \$4 + \$11 + \$13 = \$34. Tion does have enough money to pay.
2. $100 \times 0.05 = 5$. Rachel's estimate is too big, and she is unlikely to be correct.
5. This is not reasonable. He only measured the temperature to the nearest degree. The answer he has given is far too precise.
6. $0.25 \times 4 = 1$. Diega's estimate is reasonable.

Appendixes

Glossary

Symbols

$<$	is less than	$\neq$	is not equal to
$>$	is greater than	$\mathbb{W}$	the *whole numbers*
$\leq$	is less than or equal to	$\mathbb{Z}$	the *integers*
$\geq$	is greater than or equal to		

A

absolute value the absolute value of a number, n, is its distance from zero on the number line, and is written $|n|$. Absolute values are always positive, for example $|-3.6| = 3.6$.

acute triangle a triangle in which all angles are less than 90°

altitude the "height" of a triangle, measured at right angles to its base

arc part of a circle's *circumference*; can be drawn with a compass

associative properties (of addition and multiplication) for any a, b, c: $a + (b + c) = (a + b) + c$
$a(bc) = (ab)c$

B

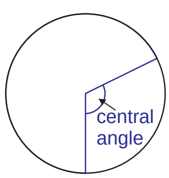
base in the *expression* b^x, the base is b

bisect divide in half

box-and-whisker plots a diagram showing the *range*, *median*, and *quartiles* of a data set against a number line.

C

central angle the angle between two *radii* of a circle, for example:



chord a straight line joining together two points on the *circumference* of a circle

circumference the distance around the outside of a circle

coefficient the number that a *variable* is multiplied by in an algebraic term. For example, in $6x$, the coefficient of x is 6

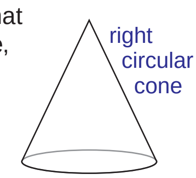
commission money earned by an agent when he or she sells a good or service, usually given as a *percent* of the sale price

common factor a number or expression that is a *factor* of two or more other numbers or expressions

common multiple a multiple of two or more different integers

commutative properties (of addition and multiplication) for any a, b: $a + b = b + a$ and $ab = ba$

cone a three-dimensional figure that has a circle or ellipse as its base, from which a curved surface comes to a point, for example:



conjecture a mathematical statement that is only an informed guess. It seems likely to be true, but hasn't been proved

congruent exactly the same size and shape

constant of proportionality a number, k, which always has the same value in an *equation* of the form $y = kx$ or $y = \frac{k}{x}$

converse of the Pythagorean theorem if a triangle has sides a, b, and c, where $c^2 = a^2 + b^2$, then it is a *right triangle*, and c is its *hypotenuse*

conversion factor the ratio of one unit to another; used for converting between units

coordinate pair an ordered pair of coordinates representing a point on the *coordinate plane*, for example, (2, –3)

coordinate plane a flat surface that extends to infinity, on which points are plotted using two *perpendicular* axes (usually x- and y-axes)

coplanar points, lines, or figures are coplanar if they lie in the same plane

correlation a relationship between two *variables*

counterexample an example that disproves a *conjecture*

cube a three-dimensional figure with six identical square *faces*

customary units the system of units that includes: inches, feet, yards, miles, ounces, and pounds

cylinder a three-dimensional figure with two parallel circular or elliptical bases and a constant cross-section

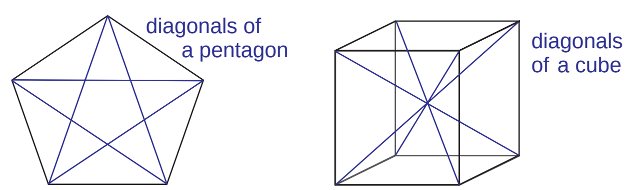
D

data set a collection of information, often numbers

decimal a number including a decimal point; digits to the right of the decimal point show parts of a whole number

denominator the bottom expression of a fraction

diagonal a straight line joining two nonadjacent corners of a two-dimensional figure, or two *vertices* of a three-dimensional figure that aren't on the same *face*, for example:



diameter a straight line from one side of a circle to the other, passing through the center

dimensional analysis a method of checking that a formula is correct by examining units

direct variation a relationship between two variables in which the *ratio* between them is always the same

distributive property (of multiplication over addition) for any a, b, c: $a(b + c) = ab + ac$

divisor the number by which another number is being divided. For example, in $12 \div 3$, the divisor is 3

E

edge on a three-dimensional figure, an edge is where two *faces* meet

equation a mathematical statement showing that two quantities are equal

equilateral triangle a triangle whose sides are all the same length

Glossary (continued)

equivalent fractions fractions are equivalent if they have the same value, for example: $\frac{1}{2} = \frac{2}{4}, \frac{2}{3} = \frac{4}{6}$

estimate an inexact judgement about the size of a quantity; an "educated guess"

evaluate find the value of an *expression* by substituting actual values for *variables*

exponent in the *expression* b^x, the exponent is x

expression a collection of numbers, *variables*, and symbols that represents a quantity

F

face a flat surface of a three-dimensional figure

factor a number or *expression* that can be multiplied to get another number or expression — for example, 2 is a factor of 6, because 2 × 3 = 6

factorization a number written as the product of its *factors*

formula an equation that relates at least two *variables*, usually used for finding the value of one variable when the other values are known. For example, $A = \pi r^2$

G

generalization a statement that describes many cases, rather than just one

greatest common factor (GCF) the largest *expression* that is a *common factor* of two or more other expressions; all other common factors will also be *factors* of the GCF

grouping symbols symbols that show the order in which mathematical operations should be carried out — such as parentheses and brackets

H

hypotenuse the longest side of a *right triangle*

I

identity properties (of addition and multiplication) for any a, $a + 0 = a$, and $a \times 1 = a$

improper fraction a fraction whose *numerator* is greater than or equal to its *denominator*, for example $\frac{7}{4}$

integers the numbers 0, ±1, ±2, ±3,...; the set of all integers is denoted $\mathbb{Z}$

interest extra money you pay back when you borrow money, or that you receive when you invest money

inverse (additive) a number's additive inverse is the number that can be added to it to give 0 — for any a, the additive inverse is $(-a)$

inverse (multiplicative) a number's multiplicative inverse is the number that it can be multiplied by to give 1 — for any a, this is $\frac{1}{a}$

inverse operation an operation that "undoes" another operation — addition and subtraction are inverse operations, as are multiplication and division

isosceles triangle a triangle with two sides of equal length

L

line of best fit a trend line on a *scatterplot* — there will be roughly the same number of points on each side of the line

linear equation an *equation* linking two *variables* that can be written in the form $y = mx + b$, where m and b are constants

least common multiple (LCM) the smallest *integer* that has two or more other integers as *factors*

M

mean a measure of *central tendency*; the sum of a set of values, divided by the number of values in the set

measure of central tendency the value of a "typical" item in a data set. *Mean*, mode and *median* are measures of central tendency

median the middle value when a set of values is put in order

metric the system of units that includes: centimeters, meters, kilometers, grams, kilograms, and liters

mixed number a number containing a whole number part and a fraction part

monomial an *expression* with a single *term*

N

net a two-dimensional pattern that can be folded into a three-dimensional figure

numerator the top expression of a fraction

numeric expression a number or an *expression* containing only numbers and operations (and therefore no *variables*)

O

obtuse triangle a triangle in which one angle is greater than 90°

origin on a number line, the origin is at zero; on the *coordinate plane*, the origin is at the point (0, 0)

P

parabola a "u-shaped" curve obtained by graphing an *equation* of the form $y = nx^2$

parallelogram a four-sided shape with two pairs of parallel sides

PEMDAS the order of operations — "Parentheses, Exponents, Multiplication and Division, Addition and Subtraction"

percent value followed by the % sign; corresponds to the *numerator* of a fraction with 100 as the *denominator*

perimeter the sum of the side lengths of a polygon

perpendicular at right angles to

power an *expression* of the form b^x, made up of a *base* (b) and an *exponent* (x)

prime factorization a *factorization* of a number where each *factor* is a *prime number*, for example 12 = 2 × 2 × 3

prime number a whole number that has exactly two factors, itself and 1

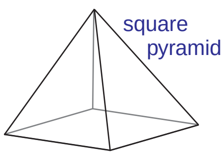
prism a three-dimensional figure with two identical parallel bases and a constant cross-section

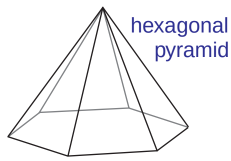
product the result of multiplying numbers or *expressions* together

proportion an *equation* showing that two ratios are equivalent

proper fraction a fraction whose *numerator* is less than its *denominator*

pyramid a three-dimensional figure that has a polygon as its base and in which all the other *faces* come to a point, for example:





Pythagorean theorem for a *right triangle* with side lengths a, b, and c, $a^2 + b^2 = c^2$

Pythagorean triple three whole numbers a, b, and c, that satisfy $a^2 + b^2 = c^2$

Q

quadrant a quarter of the *coordinate plane*, bounded on two sides by parts of the x- and y-axes

quadrilateral a two-dimensional figure with four straight sides

quartiles split an ordered data set into four equal groups — the median splits the data in half, and the upper and lower quartiles are the middle values of the upper and lower halves

quotient the result of dividing two numbers or *expressions*

R

radius the distance between a point on a circle and the center of the circle

range the difference between the lowest and highest values in a data set

rate a kind of *ratio* that compares quantities with different units

ratio the amount of one thing compared with the amount of another thing

rational number a number that can be written as a fraction in which the *numerator* and *denominator* are both *integers*, and the *denominator* is not equal to zero

reciprocal the multiplicative *inverse* of an *expression*

regular polygon a two-dimensional figure in which all side-lengths are equal, and all angles are equal, for example a square or an equilateral triangle

repeating decimal a decimal number in which a digit, or sequence of digits, repeats endlessly, for example, 3.333333333...

rhombus a two-dimensional figure with four equal-length sides in two parallel pairs

right triangle a triangle with one right (90°) angle

rounding replacing one number with another number that's easier to work with; used to give an approximation of a solution

S

scale drawing a drawing in which the dimensions of all the features have been reduced by the same *scale factor*

scale factor a *ratio* comparing the lengths of the sides of two *similar* figures

scalene triangle a triangle with three unequal sides

scatterplot a way of displaying ordered data pairs to see if the values in the pairs are related, and if so, how they are related

scientific notation a way of writing numbers (usually very large or small ones) as a product of two factors, where one factor is greater than or equal to 1, but less than 10, and the other is a *power* of 10 — for example, 5.3×10^6 (= 5,300,000)

sign of a number whether a number is positive or negative

similar two figures are similar if all of their corresponding sides are in proportion and all of their angles are equal

simplify to reduce an *expression* to the least number of terms, or to reduce a fraction to its lowest terms

skew lines nonparallel, nonintersecting lines in three-dimensional space

slope the "steepness" of a straight line on the coordinate plane, given by the ratio $\frac{\text{change in } y}{\text{change in } x}$

solve to manipulate an *equation* to find out the value of a *variable*

square root a square root of a number, n, is a number, x, that when multiplied by itself, results in n — for example, 3 and –3 are square roots of 9

stem-and-leaf plot a way of displaying numeric data, in order, so that the common values and spread of the data are easy to see

sum the result of adding numbers or *expressions* together

surface area the sum of the areas of all the *faces* of a three-dimensional figure

system of equations two (or more) *equations*, with the same *variables*, which can be solved together

T

terminating decimal a decimal that does not continue forever, for example, 0.378

terms the parts that are added or subtracted to form an *expression*

translation a transformation in which a figure moves around the *coordinate plane* (but its orientation and size stay the same)

trapezoid a four-sided shape with exactly one pair of parallel sides

U

unit rate a comparison of two amounts that have different units, where one of the amounts is "1" — for example, 50 miles per hour.

V

variable a letter that is used to represent an unknown number

vertex the point on an angle where the two rays meet on a two-dimensional figure; the point where three or more *faces* meet on a three-dimensional figure

volume a measure of the amount of space inside a three-dimensional figure

W

whole numbers the set of numbers 0, 1, 2, 3,...; the set of all whole numbers is denoted $\mathbb{W}$

Formula Sheet

Order of Operations — PEMDAS

Perform operations in the following order:

1. Anything in **parentheses** or other grouping symbols — working from the innermost grouping symbols to the outermost.
2. **Exponents**.
3. **Multiplications** and **divisions**, working from left to right.
4. **Additions** and **subtractions**, again from left to right.

Fractions

Adding and subtracting fractions with the same denominator: $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$ $\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$

Adding and subtracting fractions with different denominators: $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$ $\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$

Multiplying fractions: $a\cdot\frac{c}{d}=\frac{ac}{d}$ and $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$

Dividing fractions: $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$

Reciprocals: $\frac{d}{c}$ is the called reciprocal of $\frac{c}{d}$

Rules for Multiplying and Dividing

positive ×/÷ positive = positive

positive ×/÷ negative = negative

negative ×/÷ positive = negative

negative ×/÷ negative = positive

Axioms of the Real Number System

For any real numbers a, b, and c, the following properties hold:

Property Name	Addition	Multiplication
Commutative Property:	$a + b = b + a$	$a \times b = b \times a$
Associative Property:	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Distributive Property of Multiplication over Addition:	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	
Identity Property:	$a + 0 = a$	$a \times 1 = a$
Inverse Property:	$a + (-a) = 0$	$a \times \frac{1}{a} = 1$

Area

Area of a rectangle: $A = bh$

Area of a parallelogram: $A = bh$

Area of a triangle: $A = \frac{1}{2}bh$

Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

where b is the length of the base (for a trapezoid, b_1 and b_2 are the lengths of the bases) and h is the perpendicular height.

Formula Sheet (continued)

Slope of a Line

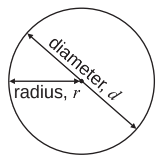
For a line passing through the points (x_1, y_1) and (x_2, y_2): $\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Circles

Diameter: $d = 2r$

Circumference: $C = \pi d$

Area: $A = \pi r^2$



Volume of a Prism

$V = Bh$ where B stands for the base area, and h stands for the height of the prism.

Powers

For any real numbers, a, m, and n:

Multiplying powers: $a^m \times a^n = a^{(m+n)}$

Dividing powers: $a^m \div a^n = a^{(m-n)}$

And, for any number, $a \neq 0$:

Zero exponent: $a^0 = 1$

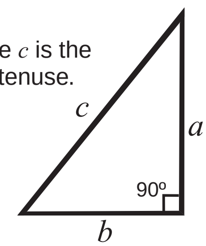
Negative exponent: $a^{-n} = \dfrac{1}{n}$

Pythagorean Theorem

For any right triangle:

$c^2 = a^2 + b^2$

where c is the hypotenuse.



Units

Lengths in Customary Units

1 foot (ft) = 12 inches (in.)
1 yard (yd) = 3 feet
1 mile (mi) = 1760 yards = 5280 feet

Capacities in Customary Units

1 cup = 8 fluid ounces (fl oz)
1 pint (pt) = 2 cups
1 quart (qt) = 2 pints
1 gallon (gal) = 4 quarts

Weights in Customary Units

1 pound (lb) = 16 ounces (oz)
1 ton = 2000 pounds

Lengths in Metric Units

1 centimeter (cm) = 10 millimeters (mm)
1 meter (m) = 100 centimeters
1 kilometer (km) = 1000 meters

Capacities in Metric Units

1 liter (l) = 1000 milliliters (ml)

Weights in Metric Units

1 gram (g) = 1000 milligrams (mg)
1 kilogram (kg) = 1000 grams

Customary-to-Metric / Metric-to-Customary Conversions

1 inch = 2.54 centimeters
1 yard = 0.91 meters
1 mile = 1.6 kilometers
1 gallon = 3.785 liters
1 liter = 1.057 quarts
1 kilogram = 2.2 pounds

Converting Between Temperatures in Fahrenheit and Celsius

$F = \frac{9}{5}C + 32$ $\quad C = \frac{5}{9}(F - 32)$

Applications Formulas

Speed

$\text{speed} = \dfrac{\text{distance}}{\text{time}}$ $\quad \text{distance} = \text{speed} \times \text{time}$ $\quad \text{time} = \dfrac{\text{distance}}{\text{speed}}$

Simple Interest

The interest (I) earned in t years when p is invested at a simple interest rate of r (as a fraction or decimal) is given by: $I = prt$

Compound Interest

The amount in an account (A) when P is invested at a compound interest rate of r is given by: $A = P(1 + rt)^n$
where t is the time (in years) between each interest payment,
and n is the total number of interest payments made.

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